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CLASSICS REVISITED

Hermann Weyl: "Raum · Zeit · Materie: Vorlesungen über allgemeine Relativitätstheorie"

Springer 1918, 234 pp. (and later editions)

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In the history of the general theory of relativity, the significance and impact of Hermann Weyl's monograph *Raum · Zeit · Materie* [33] can hardly be underestimated. From a mathematical point of view, its first publication in 1918 is perhaps of equal importance as the publication of the relativistic field equations by Einstein in late 1915 which constituted the theory in the first place. Weyl's book is, in fact, the first comprehensive mathematical exposition of the new relativistic theory of gravitation. It presented the new tensorial formulation of a physical theory in a way that not only put a rather idiosyncratic formulation on a new level of mathematical sophistication but also introduced a generation of mathematicians to an exciting new field of research in mathematical physics. It also introduced, especially in its later editions [35–37, 41], a number of conceptual and technical innovations of its own that have born fruit in the further development of the theory. The book saw five heavily revised editions in rapid succession until 1923. The final, fifth edition has recently been reprinted, with extensive historical and technical introduction and commentary, by Domenico Giulini and Erhard Scholz [43].

1 Prehistory

Hermann Weyl (1885–1955), born in Elmshorn near Hamburg, had received a classic humanistic education at the *Christianaeum*, an old tradition gymnasium in Hamburg-Altona and had then studied in Göttingen with the mathematician David Hilbert (1862–1943) and the philosopher Edmund Husserl (1859–1938), among others.[1] In 1908, he had obtained his Ph.D. with Hilbert with a thesis on "singular integral equations with special consideration of Fourier's integral theorem." Two years later he had

[1] For biographical information on Weyl, see e.g. [6, 7, 13].

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obtained his *venia legendi* at Göttingen University with a habilitation thesis "on ordinary differential equations with singularities and the corresponding developments of arbitrary functions". In 1913, he had accepted a call to the chair for mathematics at Zurich's Polytechnic school, today's ETH, where he would stay until 1930.

In his first year in Zurich, Weyl was a colleague of Albert Einstein (1879–1955) who had been appointed Professor of Theoretical Physics there in 1912. But their overlap in Zurich was only brief and ended already in spring 1914 when Einstein left for a sinecure position with the Prussian Academy in Berlin. While Einstein was still in Zurich he had worked intensely together with Marcel Grossmann (1878–1936), a student friend of his, who held then the Chair for Geometry at the Polytechnic. Together they developed a generalization of the (special) theory of relativity resulting in an "Outline of a Generalized Theory of Relativity and a Theory of Gravitation" [12]. Their joint work had pushed the development of a generally covariant theory considerably and soon after Einstein had left Zurich, he finally succeeded in completing the general theory of relativity in the fall of 1915.

In the summer of 1917, Weyl held a course on "Raum, Zeit, Materie" [13, p. 175] in Zurich, and the book *Space-Time-Matter* (*Raum · Zeit · Materie*, or RZM in the following) is a publication growing out of that set of lectures. Its manuscript was completed in the spring of 1918 when World War I was in its fourth year, and the senseless killing at a sad peak. Einstein had proofread the manuscript and by July 3 had received a printed copy from Springer [29, Doc. 579]. In the preface, Weyl thanked the publisher for "the admirably rapid printing and good layout of the book, given today's circumstances."

Hermann Weyl dedicated the book to his wife: "Meiner Frau gewidmet." Helene Weyl, née Joseph, (1893–1948), came from a secularized Jewish family. She was baptized before their marriage in order to please Weyl's Christian mother [42]. The preface of RZM was dated "Ribnitz in Mecklenburg, Ostern 1918," indicating that it was completed at the place, where his wife's family lived.

