

Perspective: Magnetic Skyrmions – overview of recent progress in an active research field^{a)}

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Within a decade, the field of magnetic skyrmionics has developed from a niche prediction to a huge and active research field. Not only do magnetic skyrmions – magnetic whirls with a unique topology – reveal fundamentally new physics, but they have risen to prominence as up-and-coming candidates for next-generation high-density efficient information encoding. Within a few years it has been possible to efficiently create, manipulate and destroy nanometer-size skyrmions in device-compatible materials at room-temperature by all electrical means. Despite the incredibly rapid progress, several challenges still remain to obtain fully functional and competitive skyrmion devices, as discussed in this perspectives article with a focus on recent results.

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I. INTRODUCTION

Due to the achievements in science and technology towards miniaturization of devices into the nano-meter length scale within the last decade, the physics of nano-structures, interfaces and surfaces is nowadays a central area of research.^{1,2} It is characterized by new experimental and theoretical challenges which are imposed due to the fact that the objects of study are in the range or even smaller than characteristic length scales of the system, such as the carrier mean free path, spin diffusion length, magnetic exchange length, excitation wavelength, etc. Therefore, in such systems new properties emerge and also the transition from classical to quantum behavior becomes apparent. In particular, novel quasi-particles might form, which have exceptional topological properties that are based on the system's effective dimensionality and its symmetry. Striking examples are relativistically behaving charge carriers originating in an effective Dirac dispersion in the complex electronic band structure and magnetic quasi-particles that arise due to chiral magnetic interactions emanating from spin-orbit coupling.

A feature that makes quasi-particles valuable for applications is their potential stability. Self-localized wave packets, so called solitons, are examples of quasi-particles with a remarkable stability. They occur in media with non-linear and dispersive constitutive laws.³⁻⁷ In mag-

netism, a variety of quasi-particle solitons has been investigated and experimentally observed, such as (i) domain walls (DWs),⁸⁻¹² (ii) vortices¹³⁻²⁰, (iii) bubbles²¹⁻²⁵, (iv) skyrmions²⁶⁻³², etc.

In this perspective article we highlight the recent progress in the field of skyrmionics and show some new results. We do not attempt to give a complete overview of the field, but rather focus on the new physics findings that have become available in the last two years. For an overview of earlier works we refer to previously published reviews³³⁻⁴⁰ and for an overview of methods we refer to Ref. 41. While many concepts are valid regardless of the precise twisting mechanism responsible for the skyrmions (e.g. induced by interfacial Dzyaloshinskii-Moriya interaction^{42,43} (DMI), bulk DMI, or frustrations), for the scope of this article our focus is on systems with interfacial DMI.

II. NEW SPIN STRUCTURES WITH TWISTS

A new type of magnetic quasi-particles that has attracted attention is the skyrmion. Although skyrmions have been predicted a long time ago,^{26,27,44,45} experimentally these chiral spin-structures have been only discovered less than ten years ago in bulk,²⁸ thin films,³⁰ and monolayers.²⁹ Since then a variety of novel (topological) magnetic textures has been observed, see Fig. 1 for a selection. Some of these will be discussed in detail later in this article.

Concerning what constitutes a skyrmion, in the community there are different definitions used. The term itself was introduced in relation to Tony Skyrme's original work in nuclear physics⁴⁵ where he investigated topologically non-trivial localized field solutions of a non-linear sigma model to describe elementary particles. Since there

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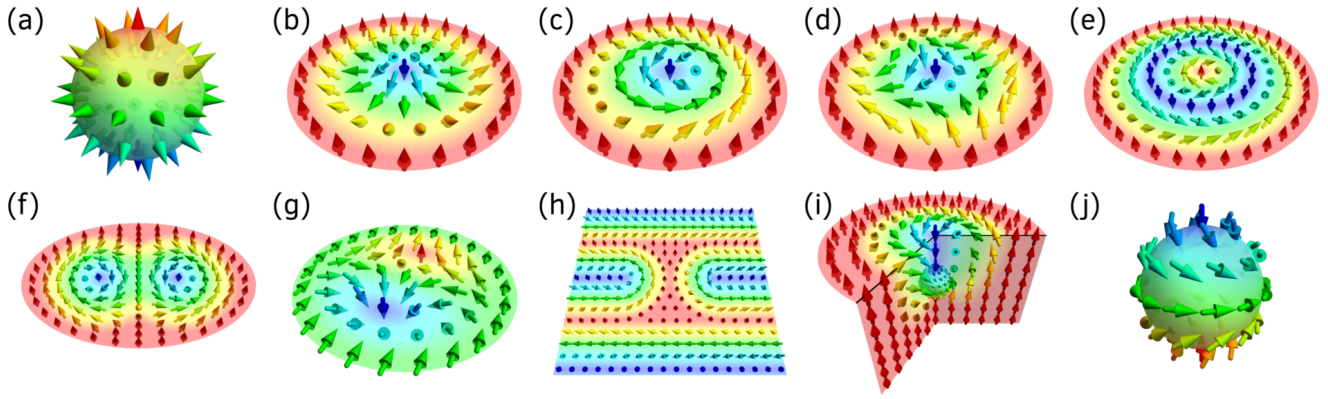


FIG. 1: Zoo of (topological) spin textures with different winding numbers. a) hedgehog, b) Néel-type skyrmion, c) Bloch-type skyrmion, d) antiskyrmion, e) skyrmionium, f) biskyrmion g) example of an in-plane skyrmion, h) skyrmion in helical background, i) chiral bobber, j) combed anti-hedgehog formed around the Bloch point in panel (i). The winding number for b), c), d), g), h) is $|\mathcal{W}| = 1$, for f) it is $|\mathcal{W}| = 2$ and e) is topologically trivial. Note that the winding number in Eq. (1) is defined for 2d structures that does not directly apply to a), i) and j).

is no obvious distinction for what should be called a skyrmion in condensed matter physics, within this article we defines a skyrmion as being any spin structure in which the center magnetization is in the opposite direction to its boundary and which can be mapped once to the sphere. In this sense, a radially symmetric skyrmion is characterized by two quantities, its radial profile and the twisting angle.

A classification of spin structures can be based on their topology, which often has direct consequences for the physical properties of magnetic textures and measurable quantities like Hall resistances. Effective two dimensional textures described by a local magnetization direction \mathbf{m} can be classified by the skyrmion winding number,

$$\mathcal{W} = \frac{1}{4\pi} \int dx dy \mathbf{m} \cdot (\partial_x \mathbf{m} \times \partial_y \mathbf{m}). \quad (1)$$

We would like to note that topology is a mathematical concept for continuous systems that defines two structures to be equivalent if a continuous mapping from one to the other exists. In contrast, real physical systems are usually discrete, for example due to the underlying atomic lattice. Naturally, this implies that topologically non-conserving transformations are allowed but have a finite, non-zero energy penalty that must be overcome, see Sec. III. In general, the understanding of convoluted spin textures is based on the idea of the interplay of different interactions and their relative strengths. For twisted magnetic textures, there is at least one interaction scale favoring a ferromagnetic alignment of neighboring spins at a certain energy scale (like exchange or anisotropy) and other interactions that prefer a twisted spin texture (like DMI or dipolar interactions), which are usually much weaker.

Experimentally it is important whether a system has a fixed chirality or whether both chiralities can exist. Therefore, in this article, we will use the notion of *chiral*

skyrmions for systems where chiral interactions are so strong that only one chirality is found. In contrast, for systems in which chiral interactions are either absent or too weak such that skyrmions with multiple chiralities are stable, we will denote those skyrmions *bubble skyrmions*. As opposed to a bubble skyrmion, a magnetic bubble can have different topologies including being topological trivial or having higher winding numbers.

