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ORIGINAL PAPER

What Have We Learned About Modelling and Assessing Mathematics Teachers' Competence?–Insights from a Literature Review

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Abstract
Sigrid Blömeke and colleagues developed a model in 2015 that describes teacher competence as a continuum moving from dispositions such as knowledge and beliefs, to situation-specific skills such as classroom decision-making to (observable) performance. In this paper, we investigate its application and continued development in research by focusing on pre- and in-service mathematics teachers. To this end, we conducted a comprehensive, systematic review of studies that further developed this model conceptually and/or empirically and expanded it to other areas. The developments concern the specification of the central components of this model such as dispositions, skills, and performance, the relationship between these components, and the inclusion of further factors such as student mediation. Through a systematic synthesis of these further developments, we create an expanded model of teachers' competence. Our systematic review reveals some strong methodological deficits in empirical studies testing the Blömeke et al. model, which need to be addressed in further research.

Keywords Teacher competence · Competence structure · Performance · Assessment · Literature review

1 Introduction and Research Objectives

Blömeke and colleagues (2015) examined approaches to conceptualizing and measuring competencies. The challenging nature of this undertaking becomes particularly evident in defining the concept of competencies. Blömeke et al. (2015) went beyond the prevailing conceptualization at the time (e.g., Koeppen et al., 2008, p. 61) and aimed at a deeper understanding and unpacking of the complex construct of competency, leading to a groundbreaking definition in this research field. They conceptualized it as "the latent cognitive and affective-motivational underpinning of domain-specific performance in varying situations" (Blömeke et al., 2015, p. 3). Building on this conceptualization, they addressed the theoretical and methodological challenges associated with measuring competencies (in higher education). As highlighted in their definition, they linked the measurement of latent constructs, which are not directly observable but must be inferred from observable behavior—hence the continuum of competence. They conceptually separated the construct into various knowledge-based, cognitive, and affective-motivational traits while simultaneously observing competencies as behavior in specific real-life situations. In this context, they consolidated various concepts and definitions into a theoretical framework that demonstrates the relationships among paradigms for modeling and measuring teacher competencies; e.g., 'personality paradigm', 'process-product paradigm', and 'expertise paradigm' (Bromme, 2001).

In addition, "[i]nstead of insisting on an unproductive dichotomy view of competence, Blömeke et al. (2015, p. 7) offered a perspective on competencies in which both *analytical* and *holistic* approaches complement one another (Zlatkin-Troitschanskaia et al., 2019a). This culminated in the development of a process-oriented path model of competence as a continuum, which provides an integrative perspective on competencies as depicted in Fig. 1. This model views competencies as both a process and product along a continuum that extends from (meta)cognitive and affective-motivational dispositions through situation-specific skills to observable teachers' behavior. Thus, Blömeke et al. (2015) aptly capture the transformative significance of their model for research.

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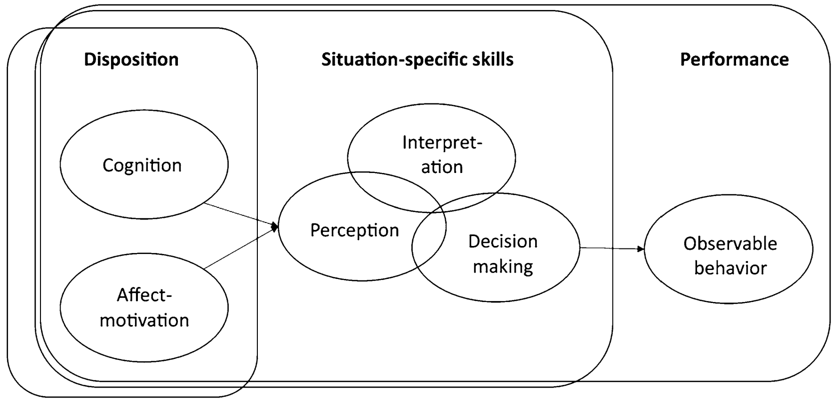

Fig. 1 The Blömeke et al. (2015) model

Blömeke et al. (2015) did not claim to offer a complete framework, or, "to [have found] 'the' one definition and statistical model for competence assessment" (p. 11). Rather, they regarded their model as an overview-type structural framework that primarily serves as inspiration for subsequent research aimed at testing and validating the fundamental tenets of this model. Thus, Blömeke et al. (2015) encouraged future studies: "Research on competence measurement, then, might work on various parts of the model and even attempt to test the entire model conceptually and statistically" (p. 11).

In the last decade, many studies have used this model within and beyond mathematics teacher education. In this paper, we investigate

(*a*) how, if at all, has empirical research addressed the challenge of testing, validating, and improving the Blömeke et al. (2015) model, and

(*b*) has this research led to an expanded, validated framework for addressing mathematics teachers' competencies.

In our literature review, we particularly focus on the following research questions (RQs):

1) What have we learned from the findings from the literature review regarding the specifications and developments of the competence model?
2) How would findings from RQ1 contribute to shaping an improved model?
3) What are the methodological challenges pertaining to measuring teacher competences?

To address these RQs, we explore the methodological background of our literature review and present its approach and search strategy. Next, we provide an overview of empirical studies on the competencies of (pre- and in-service) mathematics teachers included in this review. We then describe important developments of the Blömeke et al. (2015) model and its main components (RQ1) and present a revised model of teachers' competence (RQ2). Finally, we illustrate some measurement advances and new frontiers and conclude with implications for future research (RQ3).