As a young student, Helene Joseph had been captivated by Edmund Husserl's philosophy and had come to Göttingen in 1911 to study with him. She studied philosophy with mathematics as a minor, and she and Hermann met already in her first semester at the historian Max Lehmann's (1845–1929) house. Later in Zurich, Helene continued her mathematics studies. As Weyl recalled, in 1914 she was pregnant with their first child Fritz Joachim when they were both studying Hilbert's *Zahlbericht*, but the pregnancy interrupted their joint intellectual project:

> For now the worries of childbirth began, and soon events occurred that gave Hella's interests a different direction. In later life she was happy to listen when I told her in general terms about my own and other's mathematical ideas that were alive around us, but she no longer got involved in detailed mathematical explanations. She sometimes said that one did not like to be reminded of a building that had been abandoned after it had barely risen from its foundations and was now being washed away by the rain and is falling apart.[2]

[2]"Denn nun begannen die Kindersorgen, und bald traten Ereignisse ein, die Hellas Interessen eine andere Richtung gaben. Im späteren Leben hörte sie wohl gern zu, wenn ich ihr in allgemeinen Zügen von meinen und anderen um uns herum lebendigen mathematischen Ideen erzählte; auf eigentlich mathematis-

When the war broke out, Weyl was drafted and garrisoned in Saarbrücken without being sent to the front until the Swiss authorities succeeded to have Weyl released from the German military in May 1916. When Weyl gave his lectures on relativity in the summer of 1917, his wife was pregnant with their second son Michael, born in the fall. Later in life, it might be added, Helene Weyl became a prolific translator of, among others, the poet José Ortega y Gasset.

2 The First Edition

Let us now have a look at Weyl's RZM. The preface to the first edition begins with this paragraph:

> Einstein's Theory of Relativity has advanced our ideas of the structure of the cosmos a step further. It is as if a wall which separated us from Truth has collapsed. Wider expanses and greater depths are now exposed to the searching eye of knowledge, regions of which we had not even a presentiment. It has brought us much nearer to grasping the plan that underlies all physical happening.[3]

Weyl announced his intent to "present this great subject as an illustration of the intermingling of philosophical, mathematical, and physical thought, a study which is dear to my heart." But he then admitted that he might not have fully lived up to those ambitions: "the mathematician predominates at the expense of the philosopher." His intent, in any case was to give an introduction to the theory for readers with minimal background requirements, while nonetheless leading readers up to current research in the field.

The book is divided up into four chapters. The first chapter deals with Euclidean space, its mathematical formalization and its role in physics. It discusses basic concepts of affine and multidimensional geometry, linear algebra, quadratic forms, and only then introduces a metric structure. True to Weyl's intent to introduce readers to the new mathematics of a tensor calculus, it lays out tensor algebra and calculus, with examples from physics, including a discussion of the stationary electromagnetic field.

The second chapter gives a self-contained introduction to Riemannian geometry and tensor calculus, or 'absolute differential calculus' as it would have been called at the time with reference to Ricci's and Levi-Civita's work [24]. The chapter begins with an account, historical and systematic, of non-Euclidean geometry, i.e. Euclidean geometry with the parallel postulate replaced by a different axiom, or, in more modern terms, three-dimensional space with constant negative or positive curvature. Riemannian geometry, i.e. multidimensional geometry of variable curvature is the topic of

che Detaildarlegungen liess sie sich aber nicht mehr ein. Sie sagte manchmal, man würde nicht gern an ein Bauwerk erinnert, das verlassen wurde, nachdem es sich kaum aus den Fundamenten erhoben hätte und nun vom Regen zerwaschen wird und zerfällt." [42, p. 4].

[3]"Mit der Einsteinschen Relativitätstheorie hat das menschliche Denken über den Kosmos eine neue Stufe erklommen. Es ist, als wäre plötzlich eine Wand zusammengebrochen, die uns von der Wahrheit trennte: nun liegen Weiten und Tiefen vor unserem Erkenntnisblick entriegelt da, deren Möglichkeit wir vorher nicht einmal ahnten. Der Erfassung der Vernunft, welche dem physischen Weltgeschehen innewohnt, sind wir einen gewaltigen Schritt näher gekommen." [33, p. V], English translation from [38].

the next section. Weyl emphasizes the naturalness of Riemannian geometry. He sees a parallel between the transition from action-at-a-distance physics to field theory and the transition from Euclidean to Riemannian geometry. He credits Riemann with a programmatic emphasis on the infinitely small:

> *The approach of understanding the world from its behavior in the infinitesimally small* is the driving epistemological motive of near action physics and Riemannian geometry, but it is also the driving motive in the rest of Riemann's magnificent life's work, which is primarily directed towards complex function theory.[4]

