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Tau longitudinal and transverse polarizations from visible kinematics in (anti-)neutrino nucleus scattering

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ABSTRACT

Since the $\nu_\tau(\bar\nu_\tau)A_Z \to \tau^\mp X$ reaction is notoriously difficult to be directly measured, the information on the dynamics of this nuclear process should be extracted from the analysis of the energy and angular distributions of the tau decay visible products. These distributions depend, in addition to $d^2\sigma/(dE_\tau d\cos\theta_\tau)$, on the components of the tau-polarization vector. We give, for the first time, the general expression for the outgoing hadron (pion or rho meson) energy and angular differential cross section for the sequential $\nu_\tau A_Z \to \tau^-(\pi^-\nu_\tau, \rho^-\nu_\tau)X$ and $\bar\nu_\tau A_Z \to \tau^+(\pi^+\bar\nu_\tau, \rho^+\bar\nu_\tau)X$ reactions. Though all possible nuclear reaction mechanisms contribute to the distribution, it may be possible to isolate/enhance one of them by implementing appropriate selection criteria. For the case of the quasi-elastic reaction off oxygen and neutrino energies below 6 GeV, we show that the pion distributions are quite sensitive to the details of the tau-polarization components. We find significant differences between the full calculation, where the longitudinal and transverse components of the tau polarization vector vary with the energy and the scattering angle of the produced tau, and the simplified scheme in which the polarizations are set to one and zero, being the latter their respective asymptotic values reached in the high energy regime. In addition to its potential impact on neutrino oscillation analyses, this result can be used to further test different nuclear models, since these observables provide complementary information to that obtained by means of the inclusive nuclear weak charged-current differential cross section. We also study the effects on the cross section of the W_4 and W_5 nuclear structure functions, which contributions are proportional to the charged lepton mass, and therefore difficult to constrain in muon and electron neutrino experiments.

1. Introduction

The outgoing τ-lepton produced in ν_τ and $\bar\nu_\tau$ charged-current (CC) nuclear interactions are not fully polarized for a wide range of energies. Within the Standard Model (SM), the tau polarization component perpendicular to the lepton scattering plane is zero [1,2], while the $\tau^\mp$ longitudinal and transverse polarizations within this plane do not vanish, being sensitive to independent combinations of nuclear structure functions [3,4]. Thus, these observables can be used to further test different nuclear models, since they provide complementary information to that obtained by means of the inclusive nuclear CC differential cross section.

Longitudinal and transverse tau polarization projections have been previously computed in the quasi-elastic (QE) region, in which the single nucleon knock-out is the dominant reaction mechanism. The pioneering work of Ref. [3] considered the nucleus as an ensemble of free nucleons, and predictions were substantially improved in Ref. [5], with the inclusion of Random Phase Approximation (RPA) and effective nucleon mass effects. More robust theoretical results were presented in [4], and in particular in [6], for neutrino energies below

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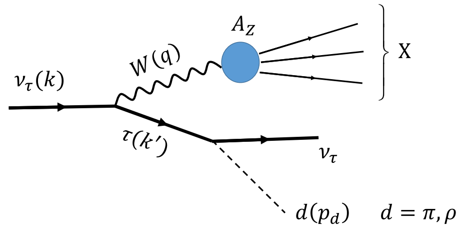

Fig. 1. Sequential $\nu_\tau A_Z \to \tau^-(\pi^-\nu_\tau, \rho^-\nu_\tau)X$ nuclear reaction.

10 GeV, by using realistic spectral functions.[1] Recently, the effects on the $\nu_\tau + n \to \tau^- p$ and $\bar\nu_\tau p \to \tau^+ n$ CCQE scattering of different parametrizations of the isovector vector, axial-vector and pseudoscalar form factors together with the use of second class currents with time-reversal invariance have been analyzed in Ref. [8]. The cross section for $\nu_\tau/\bar\nu_\tau$ scattering off nuclei and the polarization state of the outgoing $\tau^\mp$ were also studied in [3] for Δ resonance production and deep inelastic scattering (DIS) processes, within the simplified picture for the nucleus employed in that work. A more rigorous treatment of the nuclear medium effects in the DIS region is presented in Ref. [9], where the authors have obtained results for the scattering cross sections in ^{40}Ar in the energy region of interest for the proposed DUNE experiment [10].