2 Review Approach

2.1 Search and Selection Strategy

To provide a comprehensive overview of international research on the development and application of Blömeke et al.'s (2015) approach to modelling competencies, we conducted a semi-systematic literature review (Snyder, 2019) that considers both empirical and review studies. This approach was deemed suitable for our RQs, as it was specifically designed to develop systematic insight into subjects that have been conceptualized differently and have been addressed by different disciplines within research (Wong et al., 2013). This approach allows us to depict the development and evolution of research on teachers' competencies as a continuum over time and to identify research gaps.

This approach also enables us to combine two sampling methods (Lage, 2022) using Google Scholar and JSTOR library as the main literature databases. We selected these databases since they provide a particularly comprehensive coverage of various types of sources (including 'gray literature' such as internal research/technical reports as well as book articles and journals), aligning with our RQs (Jarvis, 2023; Martín-Martín et al., 2021).

In the first stage, we examined peer-reviewed articles and book chapters that have cited the publication by Blömeke et al. (2015) following the ´forward citation chasing´ technique (Hirt et al., 2023). This approach enabled us to anticipate further developments and applications of the Blömeke et al. (2015) model by tracing *citation networks* that have formed following the core publication (Haddaway et al., 2022). As over 1.700 articles have resulted from this process, we systematically searched for relevant literature within this sample by using keywords tailored specifically to our RQs (Table 3 in the Appendix; Lage, 2022; Portus et al., 2024). To ensure a selection process that is as comprehensive as possible, we have used various combinations of keywords, which we have linked using Boolean operators (Atkinson & Cipriani, 2018). Our inclusion and exclusion criteria for articles are documented in Table 4 in the Appendix.

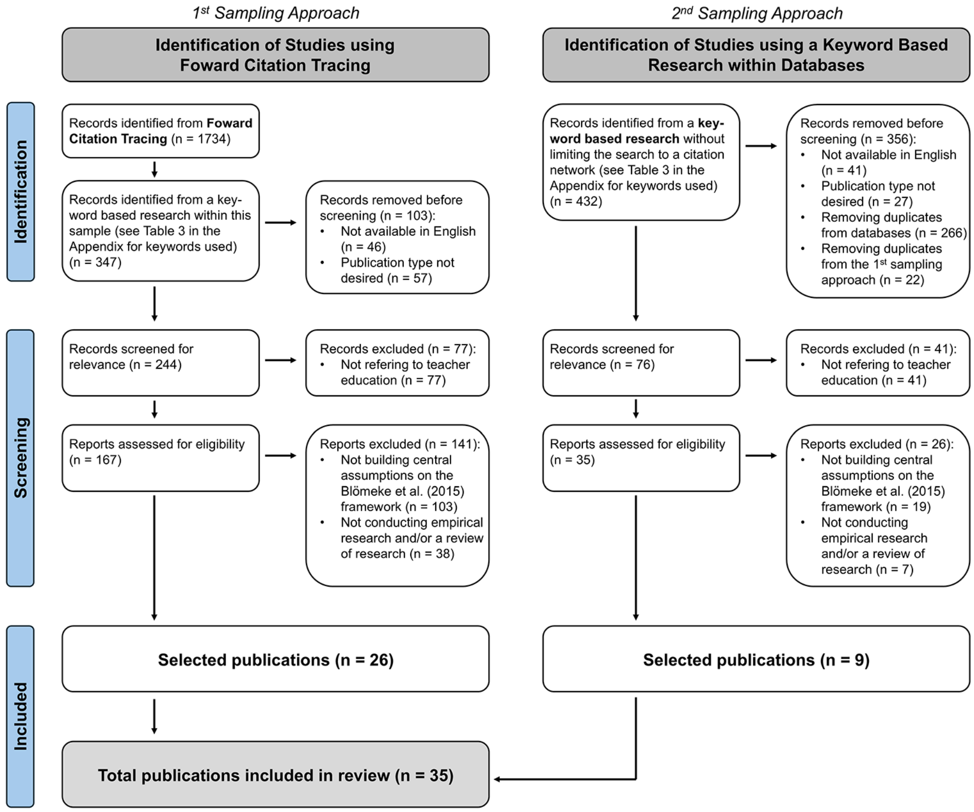

Fig. 2 Literature selection process. *Note*. This flow diagram is based on the PRISMA guidelines and adapted from the diagram proposed by Page et al. (2021). Record: A database- or website-indexed entry containing the title and/or abstract of a report. Report: A document that details information on a specific study. Examples include journal articles, preprints, conference abstracts, or dissertations

After screening the abstracts of selected literature for relevance, a sample of 26 publications resulted from this first stage for further analysis. One of the findings of the initial search was that in most of the studies reviewed, the source Blömeke et al. (2015) was only cited in the bibliography or only once as an additional reference in the text and therefore did not meet our selection criteria (Table 4).

To minimize a potential selection bias (Chigbu et al., 2023), that might result from our exclusion criteria, in the second stage, we applied an additional sampling approach. We conducted a further keyword-based literature search (Rowley & Slack, 2004) without limiting the search to a *specific citation network*. While the exclusion criteria and the Boolean operators were also applied here, the keywords used were extended by additional search terms (Table 3). After the abstracts of selected publications were evaluated in terms of relevance, 9 publications were added to the sample. As a result, a total of 35 publications were included in the analysis of this review. The selection process across the two phases is visualized in Fig. 2.

