A Behavioral Economic Analysis of Mental Health

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Chapter 1

Introduction

This dissertation analyses the relationship between mental health and individual behavior and channels through which the aggregate distribution of mental health among a population can be influenced for the purpose of increasing welfare. It consists of three autonomous chapters and a final discussion. The three chapters are interrelated to one another as regards content as well as analytical structure and methods used. This introduction should briefly explain the central ideas and provide an overview of how the three chapters are interrelated. Since Chapter 2 is linked to 3 and 4 rather methodologically and regarding the analytical structure, while Chapters 3 and 4 additionally connected contentwise, we should start with the latter connection.

Chapter 3 presents a general theory of mental health, covering several different kinds of mental illnesses, while the focus of Chapter 4 in on substance-related addictions in particular. In psychology addictions are commonly considered as a specific form of mental illness (American Psychiatric Association, 2013, p.481 ff.). Thus, Chapter 4 can be seen as a chapter that places a stronger focus on one particular kind of mental illness. Both chapters analyze within a theoretical economic context the influence of mental health on individual behavior. Through the dynamic structure of the analytical framework we are also able to understand what the consequences of individual behavior on the distribution of mental health across a population are. This allows us to simulate and analyze the effects of various public policy interventions on the individual level, i.e. on behavior and on subjective well-being but also on the aggregate level, such that we are able to understand a policy's influence on the distributional dynamics of mental health and to compute the welfare effects using a benevolent planner framework. Both models in Chapters 3 and 4 are calibrated to real world data, to quantify these effects. The goal of the frameworks is to make quantitative forecasts, because this makes it possible to derive policy recommendations and also to assess the effects of various policy interventions qualitatively and quantitatively. A great advantage of these formal approaches is that without exception all macroeconomic outcomes are micro-founded, which allows us to understand processes as well as changes on the aggregate level in detail by drawing conclusions from the individual level.

Due to the content related proximity of Chapters 3 and 4, it becomes obvious that also the analytical structures of both chapters resemble each other with regard to the following aspects. A key element in both model setups is the state-dependent utility function. The inspiration for this structure comes from a classic paper by Grossman (1972) in which he considers health as a durable capital stock. Given this, health not only allows for individual activity, it delivers beyond that an additional direct instantaneous utility stream. And conversely, bad health leads to an instantaneous dis-utility, reflecting individuals' suffering under a bad health condition. In both setups we model the evolution of mental ill-health as driven by the stochastic increments of Poisson processes with endogenous arrival rates. Therefore it makes sense to use a continuous time framework as it provides a better analytical description of the stochastic dynamic processes in the models. Yet, there are also differences between the two approaches. In contrast to the general mental health framework in Chapter 3, we decided to endogenize recovery in the addiction model in Chapter 4. More precisely, individuals are provided with the opportunity to participate in therapy to cure their physical and mental deterioration. The reason for endogenizing recovery in the addiction framework is that active participation in therapy is particularly important when it comes to addictions, while the passive control of mental health care in general is able to capture various ways of curing mental ill-health in the sense of improving mental health care quality within an economy in a more general framework.

On a methodological level there are similarities as well. We derive numerical solutions for optimal behavior and the value of optimal behavior by solving systems of maximized Hamilton-Jacobi Bellman equations. To explain the distributional dynamics we analytically derive Fokker-Planck equations from the corresponding stochastic differential equations that describe the transition of mental ill-health. This also allows us to compute the stationary distributions. We solve the systems of Fokker-Planck equations numerically. Similar structures can be used to derive the stationary equilibrium distribution of mental health. The calibration of the models to real world data allows us to make predictions regarding individual behavior on the micro level, the distributional dynamics of a populations' mental condition and its long-term distribution on the macro level and also demonstrates the explanatory power of Fokker-Planck equations.

As state-dependent utility turned out to be an appropriate structure for the utility functions, which, however constitutes a deviation from standard utility functions, it was necessary to understand the peculiarities this deviation comes along with. Therefore the first chapter of this dissertation can be seen as an auxiliary chapter in which we analyze an otherwise standard household saving problem with a status effect of wealth. The idea was to start with an optimization problem which is very familiar to economists and introduce a state-dependent utility function, such that the acquired findings regarding the special characteristics of analyzing such an optimization problem can be transferred to the analyzation of the problems in Chapters 3 and 4. Besides the above mentioned contribution of Chapter 2 to the deeper understanding of the underlying mathematical structures used in this dissertation it also provides a nice contribution to the research on economic growth and to the status effect literature.

Chapter 2

State-Dependent Utilities

by Dennis Krieger[1]

2.1 Introduction

[Motivation] In many different areas of economic research models incorporating state-dependent utility functions have proven to be suitable (or more suitable than state-independent utility functions) for explaining certain issues of economic interest. Generally speaking, a utility function is state-dependent, when it directly depends on the state of the underlying dynamic system. Thus, depending on what a theory seeks to explain, very different things can be meant by a state (e.g. classic economic quantities like the stock of wealth, human capital or resources, the size of a population, environmental pollution, but also physical states of economic agents like health or psychological states like mental health, distress, cognitive load or a subjective belief). The idea that a state not only characterizes a dynamic system and changes by its in- and outflows, but also yields a direct flow of services to economic agents has already been implemented in (Bayesian) decision theory quite a while ago. But the same idea also gave birth to other strands of economic literature like the literature on (social) status and particularly to some important parts of the emotional economic literature, e.g. the literature on anticipatory emotions, visceral factors, disappointment, regret and addiction. Finally, also this dissertation employs a state-dependent utility structure in Chapters 3 and 4.

[Problem] State-dependent preference structures constitute a deviation from standard utilities, not only in terms of the mere existence of an additional utility stream that is received and taken into consideration by economic agents. Usually, also the assumption of additive separability of the life-time utility function is implicitly relaxed. That is, state-dependence of the utility function creates an

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additional trade-off. In rational addiction models, for instance, the amount of an addictive good that is consumed at time t has an impact on the enjoyment an agent gets from consuming the same good at $\tau > t$ (in addition to the trade-off between consumption of the addictive good versus the normal good at time t). Another peculiarity of state-dependent utilities is that they can cause time inconsistent behavior. This might result in the fact that additional assumptions regarding the agents and/or the environment they live in are required to describe an optimization problem adequately and completely.

[Goal] The special aspects state-dependent utility comes along with require developing a general understanding for this utility structure and what it implies for the analytical and numerical analysis of optimization problems that employ such structures.

[Approach] In our opinion, the most convenient way of exploring the peculiarities of state-dependent utility is to start with something that is familiar for economists and then to deviate from it in a very straightforward way. Therefore we decided to start our analysis using a classic household saving model, which we extent further below to make the utility state-dependent. This leads us to a status effect of wealth model, which we consider as our main example in this chapter. In the status effect model, instead of consumption only, also the possession of wealth yields a utility stream for the individual. In traditional capital accumulation models the lower boundary condition, $r > \rho$, must hold to ensure positive consumption growth. In presence of a status effect, however, the lower boundary condition is not binding anymore. A status effect of a certain extent is necessary to ensure the existence of a balanced growth path. We derive numerical and analytical closed-form solutions. Moreover and in a more general framework we ask under which circumstances state-dependent utility functions can cause time inconsistent behavior. The latter analysis is conducted both in a discrete and in a continuous time framework.

[Results/Findings] The analysis in this chapter helps us to understand which analytical and numerical techniques are applicable when we work with state-dependent utility function and under which circumstances models with state-dependent utility functions evoke time inconsistent behavior. We learn that numerical solutions can be derived by solving initial value problems that result from the model. To pin down unique optimal solutions sufficiently many initial and terminal conditions are necessary. A balanced growth path can be determined by using binary search algorithms. Moreover, we find that time inconsistency can be caused by a state-dependent utility structure. Therefore the utility function must include an anticipatory element, which disappears when switching perspectives.

[Related Literature] A substantial part of our paper is based on the status effect of wealth literature. The general idea that wealth accumulation not only serves the function of financing future consumption, but that also the possession of wealth itself yields an additional flow of services for the owner, such as social status, goes back to Weber (1958), who also coined the term "capitalist spirit" for this phenomenon. In economic research this idea was taken up and used in various ways to gain deeper insights into economic growth, consumption and portfolio decisions, wealth inequality and the demand for social status. Here we want to provide a brief overview of some of the most important research projects in this field.

From a formal perspective the closest and most directly related paper is Zou (1994). Zou's paper follows a paper by Kurz (1968), who showed that the presence of preferences for wealth in a deterministic optimal growth model can cause a multiplicity of stationary points. Zou, however, demonstrates that in a similar framework direct preferences for wealth are able to absorb or compensate individual's impatience to a certain extent, such that an additional incentive to accumulate capital arises, which has positive growth effects. The difference to the present paper is that the focus of our analysis is centered on an individual's modified incentives to accumulate wealth in present of a status effect and on deriving analytical and numerical solutions of the underlying household problem, while Zou rather examines economic growth effects on the macro level. One of the main results of the model in our paper, that the lower boundary condition, $r > \rho$, is in the presence of a status effect not a necessary condition for the existence of a balanced growth path is perfectly in line with Zou's result. The great advantage of this paper is that we can abstain from using a resource constraint which neglects capital depreciation to create the fiction of endogenous economic growth, as in Zou's paper.

In contrast to Kurz (1968), Zou (1994) and this paper, Bakshi and Chen (1996) write *relative* social status into the utility function. Relative social status is assumed to increase in wealth holdings and decrease in the wealth holdings of an individual's reference group. Within the framework of a consumption-portfolio problem they consider three different utility specifications to examine the implications for optimal consumption and savings decision as well as their portfolio composition and stock prices. Interestingly, they find that individuals with "catching up with the Joneses" utility tend to be more frugal when it comes to the consumption decision and tend to make less risky decisions on the capital markets, because an individual's aversion to wealth risk causes the expected wealth to grow slower on the one hand, but on the other hand they have an incentive to save more, which in turn causes wealth to grow faster. In contrast to the literature on relative social status the present paper considers social status as independent of a relation to a reference group, as the effects of different wealth distributions are not in the focus of attention. However, the utility function proposed here is open for extensions, i.e. it can be thought of as the reduced form of a general utility function, $w(u(c(t)), v(a(t), x(t)))$, with a constant normalized reference point, $x(t)$.

The works by Carroll (1998), Reiter (2004) and Francis (2009) give particular attention to explaining and addressing the reasons for the empirically observable wealth inequality in the US (Díaz-Giménez et al., 1997). They find that the saving behavior especially of rich people and thus the empirical wealth distribution can be explained better, in comparison to standard life-cycle models based on consumption smoothing or infinite horizon models with a bequest motive, when wealth is included as a direct argument of the utility function.

Cole et al. (1992) investigate in which way the demand for social status shapes individual's wealth accumulation behavior. Thus it can be seen as the micro foundation for the underlying preference structures examined by the above mentioned papers as well as this paper.

On a more general and conceptional level, this paper is also related to the following strands of literature using state-dependent utility functions. Abel (1990) introduced a generic utility function which includes three different classes of utility functions (inter alia a "catching up with the Joneses" utility function) and incorporated it into an otherwise standard Lucas asset pricing model to investigate the equity premium puzzle. Also the rational addiction literature, which ranks among the most influential ones of modern economic research, uses state-dependent utility as a modelling technique. Famous representatives are amongst others Becker and Murphy (1988); Ferguson (2000); Gruber and Köszegi (2001); Orphanides and Zervos (1995). They consider current consumption of an addictive good as being accompanied by a future detrimental health effect. The latter is mapped into a state variable, the "stock of consumption capital", which has a direct negative utility effect. Another strand of theoretical literature has been devoted to analyzing ex ante, ex post and instantaneous emotions. Worth mentioning here in particular are Loewenstein (1987) and Caplin and Leahy (2001) for ex ante emotions, the regret theory going back to Loomes and Sugden (1982) and related work on disappointment undertaken by Bell (1985) as examples for ex post emotions. Further examples are the literature on guilt which includes Battgalli and Dufwenberg (2007) and Charness and Dufwenberg (2011). For instantaneous emotions Loewenstein (2000); Loewenstein et al. (2003) and Laibson (2001) are examples. What all these papers have in common is that they consider emotions as states and that individuals derive a direct utility from this state.

[Structure] In the next section we introduce the standard household saving problem in continuous time, derive its analytical solution and the lower boundary condition for infinite consumption growth. In Section 2.3 we implement the status effect of wealth. Under the assumption that $r < \rho$, we prove analytically that the existence of a balanced growth path requires that the status effect of wealth is sufficiently large. Besides the derivation of an analytical solution we also apply numerical methods of solving initial value problems to derive the numerical solution of the optimization problem. We use a binary search algorithm to find a balanced growth path. In Section 2.4 we focus on the issue of time inconsistency due to state-dependent utility functions. The analysis in this section is separated according to the time structure (discrete and continuous). Finally, in the last section we conclude.

2.2 The Standard Saving Problem

We first present the individual's saving problem and its implications for consumption growth in absence of a status effect. This model is well-known in the economic literature and follows in style of presentation and notation chapter 5.6.1 in Wälde (2012).

An individual's problem is to maximize life-time utility, $U(t)$, discounted at rate ρ by choosing the optimal time path of consumption, $c(t)$, subject to a dynamic budget constraint, given an initial endowment of capital, $a(0) = a_0$. Formally,

$$\max_{\{c(\tau)\}_{\tau=t}^{\infty}} U(t) = \max_{\{c(\tau)\}_{\tau=t}^{\infty}} \left\{ \int_t^{\infty} e^{-\rho[\tau-t]} u(c(\tau)) \, d\tau \right\}, \tag{2.1a}$$

subject to the budget constraint,

$$\dot{a}(t) = ra(t) + w - c(t), \tag{2.1b}$$

given the initial endowment of capital

$$a(0) = a_0 \geq 0, \tag{2.1c}$$

where $u(c(t))$ is the instantaneous utility function, with $u'(c(t)) > 0$ and $u''(c(t)) < 0$. $r \geq 0$ is the interest rate and $w \geq 0$ is the wage. The price of the consumption good is normalized to unity.

2.2.1 Optimality Conditions

The current-value Hamiltonian for the optimization problem (2.1) reads

$$H(c(t), a(t), \lambda(t)) = u(c(t)) + \lambda(t) \left[ra(t) + w - c(t) \right]. \tag{2.2}$$

The optimality conditions read

$$\frac{\partial H(.)}{\partial c(t)} = u'(c(t)) - \lambda(t) = 0 \tag{2.3}$$

and

$$\dot{\lambda(t)} = \rho\lambda(t) - \frac{\partial H(.)}{\partial a(t)} = [\rho - r]\,\lambda(t). \tag{2.4}$$

From the two optimality conditions we can derive the Keynes-Ramsey rule (KRR)

$$-\frac{u''(c(t))}{u'(c(t))}\dot{c(t)} = r - \rho. \tag{2.5}$$

Given utility is CRRA, the KRR becomes $\dot{c(t)}/c(t) = [r - \rho]/\sigma$ and tells us that consumption optimally grows at rate $[r - \rho]/\sigma$ (or remains constant or declines) over time, if the interest rate, r, is greater (or equal or less) the time preference rate, ρ.

2.2.2 Candidate Solutions And The Explicit Solution

The optimization problem has a unique optimal solution, $(c^{\text{opt}},\ a^{\text{opt}})$, that must solve (2.1b) and (2.5), the necessary conditions, given (2.1c). However, a solution of this two-dimensional dynamic system, $(\tilde{c},\ \tilde{a})$, not necessarily solves the optimization problem. $(\tilde{c},\ \tilde{a})$ is merely a candidate solution of the dynamic system.

For convenience, we repeat the two differential equations here. For a CRRA utility they read

$$\dot{a}(t) = ra(t) + w - c(t), \tag{2.1b}$$

given $a(0) = a_0$ and

$$\dot{c(t)} = c(t)\frac{r - \rho}{\sigma}. \tag{2.5}$$

The explicit solution to the dynamic system reads[2]

$$c(t) = c(0)e^{\frac{r-\rho}{\sigma}t} \tag{2.6}$$

and

$$a(t) = a_0 e^{rt} + \frac{1}{r}w\left[1 - e^{rt}\right] - \frac{\sigma}{\rho - [1 - \sigma]\,r}C\left[e^{\frac{r-\rho}{\sigma}t} - e^{rt}\right]. \tag{2.7}$$

The important insight we gain from the explicit solution is that it is parametrized in $c(0)$, given a_0. In other words, there is an infinite number of solutions to the system of differential equations. As economists we want to know which of the infinitely many solutions solves the optimization problem. And the answer to this question is very simple. It depends on the initial value for consumption, $c(0)$. Once we assume that $c(0)$ equals some given value c_0, we obtain an unique optimal solution. Although the answer to this question is quite obvious, for economists it

[2] A formal derivation is provided in Appendix 2.A.

might seem very unsatisfactory, because one might get the impression that optimal solutions can be generated arbitrarily by defining c_0 accordingly. In the following sections we will see that optimal solutions of standard problems (and ideally also those of non-standard problems) are not generated arbitrarily, but rather are based on well-considered economic arguments.

2.2.3 Phase Diagram Analysis

From the KRR we know already that both parameters, r and ρ, play a crucial role for individual behavior. Therefore we shall examine the following three generic cases.

a) $r < \rho$,

b) $r > \rho$,

c) $r = \rho$.

A steady state requires that $\dot{c}(t) = \dot{a}(t) = 0$. When $r \neq \rho$, i.e. in cases a) and b), it immediately follows from the KRR that there is a steady state at a consumption level of $c^* = 0$. Plugging this into the budget constraint gives us the steady state wealth level of $a^* = -w/r$.

In case c), however, we find that there is a continuum of steady states on the zero-motion line of wealth, $\dot{a} = 0 \Leftrightarrow c = ra + w$.

We illustrate the results in the phase diagram in Figure 2.1.

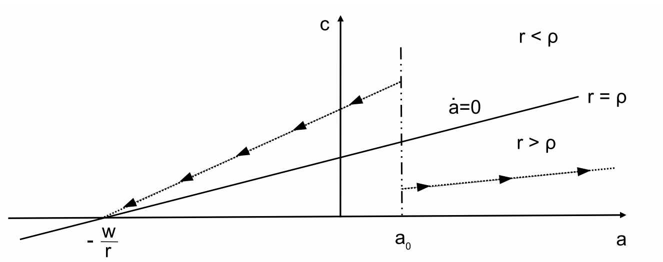

Figure 2.1: Solution paths for $r \gtreqless \rho$

The directions of motion, indicated by the small arrows on the dotted lines in each sector of the phase diagram, result from the partial derivatives of the dynamic budget constraint.

The two dotted lines represent paths that illustrate graphically the time evolution of consumption and wealth, provided the interest rate is either lower or greater than the time preference rate, given some initial capital endowment, a_0.

The graphic shows nicely what we have derived analytically in the previous section. Given the initial endowment, a_0, it depends on the level of c_0 how consumption and wealth develop over time. However, c_0 should not be chosen arbitrarily, because although this leads mathematically to a solution, such a solution might be economically not reasonable. For example c_0 could be chosen such that debt ($a(t) < 0$) grows unboundedly, meaning the individual would never pay back her debt. Therefore, in economic theory it is common practice to introduce a no-Ponzi game condition, which is a constraint that prevents over-accumulation of debt. Another form of constraint is the transversality condition, which is a condition that ensures that the individual starts on a saddle path and approaches a steady state in the long-run. Such a saddle path is drawn in Figure 2.1 for case a). We see that the individual in this case reaches a steady state debt level of $a^* = -w/r$ in the long run. This debt level is commonly known as the natural borrowing limit and describes a level of debt that can be paid back by the individual using her work income w. Thus, in this case the transversality condition as well as the no-Ponzi game condition lead to the same optimal solution. The important point is that also an adequate initial condition for consumption is appropriate to ensure that the individual is on the saddle path and / or does not violate a no-Ponzi game condition. This is supported by the fact that Wälde (2012) solves a similar optimal saving problem with a logarithmic utility function in ch. 5.6.1 using an intertemporal budget constraint, instead of the dynamic one which we employ here, and shows that it is possible to derive level information for the endogenous variables. The reason is that the intertemporal budget constraint is derived from its dynamic counterpart using a no-Ponzi game condition (Wälde, 2012, ch. 4.4.2).

2.2.4 Infinite Growth And The Lower Boundary Condition

Having analyzed case a) with $r < \rho$ in the previous section, we will now take a closer look at case b) where $r > \rho$ and we will see that in this optimal saving model, there is the possibility of infinite consumption and wealth growth, which is maybe not such a great surprise as one can certainly draw parallels to the well known $A - K-$model by Barro (1990) and Rebelo (1991). Technically, the linearity of the differential equation (2.1b) in wealth mainly generates this result, equivalently as in the $A - K-$model, where the linearity of the resource constraint in capital allows for infinite growth.

Let the instantaneous utility function be standard CRRA,

$$u(c(t)) = \frac{c(t)^{1-\sigma} - 1}{1 - \sigma}, \quad \sigma > 0. \tag{2.8}$$

Then the KRR becomes

$$\frac{\dot{c}(t)}{c(t)} = \frac{r-\rho}{\sigma} \tag{2.9}$$

and we define the growth rate of consumption as $g \equiv (r-\rho)/\sigma$.

Proposition 1. *The existence of a balanced growth path, along which both, consumption and wealth, grow with the same positive and constant rate, requires that the lower boundary condition, $r > \rho$, permanently holds.*

Proof. Suppose that $r > \rho$, then the growth rate of consumption,

$$g = \frac{\dot{c}(t)}{c(t)} = \frac{r-\rho}{\sigma} \tag{2.10}$$

is positive. If a balanced growth path exists, then wealth must grow with the same positive rate as consumption. We denote the growth rate of wealth as $h > 0$. Using the budget constraint we can write

$$h \equiv \frac{\dot{a}(t)}{a(t)} = r + \frac{w}{a(t)} - \frac{c(t)}{a(t)}. \tag{2.11}$$

Then the balanced growth path is given by

$$c(t) = (r-h)a(t) + w. \tag{2.12}$$

As consumption and wealth grow at the same rate, $c(t)/a(t)$ is a constant, which we define as χ. It follows from (2.11) that

$$h = r + \frac{w}{a(t)} - \chi \tag{2.13}$$

From $g \overset{!}{=} h$ follows that

$$\begin{aligned} \frac{r-\rho}{\sigma} &= r + \frac{w}{a(t)} - \chi \\ \Leftrightarrow \chi &= r + \frac{w}{a(t)} - \frac{r-\rho}{\sigma}. \end{aligned} \tag{2.14}$$

Plugging the last equation back into (2.13) finally gives

$$h = \frac{r-\rho}{\sigma}. \tag{2.15}$$

□

In this section we proved the existence of a balanced growth path, along which both, consumption and wealth, grow with the same positive and constant rate, g, as long as the lower boundary condition, $r > \rho$, permanently holds. The balanced growth path is also depicted in Figure 2.1. Again we can apply the insights from the analysis in the previous chapters. If we want the individual to start on the balanced growth path, we either need to ensure this by the introduction of an adequate transversality condition, or by specifying the initial value for consumption, c_0, accordingly.

2.3 The Status Effect Of Wealth

In this section we now deviate from the standard optimal saving problem by introducing what we call a status effect of wealth. Thus, consider now, apart from consumption also the possession of capital yields a utility stream to the individual.

The introduction of the status effect of wealth also generates a second dynamic trade-off. Additional to the usual trade-off between current and future consumption, now there is another trade-off between current consumption and future status.

Formally, instead of $u(c(t))$, the individual maximizes a utility function consisting of two sub-utility functions, of consumption, $u(c(t))$, and of wealth, $v(a(t))$. We assume that both sub utilities are concave in their arguments, i.e. $u'(c(t)) > 0$, $u''(c(t)) < 0$, $v'(a(t)) > 0$ and $v''(a(t)) < 0$.

An individual's optimization problem is to maximize life-time utility, $U(t)$, discounted at rate ρ by choosing the optimal time path of consumption, $c(t)$, subject to a dynamic budget constraint, given an initial endowment of capital, $a(0) = a_0$. Formally,

$$\max_{\{c(\tau)\}_{\tau=t}^{\infty}} U(t) = \max_{\{c(\tau)\}_{\tau=t}^{\infty}} \left\{ \int_t^{\infty} e^{-\rho[\tau-t]} w(c(\tau), a(\tau)) \, d\tau \right\}, \tag{2.16a}$$

subject to the budget constraint,

$$\dot{a}(t) = ra(t) + w - c(t), \tag{2.16b}$$

given the initial endowment of capital

$$a(0) = a_0 \geq 0, \tag{2.16c}$$

where $w(c(t), a(t))$ is the instantaneous utility function, $r \geq 0$ is the interest rate and $w \geq 0$ is the wage. The price of the consumption good is normalized to unity.

For concreteness sake we specify the functional form of the instantaneous utility function. We assume that both sub-utility functions are of the CRRA form such that

$$w(c(t), a(t)) \equiv u(c(t)) + \phi v(a(t)) = \frac{c(t)^{1-\sigma} - 1}{1-\sigma} + \phi \frac{a(t)^{1-\sigma} - 1}{1-\sigma}. \tag{2.17}$$

The first term is the standard CRRA utility, with the utility elasticity of intertemporal substitution, $\sigma \geq 0$, or the coefficient of relative risk aversion. Through the second term wealth enters the utility function also in CRRA form. For $\phi = 0$ we come back to the standard utility function without a status effect. If $\phi > 0$ the individual gains utility from the possession of wealth and ϕ measures the strength of the status effect.

2.3.1 Optimality Conditions

Proposition 2. *In presence of a status effect of wealth, consumption optimally grows at rate*

$$g^S = \frac{r-\rho}{\sigma} + \frac{\phi \left[\frac{c(t)}{a(t)}\right]^{\sigma}}{\sigma}. \tag{2.18}$$

Proof. The current-value Hamiltonian for the optimization problem (2.16) reads

$$H(c(t), a(t), \lambda(t)) = w(c(t), a(t)) + \lambda(t) \left[ra(t) + w - c(t)\right]. \tag{2.19}$$

The optimality conditions are

$$\frac{\partial H(.)}{\partial c(t)} = u'(c(t)) - \lambda(t) = 0 \Leftrightarrow \lambda(t) = u'(c(t)) \tag{2.20}$$

$$\begin{aligned} \dot{\lambda}(t) = \rho\lambda(t) - \frac{\partial H(.)}{\partial a(t)} &= \rho\lambda(t) - v'(a(t)) - \lambda(t)r \\ \Leftrightarrow \frac{\dot{\lambda}(t)}{\lambda(t)} &= \rho - \frac{v'(a(t))}{\lambda(t)} - r. \end{aligned} \tag{2.21}$$

Applying the logarithm on (2.20) gives

$$\ln \lambda(t) = \ln u'(c(t)). \tag{2.22}$$

Taking the time derivative yields

$$\frac{\dot{\lambda}(t)}{\lambda(t)} = \frac{u''(c(t))}{u'(c(t))}\dot{c}(t). \tag{2.23}$$

Inserting this derivative and (2.20) into (2.21) yields the KRR

$$-\frac{u''(c(t))}{u'(c(t))}\dot{c(t)} = r - \rho + \frac{v'(a(t))}{u'(c(t))}. \tag{2.24}$$

By plugging in the partial derivatives of the subutility functions we finally obtain

$$g^S = \frac{\dot{c(t)}}{c(t)} = \frac{r-\rho}{\sigma} + \frac{\phi\left[\frac{c(t)}{a(t)}\right]^{\sigma}}{\sigma}. \tag{2.25}$$

□

From the "new" KRR we learn that consumption now grows at rate $g^S = (r - \rho)/\sigma + \phi\left[c(t)/a(t)\right]^{\sigma}/\sigma$ (or remains constant or declines) over time, if the marginal rate of substitution plus the interest rate is greater (or equal or less) the time preference rate ρ.

It follows immediately from (2.25) that consumption growth can be positive even if the interest rate is lower than the time preference rate. We will discuss this phenomenon in detail further below.

2.3.2 Candidate Solutions And The Explicit Solution

Equivalent to the standard problem, the necessary conditions derived from the optimization problem constitute a two-dimensional system of differential equations, repeated here for convenience.

$$\dot{a}(t) = ra(t) + w - c(t), \tag{2.16b}$$

given the initial endowment of capital $a(0) = a_0 \geq 0$ and

$$\dot{c(t)} = c(t)\left[\frac{r-\rho}{\sigma} + \frac{\phi\left[\frac{c(t)}{a(t)}\right]^{\sigma}}{\sigma}\right]. \tag{2.25}$$

In contrast to the standard optimal saving problem we now have a non-linear system which cannot be solved analytically as straightforward as in absence of a status effect of wealth. However, the argument we made before still holds. Any explicit solution we derive (analytically or numerically) is merely a candidate solution and will depend on c_0.

2.3.3 Phase Diagram Analysis

Although there exist several different cases of parameter constellations, we focus during the following analysis mainly on the most interesting case in our opinion, which is $r < \rho$.

Setting $\dot{c}(t) = 0$ and $\dot{a}(t) = 0$ yields the stationary values, which satisfy

$$\frac{v'(a^*)}{u'(c^*)} = -(r - \rho) \tag{2.26}$$

and

$$c^* = ra^* + w. \tag{2.27}$$

From (2.26) follows first that the marginal rate of substitution is a constant in the steady state. And second that under the assumption of positive marginal utilities there exists no steady state for the case $r > \rho$.

Inserting the partial derivatives of the utility function into the steady state conditions yields the zero-motion lines of wealth and consumption,

$$f(a^*) \equiv c^* = ra^* + w \tag{2.28}$$

and

$$g(a^*) \equiv c^* = a^* \left[\frac{\rho - r}{\phi}\right]^{1/\sigma}. \tag{2.29}$$

Compared with the standard optimal saving problem, the zero-motion line of wealth remains unchanged in presence of a status effect. As mentioned above, no steady state exists, if $r > \rho$ as the zero-motion line for consumption is not defined. In the case $r < \rho$, the zero-motion line for consumption is a linear and positively sloped function of wealth. Given, $r < \rho$, there can be two cases. If $f'(a^*) < g'(a^*)$ a stationary point exists, while there is no steady state if $f'(a^*) \geq g'(a^*)$.[3] Figure 2.2 illustrates the latter case.

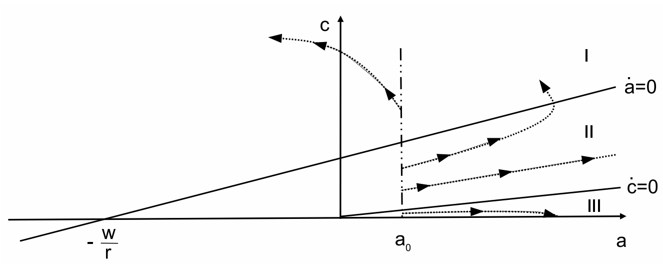

Figure 2.2: Solution paths for $f'(a^*) \geq g'(a^*)$ and $r < \rho$

[3]Note that we rule out the case that consumption is negative at any point in time.

The directions of motion, for each sector of the phase diagram are derived from the budget constraint and the KRR, respectively. The dashed lines starting at a_0 with different initial values of consumption c_0 are trajectories and illustrate the evolution of consumption and wealth over time. In the present case, we assume that $f'(a^*) \geq g'(a^*)$, such that there are only three different sectors, I, II and III, in the phase diagram. Given some initial capital endowment, $a_0 > 0$, then, graphically spoken, the trajectory can start in one of the three sectors. Only in sectors I and II there is consumption growth. In sector III consumption never grows. Even if a path, on which wealth infinitely grows while $\dot{c}(t) = 0$, exists in sector III, then at some point in time the marginal utility of wealth will fall below the marginal utility of consumption such that the individual's well-being can be improved by not following this path but increasing consumption by spending the accumulated wealth. If in sector I exists a path on which consumption increases infinitely, while wealth keeps decreasing, then this path cannot be sustainable in the sense that a no-Ponzi game condition will be violated. In sector II, however, there is the possibility that a balanced growth path exists, on which both, consumption and wealth grow at the same constant rate. We will analyse the latter case in detail in the following section.

2.3.4 Infinte Growth And The Lower Boundary Condition

We have seen that infinite growth as in the standard optimal saving problem is possible, if $r > \rho$. This result is not very exciting, we therefore focus here on the more interesting case $r < \rho$.

During the phase diagram analysis we explained that in the case $r < \rho$ it is possible that no stationary point exists and that there can be a balanced growth path. In this section we state the following proposition.

Proposition 3. *If there is a status effect of wealth, such that an individual gains utility through the possession of wealth, then the lower boundary condition $r > \rho$ is not a necessary condition for the existence of a balanced growth path, along which both, consumption and wealth grow with the same positive constant rate. A necessary condition for the existence of a balanced growth path is*

$$\phi > -[r - \rho]. \tag{2.30}$$

Moreover, the balanced growth path exists only in the long run, i.e. for $\lim_{t\to\infty}$. In the short run, consumption and wealth do not grow with a constant rate.

Proof. Suppose that $r < \rho$ and consumption grows at rate

$$g^S = \frac{\dot{c(t)}}{c(t)} = \frac{r-\rho}{\sigma} + \frac{\phi \left[\frac{c(t)}{a(t)}\right]^\sigma}{\sigma}. \tag{2.25}$$

If a balanced growth path exists, then wealth must grow with the same rate as consumption. We denote the growth rate of wealth as h^S. Using the budget constraint we can write

$$h^S \equiv \frac{\dot{a(t)}}{a(t)} = r + \frac{w}{a(t)} - \frac{c(t)}{a(t)}. \tag{2.31}$$

It follows that the balanced growth path is given by

$$c(t) = (r - h^S)a(t) + w. \tag{2.32}$$

If consumption and wealth grow with the same rate, then $c(t)/a(t)$ is a constant, which we define as χ. From (2.31) then follows

$$h^S = r + \frac{w}{a(t)} - \chi. \tag{2.33}$$

We require $g^S \overset{!}{=} h^S$ and obtain

$$\frac{r-\rho}{\sigma} + \frac{\phi\chi^\sigma}{\sigma} = r + \frac{w}{a(t)} - \chi. \tag{2.34}$$

Defining the LHS of the equation as $L(\chi)$ and the RHS as $R(\chi)$ we see that $L(\chi)$ is a monotonously increasing function of χ while $R(\chi)$ is a linear and decreasing function of χ with a positive intercept, while the intercept of $R(\chi)$ depends on $a(t)$. Thus, a necessary condition for the existence of an intersection point, χ, is $(r + \phi\chi^\sigma - \rho)/\sigma > 0 \;\forall\; \chi > 0$. It follows that $r - \rho + \phi > 0$ and finally

$$\phi > -(r - \rho). \tag{2.35}$$

The latter condition must hold such that a balanced growth path can exist. The condition requires that the status effect of wealth is sufficiently large.

We define the auxiliary function

$$z(\chi) \equiv \frac{r-\rho}{\sigma} + \frac{\phi\chi^\sigma}{\sigma} - r - \frac{w}{a(t)} + \chi, \tag{2.36}$$

which is only defined for $\chi > 0$. Thus we can always identify a unique positive root.

It is immediately clear that the root is a function of $a(t)$. However, ad infinitum $\lim_{t\to\infty} w/a(t) = 0$ and χ is constant. Thus, a balanced growth path along which the endogenous variables grow with the same constant rate, can only exist in the long-run. □

Note that given the infinite growth of consumption and wealth, the objective function $\int_t^\infty e^{-\rho[\tau-t]} w(c(\tau), a(\tau))\, d\tau$ is finite only if the boundedness condition $(1-\sigma)g^S < \rho$ holds.[4]

2.3.5 The Asymptotic Analytical Solution

In Section 2.3.2 we saw that it might be difficult or even impossible to derive a general analytical solution for the status effect of wealth problem as the underlying system of differential equations is non-linear. However, under the balanced growth assumptions, $c(t)/a(t)$ is a positive constant. Making use of this assumption we obtain a linear two-dimensional system of differential equations.

To derive an asymptotic solution, we can easily show that and equals $c(t)/a(t) = r - g^S$ by applying the condition $\lim_{t\to\infty} w/a(t) = 0$ on (2.31). [5] The linear system

[4] The integral

$$U(t) = (1-\sigma)^{-1}\left[c(t)^{1-\sigma}\int_t^\infty e^{\left((1-\sigma)g^S-\rho\right)[\tau-t]}d\tau + \frac{1}{\rho} + a(t)^{1-\sigma}\int_t^\infty e^{\left((1-\sigma)g^S-\rho\right)[\tau-t]}d\tau + \frac{1}{\rho}\right]$$

is bounded only, if $(1-\sigma)g^S < \rho$ holds.