The τ lepton decays rapidly (with a mean life of 2.9×10^{-13} s) and its disintegration involves at least one neutrino that escapes detection. This makes its clear identification very challenging posing a serious problem to the experimental measurement of the inclusive nuclear $\nu_\tau(\bar\nu_\tau)A_Z \to \tau^\mp X$ differential cross section $d^2\sigma/(dE_\tau d\cos\theta_\tau)$. Moreover, the polarization state of the outgoing tau cannot be directly measured.

The information about the τ lepton polarization should be inferred from the energy and angular distributions of its decay visible products. The study of these distributions and their relation to the components of the tau-polarization vector and to $d^2\sigma/(dE_\tau d\cos\theta_\tau)$ is precisely the main objective of this work. We give, for the first time, the general expression for the outgoing hadron (pion or rho meson) energy and angular differential cross section for the sequential $\nu_\tau A_Z \to \tau^-(\pi^-\nu_\tau, \rho^-\nu_\tau)X$ and $\bar\nu_\tau A_Z \to \tau^+(\pi^+\bar\nu_\tau, \rho^+\bar\nu_\tau)X$ reactions (see Fig. 1). We show that such a distribution, given in Eq. (10), depends on the tau inclusive nuclear CC differential cross section and the longitudinal and transverse polarization observables integrated, with certain dynamical weights, over the outgoing τ available phase space. All possible nuclear reaction mechanisms contribute to the $d^2\sigma_d/(dE_d d\cos\theta_d)$ visible distribution (where $d = \pi,\ \rho$). However, depending on the neutrino energy and implementing appropriate event selections, it may be possible to isolate/enhance the contribution of different (anti-)neutrino-nucleus reaction channels (QE, 2p2h, coherent and incoherent pion production, DIS, etc.) within some regions of the visible $(E_d, \cos\theta_d)$ phase-space.

Since in the $E_\nu \gg m_\tau$ high energy limit the outgoing τ leptons are produced in fully polarized states, the interesting energy region to learn details on the polarization observables is limited to the values of $E_\nu \lesssim 10$ GeV. This energy range can be studied by the DUNE oscillation experiment [10], although the measurement is demanding because of low statistics and the contamination of the sample by neutrino's neutral-current (NC) interactions. For these relatively moderate energies of interest, we want to illustrate the visible distributions that can be obtained and to emphasize on the relevance of a non-trivial tau-polarization vector. To this end we show results for the pion mode in oxygen, assuming a pure QE-reaction mechanism evaluated within the model derived in Ref. [11]. We find significant differences in the pion distributions between the full calculation, where the tau longitudinal and transverse polarization components depend on the nuclear model, and the simplified scheme in which they are set to one and zero (their respective asymptotic values reached in the high energy regime). Even if the direct determination of these distributions might be beyond the capabilities of next generation experiments, having their best theoretical values could be very relevant in the analysis of certain experiments. For instance, the expected sizeable contribution of low energy ν_τ's in the oscillated samples of the DUNE experiment [10] might affect the precision of the oscillation parameters since the momentum and angular distributions of the pions, but also of the muons and electrons from the tau leptonic decays, will influence the ability of the experiment to control the backgrounds at the far detector.

This work is organized as follows. In Sec. 2 we discuss the inclusive $\nu_\tau(\bar\nu_\tau)A_Z \to \tau^\mp X$ cross sections in terms of the structure functions, which provide a general parametrization of the nuclear hadron tensor. We also introduce the $\tau^\mp$ polarization vector and give the expressions of their components as a function of the structure functions. In Sec. 3, we present the master formula for the cross section of the $\nu_\tau A_Z \to \tau^-(\pi^-\nu_\tau, \rho^-\nu_\tau)X$ and $\bar\nu_\tau A_Z \to \tau^+(\pi^+\bar\nu_\tau, \rho^+\bar\nu_\tau)X$ sequential processes. Results for the CCQE contribution evaluated in oxygen at different (anti-)neutrino energies are presented in Sec. 4 and a brief summary of the main findings of our work is given in Sec. 5.