What Weyl here points out is based on a thorough knowledge of Riemann's work as witnessed by his monograph on the "Idea of the Riemann surface" [31]. But it also expresses his own take on the relevant mathematics for the general theory of relativity. In fact, this second chapter, and especially its §14, introduced the notion of parallel transport of a vector. Weyl here explored the geometric foundations of Einstein's theory, which so far had mainly been formulated in analytic terms. As Weyl pointed out in the preface, he was not the first to discover the role of parallel transport and the notion of an affine connection on a topological manifold. Levi-Civita's work [20] had just been published, but he had introduced parallel displacement still with explicit reference to a higher-dimensional embedding space even though his notion did not depend on that embedding space. Related work by Gerhard Hessenberg [15] was only briefly mentioned by Weyl as it had appeared only after completion of the manuscript. It was only used by Weyl to add to the book a proof of the symmetries of the Riemann curvature. The work of the Dutch geometer Jan Arnoldus Schouten [28] came out too late altogether and was only mentioned in annotation to later editions of RZM.[5]

With respect to the second half of the book, dealing with the physical theories of relativity, Einstein wrote in a rather favorable review:

> After these formal tools are completely mastered, the third chapter presents the theory of special relativity and the fourth chapter the general theory; the special one on 59 pages, the general one on 54 pages. It is here that Weyl not only demonstrates his easy mastery of the mathematical form, but also his deep insight into what is essential in physics.[6]

Referring to Weyl's own contributions to finding explicit solutions to the gravitational field equations, notably by giving derivations of the Schwarzschild-Droste metric and other spherically symmetric solutions using a variational formulation, Einstein continued:

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[4]"*Das Prinzip, die Welt aus ihrem Verhalten im Unendlichkleinen zu verstehen*, ist das treibende erkenntnistheoretische Motiv der Nahewirkungsphysik wie der Riemannschen Geometrie, ist aber auch das treibende Motiv in dem übrigen, vor allem auf die komplexe Funktionentheorie gerichteten grandiosen Lebenswerk Riemanns." (p. 82).

[5]See [2, 4, 22] for further historical discussion of the early history of the notion of parallel displacement.

[6]"Nachdem so das formale Rüstzeug vollständig gewonnen ist, wird im dritten Kapitel die spezielle, im vierten die allgemeine Relativitätstheorie dargestellt, die spezielle auf 59, die allgemeine auf 54 Seiten. Hier zeigt sich so recht, daß Weyl nicht nur die mathematische Form spielend meistert, sondern auch mit tiefem Blick für das physikalische Wesentliche begabt ist." [11].

> Recently, Weyl has earned considerable merits by the integration of the field equations of gravitation. The exposition of the last paragraphs exemplify how a *born* mathematician can be effective here through simplifying and clarifying. The book will be invaluably helpful to everybody who wants to work in this field, not to mention the pure joy derived from its study.[7]

When Weyl's book came out in 1918, it was the first full-scale textbook on the theory of relativity including the new theory of gravitation and general relativity, written by someone other than Einstein himself. To be sure, the creator of the theories of relativity had written a self-contained review paper of the general theory of relativity in the *Annalen der Physik* in 1916 [9], and then a popular book *Über die spezielle und allgemeine Relativitätstheorie* in 1917 [10]. But other than that, the only monograph discussing Einstein's new theory of gravitation at the time was a slim volume written by the astronomer Erwin Freundlich (1885–1964) in 1917 on *Die Grundlagen der Einsteinschen Gravitationstheorie* [14], which first appeared as a sequence of papers in the general science journal *Die Naturwissenschaften*. Another book on general relativity was the philosopher Moritz Schlick's (1882–1936) *Raum und Zeit in der gegenwärtigen Physik* which also had already come out in 1917 [25].

It turned out that Weyl was quite prescient in publishing his monograph, since its first edition came out *before* Einstein's new theory was spectacularly confirmed by a British eclipse expedition in 1919. The results of two expeditions led by Arthur S. Eddington (1882–1944) to observe the light of stars in the immediate vicinity of the sun during the total solar eclipse of May 29, 1919, in Sobral, Brazil, and on the island of Principe in the Gulf of Guinea were announced in a joint session of the Royal Society and the Royal Astronomical Society in London on November 6, 1919. The results confirmed Einstein's prediction of light bending in the gravitational field of the sun, one of the three classical tests of general relativity. [5, 16] The announcement catapulted Einstein and his theory to world fame almost over night.