[5] Note that it directly follows that under this assumption the balanced growth path is not affine linear anymore. It is given by

$$c(t) = (r - h^S)a(t). \tag{2.37}$$

Requiring that $g^S \overset{!}{=} h^S$ has to hold and plugging $c(t)/a(t) = (r - g^S)$ into (2.25) gives

$$g^S = \frac{r-\rho}{\sigma} + \frac{\phi\left[r - g^S\right]^\sigma}{\sigma}. \tag{2.38}$$

Defining the LHS of the equation as $L(g^S)$ and the RHS as $R(g^S)$ we see that $L(g^S)$ is positive $\forall\, g^S > 0$, while $R(g^S)$ is a monotonously decreasing function of g^S. Thus, a necessary condition for the existence of an intersection point, g^S, is $(r + \phi[r - g^S]^\sigma - \rho)/\sigma > 0\ \forall\, g > 0$. It follows that $r - \rho + \phi r^\sigma > 0$ and finally

$$\phi > -(r-\rho)r^{-\sigma}. \tag{2.39}$$

Rearranging condition (2.39) gives us $r > (\rho - r)/\phi$, which is a condition that nicely supports the graphical illustration following Figure 2.2. The condition requires that the slope of the zero-motion line of wealth is greater than the slope of the zero-motion line of consumption, such that they do not intersect in the positive range. Parallelism of the zero-motion lines is not a sufficient condition for the existence of a balanced growth path, as it implies a growth reads

$$\dot{a}(t) = ra(t) + w - c(t), \tag{2.16b}$$

given the initial endowment of capital $a(0) = a_0 \geq 0$ and

$$\dot{c(t)} = c(t)\left[\frac{r-\rho}{\sigma} + \frac{\phi\left[r - g^S\right]^\sigma}{\sigma}\right], \tag{2.25}$$

which shows us, in addition to the usual dependence on time preference rate and interest rate that consumption growth is higher, the higher the strength of an individual's status effect is. This reflects an individual's willingness to forego consumption in favour of status.

We can solve this dynamic system analytically.[6] The solution reads

$$c(t) = c_0 e^{\left[\frac{r-\rho}{\sigma} + \frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right]t}. \tag{2.40}$$

and

$$\begin{aligned} a(t) &= a_0 e^{rt} + \frac{1}{r} w \left[1 - e^{rt}\right] \\ &- \frac{\sigma}{\rho - [1-\sigma]\, r - \phi\left[r - g^S\right]} C \left[e^{\left[\frac{r-\rho}{\sigma} + \frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right]t} - e^{rt}\right]. \end{aligned} \tag{2.41}$$

We see that consumption grows exponentially at rate $(r-\rho)/\sigma + \phi\left[r - g^S\right]^\sigma/\sigma$. If $r > \rho$, the growth rate exceeds the growth rate in absence of a status effect, as long as $0 < g^S < r$. However, the more interesting parameter constellation is $r < \rho$. The solution shows that consumption can grow over time, even if $r < \rho$, i.e. the lower boundary condition is not binding anymore. In presence of a status effect an individual gains an additional instantaneous utility flow from the possession of wealth which gives her an incentive to optimally accumulate savings even if her subjective discount rate exceeds the market discount rate.

But is this solution economically sustainable? To answer this, we check, if the following no-Ponzi game condition holds

$$\lim_{T\to\infty} e^{-rT} a(T) = 0. \tag{2.42}$$

rate of $g^S = 0$, which directly follows from (2.38).

[6] A formal derivation is provided in Appendix 2.B.

As we know that wealth grows at rate g^S, we can write

$$\lim_{T\to\infty} a_0 e^{[g^S - r]T} = 0. \tag{2.43}$$

Since g^S is required to be lower than r, the no-Ponzi game condition holds.

2.3.6 The Numerical Illustration

In this section we present our strategy to solve the optimization problem numerically and we also present the numerical results.

The idea is as follows. From the status effect model resulted a KRR, which is an optimality condition that has to hold in the optimum. Additionally, the dynamic budget constraint has to hold. From the analysis above we know that for a given initial value for wealth, a_0, and consumption, c_0, there exists a balanced growth path in the long run in the sector between the two zero-motion lines for wealth and consumption. Mathematically, the dynamics of the model can be described by two differential equations. Given their initial conditions the underlying numerical problem is one of solving an initial value problem that consists of the dynamic budget constraint and the KRR, repeated here for convenience,

$$\dot{a}(t) = ra(t) + w - c(t) \tag{2.16b}$$

$$\dot{c}(t) = c(t)\left[\frac{r-\rho}{\sigma} + \frac{\phi\left[\frac{c(t)}{a(t)}\right]^{\sigma}}{\sigma}\right], \tag{2.25}$$

given the two initial values $a(0) = a_0$ and $c(0) = c_0$ for wealth and consumption.

Using Matlab's built-in solver *'ode45'* we can compute solutions to this initial value problem, which of course depend on the initial values we specify. To find a balanced growth path we assume a relatively high initial value a_0 for wealth, as the term $w/a(t)$ becomes quite small such that the general numerical solution approaches the analytical asymptotic solution. Although not implemented in the program, one can use the a binary search algorithm to make the numerical solution candidate approach the analytical solution. The procedure is as follows. As depicted in Figure 2.3, we fix a_0, and try a value, $c_{01} = \left(a_0\left[(\rho - r)/\phi\right]^{1/\sigma} + ra_0 + w\right)/2$ as the initial value for consumption. If the resulting solution path intersects with the zero-motion line of wealth after a given time span, we try a value $c_{02} = \left(a_0\left[(\rho - r)/\phi\right]^{1/\sigma} + ra_0 + w\right)/4$ or a value $c_{02} = 3\left(a_0\left[(\rho - r)/\phi\right]^{1/\sigma} + ra_0 + w\right)/4$ if it intersects with the zero-motion line of consumption. Then we solve the problem again and repeat the process. Once a good solution is found, we increase the time span and continue. This entire procedure can be repeated until the desired precision has been reached. Binary search algorithms are simple, efficient and deliver reliable results. Once the algorithm has stopped, we have found the trajectory of the balanced growth path.

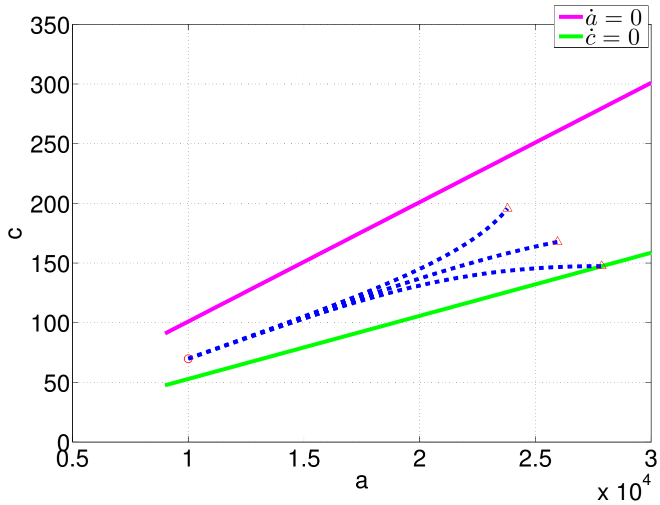

Figure 2.3: Numerical determination of the balanced growth path. Parameters: $r = 0.01$, $\rho = 0.02$, $\sigma = 0.75$, $w = 1.00$, $\phi = -(r-\rho)+0.50$, $a_0 = 10000$, time span $= 300$

As a nice side effect also the optimal consumption and wealth paths, $c^*(t)$ and $a^*(t)$, are computed by the solver. By applying logs we can easily see that consumption and wealth grow with the same constant rate. The latter is illustrated in Figure 2.4. As expected, we observe that there is a small deviation between the general numerical solution, plotted as a green solid line, and the asymptotic solutions (the asymptotic analytical solution is plotted as a red dashed line and the asymptotic numerical solution is plotted as a blue dash-dot line). Note also that, a cross-check of the asymtotic numerical and the asymptotic analytical solution yields a perfect match.

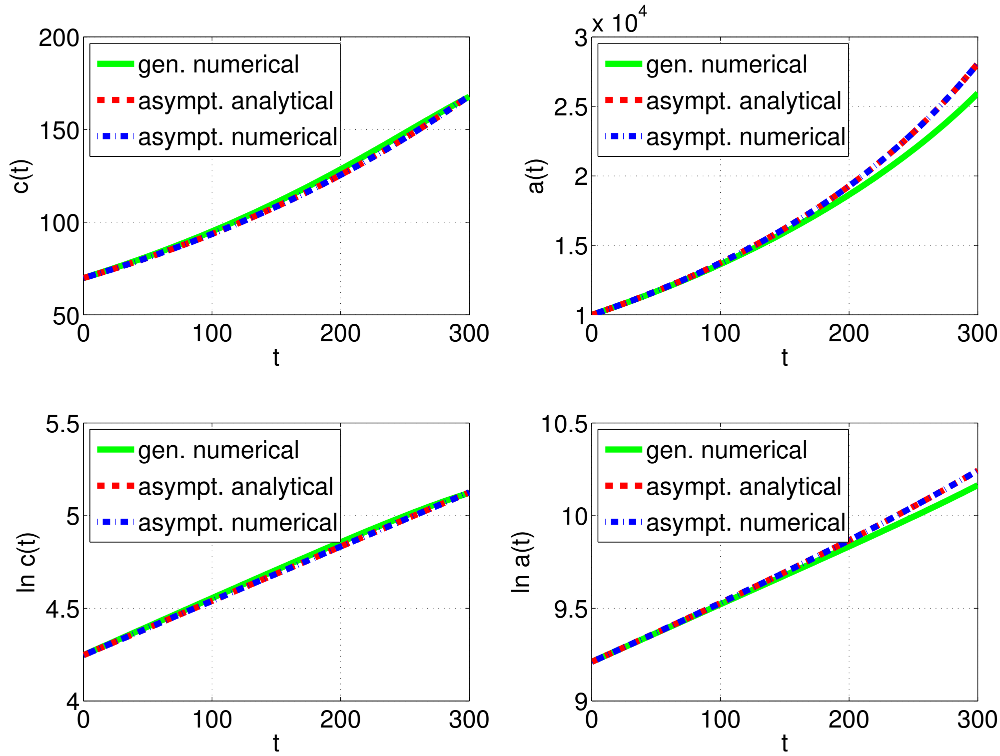

Figure 2.4: Numerical and analytical solution for optimal consumption and wealth paths. Parameters: $r = 0.01$, $\rho = 0.02$, $\sigma = 0.75$, $w = 1.00$, $\phi = -(r - \rho) + 0.50$, $a_0 = 10000$, time span $= 300$

2.4 Time Inconsistency

Time inconsistent behavior can be the result of a state-dependent utility function (Frederick et al., 2002). As time inconsistency influences the behavior of economic agents significantly, it should by no means be neglected. Moreover, the existence of time inconsistency requires additional assumptions about the agent (e.g. recognizing inconsistent behavior or not, existence of commitment devices). Therefore we devote this section to the issue of time inconsistency resulting from models with state-dependent utility functions. The goals are, first, to obtain a deeper understanding of the differences between time inconsistency resulting from state-dependent utility functions and time inconsistency resulting from time-dependent discounting functions. And second, to find out whether time-inconsistency plays a role in our framework or not. Parts of this chaper are based on an earlier work by Krieger (2011) on the link between time inconsistency and emotions.

2.4.1 Discrete Time

This section starts with a discrete time representation, before we consider a continuous time structure further below.

Standard Exponential Discounting

Let the overall utility of an individual at time $t = 0$ be

$$U(c(t), c(t+1), \ldots, c(T)) = \sum_{\tau=t}^{T} D(\tau)u(c(\tau)), \tag{2.44}$$

while consumption in period t is denoted by $c(t)$ and the individual discounts future utility with the following discount function,

$$D(\tau) = \delta^{[\tau-t]} = \left(\frac{1}{1+\rho}\right)^{[\tau-t]}, \tag{2.45}$$

while δ is commonly referred to as the discount factor and ρ as the time-preference rate or the subjective discount rate. The discount function can be seen as the relative weight an individual attaches *in period* t to her future utility in $t+1$, $t+2$, ..., T. In the standard discounted utility model the time-preference rate is assumed to be constant over time. That is, $D(\tau)$ is a function of time, while time measures the distance between the point in time at which the individual maximizes her utility and the point in time at which a future utility is experienced, but discounting between any two consecutive periods, τ and $\tau + 1$, is always the same.

Analytically, the expression for the discount factor between τ and $\tau+1$, denoted by $\varphi_{\tau,\tau+1}$, reads for $\tau = t$

$$\varphi_{t,t+1} = \frac{\partial U(.)/\partial u(c(t+1))}{\partial U(.)/\partial u(c(t))} = \delta \tag{2.46}$$

and is constant as δ is a parameter. Does this imply time consistent behavior? In the standard discounted utility model it implies time consistency (Strotz, 1955). Behavior is time consistent, if the optimal consumption plan chosen by a decision maker in period τ, is not rejected by the same decision maker at some later point in time, $\tau' > \tau$. Strotz (1955) showed that this is the case if discounting is exponential as in the standard discounted utility model.

Quasi-Hyperbolic Discounting

Let us now take a look at a quasi-hyperbolic discount function, as proposed by Laibson (1997). Let the overall utility at $t = 0$ be

$$U(c(t), c(t+1), \ldots, c(T)) = u(c(t)) + \beta \sum_{\tau=t+1}^{T} D(\tau)u(c(\tau)), \tag{2.47}$$

while $0 \leq \beta \leq 1$ is a measure for the strength of the present bias, i.e. the lower β, the less important is the future relative to today.

In this case we can write the discount function as

$$D(\tau) = \begin{cases} 1 & \text{for } \tau = t \\ \beta\delta^{[\tau-t]} & \text{for } t < \tau \leq T \end{cases}. \tag{2.48}$$

As a result, discounting between "today" and "tomorrow" differs from discounting between any other two consecutive periods.

The discount factor between τ and $\tau + 1$ for $\tau = t$ is

$$\varphi_{t,t+1} = \frac{\partial U(.)/\partial u(c(t+1))}{\partial U(.)/\partial u(c(t))} = \beta\delta, \tag{2.49}$$

while the discount factor between τ and $\tau + 1$ for $\tau = t + 1$ is

$$\varphi_{t+1,t+2} = \frac{\partial U(.)/\partial u(c(t+1))}{\partial U(.)/\partial u(c(t))} = \delta. \tag{2.50}$$

Hence, if discounting is quasi-hyperbolic like in Laibson (1997), discounting varies over time. The relative importance of utility across two periods is time dependent. Thus, an individual who has the opportunity to revise her decision made in period t at some later point in time, say at $t + 1$, would optimally reject her original plan in favour of the optimal plan from the perspective $t + 1$.[7] Hence, behavior is time inconsistent.

State-Dependent Utility and Exponential Discounting

Unlike in the previous two sections we assume now that the instantaneous utility is state-dependent. That means, we replace $u(c(\tau))$ by $u(c(\tau), s(\tau))$, while $s(t)$

[7] Without the intention of minimizing the importance of a formal proof, we refrain from providing mathematical evidence that the individuals' optimal plan in t is different from her optimal plan in $t + 1$, as this is already shown elsewhere (e.g. Laibson (1997)) and therefore can be considered as scientifically validated knowledge.

denotes the state (of the world or of the individual) in period t. Then the overall utility at $t = 0$ reads

$$U(c(t), s(t), c(t+1), s(t+1), \ldots, c(T), s(T)) = \sum_{\tau=t}^{T} \delta^{[\tau-t]} u(c(\tau), s(\tau)). \qquad (2.51)$$

We prefer this general notation, because it covers several distinct cases known from the economic literature. For instance, if $s(t)$ increases in past consumption and $u_{cs} > 0$ we can interpret the state, $s(t)$, as a habit (Abel, 1990) or as the stock of addiction (Becker and Murphy, 1988), if $s(t)$ increases in future expected consumption we can interpret it as an anticipatory emotion, like anxiety (Caplin and Leahy, 2001) when $u_s < 0$ or as a joyful anticipation (Loewenstein, 1987) when $u_s > 0$. But $s(t)$ can also be interpreted as a visceral state (Laibson, 2001; Loewenstein, 2000) or a belief (Compte and Postlewaite, 2004) that results from past behavior. Of course $s(t)$ can also be wealth or status, as in the current paper, when $s(t)$ decreases in past consumption and $u_s > 0$.

It is trivial to show that the discount factor between τ and $\tau + 1$ equals δ for all periods. However, when utility is state-dependent this does *not* allow to draw the conclusion that behavior is time consistent as in the first example. The reason is that state-dependent utility constitutes another kind of deviation from the standard discounted utility model. The current deviation consists of a change of the instantaneous utility function, while discounting remains the same as in the standard discounted utility model. Thus, to test if a model with state-dependent utility (and exponential discounting) implies time inconsistency, we must check whether the fundamental definition of time consistent behavior holds, i.e. whether an individual would reject an optimal consumption plan chosen in period τ at some later point in time $\tau' > \tau$. Therefore consider, the individual maximizes her objective function (2.51) subject to

$$s(t) = f(c(t-1)) \qquad (2.52)$$

if the individual gains (dis-)utility from past actions or

$$s(t) = g(c(t+1)) \qquad (2.53)$$

if the individual gains (dis-)utility from future actions and subject to initial or terminal conditions and maybe further constraints such that there exists a trade-off and the optimization problem is well-defined.

If the individual gains (dis-)utility from past actions and optimizes at time t the marginal utility of increasing consumption, and thus the first part of the second

FOC, reads

$$\delta\frac{\partial u(c(t+1),f(c(t)))}{\partial c(t+1)}+\delta^2\frac{\partial u(c(t+2),f(c(t+1)))}{\partial f(c(t+1))}\frac{df(c(t+1))}{dc(t+1)}, \tag{2.54}$$

whereas the second term shows that the individual takes in period t the effect of her period $t+1$ consumption decision on her instantaneous utility in period $t+2$ into account.

For simplicity, let us assume, $c^*(t)$, $c^*(t+1)$, …, $c^*(T)$ denote the individual's optimal consumption decisions from perspective t, i.e. the consumption levels that satisfy the FOCs of the underlying optimization problem and further constraints.

Imagine now, the same individual has the opportunity to revise her consumption plan in period $t+1$. In period $t+1$ the marginal utility of increasing consumption and thus the first part of the first FOC reads

$$\frac{\partial u(c(t+1),f(c^*(t)))}{\partial c(t+1)}+\delta\frac{\partial u(c(t+2),f(c(t+1)))}{\partial f(c(t+1))}\frac{df(c(t+1))}{dc(t+1)}, \tag{2.55}$$

while we tread consumption in period t as given, since it has already been realized. Let $c^{**}(t+1)$, $c^{**}(t+2)$, …, $c^{**}(T)$ denote the individual's optimal consumption plan from perspective $t+1$. Obviously, the only difference between both marginal utilities is that (2.54) is multiplied with δ^{-1} due to the change of perspective. Other than that, they are identical. Hence, also the optimal consumption plan that satisfies the underlying FOCs must be identical to the consumption plan from period t. Analytically, $c^*(t+1)=c^{**}(t+1)$, $c^*(t+2)=c^{**}(t+2)$, …, $c^*(T)=c^{**}(T)$.

We can sum up the first important result as follows. If the departure from the standard discounted utility model consists of an individual gaining (dis-)utility from her past actions, a rational individual's behavior is time consistent.

Now consider the individual gains (dis-)utility from future actions, such that her optimization problem consists of maximizing objective function (2.51), given (2.53), subject to initial or terminal conditions and maybe other constraints.[8]

From perspective t the marginal utility of increasing consumption reads

$$\frac{\partial u(c(t),g(c(t+1)))}{\partial g(c(t+1))}\frac{dg(c(t+1))}{dc(t+1)}+\delta\frac{\partial u(c(t+1),g(c(t+2)))}{\partial c(t+1)}, \tag{2.56}$$

[8] Note that we - for simplicity - assume a deterministic framework here and thus follow the spirit of a paper by Loewenstein (1987) rather than the more recent paper by Caplin and Leahy (2001), who argue that certain anticipatory emotions like anxiety are caused by uncertainty about future outcomes. With this in mind, we should be careful interpreting the (dis-)utility an individual obtains from future actions as anxiety and better imagine it e.g. as a joyful anticipation of future consumption. Nonetheless, with and without uncertainty, the results regarding time inconsistency survive in both frameworks.

The first term shows that the individual takes in period t the effect of her period $t+1$ consumption on her instantaneous utility in period t into consideration.

Again, say, $c^*(t)$, $c^*(t+1)$, ..., $c^*(T)$ constitutes the optimal consumption plan from perspective t.

From perspective $t+1$ the marginal utility of increasing consumption reads

$$\frac{\partial u(c(t+1), g(c(t+2)))}{\partial c(t+1)}. \tag{2.57}$$

To understand why the first term of the marginal utility disappeared we should write down the objective function from perspective $t+1$, given the period t decision has already been made. It reads

$$U(c^*(t), g(c^*(t+1)), c(t+1), g(c(t+2)), \ldots, c(T)). \tag{2.58}$$

What becomes clear now is that we not only have to tread period t consumption as given, but the entire the instantaneous utility in period t, even if parts of it were based on planned future actions which will not be realized.

Let the optimal consumption plan from perspective $t+1$ be $c^{**}(t+1)$, $c^{**}(t+2)$, ..., $c^{**}(T)$. As long as the first term in (2.56) is different from zero (what we can certainly assume) then $c^*(t+1) \neq c^{**}(t+1)$, $c^*(t+2) \neq c^{**}(t+2)$, ..., $c^*(T) \neq c^{**}(T)$ and thus, behavior is time inconsistent.

2.4.2 Continuous Time

In the previous section we assumed a discrete time structure. As the reminder of this paper considers a continuous time structure, the current section transfers the analysis to continuous time and asks whether the results from above will hold.

Standard Exponential Discounting

In continuous time the overall utility at time $t = 0$ reads

$$U(c(t), c(t+1), \ldots, c(T)) = \int_t^T D(\tau) u(c(\tau))\ d\tau, \tag{2.59}$$

while $D(\tau) = e^{-\rho[\tau - t]}$ denotes the discount function for exponential discounting.

Equivalent to the discrete time version, we can show that the discount factor between τ and $\tau+1$, denoted by $\varphi_{\tau,\tau+1}$, for $\tau = t$ reads

$$\varphi_{t,t+1} = \frac{\partial U(.)/\partial u(c(t+1))}{\partial U(.)/\partial u(c(t))} = e^{-\rho} \approx 1 - \rho \tag{2.60}$$

and is constant, provided that the time-preference rate, ρ, is a constant, which also implies time consistency.

Quasi-Hyperbolic Discounting

The continuous time equivalent of quasi-hyperbolic discounting in discrete time was formulated by Harris and Laibson (2002).[9] The life-time utility function reads

$$U(c(t), c(t+1), \ldots, c(T)) = \int_t^s D(\tau)u(c(\tau))\, d\tau + \beta \int_s^T D(\tau)u(c(\tau))\, d\tau, \quad (2.61)$$

where $D(\tau)$ denotes the standard discount function for exponential discounting and β is a measure for the strength of the present bias.

We can write the discount function as

$$D(\tau) = \begin{cases} e^{-\rho[\tau-t]} & \text{for } \tau \leq s \\ \beta e^{-\rho[\tau-t]} & \text{for } \tau > s \end{cases}. \quad (2.62)$$

Then the discount factor between $\tau, \tau+1$, denoted by $\varphi_{\tau,\tau+1}$, reads for $\tau = t$

$$\varphi_{t,t+1} = \begin{cases} e^{-\rho} \approx 1-\rho, & \text{for } \tau \leq s \\ \beta e^{-\rho} \approx \beta\left[1-\rho\right], & \text{for } \tau > s \end{cases}, \quad (2.63)$$

which also implies time inconsistent behavior.

State-Dependent Utility and Exponential Discounting

In a continuous time model with instantaneous emotions the overall utility at $t = 0$ is

$$U(c(t), s(t), c(t+1), s(t+1), \ldots, c(T), s(T)) = \int_t^\infty e^{-\rho[\tau-t]}u(c(\tau), s(\tau))\, d\tau. \quad (2.64)$$

Again, we consider (2.52) and (2.53) as two cases for the state variable distinguishing a backward and a forward-looking instantaneous utility function.

If the individual optimizes at a specific point in time $\tau = t$ and $s(t)$ equals (2.52), the marginal utility of increasing consumption at time $t+1$ reads

$$e^{-\rho}\frac{\partial u(c(t+1), f(c(t)))}{\partial c(t+1)} + e^{-2\rho}\frac{\partial u(c(t+2), f(c(t+1)))}{\partial f(c(t+1))}\frac{df(c(t+1))}{dc(t+1)}, \quad (2.65)$$

[9] A more general formulation of non-exponential discounting can be found in Barro (1999).

and takes the effect of increasing consumption at time $t+1$ on the instantaneous utility in $t+2$ into consideration. Assuming that $c^*(t)$, $c^*(t+1)$, ..., $c^*(T)$ denote the individual's optimal consumption decisions from perspective t, we switch perspectives now and imagine the individual has the opportunity to revise her optimal consumption plan at a later point in time, $\tau = t+1$. Then the marginal utility of increasing consumption at time $t+1$ reads

$$\frac{\partial u(c(t+1), f(c^*(t)))}{\partial c(t+1)} + e^{-\rho} \frac{\partial u(c(t+2), f(c(t+1)))}{\partial f(c(t+1))} \frac{df(c(t+1))}{dc(t+1)}, \tag{2.66}$$

treating consumption at time t as given as $c^*(t)$, because it lies in the past. Let the optimal consumption plan from perspective $t+1$ be $c^{**}(t+1)$, $c^{**}(t+2)$, ..., $c^{**}(T)$. Equivalent to the discrete time version, the difference between both marginal utilities only is the multiplication with the discount factor $e^{-\rho}$, which results from the change of perspectives but does not lead to a change of the optimal consumption plan. Thus, $c^*(t+1) = c^{**}(t+1)$, $c^*(t+2) = c^{**}(t+2)$, ..., $c^*(T) = c^{**}(T)$ which shows that behavior is time consistent in continuous time as well, if an individual gains (dis-)utility from her past actions.

Now suppose that $s(t)$ equals (2.53), while everything else remains unchanged compared to above. Then the marginal utility of increasing consumption at time $t+1$ if the individual optimizes at a specific point in time $\tau = t$ reads

$$\frac{\partial u(c(t), g(c(t+1)))}{\partial g(c(t+1))} \frac{dg(c(t+1))}{dc(t+1)} + e^{-\rho} \frac{\partial u(c(t+1), f(c(t+2)))}{\partial c(t+1)}, \tag{2.67}$$

showing in the first term that the individual respects the effect of increasing her consumption level at time $t+1$ on her instantaneous utility at time t.

Like above, let $c^*(t)$, $c^*(t+1)$, ..., $c^*(T)$ denote the optimal consumption plan from perspective t.

The marginal utility of increasing consumption at time $t+1$ from perspective $t+1$ reads

$$\frac{\partial u(c(t+1), f(c(t+2)))}{\partial c(t+1)}, \tag{2.68}$$

taking $c^*(t)$ as well as $g(c^*(t+1))$ as given, as both, past consumption and the (dis-)utility in t which resulted from the planned future actions have been realized already.

Defining $c^{**}(t+1)$, $c^{**}(t+2)$, ..., $c^{**}(T)$ as the optimal consumption plan from perspective $t+1$ we finally obtain the same result as in discrete time. $c^*(t+1) \neq c^{**}(t+1)$, $c^*(t+2) \neq c^{**}(t+2)$, ..., $c^*(T) \neq c^{**}(T)$ and thus, also in continuous time the result of this analysis is that behavior is time inconsistent.

Summing up the results of this section, we can say that time inconsistency can have (at least) two different sources. First, a time-dependent discount factor, and second, a forward-looking state-dependent utility function even if discounting is exponential. Time inconsistency does not appear if the instantaneous utility is history-dependent, i.e. backward-looking. Caplin and Leahy provide in the context of anticipatory emotions a quite intuitive explanation for this phenomenon. They explain that "as time passes, so do anticipatory emotions, and preferences may change as a result", (Caplin and Leahy, 2001, p.55). A more general explanation for this phenomenon is simply the fact that time moves only in one direction. That is, if instantaneous utility is history-dependent, past decisions or states must be taken as given by construction and it does not make a difference whether we look at past decisions or states from a distance or close up (provided discounting is exponential), whereas forward-looking elements in a utility function are "caught up by the advance in time" and may disappear or remain unchangeable when we shift perspectives. As a result these elements do not play a role in our decision process any longer which may lead to a rejection of our original decision. Both, a present bias as well as a forward-looking utility lead to time inconsistent behavior, provided individuals are "naive", i.e. they do not recognize their time inconsistency and / or do not have the opportunity to invest in commitment devices. Not only the reasons for both kinds of time inconsistency are quite different, they can also lead to an opposite behavioral effect. If the reason for time inconsistency lies in the individual's present-biased time preference structure, individuals tend to repeatedly postpone unpleasant things to a later point in time (e.g. to quit smoking). Time inconsistency due to joyful anticipation, however, may result in a repeated delay of pleasant things (e.g. saving the best clothes "for good").[10] Finally, to answer the introductory question of this section, rational individuals who are subject to a status effect of wealth will not display time inconsistent behavior.

2.5 Conclusion

In this paper we demonstrated how maximization problems with state-dependent utility functions can be analyzed analytically and numerically. Given the widespread use of state-dependent utility functions in state of the art economic research, our goal was to develop a general understanding for the peculiarities maximization problems with state-dependent utility functions come along with. As a point of departure we started with a household saving problem and added a status effect of wealth to this standard framework, such that utility becomes state-dependent.

[10] Although not topic-related, another highly interesting question in this context is whether a present bias can compensate the time inconsistency resulting from a joyful anticipation and thus can be welfare enhancing.

Using analytical methods to solve the optimization problem requires setting up the model carefully as state-dependent utility functions create an additional trade-off and the additive separability of the utility function is (often) relaxed. Since optimization problems are sometimes difficult or impossible to solve analytically, this paper offers besides the analytical solution and related proofs a nice numerical contribution. We derive a numerical solution for the status effect model by solving an initial value problem using Matlab. Here the importance of setting up the maximization problem carefully, i.e. especially with sufficiently many initial conditions, became very clear. We also demonstrated how to use a binary search algorithm to find an appropriate initial condition which ensures that the individual gets on a balanced growth path.

Moreover, this papers covers the issue of time inconsistency in models with state-dependent utility. We consider this as very important, as time inconsistency influences behavior significantly and requires also additional assumptions regarding individuals, like whether they are "naive" or "sophisticated" or about the existence of commitment devises. We examined the issue in discrete and in continuous time. Generally, inconsistent behavior can have two different sources. Either a time-dependent discount factor, as for instance in hyperbolic discounting models or – for our context the more relevant case – individuals' instantaneous utility function is state-dependent. History-dependent utility functions (like in the status effect model) do not imply time inconsistency, because the key element for time-inconsistency due to a state-dependent utility function is a forward-looking instantaneous utility through which the marginal utility of increasing the control variable changes, if the individual optimizes at different points in time.

Appendix

This appendix contains the derivations of explicit solutions of the two-dimensional systems of differential equations resulting from the underlying optimization problems.

2.A Deriving An Explicit Solution For The Standard Version

The two-dimensional dynamic system, (2.1b) and (2.5), repeated here for convenience, reads for a CRRA utility

$$\dot{a}(t) = ra(t) + w - c(t), \tag{2.1b}$$

given $a(0) = a_0$ and

$$\dot{c(t)} = c(t)\frac{r-\rho}{\sigma}. \tag{2.5}$$

We can rewrite (2.5) as

$$\frac{\dot{c(t)}}{c(t)} = \frac{r-\rho}{\sigma}. \tag{A.1}$$

Through logarithmic integration we obtain

$$\ln c(t) = \frac{r-\rho}{\sigma}t + \hat{c}. \tag{A.2}$$

Solving for $c(t)$ gives

$$c(t) = e^{\frac{r-\rho}{\sigma}t+\hat{c}} \Leftrightarrow c(t) = e^{\hat{c}}e^{\frac{r-\rho}{\sigma}t}. \tag{A.3}$$

Defining the integration constant $C = e^{\hat{c}}$ we can write

$$c(t) = Ce^{\frac{r-\rho}{\sigma}t}. \tag{A.4}$$

Next, we bring the differential equation (2.1b) in a general form, writing it as

$$\dot{a(t)} = ra(t) + b(t), \tag{A.5}$$

while $b(t) = w - c(t)$. Now it is easier to see that (2.1b) is an autonomous inhomogeneous linear differential equation with time varying coefficients. Since the initial value a_0 is given, we apply the backward solution method stated in (Wälde, 2012, p.95). The solution reads

$$a(t) = a_0 e^{\int_0^t r d\tau} + \int_0^t e^{\int_\tau^t r du} \left[w - c(\tau)\right] d\tau. \tag{A.6}$$

Rearranging yields

$$a(t) = a_0 e^{rt} + \int_0^t e^{r[t-\tau]} \left[w - c(\tau)\right] d\tau$$
$$\Leftrightarrow a(t) = a_0 e^{rt} + \int_0^t e^{r[t-\tau]} \left[w - C e^{\frac{r-\rho}{\sigma}\tau}\right] d\tau, \tag{A.7}$$

while we plugged in (A.4) to obtain the last equality. Defining $\dot{f}(\tau) \equiv e^{r[t-\tau]}$ and $g(\tau) \equiv w - C e^{\frac{r-\rho}{\sigma}\tau}$ we can apply the integration by parts rule[11] to solve the integral in the equation above. Following this rule we can write the integral as

$$\int_0^t \dot{f}(\tau) g(\tau) d\tau = \left[f(\tau)g(\tau)\right]_0^t - \int_0^t f(\tau)\dot{g}(\tau) d\tau \tag{A.8}$$

[11] see e.g. (Wälde, 2012, p.94).