Skyrmions have not only been observed in the form of lattices or in an out-of-plane FM background, either due to an applied out-of-plane field^{28,30,46–48} or due to a strong perpendicular magnetic anisotropy^{46–52}, but also with different orientations within a larger background⁵³, including the limit of in-plane skyrmions.^{54,55} Furthermore, they have been observed both as single objects or clusters in a helimagnetic background^{56–58} and in the form of skyrmion fabrics.^{57,59–61} They occur in various systems including bulk materials,^{16,28,62–65} thin films, hybrid structures or heterostructures^{12,49,50,66–69} and in frustrated magnets.^{70–74} Several materials and systems where skyrmions have been found are listed in Tab. I. In addition to the low temperature results, skyrmions have been observed in materials at elevated temperatures, even above room temperature and at zero field.^{31,48,68,75–78}

Fig. 2 shows selected images of some new multilayer materials stacks in which skyrmions have been observed in addition to other systems that have been recently reported, such as Ru/Co/W/Ru⁸⁰, IrMn/CoFeB/MgO⁷⁶ and many others.^{79,81}

The particular type of skyrmion that is realized in a system depends on the symmetry.^{82,83} The most widely existing skyrmions are Bloch or Néel type structures, see Fig. 2. The former is predominantly found in bulk materials favored by the inversion symmetry breaking "standard" bulk DMI, and the latter is characteristic for the interfacial DMI of multilayers. Generalized forms of DMI,

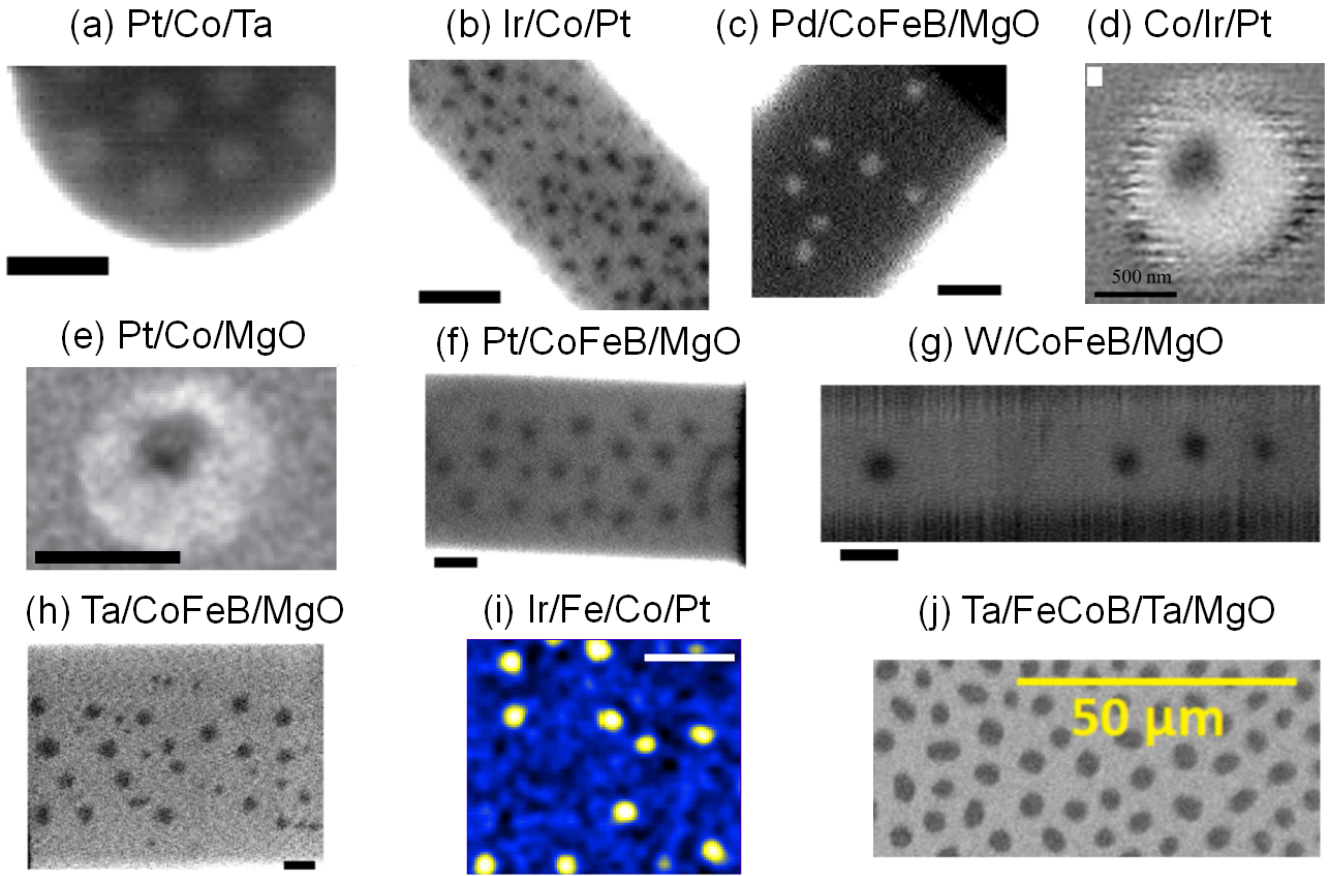


FIG. 2: Images of skyrmion spin structures in confined geometries using different multilayer materials stacks (scale bar corresponds to 500 nm for (a)-(i) and 50 μm for (j)): (a): [Pt(3)/Co(0.9)/Ta(4)] \times 15; (b): Pt(10)/Co(1)/Pt(1)/[Ir(1)/Co(1)/Pt(1)] \times 7; (c): [Pd(5)/Co60Fe20B20(0.6)/MgO(2)] \times 15; (d): [Co(0.7)/Ir(0.5)/Pt(2.3)] \times 10; (e): Pt(3)/Co(1.08)/MgO/Ta(2); (f): [Pt(3.2)/Co60Fe20B20(0.7)/MgO(1.4)] \times 15; (g): [W(5)/Co20Fe60B20(0.6)/MgO(2)] \times 15; (h): [Ta(5)/Co20Fe60B20(0.9)/MgO(1.0)] \times 5; (i): [Ir(10)/Fe(2)/Co(6)/Pt(10)] \times 20; (j): Ta(5)/Co20Fe60B20(1.0)/Ta(0.08)/MgO(2)/Ta(5), where the numbers in round brackets denote the size in nanometer. (adapted with permission from Refs. 47, 48, 78, 81, 84–86). Depending on the magnetic properties, including the anisotropy, saturation magnetization, exchange and DMI, different spin structures are stable. It is found that skyrmions can be stabilized within a wide range of sizes from a few nm (limited by the spatial resolution of the employed techniques) to micrometers in a wide range of systems.

which occur naturally in systems with reduced symmetries, also allow for more complex magnetic textures and systems.^{87–90} Not only does DMI stabilize twisted magnetic textures in infinite samples but it also modifies the properties in confined structures since DMI contributes to the boundary condition, leading to edge twists.^{88,91–93} Furthermore, in addition to DMI, the dipolar interactions also favor the formation of twisted magnetic textures.⁹⁴ Only in the limiting case of vanishing thickness do dipolar interactions not contribute. Therefore, for the true ground state, the question is to what extent each of the twisting interactions contributes. Exploiting the fact that dipolar interactions, in contrast to DMI, stabilize skyrmions with both chiralities, the different symmetries allow one to distinguish between the two contributions. By tuning the saturation magnetization (that tunes the

dipolar interactions) and the DMI as shown in Fig. 3, one can also obtain different types of skyrmions and pinning can lead to additional random deformations⁹⁵.

In summary, the improved theoretical understanding and the new experimental possibilities have brought the field of skyrmionics to a level where engineering skyrmions with desired properties via material design⁹⁶ is feasible and thus skyrmion based applications are enabled. The rest of this perspectives article is structured as follows. In Sec. III we will discuss the static properties of magnetic skyrmions and recapitulate the basis and limits of their stability. In Sec. IV we review dynamics of magnetic skyrmions. This includes their internal dynamics in the form of excitations and their particle like behavior (motion and rotation) in reaction to external stimuli as well as their creation and destruction.