In fact, other early mathematical expositions of general relativity, like Max von Laue's (1879–1960) second volume of his introduction to relativity theory [30], Max Born's (1882–1970) introduction to general relativity [1], the young Wolfgang Pauli's masterful review article on relativity in the *Encyclopädie der mathematischen Wissenschaften* [21], or Arthur Eddington's *Space, Time, and Gravitation* [8], all came out in 1920 or 1921, *after* the results of the British eclipse expedition had made the theory famous. At that time, Weyl's book had already been reprinted in a second edition in 1919, almost without any changes, and even his heavily revised third edition was prefaced "August 1919", still before the announcement of the results of the eclipse edition. With the spectacular confirmation of general relativity theory by the British expedition, the general interest in Einstein's theory boomed, and only five years later, in 1924, Maurice Lecat would compile a bibliography of publications on relativity theory, listing more than 3000 items [19].

The timely publication alone of a mathematical textbook introducing the theories of relativity thus would have secured Weyl's RZM a firm place in the history of

[7] "Die Darlegungen der letzten Paragraphen zeigen, wie vereinfachend und klärend der *geborene* Mathematiker da wirken kann. Jedem, der an dem Gebiet mitarbeiten will, wird das Buch unschätzbare Dienste leisten, abgesehen von der reinen Freude, die er beim Studium findet." (ibid.).

general relativity. But the first edition of the book turned out to be only the beginning of an intellectual development on Weyl's side, which in the sequel led to quite a few influential papers on aspects of the theory which also were incorporated in heavily revised editions of RZM until 1923.

3 A Unified Theory of the Gravitational and Electromagnetic Fields and Einstein's Objection

Weyl's interest in the project of RZM is indeed to be seen in a more general context of his interest in the foundations of mathematics [26]. It was not by chance that the mathematician Weyl not only contributed to the theory by elaborating on particular solutions of the gravitational field equations. He also clarified the geometric underpinning of the general theory of relativity. A particular focus of his arose from his interest in a purely infinitesimal geometry, which led to the clarification of the affine, metric, and conformal structures of a general manifold.

A first indication of this mathematical development was given when he corrected a mistake in the second edition of RZM. The error concerned a statement about the conformal structure of the metric, and pertained to the assertion that the coefficients of the metric tensor can be determined solely by observing locally the arrival of light signals [33, p. 183], the discussion presupposing a semi-Riemannian geometry with a semi-definite metric of Lorentz signature $(-, +, +, +)$. In the second edition, he clarified the statement to the effect that a full determination of all metric components required not only knowledge about the light cone at each point but also the geodesic information of the inertial motion of material particles.

This correction was explicitly pointed out in a footnote in a paper entitled "Purely Infinitesimal Geometry" [34, p. 400] which Weyl had submitted to the *Mathematische Zeitschrift* already on June 18, 1918, before the printed copies of the first edition had even been available. It is in this paper that Weyl fully elaborated on his notion of parallel transport, defined independently of an embedding space.[8] Weyl here clearly distinguished between the concepts of an affine, symmetric connection ∇ on a manifold $\mathcal{M}$ with connection coefficients Γ^a_{bc} and a metric field g. Metric compatibility of the connection determines the connection unique to be the usual Levi-Civita connection with its coefficients $\Gamma^a_{bc} = \left\{ \begin{matrix} bc \\ a \end{matrix} \right\}$ being identical with the Christoffel symbols of the second kind. But while Einstein had introduced the Riemann curvature in the initial formulation of general relativity directly in terms of the metric as a differential invariant, it now became clear that Riemann curvature arises independently from the metric from the connection and can be derived by looking at so-called Levi-Civita loops, i.e. by computing the deficit angle of a vector parallel transported along a closed parallelogram.

Weyl's observation now in the context of his program of a purely infinitesimal geometry was that this definition of curvature presupposes what he perceived as a

[8] A partial English translation of this paper can be found in [23, Vol. 4, pp. 1089–1107] and a complete English translation was published as a "Golden Oldie" [40] of *General Relativity and Gravitation*.

residual element of distance geometry. He emphasized that vectors at different neighboring points of the manifold cannot be compared directly without the introduction of an affine structure and connection. The local character of the connection gave rise to the non-integrability of parallel transport with respect to the direction of a vector. But, as Weyl emphasized, in this use of parallel transport the length of a vector remains unaffected, and hence two vectors at different points of a manifold could always be compared with respect to their lengths while a comparison of their direction required knowledge of the path along which a vector would be transported from one point to the other.