Next, we organized the contributions according to the pre-defined categories (Table 5 in the Appendix):

(1) we divided the studies into *empirical* and *review* analyses.
(2) we differentiated the empirical studies according to their use of quantitative and qualitative, cross-sectional and longitudinal, (quasi)experimental etc. research methods.
(3) we subdivided them in terms of whether they relate to mathematics teaching or another subject (e.g., economics).
(4) the studies were categorized according to whether they refer to empirical *or* theoretical research or include both empirical and theoretical aspects.

Specific applications of the Blömeke et al. (2015) model in intervention research as well as further developments by Blömeke herself were considered separately.

2.2 Characteristics of Selected Literature

Of the 35 publications selected, 23 were empirical studies, and 12 were review studies. Among the empirical studies, *11 focused on the context of mathematics* (see Table 2), while 12 studies also addressed other subjects such as physics (e.g., Kulgemeyer et al., 2020) or biology education (e.g., Martin et al., 2022), however, were related to mathematical content or education (e.g., Kuhn et al., 2020).

The designs of the empirical studies differed greatly, including single case studies (Hino & Funahashi, 2022), multi case studies (Santagata & Yeh, 2016), or large sample studies (e.g., Keppens et al., 2021). One study was based on a representative sample of German mathematics teachers (Krauss et al., 2020). The average sample size in the empirical studies was 222 participants, with a focus on pre-service

Table 1 Characteristics of the final sample

Type of Study	Characteristics	
Empirical studies (n = 23)	Sample size, mean (±SD):	221.74 (±346.17)
	Study design:	
	-Cross-sectional, % (n)	47.83% (11/23)
	-Longitudinal, % (n)	26.09% (6/23)
	-Quasi-longitudinal, % (n)	4.35% (1/23)
	-Quasi-experimental, % (n)	21.74% (5/23)
Review studies (n = 12)	Review focus:	
	-Meta-analysis, % (n)	33.33% (4/12)
	-Empirical and theoretical review, % (n)	58.33% (7/12)
	-Theoretical review, % (n)	8.33% (1/12)
	Country Context	
Complete Sample (n = 35)	Belgium, % (n)	5.71% (2/35)
	China, % (n)	2.86% (1/35)
	Finland, % (n)	5.71% (2/35)
	Germany, % (n)	62.86% (22/35)
	Japan, % (n)	2.86% (1/35)
	Norway, % (n)	5.71% (2/35)
	Spain, % (n)	2.86% (1/35)
	Swiss, % (n)	2.86% (1/35)
	USA, % (n)	8.57% (3/35)

teachers and fewer studies on in-service teachers (see Tables 1 and 2).

Out of 23 empirical studies, 11 studies were conducted in a cross-sectional design (e.g., Hammer & Ufer, 2023), 6 studies in a longitudinal design (e.g., Muñiz-Rodríguez et al., 2018), one study in a quasi-longitudinal design (Alwast & Vorhölter, 2021), and 5 studies in a quasi-experimental design (e.g., Jong et al., 2021). Two studies implement interventions with video-based or digital tools (e.g., Jong et al., 2021). In 3 studies, video vignettes-based assessments are applied to measure teacher competence (e.g., Krauss et al., 2020).

All the review studies drew on research findings on teachers' competencies: 4 were meta-studies (e.g., Cevikbas et al., 2024), 7 included both empirical and theoretical contributions (e.g., Wassermann et al., 2023), and one focused on theoretical perspectives (Baumgartner, 2022).

The majority of the studies (63%), originate from Germany; 37% of studies were conducted in other countries (including other EU countries, the USA, Japan and China, see Table 1). This illustrates a fairly broad cross-national testing of the Blömeke et al. model in various cultural contexts.

Table 2 provides an overview of the studies in mathematics.

3 Developments of the Competence Model

3.1 Structure of the Continuum and Underlying Dispositions, Processes and Performances

Blömeke et al. (2015) model conceptualizes competence along a continuum with three central components: dispositions, situation-specific skills, and performance. Since its publication, numerous extensions to this model have been proposed, leading to its expansion over time (e.g., König et al., 2021a). One notable contribution of these studies is the insights on the dynamics of the connection between the core elements of the model - dispositions (e.g., knowledge, beliefs), situational skills (including perception, interpretation, decision-making), and observable performance (e.g., Jong et al., 2021).

3.1.1 Student Mediation

Krauss et al. (2020) also consider competence as a continuum, with a linear effect from dispositions through situation-specific skills to observable performance and develop this causal chain further. Building on Weinert et al. (1989), they suggest that teacher behavior or observable classroom performance influences students' participation in class and use of learning opportunities (student mediation); this, in turn, affects teaching goals. Following Baumert et al. (2010) and Kunter et al. (2013), these

Table 2 Studies in mathematics teacher education providing developments of Blömeke et al. (2015) model