By plugging in $f(\tau) = -1/re^{r[t-\tau]}$ and $\dot{g}(\tau) = -C\frac{r-\rho}{\sigma}e^{\frac{r-\rho}{\sigma}\tau}$ we obtain

$$
\begin{aligned}
&\left[-\frac{1}{r}e^{r[t-\tau]}\left[w - Ce^{\frac{r-\rho}{\sigma}\tau}\right]\right]_0^t - \int_0^t \frac{1}{r}e^{r[t-\tau]}C\frac{r-\rho}{\sigma}e^{\frac{r-\rho}{\sigma}\tau}d\tau \\
\Leftrightarrow &-\frac{1}{r}\left[w - Ce^{\frac{r-\rho}{\sigma}t}\right] + \frac{1}{r}e^{rt}\left[w - C\right] - \int_0^t \frac{1}{r}e^{r[t-\tau]}C\frac{r-\rho}{\sigma}e^{\frac{r-\rho}{\sigma}\tau}d\tau \\
\Leftrightarrow &-\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{1}{r}C\left[e^{rt} - e^{\frac{r-\rho}{\sigma}t}\right] - \int_0^t \frac{1}{r}e^{r[t-\tau]}C\frac{r-\rho}{\sigma}e^{\frac{r-\rho}{\sigma}\tau}d\tau \\
\Leftrightarrow &-\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{1}{r}C\left[e^{rt} - e^{\frac{r-\rho}{\sigma}t}\right] - \frac{1}{r}C\frac{r-\rho}{\sigma}e^{rt}\int_0^t e^{\left[\frac{r-\rho}{\sigma}-r\right]\tau}d\tau \\
\Leftrightarrow &-\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{1}{r}C\left[e^{rt} - e^{\frac{r-\rho}{\sigma}t}\right] - \frac{1}{r}C\frac{r-\rho}{\sigma}e^{rt}\frac{1}{\frac{r-\rho}{\sigma}-r}\left[e^{\left[\frac{r-\rho}{\sigma}-r\right]\tau}\right]_0^t \\
\Leftrightarrow &-\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{1}{r}C\left[e^{rt} - e^{\frac{r-\rho}{\sigma}t}\right] - \frac{1}{r}C\frac{r-\rho}{\sigma}e^{rt}\frac{1}{\frac{r-\rho}{\sigma}-r}\left[e^{\left[\frac{r-\rho}{\sigma}-r\right]t} - 1\right] \\
\Leftrightarrow &-\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{1}{r}C\left[e^{rt} - e^{\frac{r-\rho}{\sigma}t}\right] + \frac{1}{r}C\frac{r-\rho}{\sigma}\frac{1}{\frac{r-\rho}{\sigma}-r}\left[e^{rt} - e^{\frac{r-\rho}{\sigma}t}\right] \\
\Leftrightarrow &-\frac{1}{r}w\left[1 - e^{rt}\right] - \left[1 - \frac{r-\rho}{\sigma}\frac{1}{\frac{r-\rho}{\sigma}-r}\right]\frac{1}{r}C\left[e^{rt} - e^{\frac{r-\rho}{\sigma}t}\right] \\
\Leftrightarrow &-\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{r\sigma}{\rho - [1-\sigma]\,r}\frac{1}{r}C\left[e^{rt} - e^{\frac{r-\rho}{\sigma}t}\right] \\
\Leftrightarrow &-\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{\sigma}{\rho - [1-\sigma]\,r}C\left[e^{rt} - e^{\frac{r-\rho}{\sigma}t}\right] \\
\Leftrightarrow &-\frac{1}{r}w\left[1 - e^{rt}\right] + \frac{\sigma}{\rho - [1-\sigma]\,r}C\left[e^{\frac{r-\rho}{\sigma}t} - e^{rt}\right]
\end{aligned}
$$

And thus, the solution for $a(t)$ reads

$$
a(t) = a_0e^{rt} - \frac{1}{r}w\left[1 - e^{rt}\right] + \frac{\sigma}{\rho - [1-\sigma]\,r}C\left[e^{\frac{r-\rho}{\sigma}t} - e^{rt}\right]. \tag{A.9}
$$

2.B Deriving An Explicit Solution For The Status Effect Model

The two-dimensional system of differential equations reads

$$
\dot{a}(t) = ra(t) + w - c(t), \tag{2.16b}
$$

given the initial endowment of capital $a(0) = a_0 \geq 0$ and

$$\dot{c(t)} = c(t)\left[\frac{r-\rho}{\sigma} + \frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right], \tag{2.25}$$

We rewrite (2.25) as

$$\frac{\dot{c(t)}}{c(t)} = \left[\frac{r-\rho}{\sigma} + \frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right]. \tag{A.10}$$

Via logarithmic integration we obtain

$$\ln c(t) = \left[\frac{r-\rho}{\sigma} + \frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right] t + \hat{c}. \tag{A.11}$$

Solving this for $c(t)$ gives

$$c(t) = e^{\left[\frac{r-\rho}{\sigma} + \frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right] t+\hat{c}} \Leftrightarrow c(t) = e^{\hat{c}} e^{\left[\frac{r-\rho}{\sigma} + \frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right] t}. \tag{A.12}$$

Defining the integration constant $C = e^{\hat{c}}$ we can write

$$c(t) = Ce^{\left[\frac{r-\rho}{\sigma} + \frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right] t}. \tag{A.13}$$

As (2.16b) equals (2.1b), the solution approach for $a(t)$ remains the same such that we can start directly with the pendant to equation (A.7). In presence of a status effect it reads

$$\Leftrightarrow a(t) = a_0 e^{rt} + \int_0^t e^{r[t-\tau]} \left[w - Ce^{\left[\frac{r-\rho}{\sigma} + \frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right]\tau}\right] d\tau, \tag{A.14}$$

where we used (A.13) to obtain this equality. Again, we define $\dot{f}(\tau) \equiv e^{r[t-\tau]}$ and $g(\tau) \equiv w - Ce^{\left[\frac{r-\rho}{\sigma} + \frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right]\tau}$ and apply the integration by parts rule to solve the integral in the equation above. We can write the integral as

$$\int_0^t \dot{f}(\tau) g(\tau) d\tau = [f(\tau)g(\tau)]_0^t - \int_0^t f(\tau)\dot{g}(\tau) d\tau \tag{A.15}$$

With $f(\tau) = -1/re^{r[t-\tau]}$ and $\dot{g}(\tau) = -C\left[\frac{r-\rho}{\sigma} + \frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right] e^{\left[\frac{r-\rho}{\sigma}+\frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right]\tau}$. For a better readability we replace the expression $\left[\frac{r-\rho}{\sigma} + \frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right]$ by Ψ. We obtain

$$\begin{aligned}
&\left[-\frac{1}{r}e^{r[t-\tau]}\left[w - Ce^{\Psi\tau}\right]\right]_0^t - \int_0^t \frac{1}{r}e^{r[t-\tau]}C\Psi e^{\Psi\tau}d\tau \\
\Leftrightarrow & -\frac{1}{r}\left[w - Ce^{\Psi t}\right] + \frac{1}{r}e^{rt}\left[w - C\right] - \int_0^t \frac{1}{r}e^{r[t-\tau]}C\Psi e^{\Psi\tau}d\tau \\
\Leftrightarrow & -\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{1}{r}C\left[e^{rt} - e^{\Psi t}\right] - \int_0^t \frac{1}{r}e^{r[t-\tau]}C\Psi e^{\Psi\tau}d\tau \\
\Leftrightarrow & -\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{1}{r}C\left[e^{rt} - e^{\Psi t}\right] - \frac{1}{r}C\Psi e^{rt}\int_0^t e^{[\Psi-r]\tau}d\tau \\
\Leftrightarrow & -\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{1}{r}C\left[e^{rt} - e^{\Psi t}\right] - \frac{1}{r}C\Psi e^{rt}\frac{1}{\Psi - r}\left[e^{[\Psi-r]\tau}\right]_0^t \\
\Leftrightarrow & -\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{1}{r}C\left[e^{rt} - e^{\Psi t}\right] - \frac{1}{r}C\Psi e^{rt}\frac{1}{\Psi - r}\left[e^{[\Psi-r]t} - 1\right] \\
\Leftrightarrow & -\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{1}{r}C\left[e^{rt} - e^{\Psi t}\right] + \frac{1}{r}C\Psi\frac{1}{\Psi - r}\left[e^{rt} - e^{\Psi t}\right] \\
\Leftrightarrow & -\frac{1}{r}w\left[1 - e^{rt}\right] - \left[1 - \Psi\frac{1}{\Psi - r}\right]\frac{1}{r}C\left[e^{rt} - e^{\Psi t}\right] \\
\Leftrightarrow & -\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{r\sigma}{\rho - [1-\sigma]r - \phi\left[r - g^S\right]^\sigma}\frac{1}{r}C\left[e^{rt} - e^{\Psi t}\right] \\
\Leftrightarrow & -\frac{1}{r}w\left[1 - e^{rt}\right] - \frac{\sigma}{\rho - [1-\sigma]r - \phi\left[r - g^S\right]^\sigma}C\left[e^{rt} - e^{\Psi t}\right] \\
\Leftrightarrow & -\frac{1}{r}w\left[1 - e^{rt}\right] + \frac{\sigma}{\rho - [1-\sigma]r - \phi\left[r - g^S\right]^\sigma}C\left[e^{\Psi t} - e^{rt}\right].
\end{aligned}$$

And thus, the solution for $a(t)$ reads

$$\begin{aligned}
a(t) = {} & a_0e^{rt} - \frac{1}{r}w\left[1 - e^{rt}\right] \\
& + \frac{\sigma}{\rho - [1-\sigma]r - \phi\left[r - g^S\right]^\sigma}C\left[e^{\left[\frac{r-\rho}{\sigma}+\frac{\phi\left[r-g^S\right]^\sigma}{\sigma}\right]t} - e^{rt}\right].
\end{aligned} \tag{A.16}$$

Chapter 3

A Theory Of Mental Health

by Dennis Krieger[1]

3.1 Introduction

[Motivation] Mental ill-health constitutes a serious burden not only on concerned individuals and their social environment, but also on the entire economy. The latest OECD reports on mental health (OECD, 2012, 2014, 2015a) show that a substantial share of the economic costs do not occur within the health sector (like hospitalization or medication), but result from side-effects of mental ill-health, which include in particular unemployment, limited labor market participation and a reduced productivity at work. Bloom et al. (2011, p.27) estimate these so called "indirect costs" of mental illness to be $1,671$ trillion US$ worldwide. This large number is a direct consequence of the high prevalence of mental illness, documented in detail in numerous epidemiological surveys (e.g. Kessler et al. (2008); Wittchen and Jacobi (2005); Wittchen et al. (2011); Special Eurobarometer 345 (2010, p.4,p.12f.) and Special Eurobarometer 248 (2006, p.2,p.8f.)). The OECD states that the "overall prevalence [across different OECD countries] found is very robust" and that on average "in every country 5% of the working-age population have a severe mental disorder and another 15% a moderate mental disorder", OECD (2012, p.20)

[Problem] However, from a theoretical standpoint, far too little is known about the relationship between mental illness and individual behavior on the one hand, and about the channels through which mental health can be improved on the other hand. "The available [empirical] evidence on mental illness and its connection with employment is partial or incomplete, and many important elements are still unknown.", OECD (2012, p.12).

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[Goal] Thus, the aim of the current paper is to investigate the causal relationship between mental illness and its effects on individual behavior. These insights should next be used to figure out how a targeted use of resources to improve mental health and subjective well-being of an economy can be achieved.

[Approach] Rational individuals maximize their life-time utility, taking the future consequences of their current actions into account. At first glance, it may seem absurd to apply the rationality paradigm to explain the behavior of individuals who suffer from mental ill-health, since the common believe is that mental ill-health is incompatible with forward-looking utility maximizing behavior. We claim that individuals affected by mental disorders indeed can act rationally, as long as they recognize their own disorder and as long as they are in full possession of their mental capabilities, i.e. their judgement is clear and unimpaired. This might seem like a very strong assumption, however, for most of the common mental disorders (CMDs) like substance use disorders (e.g. alcohol or cocaine taking to cope with stressful situations), burnout (as a consequence of excessive stress and problems to manage it), anxieties (like public speaking phobia), mood disorders, depression (e.g. missing fulfilment and success in business) agoraphobia or obsessive compulsive disorder individuals are in fact aware of their own mental illness.[2] Thus, we consider the individuals' knowledge of one's own mental state as given.[3] This implies that individuals in our model weight the benefits of their actions against the potential consequences of it, which makes it an economic problem of choice under uncertainty. An economic framework to analyze individuals' behavior helps to obtain a deeper understanding of the causal relationship between mental health and individual behavior and the evolution of mental health on the individual level as well as on the aggregate level. Therefore we consider a stochastic optimal control problem in continuous time, where individuals are faced with the trade-off between engaging in a beneficial but risky activity and mental ill-health. Given the optimal behavior of individuals we then examine the distributional dynamics of mental ill-health of an entire population using Fokker-Planck equations. We solve a system of Fokker-Planck equations numerically and derive a stationary distribution of mental ill-health which is calibrated to fit the robust empirical distribution found in numerous epidemiological studies. In different simulations we analyze the impact of a change in mental health care quality and of additional educational work on the distribution of mental health states of an entire economy and on their welfare.

[Results/Findings] We find, first that risk-averse individuals display optimal

[2] Also the Diagnostic and Statistical Manual of Mental Disorders explicitly mentions that individuals often recognize their own mental disorder, for instance specific phobia and obsessive-compulsive disorder (American Psychiatric Association (2013, p.199 and p.680)).

[3] In the "Related Literaure" section further below we refer to other works which also support this view.

avoidance behavior, i.e. individuals optimally limit their engagement in a beneficial but risky activity the worse their mental condition is. Consequently their subjective well-being also decreases with increasing mental ill-health. Risk-seeking individuals, however, compensate the pain of mental ill-health by additional engagement in the beneficial but risky activity at relatively low degrees of mental ill-health, before they start limiting their engagement at higher degrees of mental ill-health. In presence of a general life risk also risk-averse individuals compensate the pain due to their mental condition. Hence, an implication of our model is that behavioral reactions depend on individual and environmental characteristics. The results of the calibration show that a representative individual is risk averse in both, her mental health condition and also in engaging in an activity which is beneficial on the one hand - in the sense that it yields an instantaneous utility - but bears the risk to deteriorating the individual's mental health condition on the other hand. These preferences lead to an optimal avoidance behavior. At relatively high degrees of mental ill-health individuals compensate their suffering from the bad mental health condition by additional engagement in the risky activity and thus partially offset the decline in subjective well-being. Second, an improvement of mental health care (goods and services) creates an incentive for individuals to take higher risks and reduces the above mentioned compensation effect. Thus, subjective well-being increases since individuals can enjoy more instantaneous utility from the engagement in the risky activity. On the aggregate level there are two opposing forces at work. The improvement of mental health care enhances an economy's mental health since individuals recover faster, but, it also deteriorates their mental health via the reduction of the optimal avoidance behavior. In total, however, the positive effect dominates such that the mean subjective well-being increases over time due to the intervention. Third, improving educational work and raising awareness reduces the willingness to engage in a risky activity. This is accompanied by an instantaneous decline in subjective well-being. However, on the aggregate level there is just one positive effect which enhances an economy's mental health condition and thus increases the mean subjective well-being over time.

[Related Literature] Our paper is most closely related to the rational addiction (RA) literature. Famous representatives of this strand of literature are inter alia Becker and Murphy (1988), Orphanides and Zervos (1995), Ferguson (2000), Gruber and Köszegi (2001) and Adda and Cornaglia (2006). From a formal perspective, the basic idea of the mechanism behind these models is - just like in our paper - to relax the additive separability assumption of the utility function. Then today's decisions have a direct impact on future utility. More precise, in the RA literatur today's consumption of an addictive good increases today's instantaneous utility, however, it also reduces future utility, because consumption of an addictive good builds up a stock of addiction which has a negative marginal utility. The counterpart to the consumption of an addictive good in the RA literature is the engagement in the risky activity in our paper, while the stock of addiction can be seen as the analogue to mental ill-health in this paper. As is customary in the mental health literature, we also regard addictions, such as substance abuse disorders, as mental health disorders. Insofar can addiction be considered as a special case in our general framework. One of the main differences between the RA literature and this paper is that we perform our analysis in a stochastic instead of a deterministic framework. Doing this enables us to abstract from the particular problem of addiction and addictive behavior.

This paper is also related to the paper on “Confidence Enhanced Performance” by Compte and Postlewaite (2004). In their model individuals observe their own history of success and failure at a particular task. Over time, they develop a self-image which can either be positive, negative or due to a distorted self-perception, negatively or positively biased. A positive self-image makes an individual feel more self-confident, while a negative one leads her to doubt. Self-perception, which can also create and form emotions, then has an impact on the individual’s performance at a particular task. Thus, an individual, when faced with the choice whether to perform a risky activity or not, will optimally avoid it (partially), when the activity is connected with negative associations (fears, shame, failure, bad experiences), which perfectly supports our point of view. The crucial difference between Compte and Postlewaite (2004) and our approach is that in their model individuals act in the light of subjective beliefs, which are shaped through experience and can also be distorted due to misperception, whereas in our model personal experiences translate directly into future mental states without any individual assessment. Thus, our approach maybe constitutes a special case of Compte and Postlewaite (2004) with subjective probabilities equal to objective probabilities.

Moreover, this paper is related to the theoretical research on stress and coping behavior in the following way. Excessive stress caused by the daily routine or else by radical events disturbs an individual’s well-being significantly. In its most extreme form, stress can lead to a clinical mental disorder, as e.g. post-traumatic stress disorder. In Wälde (2015) reduces stress the instantaneous utility on two ways. First, directly by lowering the instantaneous utility and indirectly via a cognitive load argument, which results in a reduced performance. Stress is modelled as a stochastic state variable, similar to mental ill-heath in our model. Our model is rather designed to investigate the distributional dynamics of mental ill-health while recovery is considered as an exogenous rate. Wälde, however, has endogenized the recovery process. Thus, both papers complement each other very well.

Other than that, our paper is related in a technical sense to the following literature. We follow the basic modelling principles of the literature on instantaneous emotions. Worth mentioning here in particular are Loewenstein (2000); Loewenstein et al. (2003) and Laibson (2001). What all these papers have in common is that they consider instantaneous emotions as a state and that this state influences an individual's instantaneous utility. Another field which makes use of state-dependent utility is the RA literature mentioned above.

Last but not least, we contribute to a rather small but growing strand of economic literature that analyzes mental health disorders and other dysfunctional behaviors within an analytical framework. In addition to the RA literature, the most important representatives of this strand of literature are Yaniv (1998) who analyzes agoraphobia, Dragone (2009); Levy (2002, 2009) anorexia, Levy and Faria (2006) depression, Yaniv (2004) insomnia and Yaniv (2008) obsessive-compulsive disorder. We appear to be the first who provide a general theory of mental ill-health.

Finally, we consider our paper and also most of the above mentioned papers, too, as one that additionally offers an added value for psychological research insofar as it provides a precise formal-analytic foundation of a psychological principle (e.g. avoidance behavior) on the one hand and, on the other hand, opens up the possibility of gaining deeper insights in this field and derive new implications.

[Structure] The paper is structured as follows. In the next section we present a stochastic optimal control problem in continuous time, where individuals are faced with the trade-off between engaging in a beneficial but risky activity and mental ill-health. After setting up the optimization problem, we derive the necessary conditions and solve the model numerically. We present the general results of the model in Section 3.3. The subsequent Section 3.4 uses the results at the individual level to analyze the distributional dynamics of mental health on an aggregate level. Following this, we compute the stationary distribution in Section 3.5. Then we calibrate our model and simulate the effects of two different policy interventions on individual behavior, the distribution of mental health and aggregate welfare within Section 3.6. Finally, we conclude in the last section.

3.2 The Model

In this section we present an individual's optimization problem. Individuals in our model face a trade-off between engaging in a beneficial but risky activity and mental ill-health. Throughout this paper we assume that mental ill-health is a condition that results from the individual's current level of psychological distress. The bandwidth of psychological distress that we have in mind ranges from low levels caused by phenomena that can concern everyone of us from time to time like sorrow or exhaustion to high levels caused by a disorder in the clinical sense.

That is, we explicitly include sub-clinical mental conditions, as is customary in the mental health literature. The more an individual engages in the activity, the higher is her risk to become psychologically distressed. But the activity is not the only source of psychological distress. An individual's condition may also deteriorate by the challenges life throws at her. Difficult or dramatic experiences like an accident, an unforeseen physical illness or the death of a relative are unrelated to the engagement in the risky activity and thus out of the individual's control. Over time the individual accumulates a stock of psychological distress, can be thought of as a durable capital stock, while there is a natural load limit. This stock is reduced exogenously in the course of time through a recovery process, reflecting personal coping strategies as well as medical treatment. The model explains with which intensity an individual optimally engages in a beneficial but risky activity, taking her general life risk as well as the particular risk due to the activity into consideration.

Our model is applicable for explaining the behavior of individuals who suffer under the most common forms of mental ill-health, to which mild and moderate mental disorders (CMDs) belong, but also sub-clinical forms of psychological distress are explicitly included. Very severe forms of mental disorders, which are accompanied by an impairment of the human judgement have to be excluded due to their incompatibility with the rationality principle.

3.2.1 Model Setup

We consider an individual who decides at any moment in time about the intensity, $k(t)$, with which she engages in a risky but beneficial activity. When $k(t) > 0$ the individual gains an instantaneous benefit, however, the downside of engaging in the activity is that it entails certain risks for the individual's mental health. The reader might imagine this activity as e.g. working overtime, abusing a performance increasing substance, or placing oneself, for monetary or non-monetary purposes, intentionally in a stressful or risky situation (like a soldier deploying on operations abroad). When the individual decides to engage in the activity, she will sometimes feel very exhausted, stressed or depressed. Whenever that happens, her stock of psychological distress, $q(t)$, increases. The occurrence of these burdensome experiences follows a Poisson process, $q^A(t)$, with endogenous arrival rate, $\mu^A(k(t), q(t))$, while $d\mu^A(k(t), q(t))/dk(t) > 0$, $d^2\mu^A(k(t), q(t))/dk(t)^2 = 0$. That is, the higher the engagement in the activity, the higher is the instantaneous probability of accumulating additional psychological distress. A second Poisson process, $q^B(t)$, with an exogenous arrival rate, $\mu^B > 0 \; \forall q(t) > q(0)$ and $\mu^B = 0$ otherwise, reflects recovery. The recovery process is stochastic as well as it depends on various random factors as for instance on the individual's form of the day, her willingness to recover or whether a conducted coping strategy or a therapy is adequate or not. Moreover, individuals are subject to an idiosyncratic risk that can be though of as a general life risk of e.g. having an accident or loosing of a loved one. Incidents like the latter also have a negative psychological effect and increase the individual's level of psychological distress. Consequently, the transition law of psychological distress follows the stochastic differential equation (SDE),

$$dq(t) = dq^A(t) - dq^B(t). \tag{3.1}$$

The individual's instantaneous utility,

$$u(k(t), q(t)) = \upsilon(k(t)) - \gamma\psi(q(t)) \tag{3.2}$$

is additively defined as the sum of the "sub-utilities" of the benefit derived from working overtime, $\upsilon(.)$, and the suffering due to psychological distress, $\psi(.)$. $\gamma \geq 0$ is a preference parameter, measuring how much an individual suffers from a given level of psychological distress. The benefit derived from the engagement in the activity is assumed to be concave in its intensity, $k(t)$, while the pain due to psychological distress is convex in the mental state ($\partial\upsilon(k(t))/\partial k(t) > 0$, $\partial^2\upsilon(k(t))/\partial k(t)^2 < 0$, $\partial\psi(q(t))/\partial q(t) > 0$, $\partial^2\psi(q(t))/\partial q(t)^2 > 0$). Note that concavity in $k(t)$ and convexity in $q(t)$ both reflect risk-aversion in $k(t)$ and $q(t)$, respectively.

3.2.2 The Maximization Problem

The individual maximizes the expected life-time utility $E_tU(t)$, discounted at rate $\rho + \lambda^{\text{birth}}$, by choosing the optimal time path of $k(t)$, taking into account the evolution of psychological distress, defined in equation (3.1), given an initial value $q(0) = q_0 \geq 0$ and an additional terminal condition, to be defined further below. Formally,

$$\max_{\{k(\tau)\}_{\tau=t}^{\infty}} E_tU(t) = \max_{\{k(\tau)\}_{\tau=t}^{\infty}} \left\{ E_t \int_t^{\infty} e^{-[\rho+\lambda^{\text{birth}}][\tau-t]} u(k(\tau), q(\tau))\, d\tau \right\}, \tag{3.3}$$

subject to (3.1), given $q(0) = q_0 \geq 0$ and a terminal condition. The latter is necessary to pin down a unique optimal solution[4] and will be specified further below. Following the idea of Yaari (1965), Blanchard (1985) and D'Albis (2007), λ^{birth} is the constant instantaneous probability of "death", such that the individual's planning horizon is $1/\lambda^{\text{birth}}$, which is the expected remaining length of life.[5]

[4]See also Sections 2.2.2 and 2.3.2 for a more extensive discussion.

[5]Later on, in Section 3.4, we will look at cross-sections of an entire population of individuals at different point in times. Therefore we must take births and deaths into account. The simplifying assumption we introduce is, we assume that births and deaths happen simulta-

3.2.3 Optimality Conditions

The Hamilton-Jacobi Bellman equation (HJB) reads[6]

$$\rho V(q) = \max_k \left\{ u(k,q) + \mu^A(k,q) \left[V(q+1) - V(q)\right] + \mu^B \left[V(q-1) - V(q)\right] \right\}. \tag{3.4}$$

We have three components which build the instantaneous utility flow, $\rho V(q)$. The first component is the instantaneous utility of the benefit derived from engaging in the activity and of the pain due to the individual's mental condition. The second component describes the first random event for the individual. The random event, making a burdensome experience, occurs with arrival rate $\mu^A(k,q)$. In this case, the individual suffers the difference between the value of mental state $q+1$ and the value of mental state q. The third component describes the second random event, recovery. Recovery happens exogenously with arrival rate μ^B. Whenever the individual recovers, she enjoys the difference between the value of mental state $q-1$ and the value of mental state q.

The first-order condition (FOC) is

$$\frac{\partial u(k,q)}{\partial k} = -\frac{\partial \mu^A(k,q)}{\partial k} \left[V(q+1) - V(q)\right]. \tag{3.5}$$

The FOC shows us that additional engagement in the activity has advantages as well as disadvantages. On the one hand it leads to a higher instantaneous utility, represented here by the marginal utility $\partial u(k,q)/\partial k$, which is an advantage for the individual. On the other hand additional engagement makes a stochastic decrease of overall well-being, $V(q+1) - V(q)$, more likely. Thus, it is weighted by the increase in the arrival rate due to an increase in the intensity of additional engagement, $\partial \mu^A(k,q)/\partial k$. Optimal behavior implies that the individual chooses the intensity such that its advantages equal its potential disadvantages.

3.2.4 Defining A Terminal Condition

From a mathematical perspective it is possible to pin down a unique optimal solution for an optimization problem if sufficiently many boundary conditions exist (initial or terminal conditions, or a no Ponzi game condition).[7] Although the mere existence of these conditions is mathematically enough for deriving solutions, is

neously. Thus, the size of the population remains constant. As we consider representative agents anyway, it makes no difference, whether one individual in fact dies while another one is born or the same individual enters a "fountain of youth", which sets her current state back to the initial state. However, the latter option is formally much easier to implement.

[6] See Appendix 3.A for the detailed derivation.

[7] Also see Sections 2.2.2, 2.2.3 and 2.3.2 for a more extensive discussion of this issue.

not enough to obtain reasonable results. The conditions must be meaningful and the derived results must be interpreted against the background of the conditions.

Finding a first adequate initial condition for psychological distress is straightforward. We assume that an individual is initially endowed with $q(0) = q_0$, while $q_0 \geq 0$ captures that an individual might be born either without any cognitive preload ($q_0 = 0$) or with cognitive preload ($q_0 > 0$) that ancestors have inherited to their offspring.

However, finding and justifying a second adequate initial or terminal condition for the underlying stochastic optimal control problem is a bit more challenging. Therefore, we devote this section to explaining what the idea behind the terminal condition we will use is.

In a finite horizon optimal saving problem economists usually introduce a borrowing constraint to avoid over-accumulation of debt. Now, suppose for the current optimal control problem there exists - similarly as the borrowing constraint in optimal saving problems - a maximum amount of psychological distress, $q^{\max}$, an individual can endure. Then, in a finite deterministic world we would define a psychological distress constraint like $q(T) = q_T \leq q^{\max}$ This constraint would be sufficient to pin down an unique and optimal solution. However, in a world with uncertainty the value q_T can only be reached in expectation, as $q(t)$ is driven by the stochastic increments of two Poisson processes. Moreover, the individual's planning horizon is infinite, although she knows that her expected remaining length of life is $1/\lambda^{\text{birth}}$. For any $q^{\max}$, we can compute the probability that $q(\tau) \leq q^{\max}$ at τ. The idea of the terminal condition we use is simply that it requires that the individual at her expected end of life is in a mental condition below the threshold value $q^{\max}$. Thus, the probability of $q(T) > q^{\max}$ must equal zero at T, reversely, the probability of $q(T) \leq q^{\max}$ must equal 100%, formally

$$P(q(T) > q^{\max}, T) = 0\% \Leftrightarrow P(q(T) \leq q^{\max}, T) = 100\%. \tag{3.6}$$

We require that the individual *in no case* exceeds the limit degree of $q^{\max}$, because, if she would do so at some point in time, there would be a probability greater than zero (even if most highly improbable) to remain in this state above the threshold until her expected end of life and thus she would violate the terminal condition. This endogenously implies that $k(q^{\max}) = 0$ at any point in time.[8]

[8]Against the backdrop of the empirical application in Section 3.6 we shall understand $q^{\max}$ as a threshold level of psychological distress, which is exogenously given by the nature of human mind rather than a chosen aim in an individual's life. Thus, $k(q^{\max}) = 0$ reflects the consequence of an inevitable result, such as a mental collapse or a compulsory hospitalization, which is initiated as soon as the threshold value is hit.

3.2.5 Definition Of Optimal Behavior

Optimal behavior is defined as follows.

Definition 1 (Optimal behavior)**.** *According to the maximum principle $k^*(t)$ is an optimal time path of the intensity of engaging in a risky but beneficial activity, if the following holds at each instant in time $t \in [0, \infty]$:*[9]
The maximum condition,

$$\max_{\{k(\tau)\}_{\tau=t}^{\infty}} \left\{ E_t \int_t^{\infty} e^{-\left[\rho+\lambda^{birth}\right][\tau-t]} u(k(\tau), q(\tau)) \; d\tau \right\}, \tag{3.7a}$$

the transition equation of psychological distress with its initial condition

$$dq(t) = dq^A(t) - dq^B(t), \quad \textit{given } q(0) = q_0. \tag{3.7b}$$

Additionally, the terminal condition

$$P(q(T) \leq q^{\max}, T) = 100\% \Rightarrow k(q^{\max}) = 0 \tag{3.7c}$$

must hold.

In words, the maximum condition (3.7a) requires that at each instant in time the intensity, $k(t)$, is chosen such that it maximizes the objective function. The transition equation, $dq(t)$, characterizes the evolution of psychological distress, given its initial value. And finally, the terminal condition must hold.

The conditions specified above are necessary and sufficient as long as the objective function is twice differentiable and concave in the control variable, which is fulfilled by assumption.

3.2.6 The Solution

This section presents our approach to solve the individual's optimal control problem numerically. An analytical solution in closed-form can be derived under strong and economically untenable parameter restrictions only. The numerical solution, however, is not subject to any special parameter restrictions besides the usual and economically meaningful ones.

For an explicit solution we specify the following functional forms. Both, the benefit derived from engaging in the activity, $\upsilon(.)$, and the suffering due to psychological distress, $\psi(.)$, are assumed to be power-utility functions of $k(t)$ and $q(t)$,

[9] For details see e.g. Feichtinger and Hartl (1986, p.18f.).

respectively. We specify the instantaneous utility function as

$$u(k(t), q(t)) = k(t)^{\alpha} - \gamma q(t)^{\beta}, \tag{3.8}$$

where $0 < \alpha < 1$, $\beta > 1$ denote utility elasticities.

The endogenous arrival rate of burdensome experiences is assumed to be

$$\mu^A(k(t), q(t)) = \lambda^0(q(t)) + \lambda^A k(t), \tag{3.9}$$

while λ^A is a positive parameter of the arrival rate,

$$\lambda^0(q(t)) = \begin{cases} \lambda^0 > 0 & \forall q_0 \leq q(t) < q^{\max} \\ 0 & \text{otherwise} \end{cases} \tag{3.10}$$

and the parameter λ^0 captures the general life risk of making dramatic experiences (e.g. accidents) independently from engaging in the risky activity. Thus, the arrival rate of burdensome experiences is positive even if $k(t) = 0$, i.e. $\mu^A(0, q(t)) > 0 \; \forall q_0 \leq q(t) < q^{\max}$, it increases linearly in $k(t)$, and it equals zero if $q = q^{\max}$. The latter results from our understanding of this condition as a level of psychological distress, which is exogenously given by the nature of human mind. It has the inevitably consequence that all activities are immediately stopped, which implies $k(q^{\max}) = 0$ but also that the individual is in a condition that is safe in terms of her mental state and allows her to recover.

Given this specification, the HJB and the FOC read[10]

$$\rho V(q) = \max_k \left\{ k^{\alpha} - \gamma q^{\beta} + \mu^A(k, q) \left[V(q+1) - V(q) \right] \right. \\ \left. + \mu^B \left[V(q-1) - V(q) \right] \right\} \tag{3.11}$$

and

$$\alpha k(q)^{\alpha - 1} = -\lambda^A \left[V(q+1) - V(q) \right]. \tag{3.12}$$

The maximized HJBs for each state $q \in [q_0, q^{\max}]$ constitute a non-linear system of n equations in n unknowns, $V(q_0)$, $V(q_0 + 1)$, ..., $V(q^{\max})$), where n equals $q^{\max} - q_0$ if q_0 is a positive integer or $q^{\max} - q_0 + 1$ if $q_0 = 0$. The entire system reads as follows.

For $q = 0$ we have

$$\rho V(0) - k(0)^{\alpha} - \left(\lambda^0(0) + k(0)\lambda^A \right) \left[V(1) - V(0) \right] = 0, \tag{3.13}$$

[10] For a detailed derivation see Appendix 3.B.

with

$$k(0) = \left(-\frac{\lambda^A \left[V(1) - V(0)\right]}{\alpha}\right)^{1/(\alpha-1)}, \tag{3.14}$$

for $0 < q < q^{\max}$

$$\begin{aligned} \rho V(q) - k(q)^\alpha + \gamma q^\beta - \left(\lambda^0(q) + k(q)\lambda^A\right)\left[V(q+1) - V(q)\right] \\ -\lambda^B \left[V(q-1) - V(q)\right] = 0 \end{aligned} \tag{3.15}$$

with

$$k(q) = \left(-\frac{\lambda^A \left[V(q+1) - V(q)\right]}{\alpha}\right)^{1/(\alpha-1)} \tag{3.16}$$

and for $q = q^{\max}$

$$\rho V(q^{\max}) + \gamma q^\beta - \lambda^B \left[V(q^{\max} - 1) - V(q^{\max})\right] = 0 \tag{3.17}$$

with

$$k(q^{\max}) = 0 \tag{3.18}$$

and

$$\lambda^0(q^{\max}) = 0. \tag{3.19}$$

We use Matlab's built-in function *fsolve* to solve the system of equations numerically.[11] The results will be discussed in the following section.

3.3 Results Of The Model

In this section we present and interpret the results of the model and their implications on the individual level. To illustrate the results we use numerical examples in what follows. That is, we start with several different economically meaningful parameter constellations and ask, how and if so, why optimal behavior and subjective well-being change across the different specifications. During our numerical computations, it turned out that an individual's risk attitude towards psychological distress as well as the existence of an idiosyncratic risk crucially shapes an individual's behavior. This gives us four generic cases we should analyze,

a) risk-averse under absence of idiosyncratic risks,

b) risk-averse under presence of idiosyncratic risks,

c) risk-seeking under absence of idiosyncratic risks,

[11] See Appendix 3.C for a detailed description of the numerical approach.

d) risk-seeking under presence of idiosyncratic risks.

Figure 3.1 illustrates optimal behavior in the four different cases.

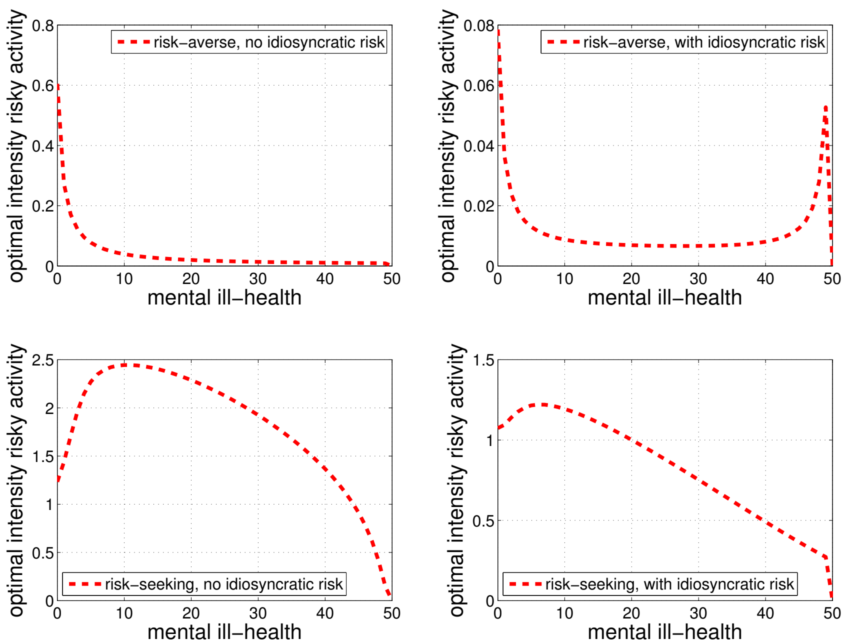

Figure 3.1: Optimal behavior in different situations and for different risk-attitudes as a function of mental state. Parameters: $\rho = 0.020$, $\alpha = 0.200$, $\beta = 1.100$ (risk-averse) / $\beta = 0.100$ (risk-seeking), $\gamma = 0.200$, $\lambda^A = 0.750$, $\lambda^B = 0.750$, $\lambda^0 = 0$ (no idiosyncratic risk), $\lambda^0 = 0.750$ (idiosyncratic risk), $\lambda^{\text{birth}} = 1/78$

The behavior of a risk-averse individual living in a world without idiosyncratic risks is shown in the upper left picture of Figure 3.1. In this environment the risky activity is the only source of mental ill-health. The individual optimally chooses a relatively high intensity of engaging in the risky activity at a perfect mental health level. In consequence, the arrival rate of burdensome experiences will also be relatively high. With increasing mental ill-health the optimal intensity decreases monotonically and therefore also the arrival rate of burdensome situations. Limiting the intensity of engaging in the risky activity is an option for the individual to prevent further harm, even entirely if desired. And the individual makes use of this option in a way, one would intuitively expect.

Suppose now, the same risk-averse individual lives in a world where she is subject to an idiosyncratic risk, i.e. she is exposed to the risk of having an accident, becoming ill and similar. The first thing to be noticed is that the individual in presence of a general life risk chooses a considerably lower engagement in the risky activity for almost all mental states. This reaction is not really surprising. It makes perfect sense to behave more cautious when the environment becomes more dangerous. But, we should keep in mind that less engagement in the risky activity also means less instantaneous utility as the individual can enjoy less benefits from the engagement in the activity. However, not only limiting the engagement in the activity, but also increasing the engagement is plausible, because the individual can also use a higher engagement in the activity to compensate the suffering from her mental ill-health. The latter is what we can observe in the upper right picture in Figure 3.1 at very high degrees of mental ill-health. The crucial difference to the situation in absence of an idiosyncratic risk is that the individual now can only limit but not actively stop her mental deterioration. Hence, if the individual would choose $k(t) = 0$, we know from (3.11) that she will suffer under her current mental ill-health, additionally she is subject to the general life risk and also has a chance to recover. Obviously, from a certain level of mental ill-health she is better off, when she engages more in the activity and uses this to compensate the dis-utility.

Now we consider an individual with risk-seeking preferences, living in a world without idiosyncratic risks. In our numerical example the lower left picture in Figure 3.1 shows that the intensity of engaging in the risky activity decreases non-monotonically in mental ill-health. With risk-seeking preferences the marginal dis-utility of mental ill-health is relatively higher for low degrees of psychological distress and diminishes with increasing psychological distress. Due to that, we can observe an interesting phenomenon in the individual's optimal behavior. Starting at perfect mental health, compensation sets in directly with increasing mental ill-health, before the individual changes course at higher levels of mental ill-health and limits the engagement in the activity, before she ultimately stops at $q = q^{\max}$.