2. Unpolarized and polarized nuclear inclusive cross section

We will first study the CC nuclear inclusive reactions,

$$\nu_\tau A_Z \to \tau^- X, \quad \bar\nu_\tau A_Z \to \tau^+ X \tag{1}$$

where a tau (anti-)neutrino, with four momentum $k^\mu = (E_\nu, \vec{k})$, exchanges a W boson with an atomic nucleus with initial momentum $P^\mu = (M_A, \vec{0})$, and a lepton τ^- (or τ^+) is detected with four-momentum $k'^\mu = (E_\tau, \vec{k}')$. In these processes, the final hadronic state is not detected, and the unpolarized differential cross section in the laboratory frame reads [11]

[1] The use of an effective mass for the nucleon is a simplified method to account for the effects due to the change of its dispersion relation inside the nuclear medium. A proper description, however, is achieved by dressing the nucleon propagators and constructing realistic particle and hole spectral functions, which incorporate dynamical effects that depend on both the energy and momentum of the nucleons [7].

$$\Sigma_0^{(\nu_\tau,\bar\nu_\tau)} \equiv \frac{d^2\sigma_{(\nu_\tau,\bar\nu_\tau)}}{dE_\tau d\cos\theta_\tau} = \frac{|\vec k'|G_F^2 M_A}{\pi} F_{(\nu_\tau,\bar\nu_\tau)}, \tag{2}$$

where G_F is the Fermi weak coupling constant, $\cos\theta_\tau \in]-1,1]$ is the angle between $\vec k$ and $\vec k'$, $m_\tau \le E_\tau \le E_\nu$[2] and F is defined as

$$F_{(\nu_\tau,\bar\nu_\tau)} = \left(2W_1 + \frac{m_\tau^2}{M_A^2}W_4\right)(E_\tau - |\vec k'|\cos\theta_\tau) + W_2(E_\tau + |\vec k'|\cos\theta_\tau) - W_5\frac{m_\tau^2}{M_A} \mp \frac{W_3}{M_A}\left(E_\nu E_\tau + |\vec k'|^2 - (E_\nu + E_\tau)|\vec k'|\cos\theta_\tau\right) \tag{3}$$

where the $\mp$ sign in the W_3 term corresponds to the case of neutrino or anti-neutrino scattering. The real Lorentz-scalar structure functions $W_i(q^0,q^2)$, depend on the four-momentum transferred to the nuclear system, with $q^2 = (q^0)^2 - |\vec q\,|^2$, $q^0 = (E_\nu - E_\tau)$ and $|\vec q\,| = \left(E_\nu^2 + \vec k'^2 - 2E_\nu|\vec k'|\cos\theta_\tau\right)^{\frac12}$, and they are obtained from the decomposition of the hadronic tensor [11]

$$\frac{W^{\mu\nu}}{2M_A} = -g^{\mu\nu}W_1 + \frac{P^\mu P^\nu}{M_A^2}W_2 + \mathrm{i}\frac{\epsilon^{\mu\nu\gamma\delta}P_\gamma q_\delta}{2M_A^2}W_3 + \frac{q^\mu q^\nu}{M_A^2}W_4 + \frac{P^\mu q^\nu + P^\nu q^\mu}{2M_A^2}W_5 + i\frac{P^\mu q^\nu - P^\nu q^\mu}{2M_A^2}W_6 \tag{4}$$

with $\epsilon_{0123} = +1$ and the metric $g^{\mu\nu} = (+,-,-,-)$. The structure functions are different for neutrino or anti-neutrino reactions because the role of protons and neutrons is exchanged. However, for simplicity in the notation, they are shown without the $(\nu_\tau,\bar\nu_\tau)$ label. The term proportional to W_6 does not contribute to the double differential cross section and, thus, it does not appear in the full expression for F in Eq. (3). As mentioned, we follow here the formalism and conventions of Ref. [11].[3]