Study	Focus
Use of quantitative methods	
Alwast and Vorhölter (2021)	Specific modeling of pre-service teachers' noticing competencies
Codreanu et al. (2021)	Development of a model explaining pre-service teachers' diagnostic abilities when assessing students' mathematical reasoning
Gerber et al. (2023)	Modeling and developing teaching skills with digital tools, considering additional dispositions of pre-service teachers: self-efficacy, expectations, and beliefs
Hammer and Ufer (2023)	Development of a model that conceptualizes the relationship between teaching skills, the planning process, and the resulting implementation of planning of pre- and in-service teachers
Jong et al. (2021)	Empirical examination of Blömeke et al. (2015) model assumptions focusing on professional noticing of pre-service teachers
König et al. (2021a)	Development of the CODE-PLAN model of effective lesson planning of pre-service teachers
Krauss et al. (2020)	Development of the "cascade model" of teaching competence of in-service teachers and quality as a sequential causal chain
Muñiz-Rodríguez et al. (2018)	Creation of an approach to develop pre-service teachers´ feedback competencies using video vignettes
Santagata and Yeh (2016)	Development of the modified model, that describes the relationship between perception, interpretation and decision making of in-service teachers as a cyclical relationship
Use of qualitative methods	
Hino and Funahashi (2022)	Development of a model that illustrates the processes of in-service teachers´ decision-making in the key interactions with students
Review studies	
Cevikbas et al. (2024)	Meta-analysis and development of a process model of effective lesson planning
Kaiser and König (2019)	Meta-analysis and development of a model viewing teachers´ competence as both a dependent and an independent variable
Leuders et al. (2018)	Review of empirical and theoretical research providing an examination of diagnostic competencies of mathematics teachers
Wassermann et al. (2023)	Review of empirical and theoretical research and development of a model illustrating the connections between teacher education and teaching practice in school
Yang and Kaiser (2023)	Meta-analysis and development of a process model that illustrates how teachers´ competence impacts instructional quality and students´ learning

goals are determined by students' achievement in mathematics, increased enjoyment, and reduced anxiety in the classroom.

A similar expansion is proposed by Yang and Kaiser (2023). They also argue that teaching performance impacts student learning outcomes, distinguishing between mathematics achievement and affective learning outcomes. Additionally, they propose that teachers' dispositions are not the starting point of the model, but rather that dispositions are influenced by aspects of teacher education or professional development programs (e.g., characteristics of institutions, opportunities to learn). Kaiser and König (2019) as well as Wassermann et al. (2023) also posit that university teacher education programs impact dispositions of pre-service and in-service teachers.

3.1.2 Recursive Nature of Competence

While the Blömeke et al. (2015) model posits a linear and one-directional relationship between the main components of their framework, Santagata and Yeh (2016) provide evidence suggesting a bi-directional relationship, i.e., "a complex interaction of situated knowledge, beliefs, and practices" (p. 164). In line with Blömeke et al. (2015), they also propose that situation-specific skills act as a mediator through which dispositions become visible in practice. They further suggest that teachers continuously reflect upon and interpret their actions through practice. This process, in turn, influences teachers' perception, interpretation, and decision-making, as well as their knowledge and beliefs. Santagata and Yeh (2016) particularly emphasize the significance of teachers' (practical) experiences, arguing that competence can "only [be understood] in the specific context in which teachers work" (p. 164). This perspective on the recursive nature of the competence process and its structure is also supported by other studies (e.g., Meschede et al., 2017). Additionally, Hino and Funahashi (2022) proposed that decisions in teachers' interactive actions are influenced by their interpretation of students' thinking.

3.1.3 Broadened Definitions of Teachers' Dispositions

Blömeke et al. (2015) define dispositions broadly as (meta)cognitive knowledge and affective-motivational facets. More specific definitions were provided, e.g., by Taylor and Wasicsko (2000), defining teachers' dispositions as the personal traits, characteristics, or tendencies individuals possess including their attitude, beliefs, values, interests, persistence, appreciation, enthusiasm, knowledge, understanding, interaction, collaboration, and ways of adapting. Several researchers (e.g., Cevikbas et al., 2024) follow a dual taxonomy and distinguish between cognitive and affective dispositions, which in parts go beyond conceptualizations proposed in the Blömeke et al. (2015) model:

(*a*) Cognitive dispositions include professional knowledge (e.g., Shulman, 1986). Based on findings suggesting that teachers' knowledge is the strongest predictor of student learning success (e.g., Hill et al., 2005), recent research, e.g., Gerber et al. (2023), attribute a central role to Pedagogical Content Knowledge (PCK) as a key component of teachers' dispositions. Hammer and Ufer (2023) define it as "knowledge about representations of mathematical content, student cognition, and the potential of instructional tasks" (p. 2). They acknowledge that not only PCK is strongly associated with domain-specific instructional quality. According to evidence, content knowledge (CK) must also be considered (e.g., Kunter et al., 2013), which Hammer and Ufer (2023, p. 2) specify as "an in-depth understanding of school mathematics". Krauss et al. (2020) also consider both PCK and CK as central components of teachers' dispositions. They further refer to pedagogical knowledge (PK), which, according to Brühwiler and Vogt (2020), relates to adjusting instructional designs to learners' prerequisites.

(*b*) Affective dispositions include according to recent studies facets such as self-efficacy expectations and beliefs (Gerber et al., 2023), as well as motivational orientations (e.g., enthusiasm for teaching) and aspects of self-regulation (e.g., burnout-preventive behavior and attitudes) (Cevikbas et al., 2024).

3.1.4 Broadened Conceptualization of Teachers' Situation-Specific Skills

Blömeke et al. (2015) consider situation-specific skills (perception, interpretation, and decision-making) as a mediating factor between teachers' dispositions and their observable performance. This assumption is also adopted by many studies (e.g., Alwast & Vorhölter, 2021). At the same time, some researchers have contributed to a more detailed understanding of the underlying processes of situation-specific skills.

Professional Vision. Referring to the origin of this concept (Goodwin, 1994) and works on professional vision (Van Es & Sherin, 2002), Seidel and Stürmer (2014) describe it as the ability to notice and reason about significant aspects of classroom situations. Hammer and Ufer (2023) explain how dispositions are processed at this stage of the model and transformed into real-life performance. They identify noticing skills and knowledge-based reasoning as the central factors of situation-specific skills. The former is more specifically defined within a mathematical context by Alwast and Vorhölter (2021), who highlight the aspects central to perception, interpretation, and decision-making.