In presence of an idiosyncratic risk, optimal behavior of a risk-seeking individual is qualitatively quite similar to the previous case. Quantitatively, we can observe a lower engagement in the risky activity for all mental states. Yet, the compensation effect remains the same.

In summary, across the four different specifications there are two opposing forces or mechanisms at work. First, avoidance behavior and second a compensation mechanism. In the following we briefly discuss these two issues to emphasize that these are not only technical results of the model, but also phenomena which are observable in the real world.

The first insight we gain from the results is that individuals in our model across all specifications display avoidance reaction, i.e. with an increasing mental deterioration all individuals become avoiders of the beneficial action, however, it depends on the individual's and the environmental characteristics, in which state and to what extent they they avoid. Before we continue, we should briefly explain what avoidance behavior is and how we understand it in the context of our model. Avoidance behavior is essential to the survival of human beings[12] as it aims to protect our physical and mental health. Basically we can distinguish two kinds of avoidance behavior, automatic and controlled avoidance behavior.[13] Automatic avoidance reactions have developed in the course of evolution. Well-known examples are shutting of the eyelids when frightened, pulling back the hand when touching something hot or escaping from (potentially) dangerous animals. Sometimes, these automatic avoidance reactions are combined with a fear reaction. However, instead of a immediate fast and automatic reaction, avoidance behavior can also be the outcome of a well-considered decision. Some of us avoid e.g. walking alone through a dark street at night time as they fear getting assaulted. And often avoidance behavior is only partial, especially when a risky activity is beneficial as well. Then individuals face a trade-off and usually try to reduce the potential risk of a negative outcome by limiting the activity. Imagine, we could take a shortcut through a dark street to be in time for the bus. Maybe some of us would rush through the dark street in the hope that nothing will happen in such a short time, others only take half of the shortcut trying to limit the potential danger this way. Independently from the actual decision, avoidance behavior in the latter examples is the outcome of a well-considered decision.[14] Optimal avoidance behavior in our model implies limiting the risky activity until the marginal benefit and the potential risk is balanced. It results from optimal choice, has a functional value and is welfare maximizing.

Additional engagement in an activity that potentially deteriorates one's mental condition, at first glance appears somewhat surprising. But, it is not uncommon that we can observe such compensation phenomena in reality as well. After the terrorist attacks of November 13th, 2015 in Paris ordinary activities like visiting a concert, a soccer game or going out to a nightclub were assessed as highly risky activities by many. People from all over the world were shocked and traumatized and changed their behavior in consequence of the attacks. While some people began to avoid (partially) certain apparently risky activities like travelling by plane

[12]For completeness sake we should mention that avoidance behavior also appears in animals and other organisms, but here we restrict ourselves to human behavior.

[13]For economic implementations of automatic and controlled individual behavior see e.g. Kahneman (2003); Kahneman and Frederick (2002); Loewenstein and O'Donoghue (2004), for a good over article on dual-process theory in psychology see Evans (2008).

[14]Usually, avoidance behavior results from previous learning, active or passive, as we rely on our experiences when making decisions. In the case of automatic avoidance reactions, which are innate, is a result of evolutionary development. As long as avoidance behavior is partial, learning takes place and ensures an organism's adaptability to its environment. Once, avoidance behavior is consistent it blocks learning and an organism looses the ability to adapt to her environment.

or subway or visiting big events attended by many people, others, however, started "now more than ever" to visit e.g. the soccer matches of their favorite teams and thereby compensated their emotional pain with additional engagement in the activities they enjoy. Of course we cannot exclude at this point that, aside from or in addition to a compensation effect, other factors influenced these decisions, too. Anyway, there is a rational explanation for both of these behavioral patterns that can be observed in reality. The crucial difference between those who make use of a compensation mechanism and those who do not is their attitude towards risk. Thus, an implication we can derive from our model is that different behavioral reactions are a direct consequence of individual and environmental characteristics.

To complete the picture we should also take a look at the corresponding values of optimal behavior, which are drawn for all four cases in Figure 3.2.

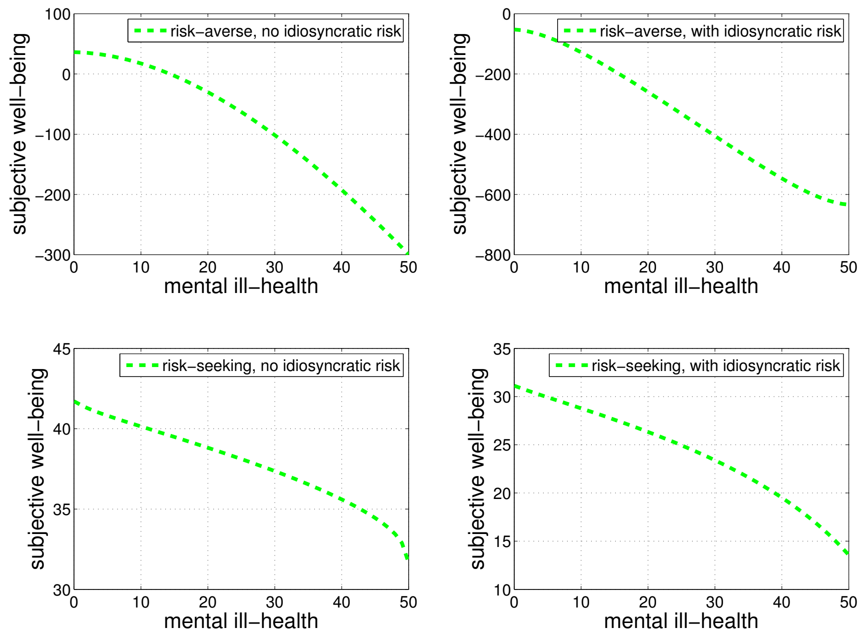

Figure 3.2: Value of optimal behavior in different situations and for different risk-attitudes as a function of mental state. Parameters: $\rho = 0.020$, $\alpha = 0.200$, $\beta = 1.100$ (risk-averse) / $\beta = 0.100$ (risk-seeking), $\gamma = 0.200$, $\lambda^A = 0.750$, $\lambda^B = 0.750$, $\lambda^0 = 0$ (no idiosyncratic risk), $\lambda^0 = 0.750$ (idiosyncratic risk), $\lambda^{\text{birth}} = 1/78$

What all value functions have in common is that they decrease monotonically in mental ill-health. That is, independently of their risk-attitude and of the presence of idiosyncratic risks, individuals in a poorer mental health condition have a lower subjective well-being than individuals in a better mental health condition. The reason for this is twofold. First, the worse their mental condition is, the more suffer individuals from their instantaneous pain due to mental ill-health and second, reducing the engagement in the beneficial but risky activity (what all individuals do at some point), lowers the instantaneous benefit derived from it.

In the case of the risk-averse individual in absence of idiosyncratic risks, the value of optimal behavior decreases with an increasing rate in mental ill-health. It starts at a positive value and becomes negative for poorer mental health conditions. The comparison with the case of a risk-averse individual living in a world with idiosyncratic risks shows that subjective well-being is significantly lower in this more dangerous environment, which is also a result of a lower optimal engagement in the activity. Interestingly, the value of optimal behavior decreases with an decreasing rate as soon as the individual starts compensating. Risk-seeking individuals, however, are better off altogether. Their subjective well-being is positive for all mental states. Here we also can observe that the compensation behavior is visible in the value function. The compensation behavior changes the sign of the second derivative of the value function. If an individual displays compensation behavior, subjective well-being decreases with a decreasing rate in the mental state, if an individual displays avoidance behavior, then subjective well-being decreases with an increasing rate in the mental state.

3.4 Distributional Dynamics Of Mental Health

Consider now, instead of looking at one individual we look at an entire population of N individuals and ask how the mental health states of individuals develop over time.

To answer this question, we first derive a system of Fokker-Planck equations (FPEs) which explains how an initial probability distribution of mental states evolves over time. To obtain a frequency distribution we apply the law of large numbers. By taking births and deaths into consideration we are able to adopt a cross-sectional perspective. Following the idea of Yaari (1965), Blanchard (1985) and D'Albis (2007), an elegant way to do this requires the assumption that births and deaths happen simultaneously. Thus, the size of the population remains constant. As we consider representative agents anyway, it makes no difference, whether one individual in fact dies while another one is born or the same individual enters a "fountain of youth", which sets her current state back to the initial state. However, the latter option is formally much easier to implement as we can describe the occurrence of such an event with a Poisson process, $q^{\text{birth}}(t)$. The idea is as follows. At some random point in time one individual with some $q(\tau)$ passes away, while simultaneously another individual is born with $q(\tau) = q^0$. Since individuals in our model differ only in their mental state, we can simply let the individuals in our population enter an imaginary "fountain of youth", which sets their current mental state back to the initial state. Thus, deaths and births can be thought of as resetting individuals to their "factory defaults". Consequently, we have to define the jump size of the Poisson process for this event as $-q(t) + q_0$, while its arrival rate is denoted by $\mu^{\text{birth}} = \lambda^{\text{birth}}$. Then the evolution of the mental condition of individuals follows

$$dq(t) = dq^A(t) - dq^B(t) - (q(t) + q_0)\, dq^{\text{birth}}(t). \tag{3.20}$$

Given the transition law in (3.20), the arrival rates $\mu^A(k)$, μ^B, μ^{birth}, the initial condition $q_0 = 0$ and optimal behavior, we can derive a system of Fokker-Planck equations,[15] which reads for $i = q_0$,

$$\frac{\partial p(0,\tau)}{\partial \tau} = \mu^B p(1,\tau) - \mu^A(0)p(0,\tau) + \mu^{\text{birth}}\Sigma_{i=1}^{\infty} p(i,\tau), \tag{3.21}$$

for a general $1 \leq i < q^{\max}$

$$\frac{\partial p(i,\tau)}{\partial \tau} = \mu^B p(i+1,\tau) - [\mu^A(i) + \mu^B + \mu^{\text{birth}}]p(i,\tau) + \mu^A(i-1)p(i-1,\tau) \tag{3.22}$$

and for $i = q^{\max}$

$$\frac{\partial p(q^{\max},\tau)}{\partial \tau} = -[\mu^B + \mu^{\text{birth}}]p(q^{\max},\tau) + \mu^A(q^{\max}-1)p(q^{\max}-1,\tau), \tag{3.23}$$

while $p(i,\tau)$ reflects the absolute number of individuals who are in mental state $q = i$ at time $t = \tau$. Note that in equation (3.23) $\mu^A(q^{\max}) = 0$ and $p(q^{\max}+1,\tau) = 0$, as $k(q^{\max}) = 0$.

Each equation in the system explains how the frequency of individuals being in state $q(t) = i$ changes over time. Equation (3.21) demonstrates that the number of individuals being in state $q = 0$ changes for three different reasons. The first term in (3.21) shows that the frequency of being in $q = 0$ increases by the number of individuals who previously where in state $q = 1$ and recovered with instantaneous probability μ^B. The second term in this equation explains that the frequency of being in $q = 0$ decreases by the number of individuals who make a burdensome experience with instantaneous probability $\mu^A(0)$ and in consequence their mental state changes to $q = 1$. The last term of the equation reflects that the frequency of being in $q = 0$ also increases by the number of individuals who previously were in any state $1 \leq i \leq q^{\max}$ but die and the same number of individuals are born

[15] See Appendix 3.D for details on the derivation.

being initially in state $q = 0$ with instantaneous probability μ^{birth}.

From the general equation (3.22) we learn that the frequency of individuals being in state $q(t) = i$ increases by the number of individuals who recover from the next higher state of mental ill-health with instantaneous probability μ^B (first term) or make an additional burdensome experience with instantaneous probability $\mu^A(i-1)$ and previously were in the next lower state $i - 1$ (last term). The frequency of individuals being in state $q(t) = i$ decreases, however, by the number of individuals who are in state i but either recover with instantaneous probability μ^B, make an additional burdensome experience with instantaneous probability $\mu^A(i)$ or die with instantaneous probability μ^{birth} (middle term).

The number of individuals in state $q = q^{\max}$ decreases by the number of individuals who are in state $q^{\max}$ but recover with instantaneous probability μ^B or die with instantaneous probability μ^{birth} (first term) and it increases by the number of individuals who previously were in state $q^{\max} - 1$ and made an additional burdensome experience with instantaneous probability $\mu^A(q^{\max} - 1)$ (second term). The possibility of a change due to recovery from state $q^{\max} + 1$ as well as the possibility of making an additional burdensome experience does not appear here, as $p(q^{\max} + 1, \tau) = 0$ and $k(q^{\max}) = 0$.

In summary, each FPE explains the change of the number of individuals in a certain state, q, by its "net flow", i.e. by the difference between the "inflow" (positive terms) into this state and the "outflow" (negative terms) out of this state. A very nice feature of FPEs is that they are ordinary differential equations, although they were derived from a stochastic process. Taken together, the system of FPEs (3.21), (3.22) and (3.23) describes the time-evolution of the frequency distribution of mental health. Mathematically, it constitutes a system of n differential equations in n unknowns, which we solve for all $p(q, \tau)$, given an initial distribution. Therefore we use Matlab's built-in solver for ordinary differential equations, *ode23*.[16]

3.5 The Stationary Distribution Of Mental Ill-Health

In the long-run all frequencies are time-invariant. By setting the LHS of the system of FPEs equal to zero, we obtain a system of equations describing the stationary distribution of mental health.[17]

For $i = 0$ we have

$$p(0) = \frac{\mu^B}{\mu^A(0) + \mu^{\text{birth}}} p(1) + \frac{\mu^{\text{birth}}}{\mu^A(0) + \mu^{\text{birth}}}, \tag{3.24}$$

[16] A detailed description of the numerical approach is provided in Appendix 3.E.

[17] See Appendix 3.F for details.

for a general $1 \leq i < q^{\max}$ we obtain

$$p(i) = \frac{\mu^B}{\mu^A(i) + \mu^B + \mu^{\text{birth}}} p(i+1) + \frac{\mu^A(i-1)}{\mu^A(i) + \mu^B + \mu^{\text{birth}}} p(i-1), \tag{3.25}$$

and for $i = q^{\max}$ we have

$$p(q^{\max}) = \frac{\mu^A(q^{\max} - 1)}{\mu^B + \mu^{\text{birth}}} p(q^{\max} - 1), \tag{3.26}$$

since $\mu^A(q^{\max}) = 0$ and $p(q^{\max} + 1) = 0$, as $k(q^{\max}) = 0$.

We solve this system of equations using Matlab's built-in function *fsolve*.[18]

3.6 Quantitative Predictions

Now we look at the average distribution of mental health in OECD countries and calibrate our model in order to meet this distribution as an initial equilibrium distribution. Then we use our calibrated model to assess the consequences and effectiveness of two different policy interventions, first an improvement of mental health care and second an improvement of educational work. We ask, how individual behavior and subjective well-being changes as a consequence of the interventions. Moreover we are interested in how the distribution of mental health will change in the course of time and how mental health will be distributed among the population in the new stationary equilibrium. Finally, we measure the overall welfare effect of the policy interventions by quantifying the effect on the probability-weighted aggregated subjective well-being.

3.6.1 Calibration

We want to understand how policy interventions affect an economy's mental health distribution. Therefore we need an initial equilibrium which reflects the real world sufficiently well. We require that the equilibrium distribution of mental ill-health fits the empirical distribution, reported in ch.1.2 in (OECD, 2012). The empirical distribution is illustrated as light blue bars in Figure 3.3.

The distribution is the result of a large-scale meta analysis on the prevalence of mental ill-health conducted by the OECD, covering 21 OECD countries. The results are based on clinical interviews as well as household surveys. Applied was a classical 5-point scale "from fully correct" to "not correct at all" with 9 items. Thus, the scoring ranges from 9 points at the lowest to 45 points at the highest.

[18]For a description of the applied numerical method, see Appendix 3.G.

Defining appropriate cut-off values, 5% of a population suffers a severe mental ill-health and another 15% a moderate mental ill-health. These numbers are very robust across different OECD countries.

The original data show a small bump roughly at the median value. Its occurrence can probably be attributed to the fact that survey participants, if they are uninterested or do not know what to answer, commonly tend to put their cross in the centre of the scale. Therefore our preparation involves the following step. We determined a normal distribution to fit the data, such that we obtain a smooth function as a target. Our target function, the fitted probability distribution function (pdf), $p^{\text{target}}(q)$, is drawn in red in Figure 3.3. We also normalized the state space from 9 to 45 to 0 to 36.

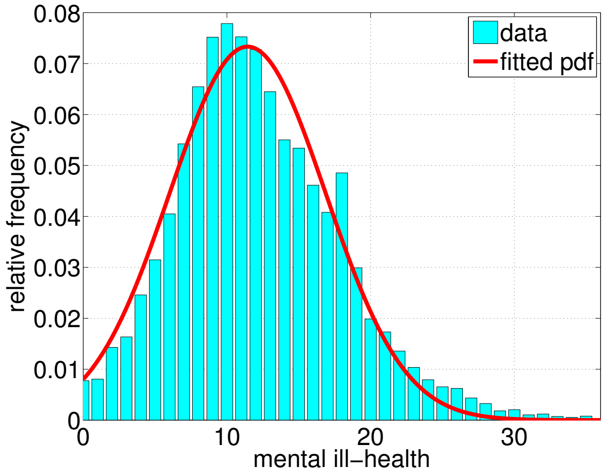

Figure 3.3: Empirical data and fitted probability distribution function. Parameters of the normal distribution: $\mu = 11.45$ and $\sigma = 5.45$.

The parameters to be calibrated are the utility elasticities, $0 < \alpha < 1$ and $0 < \beta$, the preference parameter, $\gamma > 0$, as well as the three arrival rate parameters, $\lambda^A > 0$, $\lambda^B > 0$ and $\lambda^0 > 0$. The following parameters are fixed at meaningful values. The expected length of remaining life at point t is $1/\lambda^{\text{birth}}$ which is set to 78 reflects the average total life expectancy at birth in OECD countries sufficiently well (OECD, 2010). The time preference rate, ρ, is 0.02, such that the planning horizon can be understood as measured in years as the time discount rate is $\beta = 1/[1+\rho] \approx 0.98$.

To find values for the six parameters to be calibrated, we define six targets to be met. Therefore we divided the target pdf into six parts or six sums of equal length.[19] Next, we require that the so defined targets are as equal as possible to the

[19] Since we have 37 states (0 to 36), we had to neglect the probability mass on the last state

respective sums of the stationary distribution of the model. Then we used Matlab's routine *lsqnonlin* to solve this problem numerically. The latter is a non-linear least-squares method and its idea is as follows. If we define x as a vector of n parameters to be calibrated and $f_i(x)$ as the i-th of n residual functions depending on the parameter values, then the routine solves the following minimization problem

$$\min_x \left\{ f_1(x)^2 + f_2(x)^2 + \cdots + f_n(x)^2 \right\}. \tag{3.27}$$

In other words, the routine minimizes the sum of the squared function values, i.e. the sum of the squared residuals, by choosing the parameter values for $\alpha, \beta, \gamma, \lambda^A, \lambda^B, \lambda^0$, while the residual functions are defined as

$$f_1(x) \equiv \sum_{i=0}^{5} p(i) - \sum_{i=0}^{5} p^{\text{target}}(i) \tag{3.28}$$

$$f_2(x) \equiv \sum_{i=6}^{11} p(i) - \sum_{i=6}^{11} p^{\text{target}}(i) \tag{3.29}$$

$$\vdots$$

$$f_6(x) \equiv \sum_{i=30}^{35} p(i) - \sum_{i=30}^{35} p^{\text{target}}(i) \tag{3.30}$$

After the calibration all parameters have meaningful values and are in the range one would expect. The parameters are summarized in Table 3.1. Figure 3.4 il-

Name of parameter	Notation	Value
Utility elasticity of engaging in the risky activity	α	0.720
Utility elasticity of mental ill-health	β	1.700
Preference	γ	0.001
Parameter of the arrival rate of burdensome situations	λ^A	1.510
Parameter of the arrival rate of recovery	λ^B	3.011
Instantaneous probability of life risk events	λ^0	1.080

Table 3.1: Calibrated parameter values

lustrates the quality of the calibration results. Note that the residuals also not normalized, i.e. they are not in relation to the size of the area the corresponding integral reflects. The largest deviation from zero is approximately $1e^{-2}$, which is appropriate, considering that each iteration step of the calibration procedure

$q = 36$. However, this is not dramatic, as the size is close to zero anyway.

contains solving the optimal control problem and computing the stationary distribution numerically, as explained further above.

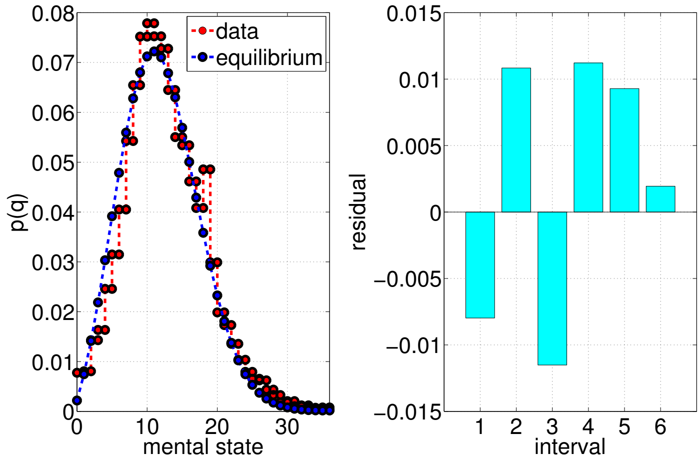

Figure 3.4: Original data and equilibrium distribution as well as residuals

This stationary distribution serves as initial equilibrium distribution and is therefore the starting point for the following policy simulations. In the next two sections we consider, first an improvement of mental health care and second, an improvement of educational work. We ask, what the effects of such policy interventions on individual behavior and on subjective well-being are. Moreover we are interested in how the distribution of mental health will change in the course of time and how mental health will be distributed among the population in the new stationary equilibrium.

3.6.2 Improving Mental Health Care Quality

Imagine, there is a policy intervention which increases mental health care quality by $x\%$. To simulate such an increase within the framework of our model we increase the arrival rate of recovery by the same percentage. Thereby we implicitly assume that an increase in mental health care quality is transferable into an increase of recovery from mental ill-health one to one.[20] For concreteness sake we assume an increase of 20%.

The results of the of the policy intervention on optimal behavior are depicted in Figure 3.5. It shows that individuals optimally increase their engagement in the

[20] Clearly, it is debatable whether the assumption of the one to one transferability of a mental health care quality improvement on recovery is realistic or not, however, for our framework, it is irrelevant where exactly a change of recovery results from.

risky activity in all mental states except of the state $q = q^{\max} - 1$ and $q^{\max}$, while the latter directly follows from the terminal condition. As the environment became safer through the policy intervention, individuals have an incentive to engage more in the risky activity now. For a broad range of mental states individuals increase their engagement by $20 - 30\%$.

As a result subjective well-being increases for all mental states, which is shown in Figure 3.6. The relative size of the increase also amounts to $20 - 30\%$, while the size increases monotonically in the degree of mental ill-health. In other words, in terms of subjective well-being, individuals in the worst mental conditions benefit the most from the policy intervention.

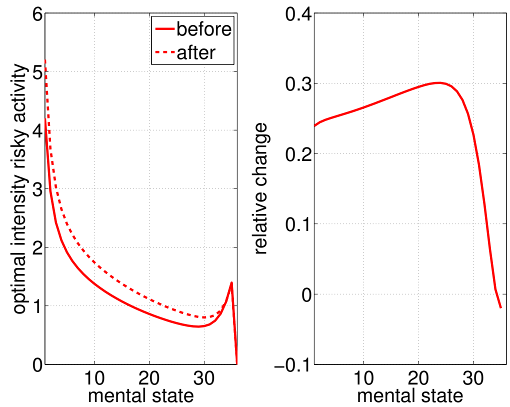

Figure 3.5: Absolute and relative change in optimal behavior after an improvement of mental health care quality by 20%

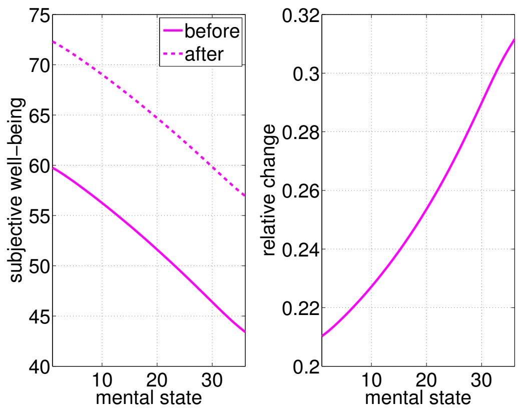

Figure 3.6: Absolute and relative change in subjective well-being after an improvement of mental health care quality by 20%

Figure 3.7 illustrates the long-run distributional effects of an improvement of mental health care quality. It shows that the equilibrium distribution moves slightly to the left over time, such that more individuals with a better mental health condition live in the economy in the new stationary equilibrium.

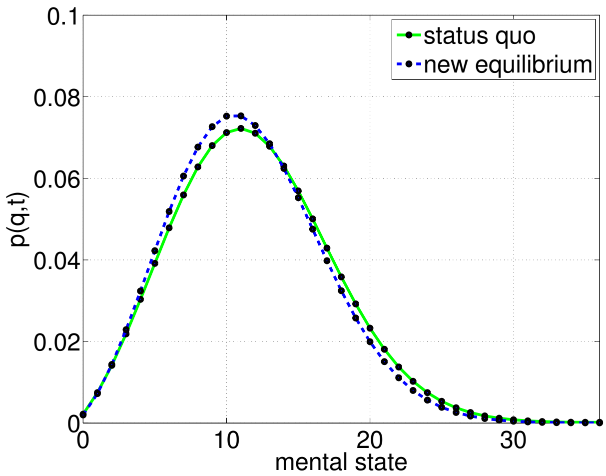

Figure 3.7: Old and new stationary equilibrium distribution after an improvement of mental health care by 20%

Interestingly, the policy intervention gives raise to two opposing effects on the equilibrium distribution. On the one hand, individuals behave riskier and thus increase the instantaneous probability of deteriorating their mental condition. This effect pushes the distribution towards higher levels of mental ill-health. On the other hand, individuals recover faster as the environment became safer (what gave them the incentive to take more risk). The latter pulls the distribution back to the opposite direction. So, what we see in Figure 3.7 is the net effect of these two opposing forces.

3.6.3 Improving Educational Work

Now we consider another policy intervention. We assume that improving educational work regarding the negative consequences of the risky activity on mental health influences an individual's view of life in the sense that she changes her risk attitude towards mental ill-health. We simulate the second policy intervention by an increase of the utility elasticity of mental ill-health, β, by 10%. We illustrate the results on optimal behavior in Figure 3.8. After the intervention individuals optimally reduce their engagement in the risky activity. While individuals in a good mental condition reduce the optimal intensity by $5-10\%$, individuals in a bad mental condition reduce it by up to 70%.

In the previous simulation we found that an improvement of mental health care quality has a positive effect on subjective well-being. So, what is the effect of an improvement of educational work on subjective well-being? We find the answer in Figure 3.9. The current policy intervention has a negative effect on subjective well-being. However, the effect is rather small. Subjective well-being is reduced by $2,5\%$ for low degrees of psychological distress and by up to 10% for the highest degrees.

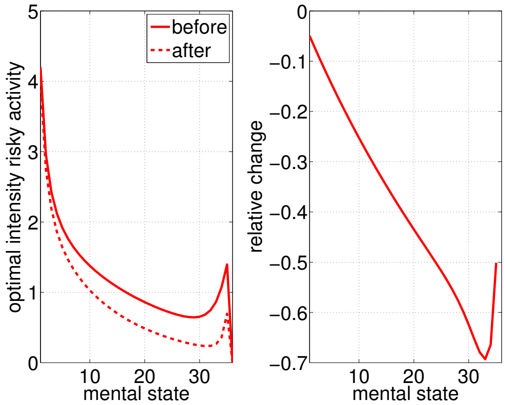

Figure 3.8: Absolute and relative change in optimal behavior after an improvement of educational work 10%

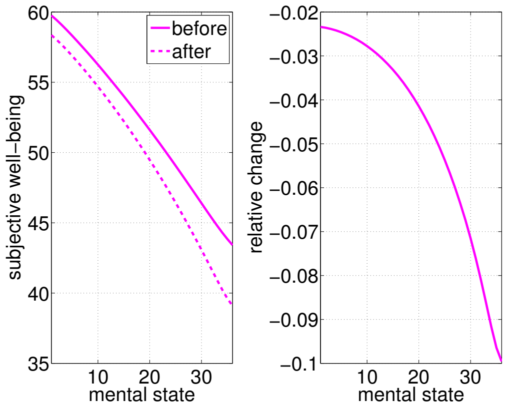

Figure 3.9: Absolute and relative change in subjective well-being after an improvement of educational work 10%

Finally, we are interested in the long-run effects of the intervention. The results, depicted in Figure 3.10 show a significant shift of the stationary distribution towards better mental health states.

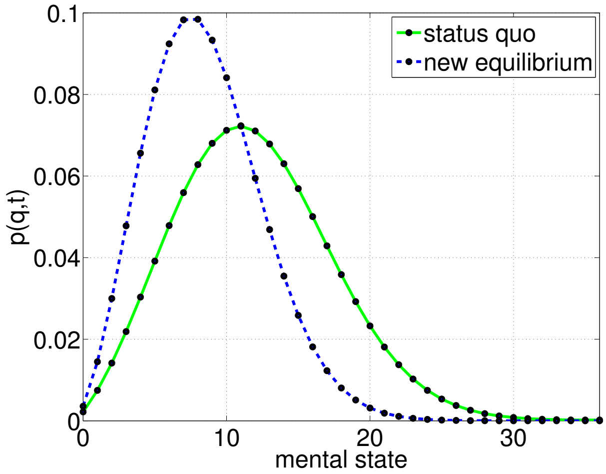

Figure 3.10: Old and new stationary distribution after an improvement of educational work by 10%

The interesting difference between both interventions is that the current intervention does not create an incentive for individuals to behave riskier as in the first simulation. Yet, the opposite is the case now. Individuals optimally engage less in the risky activity. At the same time, the environment remains unchanged. Thus, the shift of the distribution of mental health towards better mental conditions is directly attributable to the change in individuals' behavior.

In summary, we have found that both interventions have opposite effects on the optimal intensity of engaging in the risky activity and on subjective well-being. While an improvement of mental health care quality increases both of them, an improvement of educational work reduces both of them. Interestingly, either intervention affects the stationary distribution qualitatively identically. However, there are substantial differences between the quantitative impact on the long-run distribution.

3.6.4 Welfare Effects Of Policy Interventions

Once we have simulated the two different policy interventions and analyzed their effect on optimal individual behavior, subjective well-being and on the equilibrium distribution of mental health, there still remain two open questions. First, and most important, how and to what extent affect the policy interventions an economy's welfare? Or, in other words, which policy should a benevolent planner adopt? And second, how long does it take to arrive at the new equilibrium? This section is devoted to provide *first* answers to these questions and pave the way for future research on the welfare economics of mental health.

Making a quantitative statement about the extent of the welfare effects, requires several assumptions, on the one hand on the considered population and on the other hand on the applied welfare standard. We define the size of the population, N, we look at as follows. Since we calibrated the model to fit data on mental health of an average OECD country's working age population, N should be equal to the population size of an average OECD country's working age population. Another reason why we consider the working age population only is that the individuals in our model are rational and forward-looking decision makers, who are free and capable to decide to what extent they want to expose themselves to the risk of getting mentally ill. In our opinion, it is questionable whether children and senior citizens are in the position to do that. The size of an average OECD country's working age population in 2012 was $N \approx 23.7$ million people OECD (2016, p.14).

Next, we need an adequate welfare standard. Individuals in our model act fully rational, they do not have distorted beliefs and are entirely aware of all consequences of their actions. Thus, subjective well-being is an appropriate basis. And as mental health is distributed unequally among the population, we should take the distribution of mental health into consideration as well. Thus, we capture the economy's welfare at τ as the probability-weighted sum of subjective well-being, formally defined as

$$W(\tau) = N \sum_{i=0}^{q^{\max}} p(i,\tau)V(i,\tau). \qquad (3.31)$$

Note that both, the probabilities, $p(.)$, and the value function, $V(.)$, react on changes of the policy parameters, λ^B and β, respectively. Therefore the dynamic of the probabilities, described by the system of FPEs is taken into account as well.

Given, the size of the population, N, the calibrated initial equilibrium distribution, the definition of the welfare standard, $W(\tau)$, and the distributional dynamics from (3.21) - (3.23), we ask how the two considered policy interventions affect the economy's welfare.

The effect of an improvement of mental health care quality on the economy's welfare is depicted in Figure 3.11.

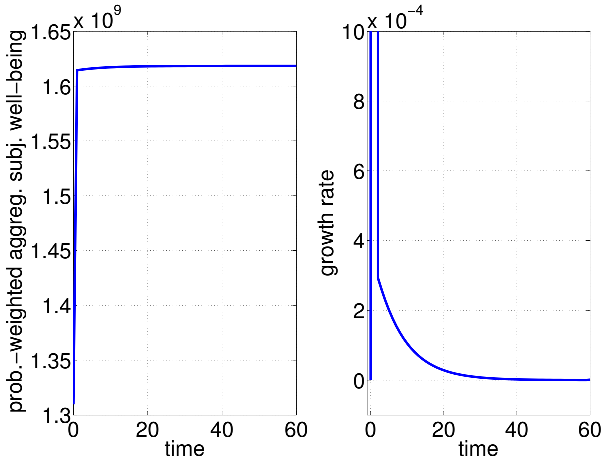

Figure 3.11: Absolute effect of an improvement of mental health care quality by 20% on the economy's welfare and growth rate of the economy's welfare over time.

At time $t = 0$ welfare measured in utility units amounts to $W(t) = 1.31e^9$. Time $t = 0$ is a point in time directly prior to the intervention. The intervention takes place at time $t = 1$. As a consequence, welfare increases by 23.3% and reaches a value of $1.62e^9$. This immediate "jump" can be explained as follows. First of all, we assumed that the policy intervention can be implemented straight away without any adjustment duration. It follows that individuals react in the moment of the implementation of the intervention by adjusting their optimal behavior to the changed environment as it is explained in Section 3.6.2. The increased optimal engagement in the beneficial but risky activity in conjunction with the increased instantaneous probability to recover enhances subjective well-being immediately. Due to the intervention the economy is out of its steady state. The changes in optimal behavior affect via the arrival rates the in- and outflows into and out of the mental states. The FPEs explain how the probability distribution evolves over time and moves towards the new stationary equilibrium distribution. This adjustment process is also visible in Figure 3.11. The adjustment process starts immediately when the jump occurs. The growth rate of the economy's welfare falls to 0.03% and converges to zero over time. After 30 years the growth rate is at approximately 0.0007% and can be considered negligible. Thus, we can consider the adjustment process as "completed" after 30 years (of course depending on what is meant by a negligible small growth rate).

Now consider an improvement of educational work. The welfare effects of this intervention are illustrated in Figure 3.12.

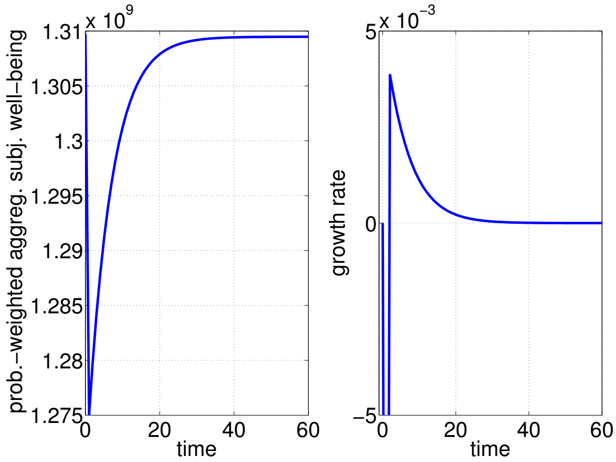

Figure 3.12: Absolute effect of an improvement of educational work by 10% on the economy's welfare and growth rate of the economy's welfare over time.

Contrary to the previous intervention an improvement of educational work has a negative effect on welfare at its moment of implementation. Welfare decreases by 2.65%, because individuals limit their engagement in the activity, as explained further above in Section 3.6.3, which has a negative effect on subjective well-being. The limited engagement, however, leads to lower arrival rates of burdensome experiences which reduces the inflows into higher states of mental ill-health. This in turn reduces the mean value and the variance of the population's mental health distribution. The adjustment process leads to positive growth rates of the economy's welfare after the immediate jump. Although the growth rate is at the beginning of the adjustment process quite small (0.4%), it is still more than 10 times as high as at the previous intervention. In the course of time it also converges to zero, while the convergence process is slower. A negligible growth rate of approximately 0.0007% is reached after 40 years.

So far we looked at the revenue side of a benevolent planner's problem only, but also the costs of mental illness and the costs of implementing a public policy play an important role and should be taken into consideration.