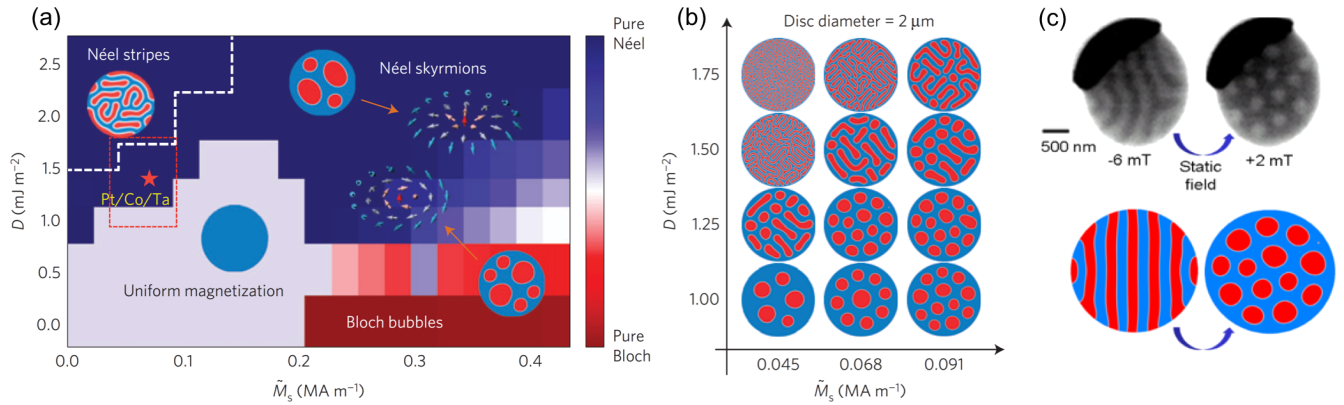


FIG. 3: Magnetic equilibrium states in Pt/Co/Ta nano discs as a function of the saturation magnetization and DMI. (A) and (B) show experimental measurements and micromagnetic simulations of the stripe phase (left) and the skyrmion lattice phase (right). Adapted from Refs. 48 and 84

In Sec. V we will review and give perspectives regarding skyrmion based applications.

III. SKYRMION GROUND STATE AND STABILITY

In systems discretized on the nanoscale, the concept for continuous magnetic textures of topological protection against arbitrarily large fluctuations for continuous magnetic textures translates into topological stabilization with a finite energy barrier. Stability itself is not a universal concept, since the size of the energy barrier depends on the transformation path that is chosen to go from one topologically distinct state to another one. As such this topic is quite complex.

The stability and associated lifetime τ of skyrmions with respect to thermal fluctuations and external fields is of particular technological importance. Regarding the life time of skyrmions at finite temperatures, several studies agree with the fact that skyrmion lifetimes as a function of temperature T can be described by an Arrhenius law $\tau \sim \tau_0 \exp \frac{\Delta E}{k_B T}$ as a function of temperature T , which allows one to calculate the rate of skyrmion creation-annihilation at any temperature for a given energy barrier ΔE is the energy barrier^{97–106}. However, reported values of the pre-exponential factor and prefactor τ_0 which are reported, are very different, even for the same systems, when calculated with different methods. Also, a temperature dependence of τ_0 is discussed. This discrepancies arise due to the fact that skyrmion creation and annihilation events are rare on the time scale of individual magnetic moment oscillations and as such stochastic modeling is challenging and different approaches, such as the transition state theory are used.

Regarding the energy barrier ΔE the creation or annihilation of skyrmions can, most generally, occur via two distinct types of transitions. On the one hand, the quantization of the topological charge associated with the

skyrmion number naively implies that a singular magnetization configuration, the so-called *Bloch point*, (see Fig. 1(i)), is necessarily formed in the process of changing the skyrmion number. Since this singular spin configuration is naturally associated with a high energy barrier, this creation or collapse path is often used as justification for the term "topological protection"^{33,57}. On the other hand, if the edge of the system is taken into account, the space manifold can no longer be mapped to a sphere anymore, thus the topological charge is no longer quantized. As a consequence, the skyrmion number can change *continuously* without the need for a singular point.

Therefore, in the following, we will split the discussion into the stability characteristics for skyrmions in infinite films and those in confined geometries, as for there two cases skyrmions can either decay through singular magnetic configurations (i.e. Bloch points or Bloch lines) or at the boundary, respectively.

A. Stability in Infinite Films

In the case of zero temperature, and based on a simple theoretical model consisting of exchange, DMI and anisotropy interaction, it has been shown that magnetic skyrmions can be stabilized above a certain critical DMI value $D_c = 4\sqrt{AK}/\pi$, above which the DW energy becomes negative.^{91,107} Here K is the anisotropy constant and A denotes the ferromagnetic exchange constant of the magnetic thin film. Beyond effects beyond the simple model have been studied in Refs.^{108,109} For multilayer systems studied in the literature also interlayer effects including interlayer magnetostatic coupling need to be taken into account^{48,67,110,111} and influence their properties⁸¹

In an infinite film skyrmions can only be created or decay through singular magnetic configurations, i.e. Bloch points. Once a skyrmion is created, the energy to destroy

it is proportional to the number of spins that need to flip their orientation to go back to the ferromagnetic state, i.e. this energy barrier is proportional to the skyrmion size, which in turn depends on the magnetic field¹¹². In this sense, skyrmions can also be annihilated by shrinking their size to zero.

A recent experimental study on $\text{Fe}_{1-x}\text{Co}_x\text{Si}$ based on Lorentz transmission electron microscopy, supporting the exponential temperature dependence on the skyrmion's lifetime, reveals that the prefactor τ_0 changes by more than 30 orders of magnitude for only slight variations of the magnetic field.¹⁰⁰ This means that the lifetime of the skyrmions is substantially reduced by entropic effects and that this system exhibits an extreme case of enthalpy-entropy compensation.

B. Stability in Confined Geometries

The crucial difference for skyrmions in confined geometries is that finite systems offer different paths to creation and annihilation of skyrmions via the boundaries. This leads to fundamentally different energy barriers and therefore to different stability properties. For example, in three dimensions skyrmion lattices can be unwound by magnetic monopoles entering through the boundary of the material which then zip skyrmion tubes together.⁵⁷ In 2D, systems decay at the boundary, as shown in Fig. 5. In general, in finite geometries, also the effects of demagnetization fields which depend on the sample shape, as well as the orientation of the spins at the boundary, also influence the magnetic ground state and for skyrmions the shape of the spin texture has been predicted to depend also on the element shape^{113,114}. In systems with DMI it additionally has been shown theoretically that there is a DMI induced edge tilting^{88,91-93} which can help to create and stabilize magnetic textures including skyrmions. Furthermore, also frustration has been shown to enhance the stability region of skyrmions.¹¹⁵ The expression for the critical DMI value D_c can be extended to consider finite size effects as well. For example, for the model of a nanodisc of finite size in an external field H_z , the critical field becomes $D_c = 4\sqrt{A(K + (H_z - 0.5N_Z\mu_0M_S)M_S)}/\pi$, where $-0.5N_Z\mu_0M_S$ describes the reduced influence of the out-of-plane demagnetizing field due to the finite size of the dot and N_Z being the out-of-plane demagnetizing factor.^{91,107} To a first approximation D_c separates two different regimes: For $D < D_c$ the diameter of the skyrmion core is almost independent of the size of the disc. For $D > D_c$ skyrmions feel the confining potential from the edges of the disc and the magnetic ground state is element size dependent.¹¹⁶⁻¹¹⁸ Increasing the disc size, at some point allows not only one skyrmion to be found in the disc, but rather multiple skyrmions are nucleated, see Fig. 3.

IV. SKYRMION DYNAMICS

The dynamics of magnetic skyrmions can be quite complex. The following requirements constitute a basis for many types of skyrmion based applications. There must be controllable and efficient ways to create, delete, manipulate and eventually excite magnetic skyrmions. In addition to skyrmions, a zoo of (topological) magnetic textures has been discovered and theoretically proposed. More complex spin structures, i.e. which break more symmetries, have potentially more complex dynamics and eigenmodes. The subfield of skyrmions with reduced symmetries⁸³ and in particular antiskyrmions^{74,87,119-122} is a branch that needs to be further explored and that shows a high potential to be useful for technical applications. In the following, we address the current state-of-the-art concerning such basic operations, mostly for magnetic skyrmions, and give perspectives on where improvement is needed and on what can be expected in the upcoming years.