Noticing skills related to *perception* involve recognizing students' difficulties in solving mathematical problems and identifying potential affective and cognitive barriers, i.e., this involves understanding typical student challenges and sensitivity to challenging situations. Regarding *interpretation*, noticing skills primarily involve possessing knowledge of multiple possible solutions to mathematical problems and supporting students in developing solutions to these problems. Finally, due to the complexity of mathematical problems, Alwast and Vorhölter (2021) consider noticing skills in the sense of instant *decision-making* to be particularly challenging for teachers. Following Stender (2016), they argue that adaptive interventions are necessary to support students' independent work. Alwast and Vorhölter (2021, p. 269) conclude that "noticing [...] requires not only perceiving and interpreting the noteworthy aspects that are specific for the teaching profession and making decisions accordingly but, moreover, being sensitive to content that is specific to and essential for mathematical modeling, such as students' modeling-specific difficulties, use of metacognitive strategies, and diverse approaches to solving modeling problems".

Another aspect of situation-specific skills, *knowledge-based reasoning*, is further elaborated by Codreanu et al. (2021), who, following Gegenfurtner et al. (2020), define it as "the act of interpreting the noticed information based on one's professional knowledge" (p. 3). They differentiate between three types of knowledge-based reasoning activities: describing, explaining, and predicting. *Describing* involves verbally expressing observed information without making any judgments. *Explaining* entails linking the described information to relevant professional concepts and knowledge. Finally, *predicting* refers to the ability to draw conclusions about students' learning.

3.1.5 Teachers´ Performance

Blömeke et al. (2015) posited that competencies manifest in real-life (observable) performance (in preparation for teaching as in planning, in classroom instruction with media or in explaining, and in reflecting on and improving instruction as in revising a lesson plan). Krauss et al. (2020) explored the specific nature of real-life performance, defining it as observable teacher behavior in the classroom. They further specify the construct as the "adequate provision of learning opportunities to students" (p. 314).

Muñiz-Rodríguez et al. (2018) emphasize the role of *feedback* as one of the most effective instructional strategies for impacting learning performance. Kulgemeyer et al. (2020) (although conducted in physics education), highlight the ability to *explain* as an essential aspect of real-life teaching performance, as it facilitates communication with students (Geelan, 2012).

Cevikbas et al. (2024) particularly emphasize the successful implementation and evaluation of *lesson plans* as teaching performance, as it is essential for delivering effective and high-quality teaching across all school subjects. Additionally, Kaiser and König (2019) incorporate the successful professional development of teachers into the conceptualization of teachers' real-life performance.

Another crucial factor regarding teachers' performance is the *instructional quality* of educators (Blömeke et al., 2022). Instructional quality can be categorized into three fundamental dimensions: *classroom management* (effective use of classroom time, maintaining structure within the classroom, and teachers' expectations of student behavior), *student support* (motivating students, creating a supportive classroom environment, and offering personalized learning support), and *cognitive activation* (the degree of cognitive engagement facilitated by the quality of teachers' instruction and selection of tasks) (König et al., 2021b). Kunter et al. (2013) further substantiated the significance of instructional quality; they identified statistically significant effects, demonstrating that teachers' professional competence influences instructional quality, in turn having a measurable impact on student learning outcomes in mathematics education. Further studies such as by Brühwiler and Vogt (2020) also provide evidence of both a direct and indirect effect of teachers' performance on student achievement.

3.2 Further Subject-Specific Expansion of the Competence Model

Beyond its fundamental structure and facet-related aspects, the Blömeke et al. (2015) model has been extended and successfully applied to specific contexts, aside from mathematical content and education. König et al. (2022) developed a concept for integrating information and communication technology (ICT) skills into teaching practice. Heine et al. (2023) developed a model for the integration and design of digital resources in teaching practice. They particularly refer to the "documentational approach to didactics (DAD)" concept by Trouche et al. (2018) and model other influencing factors of their framework. Another specific application is provided by Lakkala et al. (2020) who illustrate teachers' competencies related to *inclusive pedagogy* as well as their abilities to identify students' uniqueness and to strengthen their agency, self-efficacy, social self-image, and relatedness by integrating these concepts into the competence model.

Overall, we identified 12 empirical studies in which the Blömeke et al. (2015) model has been applied to other subjects and contents such as German (e.g., König et al., 2020), Business and Economics (e.g., Kuhn et al., 2020), Physics (Kulgemeyer et al., 2020), Biology (Martin et al., 2022) and other areas (e.g., Weyers et al., 2024). The applications in other disciplinary and study-specific contexts lead to many further specifications and differentiations of the model. For instance, Kuhn et al. (2020), following Lindmeier et al. (2013), differentiate between the dispositions, skills, and performance that are necessary for the preparation of lessons (*reflection-related*) as well as the dispositions, skills, and performance that are required for the implementation of lessons (*action-related*).

3.3 Development of an Expanded Model

Considering the recent state of research based on the Blömeke et al. (2015) model and synthesizing the studies on teachers' dispositions, situation-specific skills, real-life performance, and also studies on the structure of the model and further subject- and/or content-specific developments, we propose a further elaborated competence model. This builds on the fundamental perspective of competence as a continuum according to Blömeke et al. (2015), but also incorporates the further developments highlighted in this review. The model is illustrated in Fig. 3.