Cost-Of-Illness (COI) analyses, which have a long tradition in the health economics, usually distinguish between direct, indirect and intangible costs. Direct costs include costs of medical resources, like health care goods and services, and costs of non-medical resources such as transportation, training or research. Indirect costs, mainly comprise productivity losses e.g. due to a reduced labor supply, the inability to work, early retirement and premature death. Intangible costs contain non-monetary costs like the emotional pain and the reduced well-being of the persons concerned as well as of their relatives (see e.g. OECD (2013) for more details and Tarricone (2006) or Hodgson and Meiners (1982) for overview articles).

While the empirical methods for estimating the direct cost of (mental) illnesses are meanwhile quite sophisticated, different approaches to measure indirect costs exist and are controversially discussed (see e.g. Shiell et al. (1987)). Several recent national and international studies, highlight the great importance of the indirect costs of mental ill-health (e.g. OECD (2015a), OECD (2014), OECD (2013), the literature cited therein and Bloom et al. (2011)). Although most of these studies are difficult to compare with each other due to the different methodology applied, they arrive at the common result that indirect costs are substantial and exceed the direct costs many times over.

For the quantification of intangible costs only a few standards exist. The most common measures are quality-of-life measures such as DALY = Disability-Adjusted Life Year and QALY = Quality-Adjusted Life Year (see e.g. Sassi (2006) for more detailed informations and Loomes and McKenzie (1989) for an economic discussion on the use of QALYs in health care decision making). Many COI studies do not include intangible costs due to the difficulties to quantify them in monetary terms. Intangible costs of mental ill-health are probably far from insignificant, maybe even dominating, either in human or economic terms. Especially the stigma attached to mental diseases, leading to reject people concerned and exclude them from society pushes the loss in well-being. Due to their obvious importance, intangible costs should also not be neglected.

Against this background, what is the contribution of our framework to this strand of literature? First, the optimal avoidance behavior due to mental ill-health resulting from our model can explain the origin of and serve as a micro-foundation for (at least the majority of) indirect costs. Our value function contains the instantaneous benefits from engaging in the risky but beneficial activity as well as the welfare losses due to mental deterioration. Thus, it can (maybe jointly with existing measures) serve as a basis for the creation of a sophisticated and economically well-founded measure of intangible costs. Moreover, our model allows to explain and quantify its time evolution and its adjustment towards a new stationary equilibrium in the case of a policy intervention.

An extension of the objective function (3.31) to include first, a function for adjustment costs and second, expressions for the aggregated probability-weighted direct and indirect costs, would be a very useful next step for future research.

3.7 Conclusion

Psychological distress is a phenomenon that can concern everyone of us from time to time. It can originate from several different sources. Some of these sources are activities which we can control, like working overtime, consuming performance increasing addictive substances, engaging in a stressful activity for which one sacrifices her private life, endangers friendships and relationships. Other sources of mental illness are beyond our control, as for example getting involved in an accident or loosing a relative. Psychological distress is not a static, but a dynamic phenomenon. It increases when an individual makes burdensome experiences and declines as a result of coping strategies or medical therapy. We have found that the avoidance of a risky activity can be an outcome of optimal behavior. However, the engagement in the risky activity does not necessarily decrease monotonically in mental ill-health. If an individual is aware that there is a general life risk from which no one can escape and / or if she has a risk-seeking attitude, she optimally compensates the pain due to mental ill-health by additional engagement in the risky activity from a particular severity on.

On the aggregate level we employed FPEs for analyzing the distributional dynamics of mental health within a population. The time evolution of the frequency of individuals being in a certain mental state can be explained by the difference between the number of individuals who enter a particular state and the number of individuals who leave this state (both, due to recovery or making additional burdensome experiences). Both, in- and outflows depend on the respective arrival rates of the according events. The ratio of in- and outflows of all states characterizes the long-run distribution of mental health.

Given these insights, we calibrated the equilibrium distribution of our model to fit the empirical distribution of mental ill-health reported in OECD (2012, ch.1.2). For the calibration we divided the target distribution into six sums of equal length, such that we obtained six separate targets to be met and used non-linear least-squares methods to minimize the sum of the squared residuals by choosing six parameter values. The largest deviation of the residuals from zero is approximately $1e^{-2}$. Taking this stationary distribution as an initial equilibrium, we then simulated policy interventions of different kinds. We considered an improvement of mental health care quality on the one hand and an improvement of educational work on the other hand and asked, how both interventions influence optimal behavior, subjective well-being and the equilibrium distribution of mental health.

Both policy interventions increased the economy's mental health. The important difference is that improving the mental health care quality creates an additional incentive to engage in the risky activity, as individuals are provided with additional security with respect to their mental health. Thereby also subjective well-being increases. Improving educational work, however sensitises individuals for the possible negative consequences of the activity on their mental health. And thus, individuals optimally decrease their engagement in the activity. But this also decreases their subjective well-being. Both policy interventions improve the equilibrium distribution of mental health within a population, because in the long-run more individuals with a better mental health condition live in the economy. Our framework can also be seen as a first step towards a micro-founded welfare analysis of public policy interventions concerning mental health. From the perspective of a benevolent planner, the probability-weighted sum of subjective well-being can serve as a target quantity to be maximized. Such an optimization problem takes not only the distributional dynamics of mental health into consideration, it also accounts for the intangible costs of mental ill-health. An extension of the objective function comprising adjustment costs as well as direct and indirect costs of mental ill-health would be a useful step in future research.

Beyond the mere economic aspects of such an analysis we should not neglect, what is desirable from a societal or a moral perspective. If we assume the risky but beneficial activity as substance abuse and improve the mental health care quality, then rational individuals will have an additional incentive to abuse drugs as more security is provided. Such an incentive effect is surely undesired from a societal or moral perspective. However, if we consider the risky activity as volunteer work for civil victims in a conflict area (where one maybe risks to develop a post-traumatic stress disorder), additional engagement might be desirable. Thus, it crucially depends on which objective is pursued by the planner.

Our model is adequate for most of the mild and moderate mental disorders, frequently referred to as ”common mental disorders” (CMDs). This comprises amongst others substance use disorders (e.g. alcohol or cocaine taking to cope with stressful situations), burnout (as a consequence of excessive stress and problems to manage it), anxieties (like public speaking phobia), mood disorders or depression (e.g. missing fulfilment and success in business). CMDs represent the largest category of mental disorders (OECD, 2012, p.11f.). Moreover our model is appropriate for sub-threshold conditions of mental health problems (i.e. problems which are below the clinical threshold of a diagnosis) and severe forms of the above mentioned mental disorders. Note that, our model is not applicable to explain the behavior of individuals suffering under severe mental disorders, such as schizophrenia or psychotic disorders, which include delusions, hallucinations and a disturbed perception of reality. Such severe forms of mental illness imply an impairment of human judgement and thus it would not be adequate to apply rational optimization techniques to explain the behavior of these individuals.

Apart from the above-mentioned our paper contributes to economic research by providing a formal foundation for an important psychological concept. Avoidance behavior, which can be the outcome of optimal behavior appears in our model. In clinical psychology it is an important diagnostic criterion of many mental disorders (American Psychiatric Association, 2013). Individuals who entirely avoid performing a risky activity fulfil this criterion. Also permanent avoidance behavior can be the outcome of optimal behavior, if individuals who reach a threshold level of mental ill-health and decide to avoid the risky activity entirely. Without exogenous recovery individuals' mental state would remain constant from then on. As a consequence individuals loose their ability to adapt to their environment. But this ability is essential as it ensures survival. Intelligent organisms are capable of learning. Incorporating a learning mechanism in our model would be useful extension. For instance, arrival rates could be subjective, such that there exists a subjective belief of an activity's riskiness. A Bayesian learning mechanism could serve to update the individual's subjective believe. This would allow for a highly selective distinction between avoidance behavior which is due to a distorted belief and avoidance behavior which develops gradually by learning the objective arrival rate. When individuals learn by sampling, consistent avoidance behavior blocks learning, i.e. the ability to adapt to a changing environment. Such an analysis would be very enriching, for economic as well as psychological research.

Appendix

This appendix contains all derivations of the optimization problem and the Fokker-Planck equations.

3.A The Optimal Control Problem In General Form

The general form of the Hamilton-Jacobi-Bellman (HJB henceforth) equation reads[21]

$$\rho V(q) = \max_k \left\{ u(k,q) + \frac{1}{dt} E_t dV(q) \right\}. \tag{A.1}$$

The differential of $V(q)$ can be derived given the evolution of psychological distress in (3.1) and the change of variable formula[22]

$$dV(q) = [V(q+1) - V(q)] \; dq^A - [V(q-1) - V(q)] \; dq^B, \tag{A.2}$$

Applying expectations to the differential basically consists of applying expectations to the dq-terms. Generally speaking, $E_t dq = \mu dt$ where $\mu^A(k,q)$ and μ^B are the arrival rates of the Poisson processes q^A and q^B, respectively. Plugging this into the general HJB equation (A.1) finally gives

$$\rho V(q) = \max_k \left\{ u(k,q) + \mu^A(k,q) \left[V(q+1) - V(q) \right] + \mu^B \left[V(q-1) - V(q) \right] \right\}. \tag{A.3}$$

The first order condition defines the optimal intensity of engaging in the risky

[21] Time indices are suppressed from now on.

[22] see e.g. Wälde (2012), ch. 10.2.2: If there is a function $F(x,t)$, where variable x is driven by a stochastic process, then the differential of F is given by

$$dF(t,x) = \frac{\partial}{\partial t} F(t,x) dt + \frac{\partial}{\partial x} F(t,x) a(.) dt + \{F(t,x+b(.)) - F(t,x)\} dq.$$

activity, k,[23]

$$\frac{\partial u(k,q)}{\partial k} = -\frac{\partial \mu^A(k,q)}{\partial k}\left[V(q+1) - V(q)\right]. \tag{A.4}$$

3.B The Specific Optimal Control Problem

Instantaneous utility is specified as

$$u(k(t), q(t)) = k(t)^\alpha - \gamma q(t)^\beta, \tag{B.1}$$

Given this specification, the HJB reads

$$\begin{aligned} \rho V(q) = \max_k \big\{ & k^\alpha - \gamma q^\beta + \left(\lambda^0(q) + k\lambda^A\right)\left[V(q+1) - V(q)\right] \\ & + \lambda^B \left[V(q-1) - V(q)\right]\big\}, \end{aligned} \tag{B.2}$$

while the FOC is

$$\begin{aligned} \alpha k^{\alpha-1} &= -\lambda^A \left[V(q+1) - V(q)\right], \\ \Leftrightarrow k(q) &= \left(-\frac{\lambda^A\left[V(q+1) - V(q)\right]}{\alpha}\right)^{\frac{1}{\alpha-1}}, \end{aligned} \tag{B.3}$$

The maximized HJB equation reads

$$\rho V(q) = k(q)^\alpha - \gamma q^\beta + \left(\lambda^0(q) + k(q)\lambda^A\right)\left[V(q+1) - V(q)\right] + \lambda^B\left[V(q-1) - V(q)\right] \tag{B.4}$$

3.C The Numerical Approach Of Solving The Optimization Problem

Let $\boldsymbol{v} = [v_0, v_1, \ldots v_n]$ be a vector, its elements $v_0, v_1, \ldots v_n$ describe the values of optimal behavior in state $q = [0, 1, \ldots n]$, while n corresponds to the highest possible addictive state $q^{\max}$.

[23]For convenience, time indices are suppressed.

The general structure of the equation system we are aiming to solve reads

$$
\begin{aligned}
0 &= f(v_0, v_1) \\
0 &= g(v_0, v_1, v_2) \\
0 &= g(v_1, v_2, v_3) \\
&\vdots \\
0 &= h(v_{n-1}, v_n),
\end{aligned}
\tag{C.1}
$$

while $f(.)$, $g(.)$ and $h(.)$ are non-linear equations in their arguments. Each of these n equations is a maximized Hamilton-Jacobi Bellman equation fixed at a given state $q = [0, 1, \ldots n]$. The left-hand side of each equation is the residual and thus set equal to zero. As a whole it is a system of n equations in n unknowns. Note that we reduced the number of equations and unknowns by using maximized HJBs instead of HBJs to be maximized. The latter would double the number of unknowns and equations and makes the problem numerically less tractable and more difficult to solve.

Given an initial guess, $v^{\text{ini}} = \left[v_0^{\text{ini}}, v_1^{\text{ini}}, \ldots v_n^{\text{ini}}\right]$, we solve the system (C.1) with Matlab's built-in solver *fsolve*.

3.D The Evolution Of A Populations' Mental Ill-Health

In the following we explain the derivation of FPEs from the transition equation of mental ill-health. The procedure described here closely follows the five-step method described in Bayer and Wälde (2013) and in Nagel (2013, p.9ff.). At this point we shall also refer to Birkner and Wälde (2016, ch.8) where the peculiarities of the derivation of FPEs resulting from stochastic differential equations which are driven by Poisson processes with discrete realizations are explained in greater detail.

3.D.1 Step 1 (An Auxiliary Function And Its Time-Evolution)

The transition equation for the number of bad experiences reads

$$
dq(t) = dq^A(t) - dq^B(t) - (q(t) - q_0)dq^{\text{birth}}(t), \tag{D.1}
$$

where the first term on the RHS describes an increase in mental ill-health in consequence of the occurrence of a burdensome situation, the second term is the natural recovery process which decreases mental ill-health exogenously and the third term is the death and birth process which we take into consideration on the aggregate level.

The corresponding arrival rates are

$$\mu^A(k(q(t)), q(t)) = \lambda^0(q(t)) + k(q(t))\lambda^A, \quad \mu^B = \lambda^B \quad \text{and} \quad \mu^{\text{birth}} = \lambda^{\text{birth}}. \tag{D.2}$$

Let there be an auxiliary function $f(q(t))$ with bounded support $S \subset \mathbb{R}^{\geq 0}$, i.e. f is equal to zero outside a fixed bounded set S. Then we compute the function's expected change given the process for $q(t)$. The differentiation of f using Itô's lemma yields

$$\begin{aligned} df(q(t)) = &\{f(q(t)+1) - f(q(t))\}\, dq^A(t) + \{f(q(t)-1) - f(q(t))\}\, dq^B(t) \\ &+ \{f(q_0) - f(q(t))\}\, dq^{\text{birth}}(t). \end{aligned} \tag{D.3}$$

We are interested in the expected change of this function over time. Therefore we apply the expectation operator and divide by $d\tau$ (for clarity, time indices have been omitted),

$$\begin{aligned} \frac{1}{d\tau} Edf(q) = &\mu^A(q)\{f(q+1) - f(q)\} + \mu^B\{f(q-1) - f(q)\} \\ &+ \mu^{\text{birth}}\{f(q_0) - f(q)\}, \end{aligned} \tag{D.4}$$

where the last line was computed using $Edq^A = \mu^A(q)d\tau$ with $\mu^A(q) = \lambda^0(q) + k(q)\lambda^A$, $Edq^B = \mu^B d\tau$ with $\mu^B = \lambda^B$, and $Edq^{\text{birth}} = \mu^{\text{birth}} d\tau$ with $\mu^{\text{birth}} = \lambda^{\text{birth}}$, which are the arrival rates of the Poisson processes.

In what follows, we denote this expression by $\mathcal{A}f(q) \equiv E\frac{df(q)}{d\tau}$.

3.D.2 Step 2 (Using Dynkin's Formula)

The Dynkin formula says that $Ef(q(\tau)) = Ef(q(t)) + \int_t^\tau E\mathcal{A}f(q(s))ds$.[24] In words, the expected value of a function $f(q(\tau))$ at some future point in time $\tau \geq t$ can be written as the expected value of this function at t plus the integral of all expected future changes between τ and t.

As we are interested in expected changes, we differentiate this formula with respect to time and get

$$\frac{\partial}{\partial \tau} Ef(q(\tau)) = \frac{\partial}{\partial \tau} \int_t^\tau E\mathcal{A}f(q(s))ds = E\mathcal{A}f(q(s)), \tag{D.5}$$

since $Ef(q(t))$ can be treated as a constant, which drops when differentiating, and the differential operator can be pulled into the integral.

[24]For more detailed explanations see e.g. B.2 in Bayer and Wälde (2013)

Applying this on (D.4) yields

$$\begin{aligned}\frac{\partial}{\partial\tau}Ef(q) = E\big[&\mu^A(q)\left\{f(q+1)-f(q)\right\}+\mu^B\left\{f(q-1)-f(q)\right\}\\ &+\mu^{\text{birth}}\left\{f(q_0)-f(q)\right\}\big].\end{aligned} \tag{D.6}$$

Under the assumption that q_0 is a positive integer or zero, $q(\tau)$ as well as $q^{\max}$ are positive integers, too, and we can write the latter equation as

$$\begin{aligned}\frac{\partial}{\partial\tau}Ef(q) = \Sigma_{i=q_0}^{q^{\max}}p(i,\tau)\big[&\mu^A(i)\left[f(i+1)-f(i)\right]+\mu^B\left[f(i-1)-f(i)\right]\\ &+\mu^{\text{birth}}\left[f(0)-f(i)\right]\big]\end{aligned} \tag{D.7}$$

where $p(i,\tau)$ is the probability that q equals i in τ, i.e. $p(i,\tau)\equiv Pr(q(\tau)=i)$.

3.D.3 Step 3 (Factorizing)

Now we want to get rid of the $f(i+1)$ and the $f(i-1)$ terms in equation (D.7). By factorizing the equation we obtain

$$\begin{aligned}\frac{\partial}{\partial\tau}Ef(q) = &\Sigma_{i=q_0}^{q^{\max}}p(i,\tau)\mu^A(i)f(i+1)+\mu^B\Sigma_{i=q_0}^{q^{\max}}p(i,\tau)f(i-1)\\ &+\mu^{\text{birth}}f(0)\Sigma_{i=q_0}^{q^{\max}}p(i,\tau)\\ &-\Sigma_{i=q_0}^{q^{\max}}p(i,\tau)\left[\mu^A(i)f(i)+\mu^Bf(i)+\mu^{\text{birth}}f(i)\right],\end{aligned} \tag{D.8}$$

As

$$\Sigma_{i=q_0}^{q^{\max}}p(i,\tau)\mu^A(i)f(i+1)=\Sigma_{i=q_0+1}^{q^{\max}+1}p(i-1,\tau)\mu^A(i-1)f(i), \tag{D.9}$$

and

$$\mu^B\Sigma_{i=q_0}^{q^{\max}}p(i,\tau)f(i-1)=\mu^B\Sigma_{i=q_0-1}^{q^{\max}-1}p(i+1,\tau)f(i), \tag{D.10}$$

we can write (D.8) as

$$\begin{aligned}\frac{\partial}{\partial\tau}Ef(q) = &\Sigma_{i=q_0+1}^{q^{\max}+1}p(i-1,\tau)\mu^A(i-1)f(i)+\mu^B\Sigma_{i=q_0-1}^{q^{\max}-1}p(i+1,\tau)f(i)\\ &+\mu^{\text{birth}}f(0)\Sigma_{i=q_0}^{q^{\max}}p(i,\tau)\\ &-\Sigma_{i=q_0}^{q^{\max}}p(i,\tau)\left[\mu^A(i)f(i)+\mu^Bf(i)+\mu^{\text{birth}}f(i)\right].\end{aligned} \tag{D.11}$$

3.D.4 Step 4 (Derivation Of The Expected Value)

We now derive an expression for the change of the expected value for the LHS of the equation. Instead of Dynkin's rule we use therefore the definition of the expectation operator in order to get

$$\frac{\partial}{\partial\tau}Ef(q) = \frac{\partial}{\partial\tau}\Sigma_{i=q_0}^{q^{\max}} f(i)p(i,\tau) = \Sigma_{i=q_0}^{q^{\max}} f(i)\frac{\partial p(i,\tau)}{\partial\tau}, \tag{D.12}$$

where the last equality exploits the fact that the auxiliary function $f(q)$ is *not* a function of time. (Note that $q(\tau)$ is a function of time but not f itself.)

3.D.5 Step 5 (Collecting All Results)

Equating (D.11) and (D.12) yields

$$\begin{aligned}\Sigma_{i=q_0}^{q^{\max}} f(i)\frac{\partial p(i,\tau)}{\partial\tau} &= \Sigma_{i=q_0+1}^{q^{\max}+1} p(i-1,\tau)\mu^A(i-1)f(i) + \mu^B\Sigma_{i=q_0-1}^{q^{\max}-1} p(i+1,\tau)f(i)\\ &+ \mu^{\text{birth}} f(0)\Sigma_{i=q_0}^{q^{\max}} p(i,\tau)\\ &- \Sigma_{i=q_0}^{q^{\max}} p(i,\tau)\left[\mu^A(i)f(i) + \mu^B f(i) + \mu^{\text{birth}} f(i)\right].\end{aligned} \tag{D.13}$$

For concreteness sake we set $q_0 = 0$ and then rearranged the equation above as follows

$$\begin{aligned}\Sigma_{i=q_0}^{q^{\max}} f(i)\frac{\partial p(i,\tau)}{\partial\tau} &= p(0,\tau)\mu^A(0)f(1) + p(1,\tau)\mu^A(1)f(2) + p(2,\tau)\mu^A(2)f(3) + \dots\\ &+ p(0,\tau)\mu^B f(-1) + p(1,\tau)\mu^B f(0) + p(2,\tau)\mu^B f(1) + \dots\\ &+ p(0,\tau)\mu^{\text{birth}} f(0) + p(1,\tau)\mu^{\text{birth}} f(0) + p(2,\tau)\mu^{\text{birth}} f(0) + \dots\\ &- p(0,\tau)\left[\mu^A(0)f(0) + \mu^B f(0) + \mu^{\text{birth}} f(0)\right]\\ &- p(1,\tau)\left[\mu^A(1)f(1) + \mu^B f(1) + \mu^{\text{birth}} f(1)\right]\\ &- p(2,\tau)\left[\mu^A(2)f(2) + \mu^B f(2) + \mu^{\text{birth}} f(2)\right] - \dots\\ &= f(-1)p(0,\tau)\mu^B\\ &+ f(0)\left[-p(0,\tau)\mu^A(0) - p(0,\tau)\mu^B - p(0,\tau)\mu^{\text{birth}}\right.\\ &\left.+p(1,\tau)\mu^B + \mu^{\text{birth}}\Sigma_{i=q_0}^{q^{\max}} p(i,\tau)\right]\\ &+ f(1)\left[p(0,\tau)\mu^A(0) - p(1,\tau)\mu^A(1) - p(1,\tau)\mu^B - p(1,\tau)\mu^{\text{birth}}\right.\\ &\left.+p(2,\tau)\mu^B\right]\\ &+ f(2)\left[p(1,\tau)\mu^A(1) - p(2,\tau)\mu^A(2) - p(2,\tau)\mu^B - p(2,\tau)\mu^{\text{birth}}\right.\\ &\left.+p(3,\tau)\mu^B\right]\\ &+ \dots\end{aligned}$$

As this must hold for any $f(i)$ we obtain for $i=-1$

$$\text{n.a.} = f(-1)p(0,\tau)\mu^B, \tag{D.14}$$

for $i=q_0$,

$$\frac{\partial p(0,\tau)}{\partial \tau} = \mu^B p(1,\tau) - [\mu^A(0) + \mu^{\text{birth}}]p(0,\tau) + \mu^{\text{birth}}\Sigma_{i=q_0}^{q^{\max}} p(i,\tau), \tag{D.15}$$

Note that we can set $-\mu^B p(0,\tau) = 0$ here as the underlying auxiliary function has bounded support, i.e. it is defined for the positive numerical range only. Moreover, the sum at the end of (D.15) must equal one, as all probabilities must add up to one. For $i=1$,

$$\frac{\partial p(1,\tau)}{\partial \tau} = \mu^B p(2,\tau) - [\mu^A(1) + \mu^B + \mu^{\text{birth}}]p(1,\tau) + \mu^A(0)p(0,\tau), \tag{D.16}$$

for $i=2$,

$$\frac{\partial p(2,\tau)}{\partial \tau} = \mu^B p(3,\tau) - [\mu^A(2) + \mu^B + \mu^{\text{birth}}]p(2,\tau) + \mu^A(1)p(1,\tau), \tag{D.17}$$

for $i=3$,

$$\frac{\partial p(3,\tau)}{\partial \tau} = \mu^B p(4,\tau) - [\mu^A(3) + \mu^B + \mu^{\text{birth}}]p(3,\tau) + \mu^A(2)p(2,\tau), \tag{D.18}$$

and so on. Thus, for a general $1 \leq i < q^{\max}$

$$\frac{\partial p(i,\tau)}{\partial \tau} = \mu^B p(i+1,\tau) - [\mu^A(i) + \mu^B + \mu^{\text{birth}}]p(i,\tau) + \mu^A(i-1)p(i-1,\tau). \tag{D.19}$$

And for $i = q^{\max}$ we have

$$\frac{\partial p(q^{\max},\tau)}{\partial \tau} = -[\mu^B + \mu^{\text{birth}}]p(q^{\max},\tau) + \mu^A(q^{\max}-1)p(q^{\max}-1,\tau), \tag{D.20}$$

since $\mu^A(q^{\max}) = 0$ and $p(q^{\max}+1,\tau) = 0$, as $k(q^{\max}) = 0$.

3.E Numerical Approach Distributional Dynamics

Let $\boldsymbol{p} = [p_0, p_1, \dots p_n]$ be a vector, its elements $p_0, p_1, \dots p_n$ describe the probabilities or population shares in state $q = [0, 1, \dots n]$, at time τ, while n corresponds to the highest possible addictive state $q^{\max}$.

The general structure of the equation system we are aiming to solve reads

$$
\begin{aligned}
\dot{p}_0(\tau) &= f(p_0(\tau), p_1(\tau)) \\
\dot{p}_1(\tau) &= g(p_0(\tau), p_1(\tau), p_2(\tau)) \\
\dot{p}_2(\tau) &= g(p_1(\tau), p_2(\tau), p_3(\tau)) \\
&\vdots \\
\dot{p}_n(\tau) &= h(p_{n-1}(\tau), p_n(\tau)),
\end{aligned}
\tag{E.1}
$$

while $f(.)$, $g(.)$ and $h(.)$ are linear equations in their arguments. Each of these n equations is a Fokker Planck equation taking optimal individual behavior into account and describes the time evolution of the probability or the population shares in state $q = [0, 1, \dots n]$. As a whole it is a linear system of n differential equations in n unknowns.

Given an initial condition, $p^{\text{ini}}(t) = \left[p_0^{\text{ini}}(t), p_1^{\text{ini}}(t), \dots p_n^{\text{ini}}(t)\right]$, we solve the system (E.1) with Matlab's built-in solver *ode23* for a given time interval $(t, T]$.

3.F Deriving A System Of Equations For The Stationary Distribution

In the long-run $\frac{\partial p(i,\tau)}{\partial \tau} = 0 \quad \forall i$. It follows for $i = 0$

$$
p(0) = \frac{\mu^B}{\mu^A(0) + \mu^{\text{birth}}} p(1) + \frac{\mu^{\text{birth}}}{\mu^A(0) + \mu^{\text{birth}}}, \tag{F.1}
$$

For $i = 1$ we obtain

$$
p(1) = \frac{\mu^B}{\mu^A(1) + \mu^B + \mu^{\text{birth}}} p(2) + \frac{\mu^A(0)}{\mu^A(1) + \mu^B + \mu^{\text{birth}}} p(0), \tag{F.2}
$$

for $i = 2$ we obtain

$$
p(2) = \frac{\mu^B}{\mu^A(2) + \mu^B + \mu^{\text{birth}}} p(3) + \frac{\mu^A(1)}{\mu^A(2) + \mu^B + \mu^{\text{birth}}} p(1), \tag{F.3}
$$

and so on. Thus, for any general $1 \leq i < q^{\max}$

$$
p(i) = \frac{\mu^B}{\mu^A(i) + \mu^B + \mu^{\text{birth}}} p(i+1) + \frac{\mu^A(i-1)}{\mu^A(i) + \mu^B + \mu^{\text{birth}}} p(i-1). \tag{F.4}
$$

And for $i = q^{\max}$ we have

$$p(q^{\max}) = \frac{\mu^A(q^{\max}-1)}{\mu^B + \mu^{\text{birth}}} p(q^{\max}-1), \tag{F.5}$$

since $\mu^A(q^{\max}) = 0$ and $p(q^{\max}+1) = 0$, as $k(q^{\max}) = 0$.

3.G Numerical Approach Stationary Distribution

Let $\boldsymbol{p} = [p_0, p_1, \ldots p_n]$ be a vector, its elements $p_0, p_1, \ldots p_n$ describe the probabilities or population shares in state $q = [0, 1, \ldots n]$, at time T, while n corresponds to the highest possible addictive state $q^{\max}$.

The general structure of the equation system we are aiming to solve reads

$$\begin{aligned} 0 &= f(p_0, p_1) \\ 0 &= g(p_0, p_1, p_2) \\ 0 &= g(p_1, p_2, p_3) \\ &\vdots \\ 0 &= h(p_{n-1}, p_n), \end{aligned} \tag{G.1}$$

while $f(.)$, $g(.)$ and $h(.)$ are linear equations in their arguments. Each of these n equations is a Fokker Planck equation taking optimal individual behavior into account and under the assumption that the convergence process of the distributional dynamics is completed. Thus the time evolution of the probability or the population shares in state $q = [0, 1, \ldots n]$, i.e. the left-hand side of the equations, is zero. As a whole it is a linear system of n equations in n unknowns.

Given an initial guess, $p^{\text{ini}} = \left[p_0^{\text{ini}}, p_1^{\text{ini}}, \ldots p_n^{\text{ini}}\right]$, we solve the system (G.1) with Matlab's built-in solver *fsolve*.

Chapter 4

The Distributional Dynamics Of Addiction

by Dennis Krieger[1]

4.1 Introduction

[Motivation] Substance related addiction is a phenomenon that concerns many people worldwide. The Word Health Organization projected that in 2010 persons aged 15 and older consumed 13.5 grams of pure alcohol per day on average worldwide[2] and that 7.5% of the 15+ population (16% of the drinkers) practised "binge-drinking"[3] at least monthly (World Health Organization, 2014a, pp. 29-35). According to the results of a survey in the Special Eurobarometer on the "Attitudes of Europeans towards Tobacco", 26% of the Europeans were addicted to nicotine in the year 2014 and stated that they smoke (Special Eurobarometer 429, 2015, p.10). The United Nations Office on Drug and Crime estimated that in 2013 about 5% of the adult world population have used illicit drugs at least once within the previous year. More than ten percent of the illicit drug users are regular users and thus addicted (United Nations Office on Drugs and Crime, 2015, p. ix). These numbers clearly give cause for concern. Addictions are accompanied by substantial negative consequences on concerned person's health (OECD, 2015b, p.23ff.), on their social life as well as on the society and the economy as a whole, because addictive and drug-induced behaviors often cause negative externalities (like road fatalities, vandalism, violence, drug-related crime) and involve huge eco-

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[2]This corresponds to approximately one 330 ml bottle of beer with 4.8 vol-% alcohol.

[3]Binge-drinking is defined as the consumption of at least 60 grams of pure alcohol at one single occasion.

nomic tangible and intangible costs. In 2010 smoking and alcohol consumption were two of the “three leading risk factors for global disease burden” (Lim et al., 2013).

[Problem] The protection and improvement of public health is the primary concern of public health policy. Thus, also inventing strategies to tackle harmful consumption of addictive substances and thereby reducing the extent of economic, health and social problems is at the center of interest. This requires to understand addictive behavior on the individual level and to apply these insights on the macro level. Due to the well developed existing microeconomic theories of addiction, the former can be taken as given. But there is still a lack of contemporary theories of addiction being applied to understand the developments on the aggregate level. The current paper tackles exactly this issue.

[Goal] The goal of this paper is to provide more insights into the distributional dynamics of addiction. We want to answer the question of how and why addictive states are distributed among a population as it can be observed empirically and what determines the dynamics of addiction. Our goal is to build a bridge between state of the art theoretic research on addiction, empirical findings and public policy questions.

[Approach] Therefore, we formulate as a starting point a slightly modified version of the addiction model by Bernheim and Rangel (2004) (BR henceforth). Individuals in our model decide how much they wish to engage in two different activities, a risky activity (going out and expose oneself to environmental substance-related cues) and a safe activity (therapy participation). While going out yields an instantaneous pleasure it can also trigger uncontrolled drug consumption. The latter leads to physical and mental deterioration under which the individuals suffer. Therapy participation is an option to recover from the deterioration, but it is assumed to be instantaneously costly. We differ from the original model mainly and most importantly with regards to the following aspects. First of all, time in our framework is a continuum. This is a great advantage especially for analyzing the dynamics of addiction, as a continuous time framework provides a much better analytical description of the implemented stochastic dynamic processes than it is the case in discrete time. To keep things as simple as possible we sacrificed “cold-mode” consumption (i.e. non-triggered consumption of an addictive good). Moreover, unlike in BR’s model, the choice set in our model is not discrete, but continuous, which allows us to derive quantitative information which has much more explanatory power compared to qualitative information only. We solve our model numerically. Fokker-Planck equations, which we derive analytically from the transition equation of addiction, allow us to understand and analyze the distributional dynamics of a population’s addictive states, given the optimal behavior from the individual level. Then we apply our framework on empirical data on binge drinking behavior in Austria and simulate a public policy intervention to derive quantitative statements regarding its effects on the individual as well as the aggregate level.

BR's addiction model is to the best of our knowledge the most recent and without a doubt one of the most important contributions to state of the art economic research on addiction. Their paper represents an advance in understanding addictive behavior when substance-related cues can trigger uncontrolled drug consumption. The model is built on three key assumptions. First, addicts often consider their own drug consumption as a mistake, second drug consumption causes a sensitization to substance-related cues and third, addicts normally understand their own vulnerability to failures. From an economic perspective these assumptions are highly interesting. Therefore we shall briefly provide some background information on the findings which build the basis for these assumptions. The following insights emerged from psychological and neurological research and from clinical practice. Similar to the Pavlovian dog that starts salivating in expectation of food as soon as a bell rings, humans also can become cue-conditioned and will display the according response reactions when they are exposed to a conditioned cue. The human brain reward systems react with the release of dopamine whenever an individual is exposed to environmental cues that are associated with hedonic responses. Addictive substances however, can distort the normal functioning of these brain reward systems. Usually, addictive substances sensitize the brain reward systems, with the result that this substance-induced sensitization of the brain reward systems causes a compulsive drug pursuit in addictive individuals (Robinson and Berridge, 2003). Berridge and Aldridge (2008) consider this compulsive drug pursuit as the result of a temporary distortion of an individual's decision utility (the basis on which decisions are made). In terms of the model we understand this distortion as a temporary present bias leading to an overestimation of the hedonic reward of an addictive substance. That is, once an individual got triggered her decision utility is temporary distorted and it differs from her decision utility while not triggered and also from her experienced utility, i.e. from the actual hedonic impact which is experienced during the consumption of the substance. Sophisticated individuals are aware of the temporary distortion and can even anticipate making mistakes in the future. Thus, substance consumption is often considered as a mistake. The substance-induced sensitization of the brain reward systems maintains as long as the drug is consumed. During this time self-control is costly but sophisticated individuals understand their need for commitment devices.

[Results/Findings] On the individual level we find that rational agents behave more careful the stronger their addiction is, i.e. they engage less in risky activities that potentially increase their addictive state and instead invest more in the chance of future recovery. This, however, comes a long with a lower instantaneous benefit, whereby subjective well-being decreases with an increasing addictive state. On the aggregate level we find that the distribution of addictive states among a population crucially depends on the ratio of risk-taking to care-taking behaviors as this ratio determines the in- and out-flows into and out of each addictive state. Public policy interventions, such as printing warning labels on cigarette packs, have an welfare-enhancing effect, as they allow to engage more in less risky beneficial activities. However, due to this incentive effect they also contribute to a deterioration of a populations' health condition. To counteract this development additional steps, like improving health education, are necessary.

[Contribution] We contribute to the economic research on addiction in various ways. First, we provide a continuous time version of Bernheim Rangel's addiction model, which is far from trivial. This opens up new enriching possibilities for economic research on addiction, especially for dynamic systems, because the analytical description of stochastic dynamic processes is much better in a continuous than in a discrete time framework. Moreover individuals in our framework not only make binary choices. In our framework individuals choose frequencies. That is, individual's optimal choices provide us with level information rather than qualitative information only, which maybe serve as a mathematical foundation of behavioral patterns known through the insights from psychological research and clinical practice. Finally, we also contribute to the contemporary debate in behavioral economic welfare analysis by providing a theoretical framework that allows us to study and simulate the welfare effects of different policy interventions.

[Related Literature] The rational addiction (RA) model by Becker and Murphy (1988) is *the* most popular classic economic addiction model. Their idea consists of allocating resources between an addictive and a non-addictive good, while past consumption of the addictive good has an impact on the individual's addictive state and thus on her instantaneous utility. Moreover the marginal utility of consumption of the addictive good increases in the addictive state. Although we refrain from modelling non-triggered substance consumption as in the RA model, substance consumption in our model nevertheless has an impact on future utility and also makes future substance consumption more possible.

To keep the model as simple and tractable as possible, our main approach focusses on cue-triggered consumption of the addictive good in the spirit of BR and thus refrains from explicitly modelling non-triggered substance consumption as in the RA model.

