Correction

In Section 2.3 (Phase Behavior of TDP-43 Variants), the term "transfer free energy" should be corrected to "excess transfer free energy" to quantify the thermodynamic preference of a protein chain to reside in the dense (condensed) phase relative to the dilute phase. This quantity describes the free energy cost or gain associated with transferring a single chain into the dense phase.

The chemical potential of each phase can be split into two contributions: an ideal-gas (id) contribution and a non-ideal excess (ex) contribution [1], such that

$$\mu_{\text{dilute}} = \mu^{\text{id}}_{\text{dilute}} + \mu^{\text{ex}}_{\text{dilute}} \tag{1}$$

$$\mu_{\text{dense}} = \mu^{\text{id}}_{\text{dense}} + \mu^{\text{ex}}_{\text{dense}} \tag{2}$$

At equilibrium, the chemical potentials in the two phases are equal:

$$\mu^{\text{id}}_{\text{dilute}} + \mu^{\text{ex}}_{\text{dilute}} = \mu^{\text{id}}_{\text{dense}} + \mu^{\text{ex}}_{\text{dense}} \tag{3}$$

Rearranging gives

$$\mu^{\text{id}}_{\text{dilute}} - \mu^{\text{id}}_{\text{dense}} = \mu^{\text{ex}}_{\text{dense}} - \mu^{\text{ex}}_{\text{dilute}} \tag{4}$$

The non-ideal contribution is the excess transfer free energy, defined as

$$\Delta G^{\text{ex}}_{\text{trans}} \equiv \mu^{\text{ex}}_{\text{dense}} - \mu^{\text{ex}}_{\text{dilute}} = \mu^{\text{id}}_{\text{dilute}} - \mu^{\text{id}}_{\text{dense}} \tag{5}$$

Since the ideal chemical potential is given by

$$\mu^{\text{id}} = RT \ln C + c(T) \tag{6}$$

where C is the concentration and $c(T)$ is a temperature-dependent constant, we obtain

$$\begin{aligned} \Delta G^{\text{ex}}_{\text{trans}} &= RT \ln C_{\text{dilute}} + c(T) - (RT \ln C_{\text{dense}} + c(T)) \\ &= -RT \ln \left(\frac{C_{\text{dense}}}{C_{\text{dilute}}} \right) \end{aligned} \tag{7}$$

Equivalently, this can be written as

$$\Delta G^{\text{ex}}_{\text{trans}} = RT \ln \left(\frac{C_{\text{dilute}}}{C_{\text{dense}}} \right) \tag{8}$$

which represents the free energy change associated with transferring one chain from the dense phase to the dilute phase.

This correction is based on the discussion with Dr. Rodrique Badr.

References

[1] Ben-Amotz, D., & Widom, B. (2007). Nonideal gas solvation thermodynamics. *The Journal of Chemical Physics*, 126(10), 104502. doi:10.1063/1.2710257