A. Controlled Creation and Annihilation of Skyrmions

To the present date, numerous methods to create and annihilate skyrmions have been proposed^{29,123-138}, both regarding creating skyrmions via the boundaries and within the material. In the following, we give an overview of the different methods to create magnetic skyrmions from an experimental point of view by exploiting different stimuli. Regarding skyrmion annihilation several aspects have been discussed in Sec. III while reviewing the skyrmion stability. Below we only mention additional aspects regarding the skyrmion destruction.

1. Generation by Magnetic Fields

As described above in Sec. III, skyrmions are (meta-)stable for different magnetic field regimes depending on the magnetic properties of the system. By appropriately tuning the applied field, the skyrmion lattice phase can become the lowest energy state of the system. This then means that skyrmions are spontaneously formed, either because other states, such as stripe domains become unstable or because thermal activation overcomes the energy barrier, leading to the formation of a skyrmion lattice that is lower in energy (details, see section IV A 2).

As previously shown experimentally, we can additionally use AC field excitations to switch the magnetic configuration from stripe domains to a lower energy skyrmion lattice⁴⁸.

Finally localized fields can be used to locally write a skyrmion¹³⁹. This has been demonstrated using magnetic MFM tips as localized field sources¹⁴⁰ and a scheme based on the magnetic field from ultrashort electron pulses has been additionally suggested.^{141,142}

2. Generation by Thermal Excitations

For a system with a skyrmion lattice state being the ground state it was shown that one can generate skyrmions from stripe domains by current injection or field pulses that leads to heating which in turn generates strong magnetic excitations that allow the system to transform to its ground state configuration.^{134,143–145} This effect is currently investigated intensively experimentally as one finds a strong temperature dependence of the necessary current densities to nucleate skyrmions¹³⁴ but no complementary theoretical calculations are available so far.¹⁴⁴

3. Generation by Spin Torques

Given that spin torques and in particular spin orbit torques efficiently manipulate skyrmions (see Sec. IV B 3), different mechanisms have been put forward to use spin torques to write skyrmions. Converting DWs into skyrmions is one option^{127,146} and using spin torques in wires with geometrical variations^{147,148} to generate skyrmions is another possibility. Also by using local spin currents flowing perpendicular to a wire, the magnetization can be locally reversed forming skyrmions^{49,149–151}.

This was shown theoretically for instance in Ref. 150 where a nano-contact was used on top of a multilayer that can host single skyrmions, the nucleation of skyrmions can be induced in a controlled fashion at the nano-contact position. First experimental demonstrations of the employment of nano-pillars to generate skyrmions have also been reported¹⁵².

Furthermore, one can exploit the fundamental interplay of an inhomogeneous DC magnetization configuration with a simple homogeneous DC current to periodically create magnetic textures above a certain critical current density. Here it is not important how the magnetic inhomogeneity arises and whether it is static or dynamic. "Options" include locally modified anisotropies, local magnetic fields and Oersted induced fields resulting from current injection, etc. In 1D this mechanism can create DWs whereas in two dimensions it allows the production of skyrmion antiskyrmion pairs.^{71,133,136,153} Crucially for this mechanism, no twisting like interaction and in particular no DMI is needed. For systems with DMI the main creation mechanism is qualitatively not altered, but details including the strength of the critical current density change. Furthermore, after creating the textures, the DMI stabilizes the skyrmion and the antiskyrmion annihilates. In systems in which antiskyrmions are favored, it is theoretically predicted, that the skyrmion would annihilate and the antiskyrmion would survive. In thin films the SOTs also play a role for the dynamics. This theoretically predicted mechanism^{133,136,154} might provide an explanation for the recently observed experiments where skyrmions in systems with DMI are created^{132,134,148} and deleted by electric currents^{137,155}.

Finally skyrmions can also be annihilated by spin torques for instance by moving skyrmions into specially designed tapered geometrical ends of a wire as has been theoretically predicted in Ref. 51.

4. Generation by Electric Fields

One can influence magnetic systems by electric fields including via resulting strain in order to tune the magnetic properties (DMI, anisotropy, etc.) and thereby enable key operations such as the generation (writing) and annihilation (deleting) of chiral particles. The voltage controlled reversal of the core of a skyrmion was predicted by micromagnetic simulations using electric-field modulation of the anisotropy¹⁵⁶. The complete operation of a memory device based on skyrmions was analyzed theoretically in Ref. 157 and a modulation of the DMI with an electric field was found to be extremely efficient for writing and erasing skyrmions. Electric field-driven switching of single skyrmions was shown by spin-polarized STM^{124,158}, where both writing and deleting of skyrmions was demonstrated with electric fields of one polarity creating and the other polarity removing skyrmions. Fundamentally, the presence or absence of a skyrmion can be efficiently tuned by the electric field modulation of the magnetic properties as this effect is not limited by the time reversal symmetry that prevents easy electric-field switching of the magnetization from up to down. Typically, the formation of skyrmions in external magnetic fields is explained by a balance between Heisenberg exchange interactions, DMI interaction and magneto-crystalline anisotropy, and thus it is clear that a change of any of these parameters on the application of an electric field will modify the energy landscape. It was claimed that the dominating effect of the electric field is on the exchange¹⁵⁸, where thus one electric field direction favors the ferromagnetic state, and the other stabilizes the skyrmionic state. In contrast, Ma et al. in Ref. 159 suggest that electric fields could write skyrmions if both the anisotropy profile generated by a thickness gradient and the electric field-induced anisotropy would change. A large voltage induced DMI modulation has also been recently reported, which can modify the states.¹⁶⁰ Finally, the presence and absence of skyrmions depending on the applied electric field can also be connected to a change of the anisotropy as predicted in Ref. 161. The effect of such an anisotropy modulation was calculated recently¹⁶², showing that depending on the geometry, small anisotropy modulations, as demonstrated by electric fields, are sufficient to switch between the stable skyrmion phase and the single domain phase as confirmed for simulations of a device geometry by Li et al. where also electric field induced skyrmion pinning region was also incorporated¹⁶³. Simulations have also revealed the strain induced anisotropy change can be employed to chop chiral strip domains into individual skyrmions.¹⁶⁴ While recently the focus has been on

thin film layer systems, the stability regime and possible anisotropic shapes of skyrmionic spin structures have also been studied experimentally in bulk DMI systems^{165,166}. By selecting systems, where the changes in anisotropy, saturation magnetization and in particular DMI allow for skyrmionic spin structures to be either the stable ground state not stable, writing and deleting of skyrmionic spin structures of various skyrmion number or switching between skyrmionic spin structures with different skyrmion numbers is made feasible.

B. Moving Skyrmions

As discussed below in Sec. V, the most widely discussed skyrmion - based application is a “racetrack”⁹ type device^{49,66}. For such a device¹⁶⁷, in addition to writing (see above) and reading¹⁶⁸, the shifting of skyrmions is essential. Given the lateral translation invariance of most systems on the scale of skyrmions, one expects the lowest lying excitation simply to be the translation (shifting) of a skyrmion. In the following we will consider the displacement of skyrmions which to lowest order do not change their spin texture. Furthermore, we will differentiate between controlled displacements using a directed effective force and random thermal excitations.

1. Magnetic Field Gradients

The simplest approach to manipulate skyrmion positions is to change the energy of the skyrmion when it moves laterally by a field gradient. This was proposed theoretically in Refs. 169 and 170 where it was found that in confined geometries the resulting dynamics is more complex due to the confining potential of the edges. Moreover a gradient has been predicted to accelerate the skyrmion.¹⁷¹ Experimentally skyrmion dynamics due to field gradients was then realized in Ref. 172 where a field pulse initiated the nucleation of a domain whose spatially strongly varying stray field displaced a skyrmion. By measuring the relaxation dynamics, a gyrotropic spiraling trajectory was determined that allowed for the identification of a topologically non-trivial $\mathcal{W} = 1$ skyrmion. More recently this approach has been proposed to position and trap skyrmions¹⁷³ and to guide skyrmions on a track¹⁷⁴.