Orphanides and Zervos (1995) is an extension of the RA model, in which individuals do not know whether they have an "addictive tendency" or not, i.e. they are unsure about their own type. Thus, they have a subjective belief of being the one or the other type. As long as uncertainty persists are the individuals Bayesian learners and act in light of their belief, i.e. subjectively rational. The crucial difference to BR's idea is that neither from an individual's view, nor from a third-party's view an individuals' consumption of the addictive good can be seen as a mistake, as subjective beliefs represent an individual's best knowledge. As soon as individuals' true type is revealed to them the model becomes a more or less standard RA approach.

Gruber and Köszegi (2001) is the merger of the RA model and quasi-hyperbolic discounting according to Laibson (1997). Although not quite obvious at first glance, this paper is the most closely related work to BR and thus also our approach. In their model the behavior of an individual who does not suffer a present bias, is the analogue of BR's cold-mode behavior. Naive individuals with a strong present bias, however, act like individuals in hot mode. From the perspective of a sophisticated individual, present-biased behavior is a mistake. However, in Gruber and Köszegi's framework there are no cues which might trigger addictive consumption.

Cues, modelled as random shocks, appear in Laibson (2001). His "Cue Theory of Consumption" basically describes a model of stochastic preferences, as "cues raise the marginal utility of consumption", i.e. whenever a cue appears the individual's changes her preferences represented by one of two instantaneous utility functions. Although the individual makes decisions that are perfectly in line with her current preferences, consumption can be considered as a mistake ex post, i.e. as soon as preferences change.

Loewenstein et al. (2003) provide another interesting way to think about mistakes. In their paper on the "projection bias" they describe this bias simply as the divergence between decision utility and experienced utility. The projection bias is for example responsible for the phenomenon that some people buy too much food for their consumption within the next days when they go shopping while being hungry. Provided that decisions are made on the basis of decision utility and experienced utility reflects the hedonic reward which individuals actually experience, then individuals are ex post able to identify a decision made as a mistake. This view is among others also supported by Kahneman et al. (1997). Individuals will be aware of their own mistakes and as long as they are not naive, they will also understand their need for commitment devices.

[Structure] The paper is structured as follows. In Section 4.2 we present our model, a slightly modified continuous time version of BR's addiction model with a continuous choice set. The model is solved numerically and we illustrate optimal behavior and well-being on the individual level. Within the third section we explain the transition from the individual to the aggregate level before we describe the distributional dynamics of addictive states within a population with Fokker Planck-equations. Following that, we analyze in Section 4.4 the effects of risk in the environment in which individuals live on their welfare. In Section 4.5 follows an application of our theoretical framework on binge drinking behavior in Austria. In this context we first calibrate our model to fit the empirical distribution of addictive states and then analyze the effects of possible policy interventions. Finally, in Section 4.6 we conclude.

4.2 The Model

In the following subsections we present the setup of our model, the underlying optimization problem and derive optimal behavior as well as subjective well-being on the individual level. Individuals in our model are free to choose the intensities with which they wish to engage in one of two activities, going out and participating in therapy. Whenever the individuals choose to go out they enjoy the instantaneous pleasure of it, but they are also exposed to environmental substance-related cues which may trigger uncontrolled behavior in terms of drug consumption. Consumption of an addictive good leads to physical and mental deterioration from which individuals suffer. The decision to participate in therapy, however, is entirely risk free, instantaneously costly but also entails the possibility to recovery from the deterioration.

4.2.1 Model Setup

We consider an individual who is endowed with a maximum of $l > 0$ units of time, which she can freely allocate between two activities, going out at night, $k(t)$, and participating in a therapy, $b(t)$. While going out, the individual will be exposed to various substance-related stimuli (e.g. bottles of alcohol, cigarette smoke, drug paraphernalia), which is not the case if the individual chooses $k(t) = 0$ or while she is in treatment. On the one hand, going out yields an instantaneous hedonic reward, $v(k(t))$, on the other hand, it bears the risk that the exposure to the stimuli triggers the uncontrolled consumption of an addictive substance. Consumption is always a loss of control. The underlying idea is that the individual once she became triggered enters a state in which she suffers an infinitely large present bias so that immediate consumption is always optimal as all future consequences are blinded out. As soon as the consumption has taken place, the individual is restored to her original state. Note that, for the sake of convenience, this process is not explicitly modelled. Consumption increases the individual's addictive state, denoted by $q(t)$, by one unit. The higher this state is, the higher is her instantaneous pain, $\psi(q(t))$, which reflects the physical and mental deterioration due to addiction. Whenever the individual decides to participate in a therapy she can either choose an in-patient treatment, then $b(t) = l$ and $k(t) = 0$, which guarantees abstention, or an out-patient treatment, then $b(t) < l$ and $k(t) > 0$ which does not guarantee abstention. Whenever she participates in a therapy, the individual experiences therapeutic successes from time to time, which are accompanied by a reduction of her addictive state by one unit.

The evolution of the addictive state is described by the following stochastic differential equation,

$$dq(t) = dq^l(t) - dq^r(t), \tag{4.1}$$

given an initial state $q(0) = q^{\text{ini}} > 0$. The addictive state increases whenever the individual experiences a loss of control and consumes an addictive substance, then $dq^l(t) = 1$ (l like **l**oss of control) and the addictive state decreases due to rehabilitation, then $dq^r(t) = 1$ (r like **r**ehabilitation).

The occurrence of losses of control is driven by a Poisson process, $q^l(t)$, with an endogenous arrival rate, $\mu^l(k(t), q(t))$, which is assumed to increase in both its arguments, while $\mu^l(0, q(t)) = 0$ for $q(t) > 0$ and $\mu^l(k(t), 0) = 0$ for $k(t) > 0$. This reflects first that the instantaneous probability of a loss of control is higher, the more the individual goes out, because the more she goes out the more she is exposed to substance-related cues. Second, the instantaneous probability of a loss of control is higher, the higher the individual's addictive state is, which reflects the increasing sensitivity to substance-related cues. Third, losses of control cannot occur if the individual does not go out, since she is not exposed to substance-related cues then. And fourth, if the individual is a "blanc sheet" in terms of drug consumption, i.e. if $q(t) = 0$, then substance-related cues cannot trigger uncontrolled consumption since the neuronal links of an association between stimulus, response and outcome have not been established yet.

Rehabilitation, is explained by another Poisson process, $q^r(t)$, with endogenous arrival rate $\mu^r(b(t), q(t))$, which is an increasing function of the intensity of therapy, $b(t)$. The idea is that the participation in therapy increases the instantaneous probability that the individual experiences a therapeutic success. Note that rehabilitation not only lowers the instantaneous pain but also the sensitivity to substance-related cues via a reduction of the addictive state. That is, for a given intensity, $k(t)$, it lowers the instantaneous probability that the individual looses control when she is exposed to substance-related cues. Moreover, we assume $\mu^r(0, q(t)) = 0$ for $q(t) > 0$ and $\mu^r(b(t), q^{\text{ini}}) = 0$ for $b(t) > 0$. That is, if the individual does not participate in therapy she cannot recover and addiction cannot be entirely cured.

The participation in therapy comes along with certain instantaneous costs, $\phi(b(t))$. Note that we rather have non-monetary than monetary costs in mind here. These costs include the cognitive efforts of tackling the issue of addiction and the emotional pain of admitting that oneself has addiction problems.

Thus the individual's instantaneous utility function reads

$$u(k(t), b(t), q(t)) = \upsilon(k(t)) - \delta\psi(q(t)) - \varepsilon\phi(b(t)) \tag{4.2}$$

and is additively defined as the sum of the three "sub-utilities". $\delta, \varepsilon \geq 0$ are preference parameters measuring the relative weights assigned to mental suffering and therapy costs, respectively. The instantaneous hedonic reward of going out is assumed to be concave in $k(t)$, the mental suffering is convex in the individual's addictive state and the therapy costs are linear in the intensity of therapy ($\partial\upsilon(k(t))/\partial k(t) > 0$, $\partial^2\upsilon(k(t))/\partial k(t)^2 < 0$, $\partial\psi(q(t))/\partial q(t) > 0$, $\partial^2\psi(q(t))/\partial q(t)^2 > 0$, $\partial\phi(b(t))/\partial b(t) > 0$, $\partial^2\phi(b(t))/\partial b(t)^2 = 0$). Note that concavity in $k(t)$ and convexity in $q(t)$ both reflect risk-aversion.

4.2.2 Optimization Problem And Optimality Conditions

The individual maximizes the expected life-time utility $E_tU(t)$, discounted at rate $\rho + \lambda^{\text{birth}}$, by choosing the optimal time paths of $k(t)$ and $b(t)$, taking into account the instantaneous time restriction, the evolution of the addictive state, defined by $dq(t)$ with arrival rates $\mu^l(k(t), q(t))$ and $\mu^r(b(t), q(t))$, given an initial value $q(0) = q^{\text{ini}}$ and an additional terminal condition, to be defined further below. Formally,

$$\max_{\{k(\tau),b(\tau)\}_{\tau=t}^{\infty}} E_tU(t) = \max_{\{k(\tau),b(\tau)\}_{\tau=t}^{\infty}} \left\{ E_t \int_t^{\infty} e^{-[\rho+\lambda^{\text{birth}}][\tau-t]} u(k(\tau), b(\tau), q(\tau))\, d\tau \right\}, \tag{4.3}$$

subject to $l \geq k(t) + b(t)$, $dq(t)$ in (4.1) with $\mu^l(k(t), q(t))$ and $\mu^r(b(t), q(t))$ given $q(0) = q^{\text{ini}}$ and a terminal condition. Following the idea of Yaari (1965), Blanchard (1985) and D'Albis (2007), λ^{birth} denotes the constant instantaneous probability of "death". Thus, the individual's expected length of life at point in time t is $T = 1/\lambda^{\text{birth}}$.[4]

As a terminal condition we require that the probability of being in addictive state $q(t) > q^{\max} \geq q^{\text{ini}}$ at time T is zero,

$$P(q(T) > q^{\max}, T) = 0\% \Leftrightarrow P(q(T) \leq q^{\max}, T) = 100\%. \tag{4.4}$$

This implies endogenously that $k(q^{\max}, T) = 0$, because then losses of control will no longer appear and the addictive state will not exceed the threshold level $q^{\max}$. Beside its mathematical necessary, the basic idea underlying this terminal condi-

[4]Similar to the apporach in Section 3.4 the background of this assumption is that we want to look at cross-sections of an entire population of individuals at different point in times further below.

tion is that an individual reaches her physical and mental limits at the threshold level $q^{\max}$ and is no longer able to go out. An alternative to this interpretation is to assume the individual strives to achieve the long-run goal of the greatest possible extent of self-control which then determines $q^{\max}$.

The Hamilton-Jacobi Bellman equation (HJB) reads[5]

$$\begin{aligned}\rho V(q) = \max_{k,b} \{ & u(k,b,q) + \mu^l(k,q)\,[V(q+1) - V(q)] \\ & + \mu^r(b,q)\,[V(q-1) - V(q)]\}\,.\end{aligned} \tag{4.5}$$

There are three components which build the instantaneous utility flow, $\rho V(q)$. The first component is the instantaneous utility of the pleasure derived from going out, the costs of therapy and pain of deterioration. The second component describes the first of two possible random events, a loss of control, which occurs with rate $\mu^l(k,q)$. Whenever the individual looses control she consumes an addictive substance and consequently her addictive state increases. Thus, she suffers the difference between the value of state $q+1$ and the value of state q. The second random event is rehabilitation and it occurs with rate $\mu^r(b,q)$. In this case, the individual's addictive state decreases by one unit and she enjoys the difference between the value of state $q-1$ and the value of state q.

The first-order conditions (FOCs) are

$$\frac{\partial u(k,b,q)}{\partial k} = -\frac{\partial \mu^l(k,q)}{\partial k}\,[V(q+1) - V(q)] \tag{4.6}$$

and

$$\frac{\partial u(k,b,q)}{\partial b} = -\frac{\partial \mu^r(b,q)}{\partial b}\,[V(q-1) - V(q)]\,. \tag{4.7}$$

The individual chooses the optimal intensity, k, such that the marginal utility of going out (LHS of the first FOC) equals the potential value loss due to a stochastic increase of the addictive state, weighted by the increase in the arrival rate of a loss of control (RHS of the first FOC). And she chooses the optimal intensity, b, such that the marginal costs of therapy (LHS of the second FOC) equal the potential value gain due to a stochastic decrease of the addictive state, weighted by the increase in the arrival rate of rehabilitation (RHS of the second FOC). Put simply, she chooses both intensities such that the benefits (or costs) of additional engagement in the activity equal the potential loss (gain) of additional engagement in this activity.

[5] See Appendix 4.A for a detailed derivation of the HJB.

4.2.3 Optimal Behavior And Subjective Well-Being

To derive an explicit solution, we specify the instantaneous utility function as

$$u(k(t), b(t), q(t)) = k(t)^{\alpha} - \delta q(t)^{\beta} - \varepsilon b(t)^{\gamma}, \tag{4.8}$$

where $0 < \alpha < 1$, $\beta, \gamma > 1$ denote utility elasticities.

The occurrence of losses of control is assumed to be driven by the arrival rate

$$\mu^{l}(k(t), q(t)) = k(t)\lambda^{l}\left[q(t)\right]^{\zeta}, \tag{4.9}$$

while λ^l is a positive exogenous parameter and ζ is a parameter that controls whether the sensitivity to environmental cues increases over- or under-proportionally in the addictive state.

The arrival rate of rehabilitation is

$$\mu^{r}(b(t), q(t)) = \lambda^{r}(q(t))b(t) = \lambda^{r}(q(t))b(t) \tag{4.10}$$

with

$$\lambda^{r}(q(t)) = \begin{cases} \lambda^{r} & \text{for } q^{\text{ini}} < q(t) < q^{\max} \\ 0 & \text{otherwise,} \end{cases} \tag{4.11}$$

while λ^r is a positive exogenous parameter.

Note that the latter three specifications imply that the problem simplifies to a deterministic one for a "blanc sheet" individual, i.e. for a person with addictive state $q = 0$, because then[6]

$$\rho V(0) = k(0)^{\alpha} - \varepsilon b(0)^{\gamma}. \tag{4.12}$$

Clearly, this yields a corner solution, as we obtain from the FOCs

$$k(0) = k^{\max} \tag{4.13}$$

and

$$b(0) = 0, \tag{4.14}$$

while the intensity k is bounded from above by the instantaneous time restriction $l \geq k + b$, such that $k^{\max} = l$. If the individual is a blanc sheet, she can choose the maximum intensity of going out without running the risk of loosing control and becoming addicted.

Although our model can account for non-addictive individual's behavior, it is designed for explaining the behavior of addicted rather than non-addicted individ-

[6]See Appendix 4.B for the derivation of the HJB for the specific problem.

uals as the latter is not in the focus of our attention. Therefore the transition from being a non-addicted to being an addicted individual is not modelled explicitly.

Using the above specification the maximized HJB and the FOCs read for $q = q^{\text{ini}}$

$$\begin{aligned}\rho V(q^{\text{ini}}) = \left[k(q^{\text{ini}})\right]^{\alpha} - \delta[q^{\text{ini}}]^{\beta} - \varepsilon\left[b(q^{\text{ini}})\right]^{\gamma} \\ + k(q^{\text{ini}})\lambda^{l}\left[q^{\text{ini}}\right]^{\zeta}\left[V(q^{\text{ini}}+1) - V(q^{\text{ini}})\right],\end{aligned} \tag{4.15}$$

$$k(q^{\text{ini}}) = \left(-\frac{\lambda^{l}q^{\zeta}\left[V(q+1) - V(q)\right]}{\alpha}\right)^{-1/(1-\alpha)} \tag{4.16}$$

and

$$b(q^{\text{ini}}) = 0. \tag{4.17}$$

For $q^{\text{ini}} < q(t) < q^{\max}$ we obtain

$$\begin{aligned}\rho V(q) = k(q)^{\alpha} - \delta q^{\beta} - \varepsilon b(q)^{\gamma} + k(q)\lambda^{l}q^{\zeta}\left[V(q+1) - V(q)\right] \\ +\lambda^{r}b(q)\left[V(q-1) - V(q)\right],\end{aligned} \tag{4.18}$$

$$k(q) = \left(-\frac{\lambda^{l}q^{\zeta}\left[V(q+1) - V(q)\right]}{\alpha}\right)^{-1/(1-\alpha)} \tag{4.19}$$

and

$$b(q) = \left(\frac{\lambda^{r}\left[V(q-1) - V(q)\right]}{\gamma\varepsilon}\right)^{-1/(1-\gamma)}. \tag{4.20}$$

For an individual with addictive state $q = q^{\max} \Leftrightarrow k(q^{\max}) = 0$ the problem becomes

$$\rho V(q^{\max}) = -\delta[q^{\max}]^{\beta} - \varepsilon b(q^{\max})^{\gamma} + \lambda^{r}b(q^{\max})\left[V(q^{\max}-1) - V(q^{\max})\right], \tag{4.21}$$

$$k(q^{\max}) = 0 \tag{4.22}$$

and

$$b(q^{\max}) = \left(\frac{\lambda^{r}\left[V(q^{\max}-1) - V(q^{\max})\right]}{\gamma\varepsilon}\right)^{-1/(1-\gamma)}. \tag{4.23}$$

From a mathematical perspective is the system described in equations (4.15) until (4.23) a non-linear system of $n = q^{\max}$ equations in $n = q^{\max}$ unknowns (the values $V(.)$), which can be solved numerically.[7] For the numerical computation we use the parameter specification summarized in Table 4.1.

[7]For detailed information regarding the numerical approach see Appendix 4.C

Name of parameter	Notation	Value
Utility elasticity of going out (risky activity)	α	0.900
Utility elasticity of addictive state	β	1.100
Utility elasticity of therapy	γ	1.100
Preference parameter for addictive state	δ	0.100
Preference parameter for therapy	ε	0.900
Sensitivity parameter	ζ	0.100
Parameter of the arrival rate of losses of control	λ^l	0.130
Parameter of the arrival rate of rehabilitation	λ^r	0.100

Table 4.1: Parameter values for the numerical solution

Moreover we fixed the time preference rate, ρ, at 0.020, the instantaneous endowment with time, l, at 10.000, the instantaneous death probability, λ^{birth}, at 1/78, the initial condition, q^{ini}, at 1.000 and the highest possible addictive state, $q^{\max}$, at 48.000.

Optimal behavior is depicted in Figure 4.1. Note that we present here for the moment a numerical example using meaningful parameter values in order to gain a first impression of optimal behavior resulting from the model. Further below we calibrate the model using real world data. The solution shows us that if an individual is in addictive state $q = 1$ she optimally chooses an intensity, $k(q)$, of going out, at which the marginal utility of going out equals its expected marginal costs. At the same time she entirely renounces therapy, since she cannot improve her current state. The higher an individual's addictive state is, the less she decides to go out and expose herself to substance-related cues. Such behavior can be called optimal avoidance behavior (c.f. Section 3.3). The sharp bend in the right picture of Figure 4.1 results from the corner solution for therapy at $q = q^{\text{ini}}$. The optimal participation equals zero in this state, as therapy is, by assumption, not beneficial for an individual in this state. For $q^{\text{ini}} < q < q^{\max}$ therapy intensity increases non-monotonically in the addictive state. The reason for this non-monotony is mathematically the difference $V(q-1) - V(q)$ in equation (4.7), which also increases non-monotonically in q. Intuitively spoken, is the difference $V(q-1) - V(q)$ the potential utility gain that the individual obtains, if she has therapeutic success, i.e. if the process q^r jumps. This utility gain is the incentive to participate in therapy and it varies depending on the addictive state. In our numerical example the utility gain decreases for very low addictive states before it then increases for higher addictive states. Hence, it makes sense for the individual to limit the therapy intensity for very low addictive states and increase her therapy participation for higher addictive states. Interestingly, this non-monotony seemingly does not appear in the optimal intensity of going out, although the difference

$V(q+1)-V(q)$ appears in equation (4.6). The reason for this is the multiplication of the difference $V(q+1)-V(q)$ with $-q^{\zeta}$ in FOC of going out. The term $-q^{\zeta}$ decreases with an increasing rate in q and "overlays" the non-monotony. In other words, the development of optimal behavior as a function of the addictive state depends significantly on whether the sensitivity to environmental cues increases over- or under-proportionally in the addictive state on the one hand, and how the utility loss varies in the addictive state on the other hand.

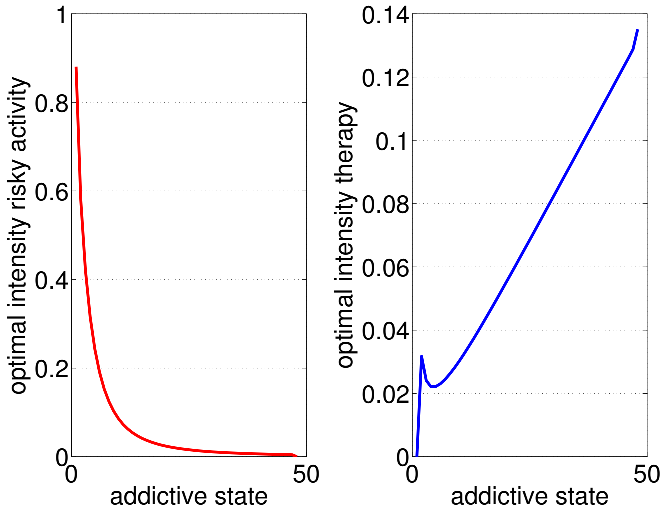

Figure 4.1: Optimal intensity of engaging in the risky activity and in therapy as functions of the addictive state

An individual's subjective well-being is illustrated in Figure 4.2. It shows subjective well-being as a function of the addictive state. Subjective well-being here is always negative (which is not a problem at all and only a matter of the choice of parameters) and decreases nearly linearly in the addictive state, reflecting the fact that an individual's well-being is lower the higher her addictive state is. There are several reasons for this. First, the individual optimally limits her engagement in the beneficial but risky activity and thus forgoes the benefits from its pleasure, second she participates more in therapy what comes a long with therapy costs, and finally the individual is very sensitive to environmental cues due to her high addictive state, what increases the risk of getting triggered. With her high participation in therapy the individual tries to increase the chance to recover from such a high deterioration.

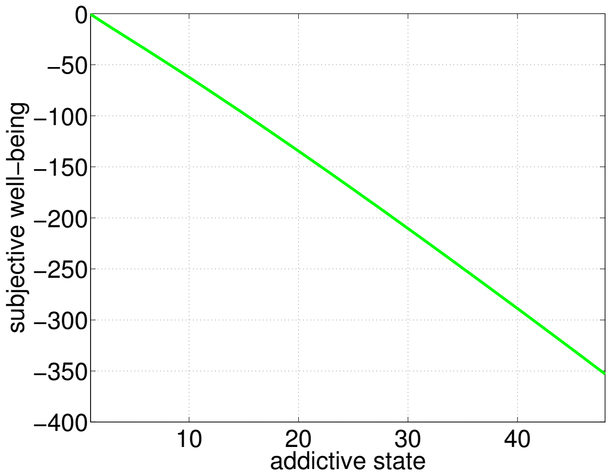

Figure 4.2: Subjective well-being as a function of the addictive state

4.3 Distributional Dynamics

Consider now, instead of looking at one individual we look at an entire population of $N > 1$ individuals. Addictive states are distributed initially among the individuals of the population according to a given initial distribution. For each individual the evolution the addictive state is driven by two stochastic processes. We want to understand, first, how addictive states are distributed across a population at any future point in time $\tau > t$ and second, what determines the evolution of the distribution of addictive states over time.

Before we tackle the first question, we have to make an important assumption. We assume that the size of the population remains constant over time. This assumption can have two different implication. Either we can consider individuals as dynastic households, where parents inherit their addictive state to their offspring. Or, we have to take deaths and births into consideration. As we imagine a population of representative agents anyway, not their age, but only their addictive state is the essential distinctive feature. Hence, instead of explicitly modelling deaths and births, it is sufficient to imagine that individuals at some random point in time step into an imaginary "fountain of youth", which resets their current addictive state to the initial state. The occurrence of this event is described by a Poisson process, $q^{\text{birth}}(t)$. We define the amplitude of the Poisson process for the "fountain of youth"-event as $-q(\tau) + q^{\text{ini}}$. Then we can formulate the evolution of the addictive state as follows.

$$dq(t) = dq^l(t) - dq^r(t) + \left(q^{\text{ini}} - q(t)\right) dq^{\text{birth}}(t), \tag{4.24}$$

Given this transition equation we can derive a system of Fokker-Planck equations (FPEs).[8] For each addictive state, $q^{\text{ini}} \leq q \leq q^{\max}$, there exists a FPE. For $q = q^{\text{ini}}$ we obtain

$$\begin{aligned} \frac{\partial p(q^{\text{ini}},\tau)}{\partial \tau} &= \mu^r(q^{\text{ini}}+1)p(q^{\text{ini}}+1,\tau) - [\mu^l(q^{\text{ini}}) + \mu^{\text{birth}}]p(q^{\text{ini}},\tau) \\ &+ \mu^{\text{birth}}\Sigma_{i=q^{\text{ini}}}^{q^{\max}} p(i,\tau), \end{aligned} \tag{4.25}$$

for a general $q^{\text{ini}} \leq q \leq q^{\max}$ the FPE reads

$$\begin{aligned} \frac{\partial p(q,\tau)}{\partial \tau} &= \mu^r(q+1)p(q+1,\tau) - [\mu^l(q) + \mu^r(q) + \mu^{\text{birth}}]p(q,\tau) \\ &+ \mu^l(q-1)p(q-1,\tau) \end{aligned} \tag{4.26}$$

and for $q = q^{\max}$ we have

$$\begin{aligned} \frac{\partial p(q^{\max},\tau)}{\partial \tau} &= -[\mu^r(q^{\max}) + \mu^{\text{birth}}]p(q^{\max},\tau) \\ &+ \mu^l(q^{\max}-1)p(q^{\max}-1,\tau), \end{aligned} \tag{4.27}$$

while $p(q,\tau)$ stands for the probability of being in state q at time τ. Via the law of large numbers we can interpret these probabilities as the share of individuals within a population who is in addictive state q at time τ, i.e. as an absolute number. Taken together the FPEs for all $n = q^{\max}$ states constitute a system of $n = q^{\max}$ equations in as much unknowns.

This system explains how and why the number of individuals who are in addictive state $q(t)$ changes over time. The LHS of equation (4.26) is the change of the number of individuals who are in state q at time τ. It is explained by the three terms on the RHS. The first term shows that the number of individuals who are in state q at time τ increases over time by the number of individuals who previously were in state $q+1$, but recovered with rate $\mu^r(q+1)$ at time τ. The second term explains why the number of individuals who are in state q at time τ decreases over time. It decreases due to three reasons. First, individuals who are in state q experience another loss of control with arrival rate $\mu^l(q)$ and thus enter addictive state $q+1$. Second, they recover with rate $\mu^r(q)$ and thus enter addictive state $q-1$. And third, they die (or step into the fountain of youth) and enter state q^{ini}. Finally, the third therm on the RHS of the FPE represents the last reason why the number of individuals in state q increases over time. Individuals who previously were in state $q-1$ experience another loss of control with arrival rate $\mu^l(q-1)$ and consequently enter addictive state q. Calling the positive terms

[8] See Appendix 4.D.1 for the detailed derivation.

on the RHS of (4.26) inflows and the negative terms outflows we can state that change of the number of individuals who are in state q at time τ is explained by the difference between the "inflows" into a state q and the "outflows" out of a state q. Consequently, $\partial p(q,\tau)/\partial\tau$ describes the "net flow". Note that in state $q = q^{\text{ini}}$ individuals cannot recover anymore as $\mu^r(b(t), q^{\text{ini}}) = 0$. This is why the expression describing the outflow is different in (4.25). Additionally there appears a sum on the RHS of this equation, which constitutes the part of the inflow into state q^{ini} due to the "fountain of youth"-event. Also in state $q = q^{\max}$ the outflow is different than for a general $q^{\text{ini}} \leq q \leq q^{\max}$, because individuals cannot reach or recover from a higher addictive state, otherwise the terminal condition would be violated.

The FPEs constitute a system of ordinary differential equations in $p(.)$, which can be solved, given a distribution $p(1,0)$, $p(2,0)$, $p(3,0)$, ..., $p(q^{\max})$ which serves as the initial condition for the system. This allows us then to predict the addictive states of all N individuals at each point in time τ.[9]

Once we are informed about the short-run distributional dynamics we should ask what in the long-run happens. Will the distribution converge to some stationary distribution or will it be degenerate? We obtain the long-run distribution by setting the change in the frequencies equal to zero, such that they are time-invariant. Thus, for $q = q^{\text{ini}}$ we obtain,

$$p(q^{\text{ini}}) = \frac{\mu^r(2)p(2,\tau) + \mu^{\text{birth}}}{\mu^l(1) + \mu^{\text{birth}}}, \tag{4.28}$$

for $q^{\text{ini}} < q < q^{\max}$,

$$p(q) = \frac{\mu^r(q+1)p(q+1,\tau) + \mu^l(q-1)p(q-1,\tau)}{\mu^l(q) + \mu^r(q) + \mu^{\text{birth}}}, \tag{4.29}$$

and for $q = q^{\max}$ we obtain

$$p(q^{\max}) = \frac{\mu^l(q^{\max}-1)p(q^{\max}-1,\tau)}{\mu^r(q^{\max}) + \mu^{\text{birth}}}. \tag{4.30}$$

We learn that in the long-run the number of individuals who are in state q converges to the ratio of inflows into state q relative to outflows away from state q.

Again, the system (4.28) - (4.30) constitutes a system of $n = q^{\max}$ equations in $n = q^{\max}$ unknowns, which we solve numerically.[10]

Figure 4.3 depicts a numerical example of the distributional dynamics of the

[9] For detailed information on the numerical solution see Appendix 4.D.2.
[10] A detailed description of the numerical approach is provided in Appendix 4.D.4.

addictive states. We considered that the individuals of our fictive population of size N are initially normally distributed across all possible addictive states, $q^{\text{ini}} \leq q \leq q^{\text{max}}$, with mean value $q = 25$ and a standard deviation of 6 at $t = 0$. This is represented by the green line in the figure. The black lines illustrate the evolution of the distribution in the course of time. Over time the distribution shifts continually towards lower addictive states. The evolution stops, as soon as the stationary distribution, drawn in blue, is reached. In the picture the adjustment process has not yet been fully completed after 100 years. In the stationary equilibrium the distribution is slimmer than the initial normal distribution and the new mean addictive state is at $q = 3$. So this example demonstrates a positive development, as the population is healthier than in the initial state. Note that the distributional dynamics are driven by the optimal behavior displayed by the individuals.

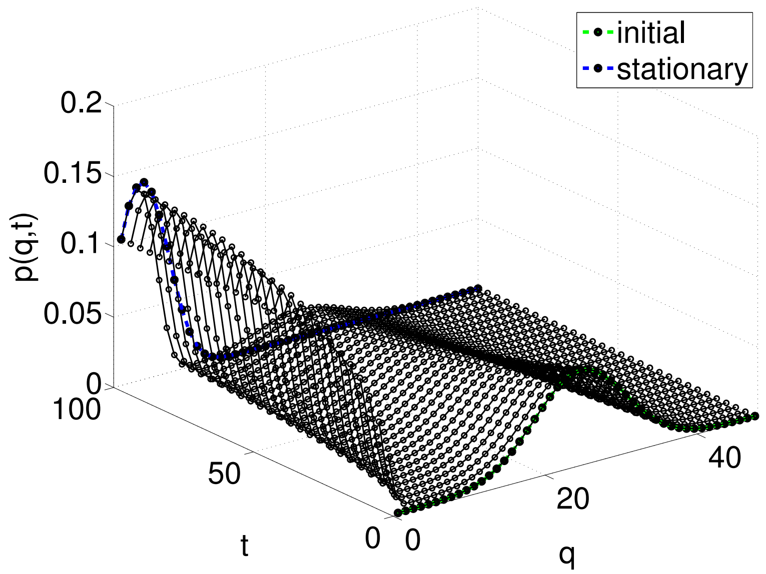

Figure 4.3: Time evolution of probability distribution

4.4 Welfare Analysis

In many countries around the world policies and legislations regarding addictive substances have been adopted. They involve e.g. prohibitions on consumption, ownership, trade, production (in most countries for heroin, cocaine, amphetamine, cannabis), age restrictions, (partial) advertising bans, non-user protection laws (smoking at the workplace and in public buildings), increased taxes on addictive substances (alcohol and tobacco), regulation on product contents and packaging (tobacco, "alcopops"), awareness-raising campaigns and prescription requirements (pharmaceuticals).[11]

So far we understand the short- and long-run dynamics of addiction. Now we can ask how and to what extent these interventions affect mental health and well-being of a population. This is why we devote this section to analyze welfare-related questions within the framework of our formal model.

The above-mentioned policy interventions can be attached to (at least) one of the following three objectives that might be pursued by a benevolent planner.

(a) protecting individuals against externalities,

(b) protecting boundedly rational or distorted individuals against suboptimal decisions,

(c) protecting individuals against mistakes.

It is a well-known fact that public policy interventions can be justified economically if objective (a) is pursued, provided that externalities exist. We do not doubt that externalities of addictive behaviors in fact exist and we also think that implementing externalities into an addiction framework would be highly interesting. However, we rather focus on the "externalities" of addictive behaviors on individuals' own mental health. Thus, we should not analyze a policy intervention with objective (a).

We should also not analyze an intervention with objective (b) since individuals in the current model are not boundedly but fully rational as they are not distorted in their perception (unlike for instance in Gruber and Köszegi (2001)) and also are fully informed about all consequences of their behavior. Thus, an economic rationale based on bounded rationality which would justify an intervention is not given. Moreover there exists a well-structured branch of literature which addresses this issue already.[12]

The novelty in BR's approach is that they consider the consumption of an addictive substance as a mistake, although the individual is unboundedly rational. So, we should focus on policy interventions with objective (c). What kind of interventions into individual behavior with the objective to prevent individuals from

[11] This list does not claim to be complete.

[12] To the most influential works of this category count, inter alia, Camerer et al. (2003) (asymmetric paternalism), Choi et al. (2003) (optimal defaults), Carroll et al. (2009) (optimal defaults and active decisions), Thaler and Sunstein (2003) (libertarian paternalism) Thaler and Sunstein (2009) (nudge) and O'Donoghue and Rabin (2006) (sin taxes). Beshears et al. (2008) explain that "human behavior is jointly determined by both normative preferences and other factors such as analytic errors, myopic impulses, inattention, passivity, and misinformation." Of course, if the distorting component is known, normative preferences can be identified and used to infer an adequate welfare norm.

failing do we know from the real world? Two quite famous examples can be found in London's traffic system. First, to prevent that tourist pedestrians accidentally get involved in accidents, bold letters were painted on the curbs telling them in which direction to look before stepping out in the street[13]. Second, the announcement "mind the gap" in the London Underground reminding passengers to mind the gap between the platform and the train, has the purpose to urge travellers to stay mindful when leaving the train to prevent serious accidents. In our model the most straightforward way to avoid mistakes is to eliminate their cause. If substance-related cues would not at all be part of an individual's environment, mistakes would not happen. But a full elimination of all environmental stimuli is practically utopian. However, beyond doubt, the introduction of advertising bans regarding certain addictive goods at least reduces the likelihood of occurrence of these cues. And not only the reduction of cues can help to make an environment "safer". Also the creation of counteracting cues (like in the examples above) can have a similar effect, given it lowers the probability of an individual to loose control. Nowadays many European countries adopted a law that requires cigarette companies to print deterrent pictures on cigarette packs. The idea is that the pictures of smoke-filled lungs and rotting teeth make the harmful effects of smoking more salient to potential consumers and thus mitigate the instantaneous probability of being triggered by seeing a cigarette packet. Further examples are awareness-raising campaigns like the German campaign "Alkohol? Kenn Dein Limit"[14] of the Federal Centre for Health Education[15] supported by the Private Health Insurance Association[16] (Bundeszentrale für gesundheitliche Aufklärung, 2016). To the actions undertaken pursuant to that campaign count inter alia the use of billboards and posters to provide warnings regarding excessive alcohol consumption and information about its harmful consequences. Moreover, there are examples of corporate social responsibility. Since May 2013 the international brewery group "Anheuser-Busch Inbev" decided voluntarily to print pictograms on their beer bottle labels signalizing the dangers of alcohol during pregnancy and of drink-driving (Anheuser-Busch InBev, 2016). Other than that, the legal prohibition of a certain addictive substance might have a similar inhibiting effect. Following BR also price increases may hinder cues of triggering individuals. Thus taxation can also be interpreted as a step to create counteracting cues.

In this context we want to ask the following questions. First, results an increased protection of individuals against mistakes from the implementation of the strategies mentioned in the previous paragraph? Second, can such an intervention be justified economically and is it welfare-enhancing? And if yes, third, what is

[13]Following BR's pedestrian example.
[14]in English: Alcohol? Know your limit.
[15]"Bundeszentrale für gesundheitliche Aufklärung" (BZgA)
[16]"Verband der Privaten Krankenversicherung e.V."

the optimal policy?

Following the argumentation above, we model exemplary the promulgation of an awareness-raising campaign as a decrease in the parameter of the arrival rate of losses of control, λ^l. As individuals in the model are entirely rational and fully aware of the consequences of their behavior including their mistakes, subjective well-being is an adequate welfare standard.