The (anti)neutrino inclusive-differential cross section for the production of an (anti-)tau with polarization $h = \pm 1$ along a certain four-vector S^μ, verifying $S^2 = -1$ and $S\cdot k' = 0$ [13], can be written, using the results of Appendix A of Ref. [6], as

$$\Sigma^{\nu_\tau} = \frac12\Sigma_0^{\nu_\tau}\left(1 + hS_\mu\mathcal{P}^\mu_{(\tau)}\right), \quad \Sigma^{\bar\nu_\tau} = \frac12\Sigma_0^{\bar\nu_\tau}\left(1 - hS_\mu\mathcal{P}^\mu_{(\bar\tau)}\right) \tag{5}$$

where $\Sigma_0^{(\nu_\tau,\bar\nu_\tau)}$, given in Eq. (2), is the unpolarized cross section, and $\mathcal{P}^\mu_{(\tau,\bar\tau)}$ is the (anti-)tau lepton polarization vector [13,14], which is given in Eq. (5) of Ref. [6] for SM neutrino and anti-neutrino nuclear inclusive reactions. Note that the anti-neutrino $\mathcal{P}^\mu_{(\bar\tau)}$ vector introduced here in Eq. (5) above differs by a minus sign from that defined in Ref. [6]. In the absence of physics beyond the SM, the relevant components of the polarization vector in the laboratory system are denoted as P_L (longitudinal, in the direction of $\vec k'$), and P_T (transverse to $\vec k'$ and contained in the neutrino-tau lepton plane),

$$P_{L,T}^{(\tau,\bar\tau)} = -\left(\mathcal{P}_{(\tau,\bar\tau)}\cdot N_{L,T}\right), \quad N_L^\mu = \left(\frac{|\vec k'|}{m_\tau}, \frac{E_\tau\vec k'}{m_\tau|\vec k'|}\right), \quad N_T^\mu = \left(0, \frac{(\vec k\times\vec k')\times\vec k'}{|(\vec k\times\vec k')\times\vec k'|}\right) \tag{6}$$

The $P_{L,T}$ components depend on the lepton kinematics and on the (anti-)neutrino structure functions, W_i, introduced in Eq. (4) [3,4,6]. For the sake of completeness and clarity, we also reproduce these formulae here,[4]

$$P_L^{(\tau,\bar\tau)} = \left\{\left(2W_1 - \frac{m_\tau^2}{M_A^2}W_4\right)(|\vec k'| - E_\tau\cos\theta_\tau) + W_2(|\vec k'| + E_\tau\cos\theta_\tau) - W_5\frac{m_\tau^2}{M_A}\cos\theta_\tau \mp \frac{W_3}{M_A}((E_\nu + E_\tau)|\vec k'| - (E_\nu E_\tau + |\vec k'|^2)\cos\theta_\tau)\right\}/F_{(\nu_\tau,\bar\nu_\tau)} \tag{7}$$

$$P_T^{(\tau,\bar\tau)} = m_\tau\sin\theta_\tau\left(2W_1 - W_2 - \frac{m_\tau^2}{M_A^2}W_4 + W_5\frac{E_\tau}{M_A} \mp W_3\frac{E_\nu}{M_A}\right)/F_{(\nu_\tau,\bar\nu_\tau)} \tag{8}$$

where the $\mp$ sign in the W_3 term corresponds to the case of tau or anti-tau polarization components. Note that the transverse polarization P_T is proportional to the charged-lepton mass, and therefore for muon or electron neutrino reactions, it is highly suppressed, as expected by conservation of chirality [6].