In order to remove this residual element of distance geometry, Weyl introduced what he called a "length connection" in complete analogy to the affine connection. In addition to the quadratic differential form $ds^2 = g_{ik} dx^i dx^k$, he defined a linear differential form $d\varphi = \varphi_i dx^i$ on $\mathcal{M}$. This differential form now governed the length transport of a vector. Simultaneously, the metric field g was reduced to a conformal class of metrics $[g]$ with two metrics g, g' conformally equivalent if they differ only by a common factor λ. Knowledge of the light cone structure determines the equivalence class $[g]$ but not the full metric, since only the ratio of the lengths of two vectors remains invariant on parallel transport. The factor φ, on the other hand, now determines the absolute length, or in physical terms, a measuring rod at each point of $\mathcal{M}$. Weyl now postulated that the theory needs to be formulated in a way that allows for an independent recalibration of length measurements at each point, a process for which he introduced the term "gauge invariance" ("Eichinvarianz"). Technically, this amounted to the simultaneous transformation $g_{ik} \to g'_{ik} = \lambda g_{ik}$ and $d\varphi \to d\varphi' = d\varphi - d\lambda/\lambda$, where λ is an arbitrary, scalar, everywhere non-vanishing C^2-function on $\mathcal{M}$.

Looking at the analog of the Levi-Civita loop for the length connection, Weyl showed that the length deficit $\Delta l = -l \Delta\varphi$ of a vector taken around a parallelogram spanned by line elements dx and δx leads to $\Delta\varphi = f_{ik} dx_i \delta x_k$ with $f_{ik} = \varphi_{i,k} - \varphi_{k,i}$ (the comma denoting partial coordinate derivative). In precise analogy to the Riemannian "vectorial curvature" ("Vektorkrümmung"), Weyl termed the second-rank tensor f the "length curvature" ("Streckenkrümmung"). Its definition implies that f is invariant under those gauge changes and that the quantity $f_{ik,l} + f_{kl,i} + f_{li,k}$ is also gauge invariant and vanishes identically. Conventional (semi-)Riemannian geometry is recovered for vanishing φ, a case that Weyl referred to as "normal gauge" ("Normal-Eichung"). In that case the length connection f vanishes everywhere.

Weyl did not stop here. Based on his elaboration of these conceptual generalizations in the differential geometry of a manifold, he boldly gave the new quantities a physical interpretation. He identified the components φ_i with the electromagnetic four-potential and hence the length connection f with the electromagnetic field. Maxwell's vacuum field equations were automatically satisfied by its definition as a curl of φ. Its vanishing on $\mathcal{M}$ implied that the space-time manifold displayed no length curvature. The presence of an electromagnetic field, therefore, would manifest itself in the presence of a length curvature.

This was a beautiful, attractive idea. It amounted to the first attempt at a true unification of the two fundamental fields of gravitation and electromagnetism and it intimately linked geometry and physics by geometrizing not only the effects of gravitation but also of the electromagnetic field. Weyl first elaborated on his idea in a paper which he asked Einstein to present to the Prussian Academy for publication in its Proceedings in April 1918 [29, Doc. 497]. Einstein agreed, and when he read Weyl's manuscript, he called it an admirable "stroke of genius" ("ein Geniestreich ersten Ranges") and found the argument "wonderfully coherent" ("Der Gedankengang ist von wunderbarer Geschlossenheit") [29, Docs. 498, 499].

Notwithstanding his admiration for Weyl's mathematical argument, Einstein had a serious objection against Weyl's unified theory. The introduction of a length curvature, he argued, would undermine the very foundation of the general theory of relativity itself. It would mean that the lengths of measuring rods or the rates of clocks would depend on their prehistory. This would imply, for instance, that the frequency of spectral lines of two atoms of the same chemical element would in general not be the same. This was never observed in nature. In spite of this objection, Einstein presented the paper to the Academy for publication. However, his Academy peers objected to the publication of a paper which Einstein would not fully endorse [29, Docs. 512, 529]. The conflict was resolved by Einstein's adding an addendum to the paper, explaining his objection and also allowing Weyl to respond to Einstein's objection [32].