The analysis of studies that either focused on the further development of specific aspects of the Blömeke et al. (2015) model or critically examined its structure contribute advancements that we have incorporated in the modified model. Since the fundamental structure of a competence continuum consisting of teachers' dispositions, their situation-specific skills, and their observable real-life performance has been conceptually and empirically supported, at least in part, we endorse this framework and integrate additional aspects to the original model.

Firstly, in line with findings by Kaiser and König (2019), Scheerens and Blömeke (2016), Wassermann et al. (2023), and Yang and Kaiser (2023), characteristics of university teacher education or professional development programs are incorporated. These characteristics include the availability and use of learning opportunities, as well as general training

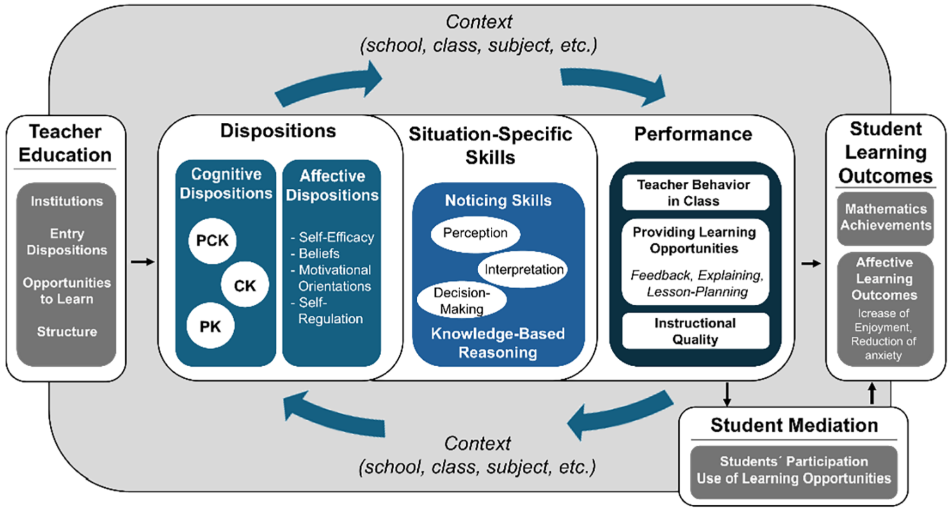

Fig. 3 Modified Blömeke et al. (2015) model of teacher competence. *Note*. Only a few examples of the possible (sub)facets are illustrated for the factors in this model. Cognitive dispositions, e.g., comprise many other facets such as technological PCK (TPACK, Angeli et al., 2016) and general cognitive aptitudes such as verbal, spatial and quantitative reasoning (Shavelson & Huang, 2003)

program structure and curricula potentially influence the entire competence continuum of (pre- and/or in-service) teachers.

Secondly, the dispositions, as argued by Cevikbas et al. (2024), can be both cognitive and affective. Gerber et al. (2023) and Hammer and Ufer (2023) provide insights into the nature of cognitive dispositions, which consist of teachers' PCK, CK, and PK. According to Cevikbas et al. (2024) and Krauss et al. (2020), affective factors such as self-efficacy, beliefs, motivational orientations, and self-regulation play a crucial role in shaping teachers' dispositions, which we included in the modified model (whereby we can only name a few facets in this Fig. 3).

Secondly, situation-specific skills, related to the perception, interpretation, and decision-making, can be divided into noticing skills and knowledge-based reasoning (Hammer & Ufer, 2023). The resulting real-life performance can be further specified as observable teacher behavior in planning and the provision of learning opportunities for students, acting in the classroom, and reflecting on and evaluating their performance (Krauss et al., 2020). This is demonstrated through practices such as providing feedback (Muñiz-Rodríguez et al., 2018), explanations (Kulgemeyer et al., 2020), and the successful implementation and evaluation of lesson plans (Cevikbas et al., 2024).

Thirdly, we extend the original model, drawing on the findings of Krauss et al. (2020) and Yang and Kaiser (2023): teachers' real-life performance influences students' active participation in class and their utilization of learning opportunities. We have included this factor as student mediation in the modified model, as it serves as a mediating factor ultimately impacting student learning outcomes. In mathematics teacher education, these outcomes can be further divided into mathematics achievements and affective learning outcomes, such as an increase in classroom enjoyment and reduced anxiety during lessons.

Fourthly, unlike Blömeke et al. (2015), we do not assume a unidirectional causal relationship between teachers' dispositions manifesting in real-life performance through the mediating factor of situation-specific skills. Following the findings of Santagata and Yeh (2016) and other studies, we suggest a bi-directional and recursive relationship.

Finally, in line with Blömeke et al. (2015), we also adopt a more holistic tradition that primarily emphasizes the significance of competencies as a real-life criterion for behavior (e.g., McClelland, 1973) and further integrate this aspect into our modified model.

4 Measuring Competence Along the Continuum - Implications and Prospects for Future Research

Blömeke et al. (2015) criticize the limited diversity of measurement approaches employed in the assessment of (teacher) competencies; they highlight the reliance on either multiple-choice items or highly complex task formats. However, this traditional approach presents significant limitations for research, as "each only taps into parts of the construct definition" (Blömeke et al., 2015, p. 13). Therefore, it's important to consider the constraints in studies that depend on single-choice items in a self-designed testing tool to evaluate aspects of teachers' knowledge (e.g., PCK; Gerber et al., 2023). Blömeke et al. (2015) instead advocate for the use of combined measurement approaches to capture competencies as complex multifaceted constructs more comprehensively and validly.