The study of the tau polarization vector in (ν_τ,τ) and $(\bar\nu_\tau,\bar\tau)$ reactions is of great theoretical interest, since both P_L and P_T display peculiar sensitivities to the ingredients of the nuclear and reaction models, which are different to the ones shown by the pure double differential inclusive cross sections. This is to say, the polarization observables of Eqs. (7) and (8), and the unpolarized function F defined in Eq. (3) are given by different linear independent combinations of the hadron-tensor $W_{1,2,3,4,5}$ structure functions, being sensitive to different $(E_\tau,\cos\theta_\tau)$ kinematics. Experimental information on $P_{L,T}^{(\tau,\bar\tau)}$ would provide additional valuable constraints to test the models used to describe (anti-)neutrino-nucleus cross sections.

[2] We are not explicitly considering the minimum energy ($\sim$ tens of MeV) to be transferred to the nuclear system to account for the mass difference between the mass of the initial nucleus and that of the ground state of the final nuclear configuration [11,12]. This should be a good approximation for E_ν, at least, in the few GeV region.

[3] We should mention that there is a typo in Eq. (10) of this latter work, which affects the W_4 term, where $\sin^2\theta_\tau$ should be $\sin^2\theta_\tau/2$.

[4] Note that the projections used in Eq. (6) to define $P_{L,T}^{(\tau,\bar\tau)}$ differ, for both neutrino (tau) and anti-neutrino (anti-tau) reactions, in a global sign to those taken in Refs. [4,6] (see Eq. (6) of Ref. [6]). In addition, the convention employed here for the polarized cross section in Eq. (5), changes the sign of the anti-tau polarization components $P^{\bar\tau}_{L,T}$ with respect to those given in Refs. [4,6]. In summary, tau $P^\tau_{L,T}$ [anti-tau $P^{\bar\tau}_{L,T}$] defined here differ in sign [are equal] to the ones introduced in Refs. [4,6], as can be seen comparing Eqs. (7) and (8) below with Eqs. (5) and (6) of Ref. [4].

3. Sequential $\nu_\tau A_Z \to \tau^-(\pi^-\nu_\tau, \rho^-\nu_\tau)X$ and $\bar\nu_\tau A_Z \to \tau^+(\pi^+\bar\nu_\tau, \rho^+\bar\nu_\tau)X$ reactions

Given the practical impossibility of making a direct measurement of the τ polarization, the information on the polarization state of the outgoing tau-lepton, encoded in its polarization vector, should be obtained from the energy and angular distributions of its visible decay products. A more sensitive measurement would be obtained from a multiple differential cross section involving also the tau-momentum variables. However, such a measurement will certainly suffer from smaller statistics, besides the fact that fully reconstructing the final τ momentum represents an experimental challenge, because it might not travel far enough for a displaced vertex and its decay involves at least one neutrino.

However, even if the three momentum of tau could not be measured, information about the $\nu_\tau(\bar\nu_\tau)A_Z \to \tau^\mp X$ inclusive nuclear processes and on the polarization state in which the tau-lepton is created can be extracted from the distribution of its charged decay products. This is also the case for the semileptonic decays of bottomed hadrons into charmed ones, driven by the $b \to c\tau\bar\nu_\tau$ transition (see for instance Refs. [14] and [15], in which this section is based). The three dominant τ decay modes $\tau \to \pi\nu_\tau,\ \rho\nu_\tau,\ \ell\bar\nu_\ell\nu_\tau$ $(\ell = e, \mu)$ account for more than 70% of the total τ decay width (Γ_τ). In this section, we will pay attention to the hadron-modes of the tau decay, and study the visible energy and angular distributions of the pion or rho mesons in the nuclear inclusive processes

$$\begin{array}{llll} \nu_\tau A_Z \to \tau^- X & & \bar\nu_\tau A_Z \to \tau^+ X & \\ & \searrow \nu_\tau\pi^-,\ \nu_\tau\rho^- & & \searrow \bar\nu_\tau\pi^+,\ \bar\nu_\tau\rho^+ \end{array} \tag{9}$$

The analog reactions induced by the leptonic tau-decay channel are less sensitive to the tau-polarization components, with their contribution being around a factor of 3 smaller than for the $\tau \to \pi\nu_\tau$ case [15]. Hence, the $\tau \to \ell\bar\nu_\ell\nu_\tau$ decay will provide less statistically meaningful results.