All of this was worked into a substantially revised chapter on Riemannian geometry of the third edition of RZM. In addition to the mathematical developments of the purely infinitesimal mathematics, the fourth chapter on general theory of relativity proper was also substantially revised, reflecting Weyl's ideas on a unified field theory. The revised third edition of RZM came out in the fall of 1919, right before the announcement of the British eclipse expedition.

Weyl begins his preface to the third edition like this:

> Although this book offers fruit of knowledge in a refractory shell, yet communications that have reached me have shown that to some it has been a source of comfort in troublous times. To gaze up from the ruins of the oppressive present towards the stars is to recognise the indestructible world of laws, to strengthen faith in reason, to realise the "harmonia mundi" that transfuses all phenomena, and that never has been, nor will be, disturbed.[9]

Einstein, for his part, was not enchanted by all of Weyl's changes. To his good friend Heinrich Zangger (1874–1957) in Zurich, he wrote:

> [Weyl] is a very remarkable mind but a little removed from reality. In the new edition of his book, he made a complete mess of relativity, I think—God forgive him. Perhaps he will eventually realize that, for all his keen perception, he has shot wide off the mark."[10]

[9] "Obschon dieses Buch die Frucht der Erkenntnis in harter Schale bietet, ist es doch manchem, wie mir verschiedene Zuschriften zeigten, ein Trostbüchlein in wirrer Zeit gewesen; ein Anblick aus dem Trümmerfeld der uns unmittelbar bedrängenden Gegenwart zu den Sternen, das ist: der unzerbrechlichen Welt der Gesetze; Bekräftigung des Glaubens an die Vernunft und eine alle Erscheinungen umspannende, nie gestörte, nie zu störende 'harmonia mundi'."

[10] "[Weyl] ist ein sehr bedeutender Kopf, aber etwas thatsachenfremd. In der neuen Auflage seines Buches hat er mir die Relativität ganz verhunzt—Gott verzeihe es ihm. Vielleicht wird er doch noch einmal einsehen, dass er da bei allem Scharfsinn daneben geschossen hat." [17, Doc. 332].

And to his colleague and friend Paul Ehrenfest (1880–1933) in Leyden, he complained:

> "Weyl has now added his electromagnetic theory to the new edition of his textbook, unfortunately, so this admittedly very ingenious nonsense will make its way into the cerebra. But I console myself that the sieve of time will do its work here as well."[11]

4 Further Revisions and Later Reception

Einstein's uneasiness about the expected success of Weyl's book was not unfounded. A fourth edition of RZM came out already in 1921. This fourth edition was published in 1922 both in English translation by Methuen [38] and in French translation by Blanchard [39], although Weyl found the French version so freely translated that he declined to take on any responsibility for it [41, p. VI]. Both translations have had considerable impact, the English one is still available as a Dover reprint, but also the French edition has had considerable influence. Among others, it introduced Élie Cartan (1869–1951) to the theory of relativity and the problem of space and, according to Giulini and Scholz in [43], it triggered Cartan's work on the representations of Lie groups in 1924.

A fifth edition of 1923 was again substantially revised in all its parts, reflecting the results and insights of recent and ongoing theoretical investigations both of Weyl and of his contemporaries. Two main strands of development over the five editions are highlighted by Giulini and Scholz. One concerns Weyl's theory of matter, the other his contribution to the contemporary debates in relativistic cosmology.

In very general terms, Weyl presented three different approaches toward an understanding of matter in the field theoretic context of a general theory of relativity [27]. The first approach might be called a classical field-theoretic approach and builds on ideas put forward by Gustav Mie (1868–1957) and David Hilbert. The idea is to conceive of elementary matter in terms of particle-like solutions to field equations derived from a variational principle with a suitably generalized Lagrangian. Such solutions are regular, spherically symmetric and static and fall off sufficiently rapidly away from the particle. The second approach conceives of elementary particles as singularities of the field, and the third approach finally takes on a more agnostic approach, leaving the nature of matter undetermined and focusses on the far-field behavior outside of tubes enclosing the world lines of particles sufficiently far away from the localized matter. Weyl's final agnosticism in this respect is expressed in his reference to the "dark cloud" of the emerging quantum mechanics. Indeed it is the fundamental changes in our understanding of matter brought about by quantum mechanics and quantum field theory, which prevented Weyl from ever attempting to bring out another revised edition after the fifth one in 1923.