Adopting such a combined measurement approach can be considered one of the key strengths of recent empirical research, as most studies use mixed-method designs and/or the combination of assessment techniques, from classic multiple-choice tests to complex video-based performance assessments. Santagata and Yeh (2016) conducted a mixed-methods study, collecting and analyzing quantitative scores alongside qualitative interviews, with the findings synthesized through a triangulation process. However, this study was based on a very limited sample ($n = 3$). Therefore, the external validity and generalizability of the results are constrained (Ferguson, 2004). Limited external validity is also evident in the study by Hino and Funahashi (2022), who conducted a case study with a single mathematics teacher. This also applies to most of the other studies, which are based on small and non-representative samples, which significantly limits the importance of the results.

Hammer and Ufer (2023) employ a variety of measurement methods, including traditional paper-and-pencil tasks, lesson preparation assignments, and open-ended questions. In this context, potential limitations due to insufficient uni-dimensionality must be considered, as "the principle is violated by the presence of method variance and multiple-expected dimensions" (Blömeke et al., 2015, p. 10). In this regard, many recent studies adhere to Blömeke et al.'s (2015) methodological recommendations by incorporating video-based assessment instruments. Drawing on Wass et al. (2001), Blömeke et al. (2015) recommend video-based assessments "so that the perception of real-life, that is unstructured situations, can be included" (p. 9). Video-based instruments have been employed in many studies to investigate the aspects of competencies. Krauss et al. (2020) and Kuhn et al. (2020) utilize video vignette instruments as measures of actual instructional quality in pre- and/or in-service teachers' lessons. Codreanu et al. (2021) use such tools to capture the diagnostic activities of pre-service teachers, Santagata and Yeh (2016) for assessing teachers' usable knowledge of mathematics teaching, Jong et al. (2021) for capturing pre-service teachers' noticing skills, Muñiz-Rodríguez et al. (2018) for evaluating feedback competencies of pre-service teachers, and Hino and Funahashi (2022) for assessing teachers' in-the-moment decision-making. This measurement approach provides an effective means "to capture the way teachers notice classroom situations" (Kaiser et al., 2015, p. 384), utilizing large sample sizes across various settings and populations (Borko, 2016).

Noteworthy, various reliability checks have been conducted within these empirical studies (e.g., $\chi 2$ tests, tests of variation of coding units, tests of assessment accuracy in Codreanu et al., 2021; tests of item fit, and sensitivity analysis in Jong et al., 2021; Coefficient alpha in Muñiz-Rodríguez et al., 2018). However, the question arises: are these statistical methods adequate for testing the assumptions of the Blömeke et al. model (2015)? Given that the assumptions propose a (reflexive) causal path from teachers' dispositions through the mediator of situation-specific skills to observable teaching performance, statistical methods related to mediation analyses are generally more suitable for empirical investigations (e.g., Zhao et al., 2010).

Krauss et al. (2020) conducted a study aimed at testing the entire Blömeke et al. (2015) model in mathematics education. Their analysis yielded evidence suggesting a statistically significant effect of teachers' dispositions (measured as CK and PCK) on their situation-specific skills (measured as teachers' subject-specific competence). However, they found no statistically significant effect of situation-specific skills on performance (measured as instructional quality). Given the consensus in recent research on the essential role of situation-specific skills, particularly noticing and reasoning skills, in teachers' performance (e.g., Blömeke et al., 2022; Jong et al., 2021), this finding suggests that the design and statistical methods used in many of the reviewed empirical studies may not be appropriate for testing a potential mediation effect in the Blömeke et al. (2015) model or our modified model.

Krauss et al. (2020) employed single- and four-predictor models predicting the respective effect paths individually. However, Hayes (2022) criticizes the separate consideration of the effect of the independent variable on the mediator and the effect of the mediator on the dependent variable, as the mediation effect is determined by the product of these two effects. Consequently, he advocates for the application of a single significance test concerning the mediation effect, arguing that this approach reduces the risk of error compared to examining them separately. Procedures such as Sobel, Aroian, or Goodman tests (e.g., Nwankwo & Igweze, 2016), the bootstrap method for calculating confidence intervals[1] (e.g., Zhao et al., 2010), or structural equation modeling (e.g., Mueller & Hancock, 2018) are recommended for this purpose.

In light of these arguments, we encourage future research to further test the mediation assumption of the Blömeke et al. (2015) and the modified model. More methodologically sophisticated research is required to achieve the goal formulated by Blömeke et al. (2015, p. 13) "to test the entire model

[1] According to Hayes (2022), this method is more suitable for testing mediation effects than Sobel, Aroian, or Goodman tests. The main limitation lies in the assumption of normal distribution used by these methods to calculate the mediation effect. Since this assumption is generally not valid in these cases, this results in incorrectly calculated p-values for the test procedure. Furthermore, these test approaches have low statistical power, leading to the calculation of confidence intervals with insufficient accuracy. To address this issue, the scientific discourse predominantly suggests the use of the bootstrap method for calculating confidence intervals (Zhao et al., 2010), as it is based on an empirical sampling distribution that does not rely on the assumption of normal distribution (Mooney & Duval, 1993).

conceptually and statistically" and to "also be able to cover the processes mediating the transformation of dispositions into performance".

5 Discussion

Blömeke et al. (2015) modelled teacher competence as a continuum moving from teacher dispositions to situation-specific skills to observable teaching performance. In this paper, we investigated, (*a*) how has research addressed the challenge of testing, validating and improving the Blömeke et al. (2015) model, and (*b*) has the research led to an expanded, validated framework for addressing mathematics teachers' competencies?