Following Sec. 3 of Ref. [14], we find that for any of these hadron modes, $(\nu_\tau, \bar\nu_\tau)A_Z \to X\tau^\mp \to Xd^\mp(\nu_\tau, \bar\nu_\tau)$ $[d = \pi, \rho]$, the laboratory double differential cross section with respect to the outgoing hadron energy (E_d) and the cosinus of the angle (θ_d) formed by the hadron ($\vec p_d$) and the incoming (anti-)neutrino ($\vec k$) three-momenta, read (see Fig. 1)

$$\begin{aligned} \frac{d^2\sigma_d^{(\nu_\tau,\bar\nu_\tau)}}{dE_d d\cos\theta_d} = {} & \mathcal{B}_d \frac{m_\tau^2}{m_\tau^2 - m_d^2}\frac{G_F^2 M_A}{\pi^2}\int_{E_\tau^-}^{E_\tau^{\rm sup}} dE_\tau \int_{\cos(\theta_d+\theta_{\tau d})}^{\cos(\theta_d-\theta_{\tau d})} \frac{d(\cos\theta_\tau) F_{(\nu_\tau,\bar\nu_\tau)}(E_\tau, \cos\theta_\tau)}{\sqrt{[\cos(\theta_d-\theta_{\tau d}) - \cos\theta_\tau][\cos\theta_\tau - \cos(\theta_d+\theta_{\tau d})]}} \\ & \times \left\{ 1 + \frac{2m_\tau}{m_\tau^2 - m_d^2} a_d \left[P_L^{(\tau,\bar\tau)}(E_\tau, \cos\theta_\tau)\left(\frac{E_d|\vec k'|}{m_\tau} - \frac{E_\tau|\vec p_d|}{m_\tau}\cos\theta_{\tau d}\right) \right.\right. \\ & \left.\left. + \frac{P_T^{(\tau,\bar\tau)}(E_\tau, \cos\theta_\tau)}{\sin\theta_\tau}|\vec p_d|\left(\cos\theta_d - \cos\theta_\tau\cos\theta_{\tau d}\right)\right]\right\} \end{aligned} \tag{10}$$

In the above expression, $\mathcal{B}_{d=\pi,\rho}$ is the branching fraction for the $\tau \to \pi\nu_\tau$ or $\tau \to \rho\nu_\tau$ decays, $m_d = m_\pi$ or m_ρ, $a_{d=\pi} = 1$ or $a_{d=\rho} = (m_\tau^2 - 2m_\rho^2)/(m_\tau^2 + 2m_\rho^2)$, and $\theta_{\tau d}$ is the angle formed by the τ and outgoing hadron three momenta,

$$\cos\theta_{\tau d} = \frac{2E_\tau E_d - m_\tau^2 - m_d^2}{2|\vec k'||\vec p_d|}\,. \tag{11}$$

In addition, we introduce

$$E_\tau^\pm = \frac{(m_\tau^2 + m_d^2)E_d \pm (m_\tau^2 - m_d^2)|\vec p_d|}{2m_d^2}, \quad E_d^{\rm max,\,int} = \frac{(m_\tau^2 + m_d^2)E_\nu \pm (m_\tau^2 - m_d^2)\sqrt{E_\nu^2 - m_\tau^2}}{2m_\tau^2} \tag{12}$$

where the $\pm$ in $E_d^{\rm max,\,int}$ correspond to the maximum energy reachable by the d-hadron (max) and to some intermediate one (int), which will be relevant in the discussion below, respectively. The minimum value of $E_d^{\rm int}$ turns out to be m_d for $E_\nu = (m_\tau^2 + m_d^2)/(2m_d)$. The lower limit of the E_τ integration is always E_τ^-, which is bigger or equal than the tau mass for any value of E_d. The upper limit could be either E_ν or E_τ^+. Actually, we have