2. Electric Fields

In addition to creating skyrmions by electric fields, very recently a number of schemes have been proposed for electric field driven skyrmion dynamics. One clear option to achieve this is via e-field induced changes in the anisotropy, as shown in micromagnetic simulations by way of anisotropy gradients from a varying thickness dielectric¹⁷⁵ or stepped magnetic anisotropy gener-

ated by multiple surface electrodes¹⁷⁶. Ma et al. were also able to demonstrate experimentally the electric field driven creation and directional motion of skyrmions in tracks with thickness and resulting anisotropy gradients via the e-field induced changes in the anisotropy¹⁵⁹. Another suggestion is based on electric-field generated surface acoustic waves for ferromagnetic materials grown on piezoelectrics, with analytical studies of both propagation along tracks and for oscillators¹⁷⁷. Yuan et al. proposed an alternative scheme based on parametric pumping of the system via alternating out-of-plane electric fields in the presence of a symmetry breaking in-plane magnetic field. Their simulations revealed a resulting so-called “rock-and-roll” motion of the skyrmion with combined excitations of breathing mode excitations and net motion along a set trajectory. The breathing mode excitations result in spin-wave emission which acts to propagate the skyrmion, resulting in the largest velocities being found theoretically for the resonance of the internal mode¹⁷⁸.

3. Spin Torques

Spin Torques induced by an effective acting spin-polarized current in the system affect the magnetic textures and allow to move them. For magnetic skyrmions it turns out that the current-induced dynamics is special as in addition to forces collinear with the current, also forces perpendicular to the current direction can occur, so called Magnus forces which originate from the skyrmion topology.^{83,179–185} This becomes evident in the quasiparticle Thiele description where a perpendicular force arises from the interplay of the current and the gyrotropic tensor. The transverse motion of skyrmions to the current direction is called the skyrmion Hall effect and recently has been experimentally observed^{181,184,185}. Overall, the details of the skyrmion dynamics depend to a large extent on the symmetries of the systems and the kind of torque that is acting. Below we discuss explicitly the two main type of torques, namely spin transfer torques and spin-orbit torques.

Despite the interesting physics surrounding the Magnus force, for many applications (see Sec. V) a vanishing skyrmion Hall angle is desired. In antiferromagnets that are fully compensated, the skyrmion Hall angle is effectively zero, as the two sub-lattices have an opposite sign of the skyrmion Hall angle and the strong antiferromagnetic coupling thus predicts a motion of the skyrmion along the direction of the current^{186,187}. The same also holds for synthetic antiferromagnets where strong spin orbit torques have been observed¹⁸⁸. The two individual antiferromagnetically coupled layers exhibit opposite skyrmion Hall angles and thus a zero net skyrmion Hall angle results¹⁸⁹. Furthermore, the skyrmion Hall angle can be reduced by using ferrimagnets, where the sub-lattices (partly) compensate each other^{190,191}. For ferrimagnets (and antiferromagnets) the order parameter

that defines the topology and impacts the skyrmion Hall angle is the difference between the sublattice magnetizations (Néel order parameter for antiferromagnets). Another advantage of antiferromagnetic skyrmions is there predicted absence of topological damping.¹⁰⁹

Another approach is to use antiskyrmions. Antiskyrmions are characterized by two high-symmetry axes along which the spin-structure twists on one axes in a left- and on the other one in a right-handed way.^{87,119,120} This reduces the full rotational symmetry of a skyrmion to a two-fold rotational symmetry around their center for an antiskyrmion. A few predictions concerning the dynamics of antiskyrmions and certain peculiarities have already been made^{74,119,192,193}. In particular by tuning the current flow direction with respect to the asymmetric in-plane spin structure, the skyrmion Hall angle can be tuned to values down to zero¹⁹³. Furthermore, in Ref. 83 it has been shown that even in ferromagnetic systems a complete elimination of the skyrmion Hall angle is possible for systems with hybrid DMI with spin-orbit torques.

a. Spin Transfer Torques: The first observation of current-driven skyrmion motion was found for bulk MnSi at very low current densities^{32,181}. Theoretically the dynamics of skyrmions due to adiabatic and non-adiabatic spin transfer torques^{194,195} was determined in Refs. 179, 182, 183, and 196. By choosing a particular geometry of the current flow perpendicular to a confining wire, a velocity enhancement has been predicted¹⁹⁷. For bulk systems spin transfer torques are a dominant contribution. There have been a range of theoretical papers that have dealt with spin transfer torque induced skyrmion dynamics^{21,83,179,182,183,196–198}. In the case of thin film systems, the ferromagnetic layers are usually very thin (few angstroms) and combined with thicker heavy metal layers so that the current flowing in the ferromagnet is small. This then results in low spin transfer torques, which are usually superseded in experiment by the spin orbit torques.

b. Spin Orbit Torques: In materials with spin-orbit torques, first measurements using photoemission electron microscopy (PEEM) have shown that DWs can be moved in opposite directions for Pt/CoFeB/MgO and Ta/CoFeB/MgO stacks and very fast walls have been observed with AlOx top layers due to spin-orbit torques^{10,199,200}. Results on the sign and strength of the DMI as well as the spin accumulation due to the SHE in Tantalum and Platinum have been obtained and a range of materials with chiral DWs including Pt/CoFeB/MgO, Ta/CoFeB/MgO and Pt/Co/Ir and other multilayer stacks have been reported.^{48,184,199–201} Furthermore, recently a fast domain wall motion in compensated ferrimagnets has been experimentally observed.²⁰² Spin orbit torques were shown to displace skyrmions in Ta/CoFeB/TaOx films, albeit with relatively low velocities < 10 m/s.^{127,167} Faster skyrmions > 100 m/s were observed in Pt/CoFeB/MgO^{48,184} possibly due to the larger spin Hall angle in Platinum compared to Tantalum. Other observations of current-driven skyrmion

motion due to spin orbit torques include work on multilayers¹³² and recent first dynamic measurements have confirmed that Néel skyrmions display a skyrmion Hall effect.^{184,185} In the measurements it was observed that the skyrmion Hall angle does not only depend on the size of a skyrmion but also on its velocity, which cannot be explained within a rigid skyrmion model. A dependence of the skyrmion Hall angle on the velocity occurs due to pinning as well as deformations of the skyrmion originating in current induced torques. Depending on the applied current strength and thus the different regimes, one of the effects plays the dominant role. In the slow creep regime the interplay of pinning and the Magnus force term, resulting from the skyrmion topology^{203–205}, is predominantly responsible for the velocity dependent skyrmion Hall angle. At higher drives, in the flow regime, the current induces torques deform the skyrmions. This deformation combined with the effect of a field-like spin orbit torque can also lead to a drive dependence of the skyrmion Hall angle¹⁸⁴. Very recently, the full dynamic range from the pinning-dominated creep regime to the viscous flow regime was probed experimentally⁸⁴ and it was indeed found that there are different regimes where the slope of the skyrmion Hall angle with velocity is different as predicted theoretically.²⁰⁶ For low velocities there is a larger slope than for high velocities and the intersection between the two slopes marks roughly the transition from the creep to the viscous flow regime⁸⁴. Overall this skyrmion Hall effect can lead to the expulsion of skyrmions from a wire²⁰⁷, making an understanding of the energy barrier and the expulsion process key for device operation. However as shown for spin transfer torque, the motion of skyrmions along the edge might actually benefit the dynamics¹⁹⁷, if this motion is sufficiently robust.

4. Magnons

Magnons, the quanta of spin wave excitations have been shown to displace spin structures, such as DWs and recently skyrmions have moved to the focus. The interaction between magnons and skyrmions was studied in Refs. 208–210, where it was found that magnons can excite the eigenmodes of skyrmions (see Sec. IV D) and that a magnon current can induce skyrmion motion^{211–213}. As the scattering of magnons from skyrmions is not isotropic due to the topology of the skyrmion, there is an associated skyrmion Hall effect due to the incoming magnon propagation direction. The theory of magnon-skyrmion interaction was further developed in Refs.^{211,214–217} and it was found that the interaction depends strongly on the magnon wave number and there is a finite momentum associated with the skyrmion. Finally the impact of skyrmions on magnon properties was also ascertained²¹⁸.

5. Temperature Gradients

One particular approach to displace spin textures that involves magnons is the use of temperature gradients. Here, thermal spin currents can transfer angular momentum and additionally entropic effects can also lead to a motion of spin structures as well. Overall the physics is quite complex and both effects can displace magnetic textures towards the colder as well as the hotter part in certain regimes. This approach has been extensively discussed for DWs theoretically^{219–222} and experimentally there are reports of thermal effects on DWs^{223,224}. This approach was also proposed for skyrmions and in Refs. 179, 196, 225, and 226 the dynamics of a skyrmion due to a temperature gradient was calculated.