[11] "Weyl hat in der neuen Auflage seines Lehrbuchs nun seine elektromagnetische Theorie leider angefügt, sodass dieser allerdings sehr geistreiche Unfug seinen Weg in die Gehirne nehmen wird. Aber ich tröste mich damit, dass das Sieb der Zeit seine Arbeit auch an dieser Stelle thun wird." [17, Doc. 294].

Weyl's contributions to cosmology, on the other hand, might appear to us in a very different light. Weyl was a major contributor to the early development of relativistic cosmology. This discussion began with Einstein's first application of the general theory of relativity to the large scale structure of the cosmos and the introduction of the cosmological constant in 1917 into the gravitational field equations. The debate soon focussed on two rival solutions and interpretations: one was a static closed universe of constant, non-vanishing spatial curvature and finite homogenous distribution of matter (Einstein's "cylinder world"), the other was de Sitter's solution for a massless universe, which, however, was time-dependent. The clarification of the intricacies and implications of cosmological solutions of the gravitational field equations involved notable astronomers, physicists and mathematicians, like Willem de Sitter (1872–1934), Einstein himself, Felix Klein (1849–1925) and Weyl.

In the first edition of RZM, Weyl still sided with Einstein in a chapter on cosmological solutions, where he very explicitly pointed out that the large scale, topological structure of the universe does not follow from the field equations alone. In the fifth edition, he distanced himself from the earlier interpretation and advanced a cosmological model of his own, based on certain time-like congruences on de Sitter's five-dimensional hyperboloid manifold. In this context, Weyl introduced an appendix to the fifth edition, in which he discussed in very clear and general terms the problem of cosmological redshift in de Sitter space time, thus laying the ground for an analytic conception of Hubble's constant. As just one example of Weyl's relevance for our modern understanding, Weyl's discussion, which does not rely on wave optics, can, in fact, be further generalized, as Giulini and Scholz point out, and they provide an explicit proof for this analytic treatment, spelling out a proof by Brill [3], which is rarely discussed in the literature.

With the revisions made necessary by Weyl's evolving views on such issues as the conception of matter or the cosmological implications, the fifth edition was again substantially revised and augmented. It contained new paragraphs and three appendices, and the book had grown to some 338 pages from 234 pages of the first edition. Weyl refrained from editing another revision due to the fundamental changes brought about by the "dark clouds" of quantum mechanics, but perhaps also because he thought that the mathematical parts had sufficiently been evolved and could stand.

The fifth edition of 1923 was only reprinted, posthumously and without changes, some fifty years later by Springer in 1970. A seventh edition was prepared in 1988 by the theoretical physicist and relativist Jürgen Ehlers (1929–2008). It presented the text of the fifth edition together with all the previous prefaces in their entirety and also included some annotation to the text from a modern perspective. In his reflections on RZM, Ehlers gave two reasons why *Raum · Zeit · Materie* can still claim interest even in a modern context. For one, he argued, it was the first mathematical textbook about general relativity and therefore it gives special emphasis and is more detailed about the historical roots and physical motivations behind the then new notions of connection and curvature. Second, since Weyl was one of last mathematical "universalists", he could see and motivate many connections between different and remote aspects of the theory, that are no longer made explicit in the same manner.

The new reprint was edited by Domenico Giulini and Erhard Scholz in Springer's series *Klassische Texte der Wissenschaft* [43]. It again presents Weyl's fifth edition in facsimile together with all earlier prefaces but also with some selected passages from the earlier four editions. More importantly, Giulini and Scholz have added extensive commentary and annotation. In fact, more than half of the 900 pages of the volume were written by the editors. Both Weyl's reprint as well as the editors' commentary are presented in German language. They provide an introduction to Weyl's book with biographical background and an overview over the changes of the various editions, and they provide extensive commentary to the text, elaborating on the historical context and explaining later developments. With their excellent commentary in hand, Weyl's magisterial *Raum · Zeit · Materie* can indeed be read not only as a historical document of major significance but also as an introduction to our modern understanding of the theory of general relativity, to its history, to its philosophical implications, and, in short, to the fascination it still evokes.

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