Our review led to an expanded model (compare Figs. 1 and 3) that incorporated more facets of dispositions (cognitive–e.g., PCK, and non-cognitive, e.g., affective–self efficacy), more detailed specifications of situation-specific skills (e.g., teacher vision), and more specification of performances (e.g., lesson planning). Moreover, the modified model moves beyond a linear model to include feedback loops and student mediation.

5.1 Study Limitations

We noted that the Blömeke et al. (2015) model, our modified model, or other alternative models need to be tested with more sophisticated methodological approaches than has been done in research reviewed. We further noted limitations in methodological quality (e.g., design, statistical analysis). Hence, future investigations of the Blömeke et al. (2015) or the modified model will necessitate research using advanced technological methods (e.g., Foster, 2023). In addition, our review might also be limited by the inadequate presentation of study methods, results, and interpretation in the reviewed publications. Often the studies do not document key information and necessary statistical details required by internationally established reporting (e.g., AERA et al., 2014).

This review is limited in its scope, as we could only touch on a few selected central results on testing and modification of the Blömeke et al. (2015) model presented in the studies. This means that many other important perspectives remain to be addressed. Such perspectives include modeling and measuring competence levels and development (on which there is a lot of research), or the differentiation between competencies of teacher novices (students in university education) versus experts (practicing teacher in the continued training). Moreover, our review focuses on English-language studies that provide perspectives on potential advancements of the Blömeke et al. (2015) model. In this context, we predominantly integrated studies from German research into our analysis (see Table 1). Studies from other counties (published in other languages) might be under representative due to our review approach.

A further limitation is that we only briefly address selected methodological aspects and perspectives in this review. Many important developments and more recent findings, such as those from eye-tracking studies with (trainee) teachers, are barely considered (e.g., Brückner & Zlatkin-Troitschanskaia, 2024).

5.2 Conclusion

Despite these limitations, our review highlights the significant role that the Blömeke et al. (2015) model has played in the last decade in mathematics teacher education and beyond. We (at least) demonstrate how this model has been applied and further developed in mathematics teaching research and other subjects and content areas, and what implications can be inferred from the evidence. Despite this rich research related to the Blömeke et al. (2015) model, valid measurement of the components along the teacher competence continuum including teaching performance as a holistic construct remains still a key desideratum. In this regard, the specific potential of authentic performance assessments in mathematics teacher education and beyond should be considered in future research (e.g., Zlatkin-Troitschanskaia et al., 2019b; Albu & Lindmeier, 2023).

Appendix

Table 3 Keywords used for the literature review

1^{st} sampling approach Forward citation chasing		2^{nd} sampling approach Keyword-based search	
Used keywords	Boolean operators	Used keywords	Boolean operators
Competence	AND	Competence	AND
Continuum	OR	Continuum	OR
Teacher education		Teacher education	
Teacher training		Teacher training	
Mathematics		Mathematics	
Students		Students	
Dispositions		Dispositions	
Situation-specific skills		Situation-specific skills	
Performance		Performance	
		Blömeke	
		Beyond dichotomies	
		PID model	

Table 4 Criteria for inclusion and exclusion of studies

Inclusion Criteria	Exclusion Criteria
The study - is written in English. - refers to teacher education. - builds its central assumptions on the Blömeke et al. (2015) model. - conducts empirical research and/or a review of research.	The study - is not written in English. - does not refer to teacher education. - does not build its central assumptions on the Blömeke et al. (2015) model. - does not conduct empirical research and/or a review of research.

Table 5 A Categorization of selected literature

Empirical studies	
Use of quantitative methods	*Mathematics teaching* *Alwast and Vorhölter (2021)* *Codreanu et al. (2021)* *Gerber et al. (2023)* *Hammer and Ufer (2023)* *Jong et al. (2021)* *König et al. (2021a)* *Krauss et al. (2020)* *Muñiz-Rodríguez et al. (2018)* *Santagata and Yeh (2016)*
	Physics teaching *Kulgemeyer et al. (2020)*
	Biology teaching *Kramer et al. (2021)* *Martin et al. (2022)*
	"Sachunterricht" (social studies and science) teaching *Meschede et al. (2017)*
	B&E teaching/vocational training: *Weyers et al. (2024)*
	Language teaching *König et al. (2020)* *Stadler-Heer (2021)*
	Teaching context not specified *Griful-Freixenet et al. (2020)* *Keppens et al. (2021)*
Use of qualitative methods	*Mathematics teaching* *Hino and Funahashi (2022)*
	B&E teaching/vocational training *Zlatkin-Troitschanskaia et al. (2019b)*
Review studies	
Review studies on empirical research	*Cevikbas et al. (2024)* *Kaiser and König (2019)* *Yang and Kaiser (2023)*
Review studies on both empirical and theoretical research	*Herppich et al. (2018)* *Leuders et al. (2018)* *Metsäpelto et al. (2021)* *Wassermann et al. (2023)*
Review studies on theoretical research	*Baumgartner (2022)*
Applications of the model by Blömeke et al. (2015)	
	Heine et al. (2023) *(Review study on both empirical and theoretical research)*
	König et al. (2022) *(Review study on empirical research)*
	Lakkala et al. (2020) *(Quantitative empirical study)*

Table 5 (*Continued*)

Further developments by S. Blömeke	
	Blömeke et al. (2016) *(Quantitative empirical study)*
	Blömeke and Kaiser (2017) *(Review study on both empirical and theoretical research)*
	Jenßen et al. (2021) *(Quantitative empirical study)*
	Kaiser et al. (2017) *(Review study on both empirical and theoretical research)*

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