6. Thermal Fluctuations

Even without any external drives, skyrmions can change their positions randomly due to thermal fluctuations. This was seen first in Ref. 127 in the supplementary movies, but not discussed and later also seen in Ref. 68. Another signature was observed in Ref. 184 where in the supplementary information (Fig. S2) it was seen that a skyrmion can move by thermal fluctuations from one position to another in the absence of any external driving force. Recently this was studied in detail²²⁷. In low pinning materials, strong thermal skyrmion diffusion was identified by measuring the mean square displacement with time. While the diffusion is in qualitative agreement with analytical predictions,²⁰⁹ it was found that the skyrmion diffusion depends on the skyrmion size and exhibits an exponential scaling with temperature. This surprising behavior can be explained by diffusion in a non-flat potential energy landscape leading to locally different pinning properties. The average energy barriers were extracted quantitatively from the experiments. Analogous to the diffusion in solids, this behavior shows that in the optimized systems, skyrmions exhibit thermally activated dynamics that can be uniquely used to quantify key unknown properties, such as energy landscapes, ascertain interaction potentials, etc. Finally, short timescale fluctuations have recently been probed by free electron laser measurements²²⁸.

C. Rotating Skyrmions

The first work revealing the strong coupling of skyrmions³² to electric currents actually reported a rotation of the skyrmion lattice by a finite angle due to the applied current. In general, e.g. in classical mechanics an object experiences a rotation if a net torque is acting on it. In the case of the skyrmion lattice in Ref. 32, the rotation in the experiment originates from an existing temperature gradient that leads to greater dissipative and

Magnus forces and thus torques on one side of the sample than on the other.¹⁹⁶ Due to the gyrotropic nature of the skyrmion dynamics the rotation is perpendicular to the temperature gradient. Up to now several experimental and theoretical works on the (dynamical) rotation of the skyrmion lattice appeared^{179,182,196,215,229–233} studying different forms of symmetry breaking (thermal gradient, magnetic field gradient, electric field...) to induce a rotation.

D. Skyrmion Excitations

The interplay of skyrmions and magnons as well as their excitation spectra is complex. The frequency range of fundamental excitations of magnetic textures are mainly determined by their strongest energy scale which for magnetic skyrmions is in the GHz range. Overall the eigenmodes of the skyrmions are related to their inertia or mass.^{209,210,217} In the following we will briefly describe the excitations for the limiting fundamental skyrmion structures: hexagonal skyrmion lattices and single skyrmions followed by a discussion about magnon skyrmion scattering. This topic has been addressed in more detail in a recent review³⁷.

1. Skyrmion Lattices:

Shortly after the theoretical prediction of skyrmion lattices^{208,234}, the clockwise, the counterclockwise and the breathing mode were observed by means of microwave absorption spectra in Cu_2OSeO_3 ^{235,236} and optical pump-probe techniques²³⁷. Since then by means of magnetic resonance techniques or inelastic neutron scattering skyrmion lattice excitations were observed in several compounds including metals, semiconductors and insulators. It has been shown that these excitations have a universal character²³⁸, see Fig. 4a), and can be quantitatively modeled by only two material specific quantities characterizing the DMI energy and the critical field value for the field polarized phase.³⁷ In addition to the thus far observed excitations, which symmetry wise couple to homogeneous excitations, there is an ongoing effort to experimentally access new theoretically predicted modes.

2. Single Skyrmions:

Numerical simulations and analytical calculations have identified a number of different internal modes, such as the gyrotropic, uniform breathing and polygon-like distortion modes^{209,210,239}. For a thin film, below the magnon gap there exist a few magnon-skyrmion bound states (see Fig. 4b), which are labeled according to their angular momentum quantum number m . Note that due to the translational invariance, the translational mode has zero frequency²⁴⁰. The breathing mode,

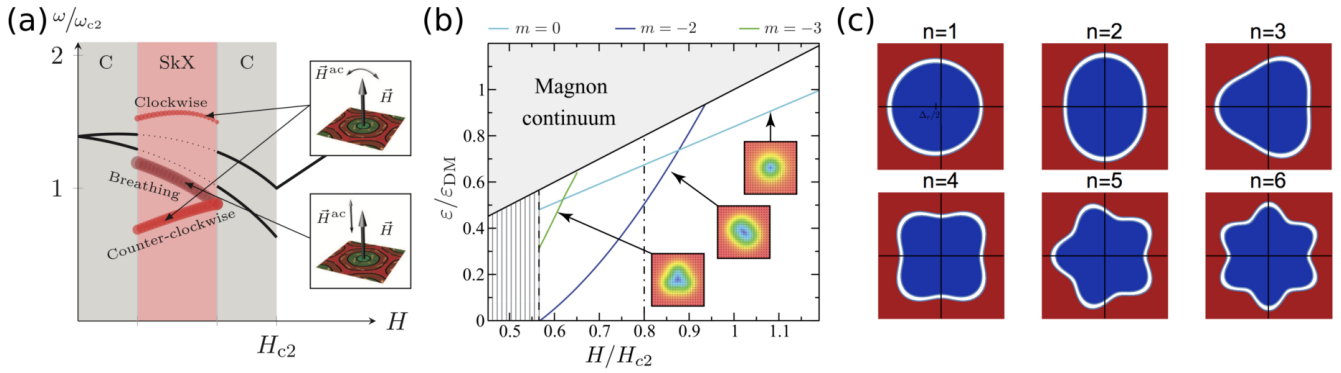


FIG. 4: a) Calculated field dependences of the resonance frequencies of the observed skyrmion crystal (SkX) modes (red background). Reprinted figure with permission from Ref. 37, Copyright 2017 IOP Publishing. (b) Magnon spectrum of a single skyrmion in a ferromagnetic background. Adapted figure with permission from Ref. 209, Copyright 2014 by the American Physical Society. (c) Sketch of skyrmion eigenmodes as a solution to a string equation. Parts of Fig. 4c) are reprinted with permission from Ref. 241, Copyright 2014 by the American Physical Society.

parametrized by the angular momentum $m = 0$, can for example be activated by fields that are directed out-of-plane and which do not affect the radial symmetry of the skyrmions^{240,242}. Imaging of skyrmion breathing modes induced by spin-orbit torques has also been achieved experimentally²⁴³ and the gyrotropic resonance has also been measured²⁴⁴. In-plane magnetic fields, on the other hand, do break the radial symmetry and can potentially excite modes that affect the shape of the skyrmion^{240,242}. The quadrupolar mode with $m = 2$ indicates the elliptical instability of the skyrmion⁴⁴ and also a sextupolar mode with $m = 3$ is realized only in a small range above the elliptical instability. For more bubble-like skyrmions also more modes are expected to exist^{24,241,245} see Fig. 4c). Experimentally those modes are interesting for microwave generation. So far, the polygon-like distortion modes are still to be observed.

In confined samples, e.g. in ferromagnetic nanodiscs, the translational mode turns into a gyrotropic mode with a non-zero frequency.^{172,240,242,246,247} Furthermore in asymmetric confining potentials new modes emerge for increased aspect ratios of the elements.¹⁷⁶

V. APPLICATIONS

Due to the fast experimental progress in observing skyrmions at room temperature and engineering their properties, the use of skyrmions for applications has become an emerging field going beyond theoretical predictions. Here we discuss both the use of skyrmions in technologies that have been conceptually put forward already and where skyrmions show improved properties over existing approaches and also discuss applications that exploit the unique skyrmion properties, which have inspired the community to come up with completely new application ideas. In general skyrmions promise low operating

powers and data non-volatility leading to a small device footprint.