4.4.1 Comparative Statics

To obtain a first impression of the effects of an adjustment of λ^l on individual behavior and on subjective well-being, we analyze the comparative statics.

Within the framework of our numerical example we consider a decrease of the parameter of the arrival rate of losses of control, $\lambda^l_{\text{low}} = 0.120$, as well as an increase $\lambda^l_{\text{high}} = 0.140$, relative to the status quo, $\lambda^l = 0.130$ and ask, how the optimal intensity of going out, the optimal therapy intensity, subjective well-being and the stationary distribution of addictive states change, ceteris paribus.

Figures 4.4 and 4.5 illustrate the effects of the interventions on optimal behavior. The solid lines show the status quo for comparison. The dashed lines represent the effects of a reduction of the parameter of the arrival rate of losses of control (safer environment) on optimal behavior. Finally, the dotted lines illustrate the effects of the opposite scenario, an increase of the arrival rate of losses of control (riskier environment). The results show that individuals decide to go out more frequently, i.e. to take more risk, when they live in a safer environment. To understand the reason for this change in behavior we should remind ourselves that individuals are confronted with the trade-off between going out and enjoy the related pleasure and risking to loose control and increase their addictive state due to their exposure to substance-related cues. In consequence of a decrease in λ^l a utility loss due to a stochastic increase of the addictive state becomes, ceteris paribus, less likely (the RHS of the FOC (4.6) decreases). Through the endogeneity of the arrival rate, μ^l, individuals can actively influence the likelihood of a potential loss. They optimally increase the engagement in the risky activity until the FOC (4.6) holds. In the new optimum the arrival rate (4.9) is now higher for all q than it was in the status quo. Individuals compensate the additional risk through a higher participation in therapy. Participation in therapy serves as a means for preventing that individuals' addictive state climbs too high.

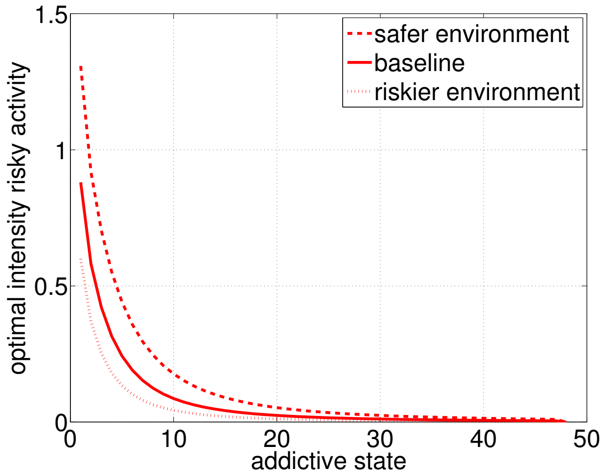

Figure 4.4: Optimal intensity of engaging in the risky activity depending on environment

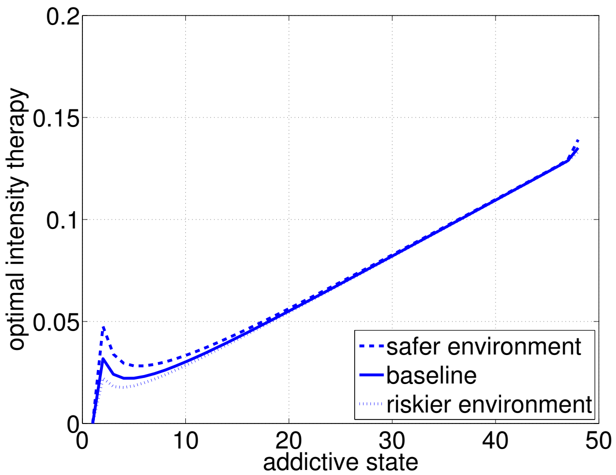

Figure 4.5: Optimal therapy intensity depending on environment

By contrast, a promotional campaign which propagates substance consumption has the opposite effect on individuals' optimal behavior, i.e. it leads to a lower intensity of both, going out and participation in therapy. The explanations above conversely.

Providing a safer environment obviously creates an incentive to stay more frequently in a potentially triggering environment and simultaneously to invest more time in safety behaviors and vice versa. The increasingly pressing question is what effects such public policy interventions have on subjective well-being. The effects of both public policy interventions on subjective well-being are depicted in Figure 4.6, which shows the differences in subjective well-being relative to the status quo (baseline). Living in a safer environment leads to an increase in subjective well-being for $q^{\mathrm{ini}} \leq q \leq q^{\max}$ while living in a riskier environment leads to a decrease in subjective well-being for $q^{\mathrm{ini}} \leq q \leq q^{\max}$. The increase in subjective well-being results from the possibility to enjoy more pleasure of going out in a safer environment and relatively less additional need for spending time in the disliked therapy sessions. Vice versa, a riskier environment decreases subjective well-being as individuals are forced to decrease the intensity of going out and to enjoy its related pleasures. This loss overweights the positive utility effect of a decrease in the participation in therapy. Hence, policy interventions that manage to reduce the capability of substance-related cues to trigger individuals enhance subjective well-being and are therefore economically justifiable, given subjective well-being is used as welfare standard. Note that the differences in subjective well-being are greater in lower addictive states than in higher ones. This follows directly from the fact that the differences in optimal behavior are also greater in lower than in higher states.

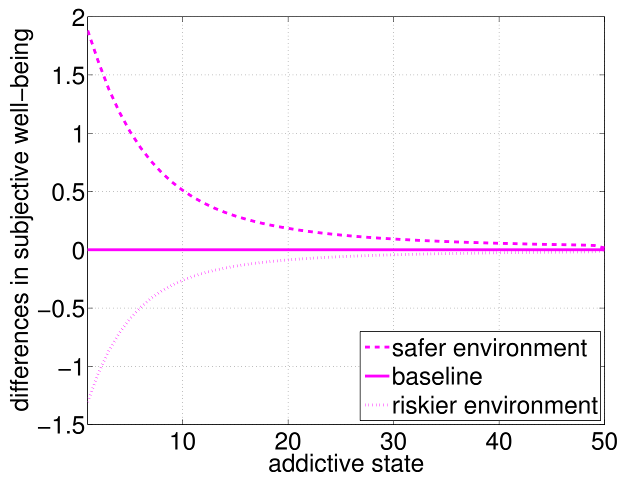

Figure 4.6: Subjective well-being depending on environment

The next question to be answered is what distributional effects the considered policy interventions have. We tackle this question by means of the methods described in Section 4.3. Given the optimal behavior in the different environments, we can derive the stationary distributions which result from the interventions. The results are depicted in Figure 4.7. The solid line in this figure represents the stationary distribution of addictive states across a population in the status quo, i.e.

prior to a policy intervention. Although a policy intervention that creates a safer environment increases subjective well-being for all addictive states, it leads to a deterioration of a population's distribution of addictive states on the aggregate level and vice versa for a policy intervention that creates a riskier environment. In the new stationary equilibrium of the safer environment, depicted by the dashed line, the share of individuals in lower addictive states decreased, while the share of individuals in higher addictive states increased. At first glance, this development seems counter-intuitive. But the direction of these distributional effects becomes obvious, when one considers the change of optimal behavior in reaction to the policy intervention. In response to the intervention, individuals go out more often such that the absolute frequency of losses of control increases (although the relative frequency decreased). The latter effect overweights the opposite effect of additional participation in therapy after the intervention. This explains the shift of the distribution towards higher addictive states. The distributional effects are reversed in case of a riskier environment. The resulting stationary distribution in a riskier environment, depicted by the dotted line in Figure 4.7, shifts towards lower addictive states.

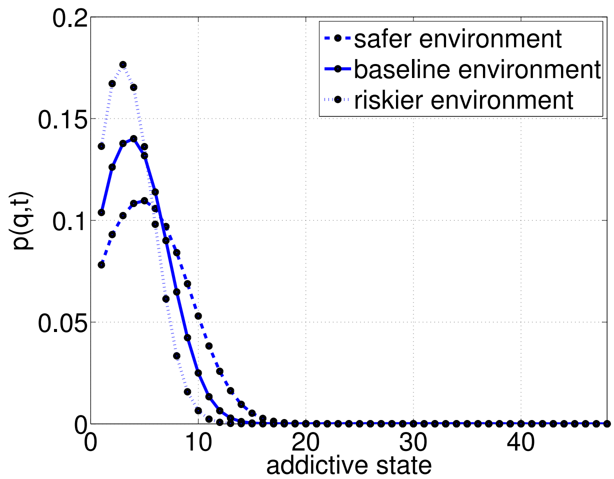

Figure 4.7: Impact of environment on stationary distribution

So far, we have found that a safer environment, ceteris paribus, increases subjective well-being for all addictive states $q^{\text{ini}} \leq q \leq q^{\max}$, however, on the aggregate level it leads to a situation in which the share of individuals with a higher addictive state is greater and the share of individuals with a lower addictive state is lower.

4.4.2 A Benevolent Planner's Problem

This subsection's purpose is to present an approach to deriving an optimal public policy decision regarding the environment in which individuals live. Although we do not explicitly derive the optimal public policy here, we believe that a description of a benevolent planner's decision problem is a good starting point for future research.

We consider a benevolent planner who maximizes, by choosing λ^l, the probability weighted[17] sum of aggregate subjective well-being (which takes individuals optimal behavior as given) minus the aggregate adjustment costs, $\Phi(\lambda^l) = |\lambda_0^l - \lambda^l|$, while λ_0^l reflects the status quo and κ is a weighting factor reflecting the cost intensity of interventions. Note that we would obtain a corner solution ($\lambda_{\text{opt}}^l = 0$) in absence of adjustment costs. It is hard to imagine that substance-related cues can be entirely and at no costs suppressed, therefore adjustment costs reflect the costs of the implementation of the planner's decision which increase in the extent of the implemented adjustment. Formally,

$$\max_{\lambda^l} W(t) = \max_{\lambda^l} \left\{ N \sum_{i=1}^{q^{\max}} p(i,t) V(\lambda^l, i, t) - \kappa \Phi(\lambda^l) \right\}. \tag{4.31}$$

The FOC reads

$$\begin{aligned} N \sum_{i=1}^{q^{\max}} \Big[p(i,t) \Big[k'(\lambda^l, q) + \mu'^l(\lambda^l, q) \left[V(\lambda^l, i+1, t) - V(\lambda^l, i, t) \right] \\ + \mu^l(\lambda^l, q) \left[V'(\lambda^l, i+1, t) - V'(\lambda^l, i, t) \right] \Big] \Big] - \kappa \Phi'(\lambda^l) = 0 \end{aligned} \tag{4.32}$$

The benevolent planner optimally chooses λ^l such that the marginal gain in subjective well-being equals the marginal costs of adjustment. Note that this optimization problem is formulated in a static fashion. The solution crucially depends on the current probability distribution of addictive states. As we have found out that the probability distribution reacts to changes in the arrival rates of losses of control λ_{opt}^l would need to be adjusted at each instant in time.

Thus, we formulate a more general version of an optimization problem which does not neglect the time evolution of the probability distribution of addictive

[17] Note that we do not apply the law of large numbers here and thus interpret $p(i, \tau)$ as the probability for an individual to be in state q at time τ.

states. It reads as follows

$$\max_{\{\lambda^l(\tau)\}_{\tau=t}^{\infty}} E_t \int_t^{\infty} e^{-\rho[\tau-t]} W(p(\tau)) d\tau =$$
$$\max_{\{\lambda^l(\tau)\}_{\tau=t}^{\infty}} \left\{ E_t \int_t^{\infty} e^{-\rho[\tau-t]} \left[N \sum_{i=1}^{q^{\max}} f(p(\lambda^l(\tau), i, \tau), V(\lambda^l(\tau), i, \tau)) - \kappa \Phi(\Delta\lambda^l(\tau)) \right] d\tau \right\}, \tag{4.33}$$

subject to

$$\begin{aligned} \frac{\partial p(\lambda^l(\tau), q, \tau)}{\partial \tau} &= \mu^r(q+1) p(\lambda^l(\tau), q+1, \tau) \\ &- [\mu^l(\lambda^l(\tau), q) + \mu^r(q) + \mu^{\text{birth}}] p(\lambda^l(\tau), q, \tau) \\ &+ \mu^l(\lambda^l(\tau), q-1) p(\lambda^l(\tau), q-1, \tau), \end{aligned} \tag{4.34}$$

and given an the initial conditions $\lambda^l(t) = \lambda^l_t$ and for the initial probability distribution $p(\lambda^l(t), q, t) = p_t^{\text{ini}}$.

Function $f(.)$ denotes the objective function and the change in the arrival rate of losses of control at time t is $\Delta\lambda^l(t)$. The second problem is a highly complex optimization problem and its solution is the optimal policy function, i.e. the optimal time path of the arrival rate of losses of control as a function of the probability distribution of addictive states, $\lambda^l(p(\tau))$. As mentioned above is the intention of this section not to solve such optimization problems explicitly but rather to make interesting suggestions for future research. Therefore we leave it at that for now.

4.5 An Application: Binge Drinking In Austria

In order to assess the quantitative usefulness of our framework, we apply it on empirical data on "binge drinking" behavior in Austria. Binge drinking is a form of hazardous alcohol consumption explicitly aiming at the drunkenness of the consumer. We decided to use data on binge drinking behavior rather than data on alcohol consumption in general, because binge drinking has nothing to do with thoughtful and cautious alcohol consumption but unambiguously constitutes a misuse of alcoholic beverages and thus matches the interpretation of substance consumption as a loss of control in our theoretical framework very well. The reason why we picked Austria is twofold. First, according to the individual country profiles of the "Global status report on alcohol and health" (World Health Organization, 2014b, p.198) Austria counts worldwide to the nations with the highest amounts of alcohol consumption per capita of the 15+ population (10.3 litres of pure alcohol in total, 13.8 litres of pure alcohol for drinkers only, both in 2010), a relatively low share of abstainers (22.6% of the overall 15+ population within the last 12 month) and an alarmingly high share of binge drinkers (40.5% of the overall 15+ population, 52.4% drinkers only, both in 2010). At the same time in Austria exists no legally required health warning labels on alcohol, which makes the simulations of public policy interventions further below even more interesting. The second reason for picking Austria is rather trivial as it simply concerns the availability of good data, which we will briefly describe in the following subsection.

4.5.1 The Data

The data we use were mainly collected through "Computer Assisted Telephone Interviewing" between October 2013 until June 2015 by STATISTIK Austria (2015, p.186 f.) on behalf of the Austrian Federal Ministry of Health and are part of the main results of the Austrian Health Interview Survey (ATHIS). In total $15,771$ randomly selected persons aged 15 and over of the resident population in Austria were interviewed. In the course of the survey the frequency of the respondents' binge drinking behavior during the last twelve months was recorded. In doing so, binge drinking was measured (as usual) as the consumption of 60 grams of alcohol or more (or six or more alcoholic beverages) on a single occasion. Table 4.2 summarizes the results. We truncated the original data to the left for persons who never did binge drinking in their lives and normalized the data afterwards such that the resulting proportions refer to the total number of all persons, N, who have indicated that they consumed alcohol within the last 12 months and did binge drinking at least once in their lives. The reference quantity projected to the entire Austrian population is then $N = 5,095,300$ persons.

Frequency of binge drinking	Proportion
Not within the last 12 months	42.34%
Less than once a month	31.35%
Once a month	14.97%
On two to three days a month	7.95%
On one to two days a week	2.69%
On three to four days a week	0.35%
On five to six days a week	0.12%
Almost every day or every day	0.23%

Table 4.2: Binge drinking frequency in Austria

4.5.2 Calibration

The first step of the empirical application is the calibration. Once the parameters are calibrated we can then predict the effects of a policy intervention in a second step. The empirical distribution of binge drinking frequencies gives us eight targets to be met. In our opinion it is very likely that the reported frequencies are positively correlated to the individual's addictive states. Moreover we set $q^{\max} = 48$ and assume that each of the eight drinking bahaviors can be assigned to a range of six addictive states, i.e. $\Sigma_{q=q_{\text{ini}}}^{6} p(q)$ corresponds to the proportion of persons who did not do binge drinking within the last 12 months, $\Sigma_{q=6}^{12} p(q)$ corresponds to the proportion of persons who did binge drinking less than once a month, etc.

We have eight parameters to be calibrated and as many targets. The calibration method is as follows. We require that each of the eight above mentioned sums over six addictive states equals the value of its corresponding proportion of persons displaying a particular binge drinking frequency. Defining x as a vector of n parameters to be calibrated and $f_i(x)$ as the i-th of n residual functions depending on the parameter values, this gives us the following minimization problem to be solved

$$\min_x \left\{ f_1(x)^2 + f_2(x)^2 + \cdots + f_n(x)^2 \right\}. \tag{4.35}$$

As in Section 3.6.1 we use Matlab's routine *lsqnonlin* to solve this problem numerically. The latter is a non-linear least-squares method. In other words, the routine minimizes the sum of the squared function values, i.e. the sum of the squared residuals, by choosing the parameter values for $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \lambda^l, \lambda^r$, while the residual functions are defined as

$$f_1(x) \equiv \sum_{i=1}^{6} p(i) - \sum_{i=1}^{6} p^{\text{target}}(i) \tag{4.36}$$

$$f_2(x) \equiv \sum_{i=7}^{12} p(i) - \sum_{i=7}^{12} p^{\text{target}}(i) \tag{4.37}$$

$$\vdots$$

$$f_6(x) \equiv \sum_{i=43}^{48} p(i) - \sum_{i=43}^{48} p^{\text{target}}(i) \tag{4.38}$$

After the calibration all parameters have meaningful values and are in the range one would expect. The parameters are summarized in Table 4.3.

Name of parameter	Notation	Value
Utility elasticity of going out (risky activity)	α	0.780
Utility elasticity of addictive state	β	1.089
Utility elasticity of therapy	γ	1.085
Preference parameter for addictive state	δ	0.086
Preference parameter for therapy	ε	0.914
Sensitivity parameter of the arrival rate of losses of control	ζ	0.082
Parameter of the arrival rate of losses of control	λ^l	0.116
Parameter of the arrival rate of rehabilitation	λ^r	0.085

Table 4.3: Calibrated parameter values

Figure 4.8 illustrates the calibration results and their quality of fit. Note that the residuals are expressed in absolute values and are not normalized, i.e. they are not in relation to the size of the area the corresponding integral reflects. The largest deviations from zero are smaller than $1.4e^{-2}$, which is appropriate for testing the quantitative usefulness of the model.

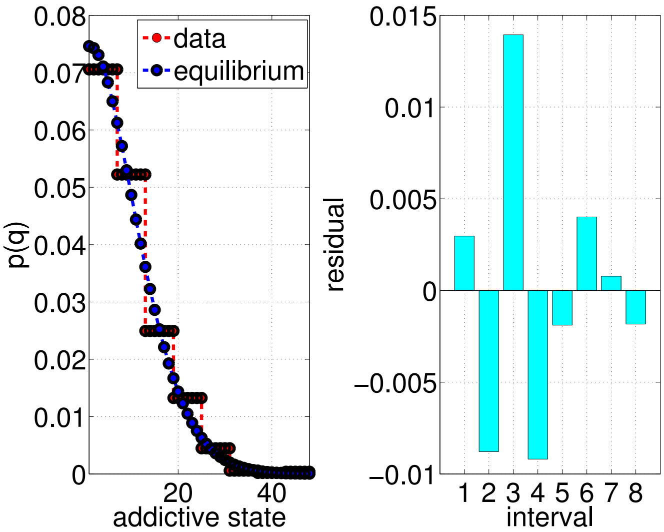

Figure 4.8: Target and equilibrium distribution as well as residuals

4.5.3 Policy Intervention

As a policy intervention we consider an awareness-raising campaign on the risks associated with binge-drinking. In accordance to Section 4.4, the campaign's objective is to protect individuals against mistakes. We assume that the campaign eliminates 20% of all substance-related cues and thus makes individual's environment safer. Formally, we assume a decrease in the arrival rate of losses of control by 20%. The questions we ask are as follows. First, to what extent will individuals engage more in going out and in therapy participation? Second, to what extent increases subjective well-being? Third, how affects the intervention the stationary distribution of addictive states? And finally, how long does it take to arrive at the new stationary distribution?

Figure 4.9 illustrates the absolute and relative changes in the optimal intensities of going out and therapy participation in consequence of an awareness-raising campaign. The graphic shows that relative change in the intensity of going out is hump shaped and varies between approximately 77% and 156%. That is, for a wide range of addictive states, the individual optimally goes out more than twice as frequently than before. At $q = 48$, the optimal intensity equals zero before and after the intervention due to the terminal condition. The relative change in the optimal therapy intensity is inversely hump shaped and the size of the relative change varies between an increase of 20% and 216%. At $q = 0$ both therapy intensities, before and after the campaign, are zero, as the campaign has no effect on these intensities.

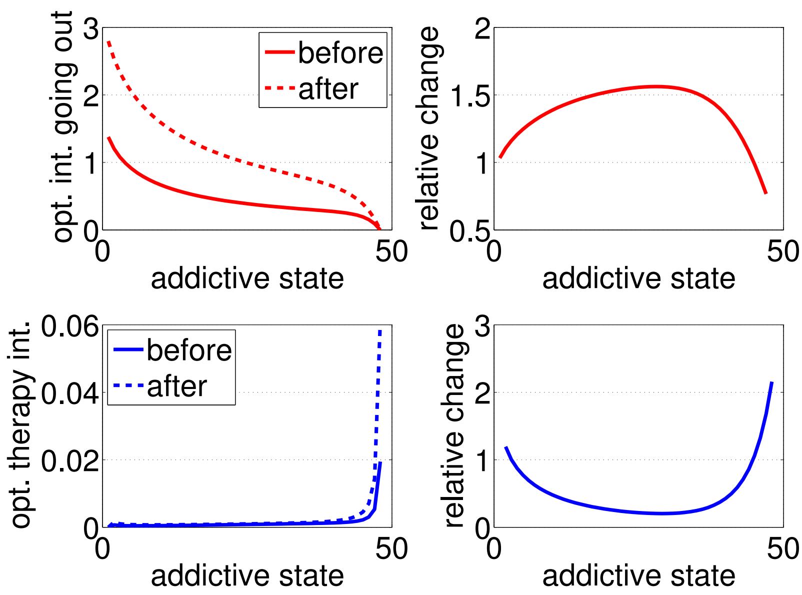

Figure 4.9: Changes in optimal intensities of engaging in the risky activity and therapy in consequence of an awareness-raising campaign

What effect have these behavioral changes on subjective well-being? The absolute and relative changes in subjective well-being are depicted in Figure 4.10. Interestingly, from a relative perspective the gain in subjective well-being is distributed quite unequally across the addictive states. While individuals with addictive states greater and equal 6 gain less than 50% in subjective well-being, individuals in state $q = 3$ benefit the most from the campaign. They gain enormous 373% in their subjective well-being. The reason for that is that individuals in state $q = 3$ are borderline cases whose subjective well-being is near zero before the intervention and significant different from zero afterwards. In general, we can state that individuals in lower addictive states tend to benefit more than individuals in higher addictive states.

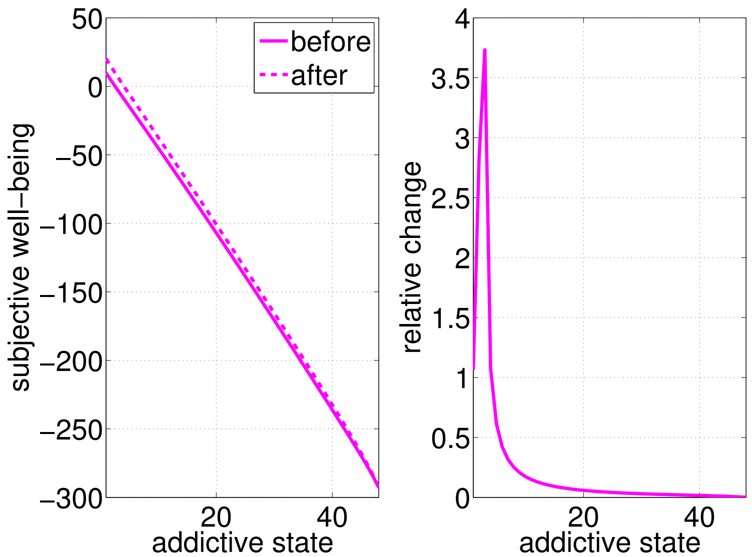

Figure 4.10: Absolute and relative change in subjective well-being as a result of an awareness-raising campaign

Figure 4.11 represents the new equilibrium distribution of addictive states (blue) compared to the old one (green). The comparison clearly shows that the share of individuals with addictive states below 14 diminishes, while the share of individuals in higher addictive states, $q \geq 14$, increases. Note that also the share of individuals in very high addictive states increases. In the long run the share of individuals in addictive states 1 until 13 shrinks from initially 77% to 56% and accordingly the share of individuals in addictive states 14 until 48 grows from 23% to 44%.

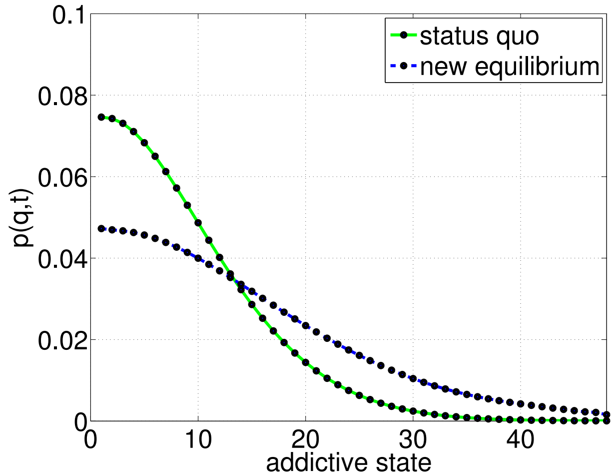

Figure 4.11: New and old stationary equilibrium distribution

Figure 4.12 yields information about the duration of the adjustment process from the old to the new stationary equilibrium. The new equilibrium distribution is reached not earlier than after 300 years.[18] At first glance this seems to be extremely long. However, when assessing this we should take the relative difference of the distribution of addictive states at time t to the new stationary distribution into consideration. This is drawn in Figure 4.12. The graphic shows that at time $t = 0$ the difference is quite high at high addictive states (up to -100%) and about half as large at medium and low addictive states. The reason for that is that in the status quo the share of individuals in high addictive states is extremely small such that the relative difference to the new stationary distribution becomes very large. Vice versa, this holds for medium and low addictive states. Thus, the development from the status quo to the new stationary equilibrium distribution is characterized by building up a certain mass at the very high addictive states. The latter can be explained by the FPE (4.26). Applied on a high addictive state, say $q = 45$, we see that the inflow into this state is very small, for two reasons, first individuals keep the arrival rate of losses of control very low by choosing to go out rather seldom at $q = 44$ and second, since there is nearly no mass at $q \geq 46$, there are only few individuals who can recover and enter $q = 45$. At lower addictive states, however, exists more mass, which underlies the dynamics and the arrival rates of losses of control and recovery have higher values due to individual's optimal choice of the control variables.

[18]In an analytical context, the stationary equilibrium will be reached ad infinitum. At this point, however, we argue in a numerical context. Thus "reaching" means that the deviation of the distribution at time t from the stationary distribution is adequately small.

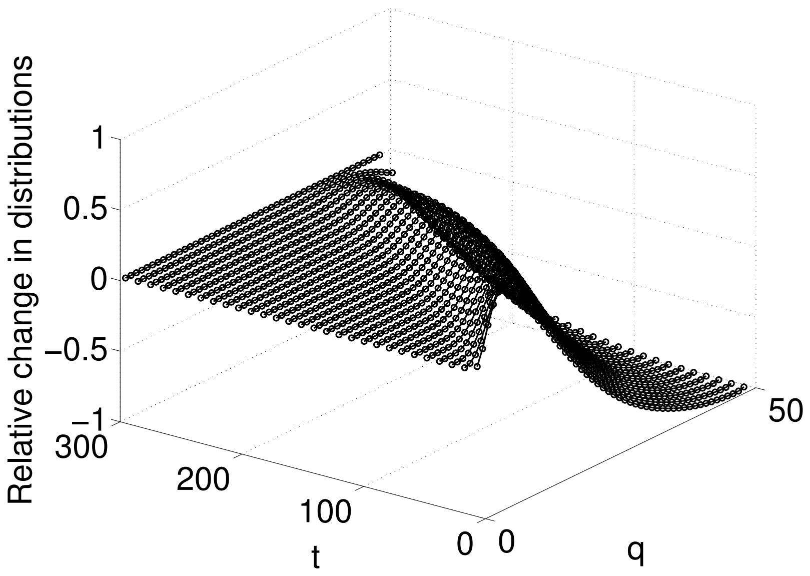

Figure 4.12: Adjustment process from the old to the new stationary equilibrium distribution

4.6 Conclusion

In many societies substance-related addictions are a problem which cannot be ignored. Some addictive substances like alcohol and tobacco are more popular (alone because they are not illegal in many countries), whereas others, mainly illicit substances, e.g. heroin or cocaine, are consumed by a rather small share of the society but often unfold their destructive or fatal effects even quicker. Those who are concerned suffer sooner or later under severe physical and mental health problems, which is associated with tangible and intangible costs for the individuals and the society as a whole.

Theoretical economic research on addiction can look back on a history of almost 30 years (considering Becker and Murphy (1988) as its starting point). While the most recent model of addiction developed by Bernheim and Rangel (2004) explains addictive behavior on the micro level very well, it has not yet been applied to understanding the developments on the macro level. The latter is the main concern of the present paper. Our primary goal was to understand how addiction is distributed among a population and what the main drivers behind the distributional dynamics of addiction are. Beyond that we applied our findings to analyze the effects of current public policy strategies with the aim of tackling addiction and reducing individual and societal pain. In order to achieve our goal, we formulated a continuous time version of BR's model, in which individuals not only make binary choices when deciding how to allocate their time. In our framework individuals choose frequencies. That is, individual's optimal choices provide us with level information rather than qualitative information only. On the individual level we have found that individuals become more carefully the higher their addictive state is, i.e. they limit their engagement in risky activities that might increase their addictive state and invest instead in the chance of future recovery. Subjective well-being decreases with an increasing addictive state as individuals enjoy less instantaneous pleasure from engaging in the risky activities and suffer more instantaneous pain from physical and mental deterioration.

On the aggregate level we learn that the distribution of addictive states as well as their dynamics crucially depend on how much risks individuals take and how much they engage in health care behaviors. The ratio of risk taking to care taking behavior influences the in- and outflows of individuals into and out of each addictive state. In the long-run the ratio of in- and outflows determines the stationary distribution.

Possible approaches to intervene and to improve the current situation are much debated in research, policy and media. Public policy interventions normally change the environment individuals live in. Many of them which are currently debated or have already been put into practice make environments safer, like advertising bans for addictive substances, health warnings on cigarette packages, awareness raising campaigns. A safer environment allows individuals to adjust the ratio of risk taking to care taking behavior, such that they can enjoy more instantaneous pleasure. Thus, public policy interventions which create safer environments enhance subjective well-being.

However, such interventions contribute to a deterioration of a populations' health condition. This result is quite counter-intuitive and at first glance it sounds even impossible. But we should understand that the absolute number of individuals who are in higher addictive state increases, but also the frequency with which individuals engage in the risky activity. At this point, it is important to mention that it would be wrong to conclude that all advertising bans etc. are inappropriate policies. On the contrary, in fact they bring the desired result as they decrease the chance that an individual exposed to a substance-related cue yields to temptation. From the adjustment of individuals' behavior results then a change in the distribution of addictive states on the aggregate level. The latter might be undesired. This reveals, however, the necessity of additional steps to be undertaken to steer this development into a desired direction.

Moreover our results are particularly interesting in the light of the debate about printing warning labels on tobacco and alcohol products. While some of the producers vehemently resist against the introduction of regulations, others, like the brewery group Anheuser-Busch Inbev deliberately and voluntarily decide in favour of labelling their own products and potentially benefit from this decision beyond the pure image improvement through the individuals' additional willingness to take risks.

To receive an impression of the quantitative usefulness of the model we applied it to binge drinking behavior in Austria. Austria belongs worldwide to the countries with one of the highest amounts of alcohol consumption per capita and has not yet implemented any legal requirements for health warning labels on alcohol. We calibrated our model such that the stationary distribution of addictive states matches the empirical distribution measured by STATISTIK Austria (2015, p.186f.). The stationary distribution was then taken as an initial equilibrium distribution and thus as a starting point to simulate an awareness-raising campaign (or a similar policy intervention with the aim of protecting individuals against mistakes). The results show that on the individual level the intensity of engaging in risky activities increases by $77 - 156\%$, while optimal participation in therapy increases by $20 - 216\%$ (depending on the addictive state). The gain in subjective well-being strongly depends on the individuals' addictive state. Individuals in low addictive states tend to gain more (up to 373%) subjective well-being than individuals in higher addictive states (only up to 50%). Due to the intervention in the long term the share of individuals in lower addictive states shrinks from initially 77% to 56% and accordingly the share of individuals in higher addictive states grows from 23% to 44%. The adjustment process after an intervention takes as a whole about 300 years, which is in deed extremely long. However and against a widely-held view, this computation shows that public policy interventions do not necessarily develop their full effect immediately or within a very short horizon of a few years but, in fact, only within a very long time horizon.

To provide an outlook on possible future research, we outlined a benevolent planner's problem and demonstrated how the continuation the welfare analysis may look like. The idea is that the benevolent planner maximizes the probability weighted expected aggregate subjective well-being of a population minus the aggregate adjustment costs, which result from an intervention, taking the distributional dynamics of the addictive states into consideration. The solution of the planner's problem yields the time-path of the optimal policy. Future approaches could also consider other forms of policy interventions, e.g. a reduction of the amplitude of a Poisson jump, which corresponds to a reduction of the extent of losses of control. According regulations in the real word are the introduction of a blood alcohol limit, a prohibition of consumption in public, the provision of supervised consumption rooms for heroin addicts or the establishment of methadone programmes.

Our approach is limited to explain cue-elicited substance consumption and does not explain non-triggered substance consumption. Certainly, our approach looses some explanatory power due to this limitation, but since we focus on behaviors that constitute a serious societal issue we consider this limitation as appropriate. Other than that there are existing well-developed approaches in the economic literature, discussed in greater detail in the related literature section, which focus on non-triggered substance consumption. However, in this context we should address another important issue. Due to our focus on cue-elicited substance consumption our model does not explain the genesis of addictions. Although we think that this would be highly interesting, it goes far beyond the scope of this paper. The genesis of addictions often has deeper psychological reasons which are not taken into account in the current framework. But we think this would be a very interesting and enriching extension, which we leave for future research.

Appendix

This appendix contains all analytical derivations and descriptions of the numerical approaches.

4.A The Optimal Control Problem In General Form

The general form of the Hamilton-Jacobi-Bellman (HJB henceforth) equation reads[19]

$$\rho V(q) = \max_{k} \left\{ u(k, b, q) + \frac{1}{dt} E_t dV(q) \right\}. \tag{A.1}$$

The differential of $V(q)$ can be derived given the transition equation of addiction in (4.1) and the change of variable formula[20]

$$dV(q) = [V(q+1) - V(q)] \;\; dq^l - [V(q-1) - V(q)] \;\; dq^r, \tag{A.2}$$

Applying expectations to the differential basically consists of applying expectations to the dq-terms. Generally speaking, $E_t dq = \mu dt$ where $\mu^l(k, q)$ and $\mu^r(b, q)$ are the arrival rates of the Poisson processes q^l and q^r, respectively. Plugging this into the general HJB equation (A.1) finally gives

$$\rho V(q) = \max_{k,b} \left\{ u(k, b, q) + \mu^l(k, q) \left[V(q+1) - V(q)\right] + \mu^r(b, q) \left[V(q-1) - V(q)\right] \right\}. \tag{A.3}$$

The first order conditions define the optimal intensities of engaging in going out,

[19] Time indices are suppressed from now on.

[20] see e.g. Wälde (2012), ch. 10.2.2: If there is a function $F(x, t)$, where variable x is driven by a stochastic process, then the differential of F is given by

$$dF(t, x) = \frac{\partial}{\partial t} F(t, x) dt + \frac{\partial}{\partial x} F(t, x) a(.) dt + \{F(t, x + b(.)) - F(t, x)\} dq.$$

k, and participating in therapy, b,

$$\frac{\partial u(k,b,q)}{\partial k} = -\frac{\partial \mu^l(k,q)}{\partial k}\left[V(q+1) - V(q)\right] \tag{A.4}$$

and

$$\frac{\partial u(k,b,q)}{\partial b} = -\frac{\partial \mu^r(b,q)}{\partial b}\left[V(q-1) - V(q)\right]. \tag{A.5}$$

4.B The Specific Optimal Control Problem

Instantaneous utility is specified as

$$u(k(t),b(t),q(t)) = k(t)^\alpha - \delta q(t)^\beta - \varepsilon b(t)^\gamma \tag{B.1}$$

Given this specification, the HJB reads[21]

$$\begin{aligned}\rho V(q) = \max_{k,b} \big\{ & k^\alpha - \delta q^\beta - \varepsilon b(t)^\gamma + k\lambda^l q\left[V(q+1) - V(q)\right] \\ & + b\lambda^r(q)\left[V(q-1) - V(q)\right]\big\},\end{aligned} \tag{B.2}$$

while the FOC are

$$k(q) = \left(-\frac{\lambda^l q\left[V(q+1) - V(q)\right]}{\alpha}\right)^{-1/(1-\alpha)} \tag{B.3}$$

and

$$b(q) = \left(\frac{\lambda^r(q)\left[V(q-1) - V(q)\right]}{\gamma\varepsilon}\right)^{-1/(1-\gamma)}. \tag{B.4}$$

The maximized HJB equation reads

$$\rho V(q) = k(q)^\alpha - \delta q^\beta - \varepsilon b(q)^\gamma + k(q)\lambda^l q\left[V(q+1) - V(q)\right] + b(q)\lambda^r(q)\left[V(q-1) - V(q)\right] \tag{B.5}$$

4.C The Numerical Approach Of Solving The Optimization Problem

Let $\boldsymbol{v} = [v_1, v_2, \dots v_n]$ be a vector, its elements $v_1, v_2, \dots v_n$ describe the values of optimal behavior in state $q = [1, 2, \dots n]$, while n corresponds to the highest possible addictive state $q^{\max}$.