A. Racetrack

The most prominent application of skyrmions is the racetrack device where demonstrators are already experimentally realized and many of the key operations such as reading, writing and shifting have been shown. Racetrack memories are based on the idea of storing data by aligning objects like beads on an abacus, exploiting the quasi-particle nature of magnetic spin states (see Fig. 5). While originally suggested for DWs⁹ using spin transfer torque¹⁹⁴, spin-orbit torques have proven to be a major advance¹¹ and using synthetic antiferromagnets, even higher velocities were reported¹⁸⁸. While fundamentally a racetrack memory can be realized using DWs, these also have certain disadvantages. Since they cover the full width of a wire touching both wire edges, their displacement can be prone to pinning at edge defects such as edge roughness. Replacing DWs by skyrmions^{50,51,66,248} and storing data by the presence or absence of skyrmions can potentially overcome the problem of edge roughness related pinning. Additionally, compared to DWs, skyrmions can move also in the transverse direction within a wire, which means that they can potentially move around defects or other obstacles as previously investigated theoretically.^{249,250} Furthermore, compared to non-chiral DWs, their chiral nature results in a certain topological stabilization, as discussed in Sec. III.

In addition to a range of theoretical investigations of skyrmion racetracks^{50,51,66,104,251–254}, recently experimental results have been made available showing that skyrmions can be moved by spin orbit torques^{48,127,167}. In these systems also the skyrmion-skyrmion interactions

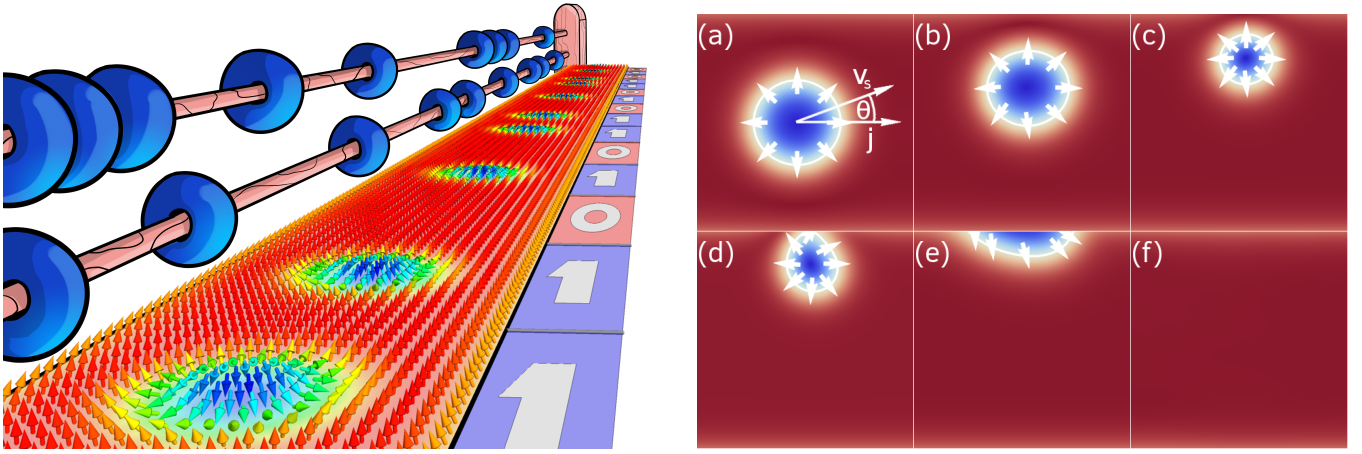


FIG. 5: Left: Schematic of a skyrmion racetrack memory storing data by aligning objects like beads on an abacus (courtesy of Marco Armbruster and Jan Masell). Right: Due to the skyrmion Hall effect, the skyrmion has a perpendicular velocity component to the direction of the current, which pushes it to the edge of the racetrack (upper and lower edge of the panels). The skyrmion is finally expelled from the racetrack if the confining potential is not strong enough. Here the simulations are performed for the example of spin-transfer torque driven skyrmion motion.

become relevant, which usually are repulsive,⁵¹ but might be attractive under certain conditions.²⁵⁵ However, it was shown that skyrmions still suffer from interactions with defects and they can even annihilate at defects^{48,250}. By improving the material, fast and completely reproducible and reliable skyrmion motion was established¹⁸⁴ with velocities that are comparable to single layer spin-orbit torque DW devices for similar current densities¹¹.

Due to the topology of the spin structure, skyrmions usually do not move along the current flow direction as discussed in Sec. IV B. While the particle like nature allows skyrmions to move around obstacles, the skyrmion Hall effect hampers the easy realization of skyrmion racetracks. Although skyrmions experience a repulsive interaction with the edges, for sufficiently strong driving current, this will be overcome and skyrmions will be expelled from the wire, (see right panel of Fig. 5), thus generating an operational error (data is lost). In order to keep skyrmions inside the wire, different approaches have been put forward. One can fabricate tapered edges with different thickness²⁵⁶ or vary magnetic properties, such as the anisotropy at the edges to increase the edge repulsion²⁵³. While these approaches mitigate some of the risk of skyrmion expulsion, they do not tackle the underlying problem of the skyrmion Hall angle. A relative straight forward cancellation of the skyrmion Hall angle occurs when using Néel skyrmions with opposite polarity (orientation of the center) since they exhibit an opposite skyrmion Hall angle. Then by either strongly coupling two skyrmions with opposite polarities like in synthetic antiferromagnets^{189,257} or directly employing antiferromagnetic skyrmions^{186,187}, the skyrmion Hall angle can be eliminated. While the generation of completely antiferromagnetic skyrmions is still challenging, a reduction of the skyrmion Hall angle was already experimentally reported for ferrimagnets with partially compensated sub-

lattices¹⁹⁰. A completely new approach uses an additional symmetry breaking to eliminate the skyrmion Hall angle. In Ref. 83 it has been shown that by the interplay of hybrid DMI (a mix of bulk and interfacial DMI) and spin-orbit torques, a complete cancellation of the skyrmion Hall angle can be achieved.

A second challenge that has been addressed much less is that one needs to move all skyrmions synchronously to keep their distances and thus the data intact. This problem is shared with DWs, where it was proposed²⁵⁸ to use regular notches to control the motion of the walls by pinning the DWs at these locations^{259,260}. For skyrmion racetracks, however, it is challenging to achieve reliable operation with this approach²⁶¹, in part because skyrmion motion is less susceptible to notches. Alternatives include either locally vary the magnetic properties to generate a pinning site across the whole wire as shown for DWs²⁶² or actively using voltages to control the magnetic anisotropy²⁶³. Another suggestion that uses the unique skyrmion quasi-particle properties, is to encode the information not in the presence or absence of the skyrmion in a wire but by using two positions for the skyrmion in effectively a two lane racetrack^{251,256,264}. A similar scheme has suggested the use of skyrmions in a second lane as dynamic pinning sites²⁶⁵ and more complex geometries with more lanes have been put forward²⁶⁶.

Note that the readout will likely to be realised with established magnetoresistive techniques such as GMR or TMR pillars on top of the race track.^{267–269} In particular for skyrmions an additional readout mechanism is, in principle, provided by the topological Hall effect.¹⁸¹ The details of the applicability of this readout mechanism are, however, still to be established.^{77,78,270–272}

As can be seen, while fundamentally the generation of a skyrmion racetrack is feasible, there are still challenges

to be overcome before a device might become a reality.

B. Skyrmion Applications Beyond the Racetrack

Due to their small size, topological stability and low critical depinning current density, magnetic skyrmions and their relative have also been suggested for various other applications.

For example, it has been proposed, primarily based on micromagnetic simulations, to build skyrmion based conventional logic devices^{273–276} which are compatible with racetrack memories. The logic functions exploit the interplay of magnetic interactions and current induced spin-torques. They are implemented via patterned nanowire structures^{273,274} of different widths where the conversion between skyrmions and DW pairs is used.^{125,127} In principle this conversion mechanism allows the controlled nucleation or merger of skyrmions through the design of specific nano-structures, and thus one can perform basic logic operations. Recently, also a novel reconfigurable skyrmion logic implementing the complete logic functions based on voltage control has been demonstrated by means of micromagnetic simulations.²⁷⁶

A skyrmion based transistor²⁷⁷ has additionally been proposed. In the prediction, magnetic tunnel junctions are used to create and detect the skyrmion¹⁵¹. Its motion from the source to the drain is induced via an effective spin current induced by the spin Hall effect of the underlying heavy metal. It can be controlled via a voltage changing the perpendicular magnetic anisotropy at the gate.

The above described applications exploit the particle-like nature of skyrmions. Predictions of applications using the internal degrees of skyrmions include skyrmion based spin transfer nano-oscillators.^{278,279} These are nanoscale devices with self-sustained oscillations of the magnetization, whereby the intrinsic magnetic damping is on average compensated by spin transfer torques. The working frequencies of skyrmion based oscillators are in the GHz range represented by their internal eigenmodes. Such oscillators are attractive for applications as wide-band nanoscale electrical oscillators, sensitive magnetic field sensors and on-chip microwave signal sources.

An emerging field in spintronics is the new and very diverse area of unconventional computing which aims to go beyond traditional von Neumann architectures. A promising example which has been theoretically predicted²⁸⁰ and recently also experimentally realized²²⁷ is to use a skyrmion gas to reshuffle a random signal into an uncorrelated copy of itself. This device is a key missing component in stochastic computing in which tolerable loss of precision is traded for speed and efficiency. The skyrmion reshuffler serves as a means to obtain an uncorrelated signal, which is a prerequisite for the correctness of stochastic computations.

A field within unconventional computing that has attracted significant attention is neuromorphic

computing,²⁸¹ inspired by the brain’s ability to work on a large multitude of tasks in parallel, with low power and high efficiency. Several skyrmion-based components have been proposed in this context. For example, it has been demonstrated by micromagnetic simulations that skyrmion based devices can emulate synapses.²⁸² Biological synapses regulate the signal transmission between neurons, the primary components of the central nervous system. A prerequisite for learning is the ability of the brain to adjust denoted as neuroplasticity which originates in the fact that the weight of a signal through a synapse adapts over time based on the temporal correlation between the spiking activities of the interconnecting neurons. The artificial skyrmion synapse device in Ref. 282 consists of a two terminal device with a gated barrier separating the pre-synapse from the post-synapse region. In the initialization process skyrmions are created in the pre-synapse region such that this region is saturated with skyrmions. Via different applied currents, skyrmions can traverse the barrier region into the post-synapse region. The magnetoresistance signal of the post-synapse region can be interpreted as the weight of the artificial synapse and readout through magnetoresistance effects. Exploiting the interplay of a constant driving force and the individual repulsive interactions between skyrmions the device will adjust its weight according to the applied current strength and direction. Additionally, neuromorphic computing elements based on skyrmions have also been proposed and simulated, including devices with both so-called “leaky integrate and fire” functionality based on current-induced skyrmion motion along a nanotrack with graded PMA²⁸³ or skyrmions interacting with a DW pair²⁸⁴ as well as “resonate-and-fire” neurons based on the dynamics of a skyrmion in the free layer of a magnetic tunnel junction²⁸⁵.

A different suggestion is to effectively employ skyrmion based magnetic textures for the implementation of a functional reservoir computer.^{60,61} Reservoir computing systems are based on recursive neural networks where the recurrent part of the network, referred to as the reservoir, is treated differently from the read-in and the read-out layers. Only the output weights are trained in reservoir computing systems, resolving the difficulty to train the recurrent networks at bifurcation points. In the devices suggested in Refs. 60 and 61, complex time-varying current signals injected via contacts into the magnetic substrate are modulated by the anisotropic resistance associated with the magnetic texture in a nonlinear fashion, due to the current distribution following paths of least resistance as it traverses the geometry. Reminiscent of Atomic Switch Networks skyrmion based reservoirs effectively carry temporally correlated information about the injected signal. This in turn allows for applications like pattern recognition.

VI. OUTLOOK: SKYRMIONS AND BEYOND

The field of magnetic skyrmions is very active and in a very short time the field has made enormous progress. Still several fundamental questions and particular challenges remain, but there is promise that they will be resolved within the future. These include the interaction of skyrmions with other magnetic textures, the particle wave duality of skyrmions, in particular regarding skyrmion lattice phase transitions^{286,287} and the suggestion to use skyrmion lattices as magnonic crystals.²⁸⁸ The interaction and coupling of magnetic skyrmions with light^{237,289} and other topological excitations such as superconducting vortices²⁹⁰ or Majorana fermions^{291–294} are directions in which we expect to find new exotic states of matter. While it is clear that in this concise perspectives article not all work can be covered, the selected topics however show that magnetic skyrmions have become an exciting and very active field of research.

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Material	Sample	Conduction	$\Delta T_{\text{sky}}/\text{K}$	$\lambda_{\text{H}}/\text{nm}$	Type	Refs.
MnSi	bulk	metal	28 - 29.5	18	Bloch	28,295–297
MnSi (press.)	bulk	metal	5 - 29	18	Bloch	166,298–300
MnSi	film ($\sim 50\text{nm}$)	metal	$<5 - 23$	18	Bloch	301,302
$\text{Fe}_{1-x}\text{Co}_x\text{Si}$	bulk	semi-metal	25 - 30	37	Bloch	303,304
$\text{Fe}_{0.5}\text{Co}_{0.5}\text{Si}$	film ($\sim 20\text{nm}$)	semi-metal	5 - 40	90	Bloch	30
FeGe	bulk	metal	273 - 278	70	Bloch	305
FeGe	film ($\sim 75\text{nm}$)	metal	250 - 270	70	Bloch	31,305,306
FeGe	film ($\sim 15\text{nm}$)	metal	60 - 280	70	Bloch	31
Cu_2OSeO_3	bulk	insulator	56 - 58	60	Bloch	64,65,307–310
Cu_2OSeO_3	film ($\sim 100\text{nm}$)	insulator	$<5 - 57$	50	Bloch	64,311
$\text{Co}_8\text{Zn}_8\text{Mn}_4$	bulk	metal	284 - 300	125	Bloch	286
$\text{Co}_8\text{Zn}_9\text{Mn}_3$	bulk	metal	311 - 320	>125	Bloch	312
$\text{Co}_8\text{Zn}_9\text{Mn}_3$	film ($\sim 150\text{nm}$)	metal	300 - 320	>125	Bloch	312
GaV_4S_8	bulk	semi-metal	9 - 13	17.7	Néel	313
Co/Ru(0001)	monolayer	—	~ 4.2	20	Néel	108
Fe/Ir(111)	monolayer	—	< 28	—	Néel	29,34,314,315
PdFe/Ir(111)	bilayer	—	< 8	6 – 7	Néel	34,124,314,316–318
Fe/Ir(111)	trilayer	metal	8	4	Néel	158
(Ir/Co/Pt) ₁₀	multilayer	metal	≤ 300	30 – 90	Néel	46
(Pt/Co/MgO)	single layer	metal	≤ 300	~ 500	Néel	47
Pt/CoFeB/MgO	multilayer	metal	≤ 300	480	Néel	48
Pt/Co/Ta	multilayer	metal	≤ 300	480	Néel	48
Pt/Co ₆₀ Fe ₂₀ B ₂₀ /MgO	multilayer	metal	≤ 300	344	Néel	48,184
W/Co ₂₀ Fe ₆₀ B ₂₀ /MgO	multilayer	metal	≤ 300	460	Néel	86
Pd/Co ₆₀ Fe ₂₀ B ₂₀ /MgO	multilayer	metal	≤ 300	300	Néel	84
Ta/Co ₂₀ Fe ₆₀ B ₂₀ /MgO	single layer	metal	> 300	4200	Néel	227
Ta/Co ₂₀ Fe ₆₀ B ₂₀ /MgO	multilayer	metal	≤ 300	< 900	Néel	84
Ir/Fe/Co/Pt	multilayer	metal	5 - ~ 300	~ 150	Néel	78,79
Pt/Gd ₂₅ Fe _{65.6} Co _{9.4} /MgO	multilayer	metal	≤ 300	440	Néel	190

TABLE I: Selection of materials that are known to host skyrmion lattices or single skyrmions. The table covers bulk materials, thin films, and layered-materials, see second column. The conductive behavior - if known - is given in the third column. The following columns provide the temperature range ΔT_{sky} at which skyrmions were observed, the wavelength λ_{H} of the helical/stripe phase, and the symmetry of the skyrmion (Bloch/Néel). The References are listed in the last column. Table partly taken from PhD theses of J. Masell²⁶⁴ and K. Litzius⁸⁴.

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