[21] Again, time indices are suppressed from now on.

The general structure of the equation system we are aiming to solve reads

$$\begin{aligned} 0 &= f(v_1, v_2) \\ 0 &= g(v_1, v_2, v_3) \\ 0 &= g(v_2, v_3, v_4) \\ &\vdots \\ 0 &= h(v_{n-1}, v_n), \end{aligned} \tag{B.6}$$

while $f(.)$, $g(.)$ and $h(.)$ are non-linear equations in their arguments. Each of these n equations is a maximized Hamilton-Jacobi Bellman equation fixed at a given state $q = [1, 2, \ldots n]$. The left-hand side of each equation is the residual and thus set equal to zero. As a whole it is a system of n equations in n unknowns. Note that we reduced the number of equations and unknowns by using maximized HJBs instead of HBJs to be maximized. The latter would double the number of unknowns and equations and makes the problem numerically less tractable and more difficult to solve.

Given an initial guess, $v^{\text{ini}} = \left[v_1^{\text{ini}}, v_2^{\text{ini}}, \ldots v_n^{\text{ini}}\right]$, we solve the system (B.6) with Matlab's built-in solver *fsolve*.

4.D The Evolution Of A Populations' Addictive States

In general the method applied here closely follows the five-step procedure described in Bayer and Wälde (2013) and in Nagel (2013, p.9ff.).

4.D.1 Derivation Of The FPE

Step 1 (An Auxiliary Function And Its Time-Evolution)

The time-evolution of addictive states is described by

$$dq(t) = dq^l(t) - dq^r(t) + \left(q^{\text{ini}} - q(t)\right) dq^{\text{birth}}(t), \tag{B.1}$$

where the first term on the RHS describes an increase in the addictive state in consequence of the occurrence of a loss of control, the second term is the rehabilitation process that decreases the addictive state endogenously through therapy and the third term is the "fountain of youth"-process which we take into consideration to keep the size of the population constant on the aggregate level. The evolution of the state variable is driven by stochastic processes with discrete realizations, which leads to a special class of FPEs, which describe the time-evolution of probability distributions (instead of the dynamics of densities in case of continuous realizations). More detailed explanations regarding the different classes of FPEs and particular examples of FPEs resulting from Poisson processes with discrete realizations are provided in Birkner and Wälde (2016, ch.8). Therefore this derivation also follows the approach outlined there.

Given optimal behavior, i.e. $k(t) = k(q(t))$ and $b(t) = b(q(t))$, the corresponding arrival rates are as follows.

$$\mu^l(q(t)) = k(q(t))\lambda^l q(t), \tag{B.2}$$

with $\lambda^l > 0$,

$$\mu^r(q(t)) = \lambda^r(q(t))b(q(t)) = \lambda^r(q(t))b(q(t)) \tag{B.3}$$

with

$$\lambda^r(q(t)) = \begin{cases} \lambda^r & \text{for } q^{\text{ini}} < q(t) < q^{\max} \\ 0 & \text{otherwise,} \end{cases} \tag{B.4}$$

while $\lambda^r > 0$ and

$$\mu^{\text{birth}} = \lambda^{\text{birth}}. \tag{B.5}$$

Let there be an auxiliary function $f(q(t))$ with bounded support $S \subset \mathbb{R}^{\geq 0}$, i.e. f is equal to zero outside a fixed bounded set S. Then we compute the function's expected change given the process for $q(t)$. The differentiation of f using Itô's lemma yields

$$\begin{aligned} df(q(t)) = &\{f(q(t)+1) - f(q(t))\}\, dq^l(t) + \{f(q(t)-1) - f(q(t))\}\, dq^r(t) \\ &+ \left\{f(q^{\text{ini}}) - f(q(t))\right\} dq^{\text{birth}}(t). \end{aligned} \tag{B.6}$$

We are interested in the expected change of this function over time. Therefore we apply the expectation operator and divide by $d\tau$ (for clarity, time indices have been omitted),

$$\begin{aligned} \frac{1}{d\tau} Edf(q) = &\mu^l(q)\{f(q+1) - f(q)\} + \mu^r(q)\{f(q-1) - f(q)\} \\ &+ \mu^{\text{birth}}\left\{f(q^{\text{ini}}) - f(q)\right\}, \end{aligned} \tag{B.7}$$

where the last line was computed using $Edq^l = \mu^l(q)d\tau$ with $\mu^l(q) = k(q)\lambda^l q$, $Edq^r = \mu^r(q)d\tau$ with $\mu^r(q) = \lambda^r$ for $q^{\text{ini}} < q(t) < q^{\max}$ and 0 otherwise, and $Edq^{\text{birth}} = \mu^{\text{birth}}d\tau$ with $\mu^{\text{birth}} = \lambda^{\text{birth}}$, which are the arrival rates of the Poisson processes.

In what follows, we denote this expression by $\mathcal{A}f(q) \equiv E\frac{df(q)}{d\tau}$.

Step 2 (Using Dynkin's Formula)

The Dynkin formula says that $Ef(q(\tau)) = Ef(q(t)) + \int_t^\tau E\mathcal{A}f(q(s))ds$.[22] In words, the expected value of a function $f(q(\tau))$ at some future point in time $\tau \geq t$ can be written as the expected value of this function at t plus the integral of all expected future changes between τ and t.

As we are interested in expected changes, we differentiate this formula with respect to time and get

$$\frac{\partial}{\partial \tau} Ef(q(\tau)) = \frac{\partial}{\partial \tau} \int_t^\tau E\mathcal{A}f(q(s))ds = E\mathcal{A}f(q(s)), \tag{B.8}$$

since $Ef(q(t))$ can be treated as a constant, which drops when differentiating, and the differential operator can be pulled into the integral.

Applying this on (B.7) yields

$$\begin{aligned}\frac{\partial}{\partial \tau} Ef(q) = E\big[&\mu^l(q)\{f(q+1) - f(q)\} + \mu^r(q)\{f(q-1) - f(q)\} \\ &+\mu^{\text{birth}}\left\{f(q^{\text{ini}}) - f(q)\right\}\big].\end{aligned} \tag{B.9}$$

Under the assumption that $q^{\text{ini}} = 1$ we can write the latter equation as

$$\begin{aligned}\frac{\partial}{\partial \tau} Ef(q) = \Sigma_{i=q^{\text{ini}}}^{q^{\max}} p(i,\tau)\big[&\mu^l(i)[f(i+1) - f(i)] + \mu^r(i)[f(i-1) - f(i)] \\ &+\mu^{\text{birth}}[f(1) - f(i)]\big]\end{aligned} \tag{B.10}$$

where $p(i,\tau)$ is the probability that q equals i in τ, i.e. $p(i,\tau) \equiv Pr(q(\tau) = i)$.

Step 3 (Factorizing)

Now we want to get rid of the $f(i+1)$ and the $f(i-1)$ terms in equation (B.10). By factorizing the equation we obtain

$$\begin{aligned}\frac{\partial}{\partial \tau} Ef(q) &= \Sigma_{i=q^{\text{ini}}}^{q^{\max}} p(i,\tau)\mu^l(i)f(i+1) + \Sigma_{i=q^{\text{ini}}}^{q^{\max}} p(i,\tau)\mu^r(i)f(i-1) \\ &\quad + \mu^{\text{birth}} f(0) \\ \Sigma_{i=q^{\text{ini}}}^{q^{\max}} p(i,\tau) & \\ &\quad - \Sigma_{i=q^{\text{ini}}}^{q^{\max}} p(i,\tau)\left[\mu^l(i)f(i) + \mu^r(i)f(i) + \mu^{\text{birth}} f(i)\right],\end{aligned} \tag{B.11}$$

As

$$\Sigma_{i=q^{\text{ini}}}^{q^{\max}} p(i,\tau)\mu^l(i)f(i+1) = \Sigma_{i=q^{\text{ini}}+1}^{q^{\max}+1} p(i-1,\tau)\mu^l(i-1)f(i), \tag{B.12}$$

[22] For more detailed explanations see e.g. B.2 in Bayer and Wälde (2013)

and

$$\Sigma_{i=q^{\text{ini}}}^{q^{\max}} p(i,\tau)\mu^r(i)f(i-1) = \Sigma_{i=q^{\text{ini}}-1}^{q^{\max}-1} p(i+1,\tau)\mu^r(i+1)f(i), \tag{B.13}$$

we can write (B.11) as

$$\begin{aligned}\frac{\partial}{\partial\tau}Ef(q) = {} & \Sigma_{i=q^{\text{ini}}+1}^{q^{\max}+1} p(i-1,\tau)\mu^l(i-1)f(i) + \Sigma_{i=q^{\text{ini}}-1}^{q^{\max}-1} p(i+1,\tau)\mu^r(i+1)f(i) \\ & + \mu^{\text{birth}} f(0)\Sigma_{i=q^{\text{ini}}}^{q^{\max}} p(i,\tau) \\ & - \Sigma_{i=q^{\text{ini}}}^{q^{\max}} p(i,\tau)\left[\mu^l(i)f(i) + \mu^r(i)f(i) + \mu^{\text{birth}} f(i)\right].\end{aligned} \tag{B.14}$$

Step 4 (Derivation Of The Expected Value)

We now derive an expression for the change of the expected value for the LHS of the equation. Instead of Dynkin's rule we use therefore the definition of the expectation operator in order to get

$$\frac{\partial}{\partial\tau}Ef(q) = \frac{\partial}{\partial\tau}\Sigma_{i=q^{\text{ini}}}^{q^{\max}} f(i)p(i,\tau) = \Sigma_{i=q^{\text{ini}}}^{q^{\max}} f(i)\frac{\partial p(i,\tau)}{\partial\tau}, \tag{B.15}$$

where the last equality exploits the fact that the auxiliary function $f(q)$ is *not* a function of time. (Note that $q(\tau)$ is a function of time but not f itself.)

Step 5 (Collecting All Results)

Equating (B.14) and (B.15) yields

$$\begin{aligned}\Sigma_{i=q^{\text{ini}}}^{q^{\max}} f(i)\frac{\partial p(i,\tau)}{\partial\tau} = {} & \Sigma_{i=q^{\text{ini}}+1}^{q^{\max}+1} p(i-1,\tau)\mu^l(i-1)f(i) \\ & + \Sigma_{i=q^{\text{ini}}-1}^{q^{\max}-1} p(i+1,\tau)\mu^r(i+1)f(i) + \mu^{\text{birth}} f(0)\Sigma_{i=q^{\text{ini}}}^{q^{\max}} p(i,\tau) \\ & - \Sigma_{i=q^{\text{ini}}}^{q^{\max}} p(i,\tau)\left[\mu^l(i)f(i) + \mu^r(i)f(i) + \mu^{\text{birth}} f(i)\right].\end{aligned} \tag{B.16}$$

Then the equation above can be rearranged as follows

$$\begin{aligned}
\Sigma_{i=q^{\text{ini}}}^{q^{\max}} f(i)\frac{\partial p(i,\tau)}{\partial \tau} &= p(1,\tau)\mu^l(1)f(2) + p(2,\tau)\mu^l(2)f(3) + p(3,\tau)\mu^l(3)f(4) + \dots \\
&+ p(1,\tau)\mu^r(1)f(0) + p(2,\tau)\mu^r(2)f(1) + p(3,\tau)\mu^r(3)f(2) + \dots \\
&+ p(1,\tau)\mu^{\text{birth}}f(1) + p(2,\tau)\mu^{\text{birth}}f(1) + p(3,\tau)\mu^{\text{birth}}f(1) + \dots \\
&- p(1,\tau)\left[\mu^l(1)f(1) + \mu^r(1)f(1) + \mu^{\text{birth}}f(1)\right] \\
&- p(2,\tau)\left[\mu^l(2)f(2) + \mu^r(2)f(2) + \mu^{\text{birth}}f(2)\right] \\
&- p(3,\tau)\left[\mu^l(3)f(3) + \mu^r(3)f(3) + \mu^{\text{birth}}f(3)\right] - \dots \\
&= f(0)p(1,\tau)\mu^r(1) \\
&+ f(1)\Big[-p(1,\tau)\mu^l(1) - p(1,\tau)\mu^r(1) + p(2,\tau)\mu^r(2) \\
&\qquad -p(1,\tau)\mu^{\text{birth}} + \mu^{\text{birth}}\Sigma_{i=1}^{q^{\max}} p(i,\tau)\Big] \\
&+ f(2)\Big[p(1,\tau)\mu^l(1) - p(2,\tau)\mu^l(2) - p(2,\tau)\mu^r(2) + p(3,\tau)\mu^r(3) \\
&\qquad -p(2,\tau)\mu^{\text{birth}}\Big] \\
&+ f(3)\Big[p(2,\tau)\mu^l(2) - p(3,\tau)\mu^l(3) - p(3,\tau)\mu^r(3) + p(4,\tau)\mu^r(4) \\
&\qquad -p(3,\tau)\mu^{\text{birth}}\Big] \\
&+ \dots
\end{aligned}$$

As this must hold for any $f(i)$ we obtain for $i = 0$,

$$\frac{\partial p(0,\tau)}{\partial \tau} = \mu^r(1)p(1,\tau) = 0, \tag{B.17}$$

where the last equality results from the fact that $\mu^r(1) = 0$.

Further we obtain for $i = 1$

$$\begin{aligned}
\frac{\partial p(1,\tau)}{\partial \tau} &= \mu^r(2)p(2,\tau) - [\mu^l(1) + \mu^r(1) + \mu^{\text{birth}}]p(1,\tau) + \mu^{\text{birth}}\Sigma_{i=1}^{q^{\max}} p(i,\tau) \\
&= \mu^r(2)p(2,\tau) - [\mu^l(1) + \mu^{\text{birth}}]p(1,\tau) + \mu^{\text{birth}}\Sigma_{i=1}^{q^{\max}} p(i,\tau),
\end{aligned} \tag{B.18}$$

Note that the sum at the end of (B.18) must equal one, as all probabilities must add up to one. For $i = 2$ we obtain

$$\frac{\partial p(2,\tau)}{\partial \tau} = \mu^r(3)p(3,\tau) - [\mu^l(2) + \mu^r(2) + \mu^{\text{birth}}]p(2,\tau) + \mu^l(1)p(1,\tau), \tag{B.19}$$

for $i = 3$,

$$\frac{\partial p(3,\tau)}{\partial \tau} = \mu^r(4)p(4,\tau) - [\mu^l(3) + \mu^r(3) + \mu^{\text{birth}}]p(3,\tau) + \mu^l(2)p(2,\tau), \quad \text{(B.20)}$$

for $i = 4$,

$$\frac{\partial p(4,\tau)}{\partial \tau} = \mu^r(5)p(5,\tau) - [\mu^l(4) + \mu^r(4) + \mu^{\text{birth}}]p(4,\tau) + \mu^l(3)p(3,\tau), \quad \text{(B.21)}$$

and so on. Thus, for a general $1 \leq i < q^{\max}$

$$\frac{\partial p(i,\tau)}{\partial \tau} = \mu^r(i+1)p(i+1,\tau) - [\mu^l(i) + \mu^r(i) + \mu^{\text{birth}}]p(i,\tau) + \mu^l(i-1)p(i-1,\tau). \quad \text{(B.22)}$$

And for $i = q^{\max}$ we have

$$\frac{\partial p(q^{\max},\tau)}{\partial \tau} = -[\mu^r(q^{\max}) + \mu^{\text{birth}}]p(q^{\max},\tau) + \mu^l(q^{\max}-1)p(q^{\max}-1,\tau), \quad \text{(B.23)}$$

since $\mu^l(q^{\max}) = 0$ and $p(q^{\max}+1,\tau) = 0$, as $k(q^{\max}) = 0$.

4.D.2 Numerical Approach Distributional Dynamics

Let $\boldsymbol{p} = [p_1, p_2, \dots p_n]$ be a vector, its elements $p_1, p_2, \dots p_n$ describe the probabilities or population shares in state $q = [1, 2, \dots n]$, at time τ, while n corresponds to the highest possible addictive state $q^{\max}$.

The general structure of the equation system we are aiming to solve reads

$$\begin{aligned}
\dot{p}_1(\tau) &= f(p_1(\tau), p_2(\tau)) \\
\dot{p}_2(\tau) &= g(p_1(\tau), p_2(\tau), p_3(\tau)) \\
\dot{p}_3(\tau) &= g(p_2(\tau), p_3(\tau), p_4(\tau)) \\
&\vdots \\
\dot{p}_n(\tau) &= h(p_{n-1}(\tau), p_n(\tau)),
\end{aligned} \quad \text{(B.24)}$$

while $f(.)$, $g(.)$ and $h(.)$ are linear equations in their arguments. Each of these n equations is a Fokker Planck equation taking optimal individual behavior into account and describes the time evolution of the probability or the population shares in state $q = [1, 2, \dots n]$. As a whole it is a linear system of n differential equations in n unknowns.

Given an initial condition, $p^{\text{ini}}(t) = \left[p_1^{\text{ini}}(t), p_2^{\text{ini}}(t), \dots p_n^{\text{ini}}(t)\right]$, we solve the system (B.24) with Matlab's built-in solver *ode23* for a given time interval $(t, T]$.

4.D.3 Deriving The Stationary Distribution

Since in the long-run the frequencies with which individuals are in state q are time-invariant, we set $\partial p(q,\tau)/\partial\tau = 0$ for all $q^{\text{ini}} \leq q \leq q^{\max}$. Then we obtain for $q = q^{\text{ini}}$,

$$p(q^{\text{ini}}) = \frac{\mu^r(2)p(2,\tau) + \mu^{\text{birth}}}{\mu^l(1) + \mu^{\text{birth}}}, \tag{B.25}$$

for $q^{\text{ini}} < q < q^{\max}$,

$$p(q) = \frac{\mu^r(q+1)p(q+1,\tau) + \mu^l(q-1)p(q-1,\tau)}{\mu^l(q) + \mu^r(q) + \mu^{\text{birth}}}, \tag{B.26}$$

and for $q = q^{\max}$ we have

$$p(q^{\max}) = \frac{\mu^l(q^{\max}-1)p(q^{\max}-1,\tau)}{\mu^r(q^{\max}) + \mu^{\text{birth}}}. \tag{B.27}$$

4.D.4 Numerical Approach Stationary Distribution

Let $\boldsymbol{p} = [p_1, p_2, \ldots p_n]$ be a vector, its elements $p_1, p_2, \ldots p_n$ describe the probabilities or population shares in state $q = [1, 2, \ldots n]$, at time T, while n corresponds to the highest possible addictive state $q^{\max}$.

The general structure of the equation system we are aiming to solve reads

$$\begin{aligned} 0 &= f(p_1, p_2) \\ 0 &= g(p_1, p_2, p_3) \\ 0 &= g(p_2, p_3, p_4) \\ &\vdots \\ 0 &= h(p_{n-1}, p_n), \end{aligned} \tag{B.28}$$

while $f(.)$, $g(.)$ and $h(.)$ are linear equations in their arguments. Each of these n equations is a Fokker Planck equation taking optimal individual behavior into account and under the assumption that the convergence process of the distributional dynamics is completed. Thus the time evolution of the probability or the population shares in state $q = [1, 2, \ldots n]$, i.e. the left-hand side of the equations, is zero. As a whole it is a linear system of n equations in n unknowns.

Given an initial guess, $p^{\text{ini}} = \left[p_1^{\text{ini}}, p_2^{\text{ini}}, \ldots p_n^{\text{ini}}\right]$, we solve the system (B.28) with Matlab's built-in solver *fsolve*.

Chapter 5

Final Discussion

Psychological distress is a phenomenon that can concern everyone of us from time to time, as we are all subject to uncontrollable idiosyncratic risks (like accidents, physical illness etc.) as well as controllable risks (e.g. working overtime, substance abuse). In every OECD country suffer on average 15% of the overall population from a mild mental disorder and another 5% from a severe one. Addiction is a special form of mental disorder. Both, mental ill-health in general and also addiction in particular cause a huge economic burden. A substantial proportion of the economic burden arises from the fact that mental states influence individual behavior. There is a general agreement, based on empirical facts that mental ill-health leads to a limited labor market participation, a reduced productivity at work and unemployment. But also subjective well-being is negatively affected by mental ill-health as it implies negative physical and social consequences to the people concerned and to their social environment.

The improvement of mental health and subjective well-being is a goal in many societies. This is also reflected in the strong public interest in this topic (OECD, 2012, 2014, 2015a). This, however, requires a solid understanding of how mental states influence individual behavior. So far the literature has identified relationships between mental states and individual behavior empirically, but, there is still a lack of a formal framework that explains this relationship with analytical stringency.

The contribution of this dissertation can be separated into the following aspects. First, we provide an economic explanation for the relationship between mental health and individual behavior. This can be seen as a micro-foundation for the dynamics on the aggregate level, because it helps us to understand through which channels, why and how a distribution of mental health and also subjective well-being can be influenced on the aggregate level. Interestingly, it turned out that improving the distribution of mental health and subjective well-being are two goals which are which are not necessarily compatible, because making the environment safer, creates incentives to behave riskier in terms of mental health and thereby increases subjective well-being on the one hand, but also deteriorates the mental health distribution on the other hand.

Furthermore, the second chapter of this dissertation provides a nice contribution regarding optimization problems with state-dependent utility functions, a structure that is used in the other two chapters. In this chapter we demonstrate how to apply numerical methods adequately to analyze optimization problems with state-dependent utility functions. Moreover, we also address the issue of time inconsistency due to state-dependent utility functions.

References

Abel, A. B., 1990. Asset Prices under Habit Formation and Catching up with the Joneses. American Economic Review, 38–42.

Adda, J., Cornaglia, F., 2006. Taxes, Cigarette Consumption, and Smoking Intensity. American Economic Review 96 (4), 1013–1028.

American Psychiatric Association, 2013. Diagnostic and Statistical Manual of Mental Disorders, fifth edition Edition. American Psychiatric Association.

Anheuser-Busch InBev, 2016. „Geklärt, wer fährt!“: Eine Initialtive von Anheuser-Busch InBev. (accessed: 15th July 2016).
URL http://www.ab-inbev.de/fileadmin/pdf/GWF_Broschuere.pdf

Bakshi, G. S., Chen, Z., 1996. The Spirit of Capitalism and Stock-Market Prices. American Economic Review, 133–157.

Barro, R. J., 1990. Government Spending in a Simple Model of Endogeneous Growth. Journal of Political Economy, S103–S125.

Barro, R. J., 1999. Ramsey meets Laibson in the Neoclassical Growth Model. Quarterly Journal of Economics, 1125–1152.

Battgalli, P., Dufwenberg, M., 2007. Guilt in Games. American Economic Review 97 (2), 170–176.

Bayer, C., Wälde, K., 2013. The Dynamics of Distributions in Continuous-Time Stochastic Models. Unpublished.

Becker, G. S., Murphy, K. M., 1988. A Theory of Rational Addiction. Journal of Political Economy, 675–700.

Bell, D. E., 1985. Disappointment in Decision Making under Uncertainty. Operations Research 33 (1), 1–27.

Bernheim, D., Rangel, A., 2004. Addiction and Cue-Tiggered Decision Processes. American Economic Review 94 (5), 1558–90.

Berridge, K. C., Aldridge, J. W., 2008. Decision Utility, the Brain, and Pursuit of Hedonic Goals. Social Cognition 26 (5), 621.

Beshears, J., Choi, J. J., Laibson, D., Madrian, B. C., 2008. How are Preferences revealed? Journal of Public Economics 92 (8), 1787–1794.

Birkner, M., Wälde, K., 2016. The Dynamics of Distributions: Promoting Kolmogorov-Fokker-Planck Equations for the Social Sciences. mimeo Gutenberg University Mainz.

Blanchard, O. J., 1985. Debt, Deficits, and Finite Horizons. Journal of Political Economy 93 (2), 223–247.

Bloom, D. E., Cafiero, E., Jané-Llopis, E., Abrahams-Gessel, S., Bloom, L. R., Fathima, S., Feigl, A. B., Gaziano, T., Hamandi, A., Mowafi, M., et al., 2011. The Global Economic Burden of Non-communicable Diseases. Tech. rep., Geneva: World Economic Forum.

Bundeszentrale für gesundheitliche Aufklärung, 2016. Alkohol? Kenn Dein Limit. (accessed: 15th July 2016).
URL http://www.kenn-dein-limit.info/

Camerer, C., Issacharoff, S., Loewenstein, G., O'donoghue, T., Rabin, M., 2003. Regulation for Conservatives: Behavioral Economics and the Case for Asymmetric Paternalism. University of Pennsylvania Law Review 151 (3), 1211–1254.

Caplin, A., Leahy, J., 2001. Psychological Expected Utility Theory and Anticipatory Feelings. Quarterly Journal of Economics, 55–79.

Carroll, C. D., May 1998. Why Do the Rich Save So Much? Working Paper 6549, National Bureau of Economic Research.

Carroll, G. D., Choi, J. J., Laibson, D., Madrian, B. C., Metrick, A., 2009. Optimal Defaults and Active Decisions. Quarterly Journal of Economics 124 (4), 1639.

Charness, G., Dufwenberg, M., 2011. Participation. American Economic Review 101 (4), 1211–1237.

Choi, J. J., Laibson, D., Madrian, B. C., Metrick, A., 2003. Optimal defaults. American Economic Review 93 (2), 180–185.

Cole, H. L., Mailath, G. J., Postlewaite, A., 1992. Social Norms, Savings Behavior, and Growth. Journal of Political Economy, 1092–1125.

Compte, O., Postlewaite, A., 2004. Confidence-Enhanced Performance. American Economic Review, 1536–1557.

Díaz-Giménez, J., Quadrini, V., Ríos-Rull, J.-V., 1997. Dimensions of Inequality: Facts on the US Distributions of Earnings, Income, and Wealth. Federal Reserve Bank of Minneapolis. Quarterly Review-Federal Reserve Bank of Minneapolis 21 (2), 3.

Dragone, D., 2009. A Rational Eating Model of Binges, Diets and Obesity. Journal of Health Economics 28 (4), 799–804.

D'Albis, H., 2007. Demographic Structure and Capital Accumulation. Journal of Economic Theory 132 (1), 411–434.

Evans, J. S. B., 2008. Dual-Processing Accounts of Reasoning, Judgment, and Social Cognition. Annual Review of Psychology 59, 255–278.

Feichtinger, G., Hartl, R. F., 1986. Optimale Kontrolle ökonomischer Prozesse: Anwendungen des Maximumprinzips in den Wirtschaftswissenschaften. Walter de Gruyter.

Ferguson, B. S., 2000. Interpreting the Rational Addiction Model. Health Economics 9 (7), 587–598.

Francis, J. L., 2009. Wealth and the Capitalist Spirit. Journal of Macroeconomics 31 (3), 394–408.

Frederick, S., Loewenstein, G., O'Donoghue, T., 2002. Time Discounting and Time Preference: A Critical Review. Journal of Economic Literature 40 (2), 351–401.

Grossman, M., 1972. On the Concept of Health Capital and the Demand for Health. Journal of Political Economy, 223–255.

Gruber, J., Köszegi, B., 2001. Is Addiction "Rational"? Theory and Evidence. Quarterly Journal of Economics 116 (4), 1261–1303.

Harris, C. J., Laibson, D., 2002. Hyperbolic Discounting and Consumption. Eighth World Congress, pp. 258–298.

Hodgson, T. A., Meiners, M. R., 1982. Cost-of-illness Methodology: A Guide to Current Practices and Procedures. The Milbank Memorial Fund Quarterly. Health and Society, 429–462.

Kahneman, D., 2003. Maps of Bounded Rationality: Psychology for Behavioral Economics. American Economic Review 93 (5), 1449–1475.

Kahneman, D., Frederick, S., 2002. Representativeness revisited: Attribute Substitution in Intuitive Judgment. In: Heuristics and Biases: The Psychology of Intuitive Judgment.

Kahneman, D., Wakker, P. P., Sarin, R., 1997. Back to Bentham? Explorations of Experienced Utility. Quarterly Journal of Economics, 375–405.

Kessler, R. C., Üstün, T. B., et al., 2008. The WHO World Mental Health Surveys: Global Perspectives on the Epidemiology of Mental Disorders. Cambridge University Press New York.

Krieger, D., 2011. Zeitinkonsistente Präferenzen und Emotionen. Master's thesis, Gutenberg School of Management and Economics, Mainz.

Kurz, M., 1968. Optimal Economic Growth and Wealth Effects. International Economic Review 9 (3), 348–357.

Laibson, D., 1997. Golden Eggs and Hyperbolic Discounting. Quarterly Journal of Economics, 443–477.

Laibson, D., 2001. A Cue-Theory of Consumption. Quarterly Journal of Economics, 81–119.

Levy, A., 2002. Rational Eating: Can it lead to Overweightness or Underweightness? Journal of Health Economics 21 (5), 887–899.

Levy, A., 2009. Rational Eating: A Proposition revisited. Journal of Health Economics 28 (4), 908–909.

Levy, A., Faria, J. R., 2006. Depression and Substance Abuse: A Rationalization of a Vicious Cycle. Working Paper 06-16, Department of Economics, University of Wollongong.

Lim, S. S., Vos, T., Flaxman, A. D., Danaei, G., Shibuya, K., Adair-Rohani, H., AlMazroa, M. A., Amann, M., Anderson, H. R., Andrews, K. G., et al., 2013. A Comparative Risk Assessment of Burden of Disease and Injury attributable to 67 Risk Factors and Risk Factor Clusters in 21 Regions, 1990–2010: A systematic Analysis for the Global Burden of Disease Study 2010. The Lancet 380 (9859), 2224–2260.

Loewenstein, G., 1987. Anticipation and the Valuation of Delayed Consumption. The Economic Journal 97 (387), 666–684.

Loewenstein, G., 2000. Emotions in Economic Theory and Economic Behavior. American Economic Review, 426–432.

Loewenstein, G., O'Donoghue, T., Rabin, M., 2003. Projection Bias in Predicting Future Utility. Quarterly Journal of Economics 118 (4), 1209–1248.

Loewenstein, G. F., O'Donoghue, T., 2004. Animal Spirits: Affective and Deliberative Processes in Economic Behavior. SSRN Working Paper Series.

Loomes, G., McKenzie, L., 1989. The Use of QALYs in Health Care Decision Making. Social Science & Medicine 28 (4), 299–308.

Loomes, G., Sugden, R., 1982. Regret Theory: An Alternative Theory of Rational Choice under Uncertainty. The Economic Journal, 805–824.

Nagel, T., 2013. Poverty and Labour Market Policies in a Stochastic and Dynamic World. Ph.D. thesis, Gutenberg School of Management and Economics, Mainz.

O'Donoghue, T., Rabin, M., 2006. Optimal Sin Taxes. Journal of Public Economics 90 (10), 1825–1849.

OECD, 2010. Life expectancy. In: OECD Factbook 2010: Economic, Environmental and Social Statistics. OECD Health Policy Studies. Paris: OECD Publishing.

OECD, 2012. Sick on the Job? Myths and Realities about Mental Health and Work. OECD Health Policy Studies. Paris: OECD Publishing.

OECD, 2013. Extension of Work on Expenditure by Disease, Age and Gender. OECD Health Policy Studies. Paris: OECD Publishing.

OECD, 2014. Making Mental Health Count: The Social and Economic Costs of Neglecting Mental Health Care. OECD Health Policy Studies. Paris: OECD Publishing.

OECD, 2015a. Fit Mind, Fit Job: From Evidence to Practice in Mental Health and Work. OECD Health Policy Studies. Paris: OECD Publishing.

OECD, 2015b. Tackling Harmful Alcohol Use Economics and Public Health Policy. OECD Health Policy Studies. Paris: OECD Publishing.

OECD, 2016. OECD Labour Force Statistics 2015. OECD Publishing.

Orphanides, A., Zervos, D., 1995. Rational Addiction with Learning and Regret. Journal of Political Economy, 739–758.

Rebelo, S., 1991. Long-Run Policy Analysis and Long-Run Growth. Journal of Political Economy 99 (3), 500–521.

Reiter, M., 2004. Do the Rich save too much? How to explain the Top Tail of the Wealth Distribution. Memeo, UniversitatPompeuFabra.

Robinson, T. E., Berridge, K. C., 2003. Addiction. Annual Review of Psychology 54 (1), 25–53.

Sassi, F., 2006. Calculating QALYs, Comparing QALY and DALY Calculations. Health Policy and Planning 21 (5), 402–408.

Shiell, A., Gerard, K., Donaldson, C., 1987. Cost of Illness Studies: An Aid to Decision-Making? Health Policy 8 (3), 317–323.

Special Eurobarometer 248, 2006. Mental Well-Being. European Commission, Brussels.

Special Eurobarometer 345, 2010. Mental Health. European Commission, Brussels.

Special Eurobarometer 429, 2015. Attitudes of Europeans Towards Tobacco. European Commission, Brussels.

STATISTIK Austria, 2015. Österreichische Gesundheitsbefragung 2014. Hauptergebnisse des Austrian Health Interview Survey (ATHIS) und methodische Dokumentation, Wien.

Strotz, R. H., 1955. Myopia and Inconsistency in Dynamic Utility Maximization. Review of Economic Studies 23 (3), 165–180.

Tarricone, R., 2006. Cost-of-Illness Analysis: What Room in Health Economics? Health Policy 77 (1), 51–63.

Thaler, R. H., Sunstein, C. R., 2003. Libertarian Paternalism. American Economic Review 93 (2), 175–179.

Thaler, R. H., Sunstein, C. R., 2009. Nudge: Improving Decisions about Health, Wealth, and Happiness. Penguin Books, New York, Toronto, London.
URL http://opac.inria.fr/record=b1134761

United Nations Office on Drugs and Crime, 2015. World Drug Report 2015. United Nations Publication Sales No. E.15.XI.6.

Wälde, K., August 2012. Applied Intertemporal Optimization, Edition 1.2 plus: Textbook and Solutions Manual Edition. Know Thyself.

Wälde, K., 2015. Stress and Coping: An Economic Approach. mimeo Gutenberg University Mainz, available at www.waelde.com/pub.

Weber, M., 1958. The Protestant Ethic and the Spirit of Capitalism. New York: Charles Scribner and Sons.

Wittchen, H.-U., Jacobi, F., 2005. Size and Burden of Mental Disorders in Europe— A critical Review and Appraisal of 27 Studies. European Neuropsychopharmacology 15 (4), 357–376.

Wittchen, H.-U., Jacobi, F., Rehm, J., Gustavsson, A., Svensson, M., Jönsson, B., Olesen, J., Allgulander, C., Alonso, J., Faravelli, C., et al., 2011. The Size and Burden of Mental Disorders and other Disorders of the Brain in Europe 2010. European Neuropsychopharmacology 21 (9), 655–679.

World Health Organization, 2014a. Global Status Report on Alcohol and Health. World Health Organization.

World Health Organization, 2014b. Global Status Report on Alcohol and Health - Country Profiles. World Health Organization.

Yaari, M. E., 1965. Uncertain Lifetime, Life Insurance, and the Theory of the Consumer. Review of Economic Studies 32 (2), 137–150.

Yaniv, G., 1998. Phobic Disorder, Psychotherapy, and Risk-Taking: An Economic Perspective. Journal of Health Economics 17 (2), 229–243.

Yaniv, G., 2004. Insomnia, Biological Clock, and the Bedtime Decision: An Economic Perspective. Health Economics 13 (1), 1–8.

Yaniv, G., 2008. Obsessive–Compulsive Disorder and Behavioral Therapy: A Rational-Choice Perspective. Mathematical Social Sciences 55 (3), 405–415.

Zou, H.-f., 1994. The Spirit of Capitalism and Long-Run Growth. European Journal of Political Economy 10 (2), 279–293.

Statutory Declaration

Bei der vorliegenden Arbeit handelt es sich um eine kumulative Dissertation, welche aus drei wissenschaftlichen Aufsätzen besteht, die in alleiniger Autorenschaft verfasst wurden. Ich habe nur die von mir angegebenen Quellen und Hilfsmittel benutzt. Alle Textstellen, die wörtlich oder sinngemäß aus veröffentlichten oder nicht veröffentlichten Schriften entnommen wurden, sind als solche kenntlich gemacht.

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