$$\begin{aligned} & E_\nu \le \frac{m_\tau^2 + m_d^2}{2m_d} \Rightarrow E_d^{\rm int} \le E_d \le E_d^{\rm max}, \quad E_\tau^{\rm sup} = E_\nu \\ & E_\nu > \frac{m_\tau^2 + m_d^2}{2m_d} \Rightarrow m_d \le E_d \le E_d^{\rm max}, \quad E_\tau^{\rm sup} = H(E_d^{\rm int} - E_d)E_\tau^+ + H(E_d - E_d^{\rm int})E_\nu \end{aligned} \tag{13}$$

with $H(\ldots)$, the step function. Finally, there is no limitation for the outgoing hadron angle and $\cos\theta_d \in]-1, 1]$.

In Eq. (10), it is assumed that the τ lepton exits the nucleus before decaying, and thus the outgoing pion or rho meson does not suffer any further final state interaction.

As shown in Ref. [14], the contribution in Eq. (10) which depends on the polarization components vanishes after integrating over the energies and angles of the visible hadron, while the independent one is properly normalized such that

$$\int dE_d\, d\cos\theta_d \frac{d^2\sigma_d^{(\nu_\tau,\bar\nu_\tau)}}{dE_d d\cos\theta_d} = \mathcal{B}_d \int_{m_\tau}^{E_\nu} dE_\tau \int_{-1}^{+1} d\cos\theta_\tau \frac{d^2\sigma_{(\nu_\tau,\bar\nu_\tau)}}{dE_\tau d\cos\theta_\tau} \equiv \mathcal{B}_d\, \sigma_{(\nu_\tau,\bar\nu_\tau)}(E_\nu), \quad E_\nu \ge m_\tau \tag{14}$$

with $\sigma_{(\nu_\tau,\bar\nu_\tau)}(E_\nu)$ the unpolarized tau (anti-)neutrino nuclear inclusive cross section in the laboratory frame obtained from the integration of the differential distribution of Eq. (2).

Note that the maximal information that one can in principle extract would be the four dimensional differential cross section $d^4\sigma_d^{(\nu_\tau,\bar\nu_\tau)}/(dE_d d\cos\theta_d dE_\tau d\cos\theta_\tau)$, which would require the measurement of the tau momentum. Such unfolded distribution might allow to obtain all three $F_{(\nu_\tau,\bar\nu_\tau)}(E_\tau,\cos\theta_\tau)$, $P_L^{(\tau,\bar\tau)}(E_\tau,\cos\theta_\tau)$ and $P_T^{(\tau,\bar\tau)}(E_\tau,\cos\theta_\tau)$ observables, which otherwise appear in Eq. (10) within an integral over the tau kinematical variables.

The expression of Eq. (10) for the energy and angular distribution of the hadron product after the decay of the virtual τ is general, and it implies a sum over all possible nuclear reaction mechanisms. Nevertheless, depending on the neutrino energy and implementing some cuts, it may be possible to isolate/enhance the contribution to the inclusive $d^2\sigma_d^{(\nu_\tau,\bar\nu_\tau)}/(dE_d d\cos\theta_d)$ cross section of different (anti-)neutrino-nucleus reaction channels (QE, 2p2h, coherent and incoherent pion production, DIS, etc.) for some regions of the visible $(E_d,\cos\theta_d)$ phase-space.

The detailed study of $d^2\sigma_d^{(\nu_\tau,\bar\nu_\tau)}/(dE_d d\cos\theta_d)$ provides some additional observables, which will serve to further test the nuclear and reaction models used to describe the CC (anti-)neutrino-nucleus interactions. Moreover and as mentioned above, the $(E_d,\cos\theta_d)$ distribution should be much easier to measure experimentally than the polarized $\nu_\tau A_Z\to\tau^- X$ and $\bar\nu_\tau A_Z\to\tau^+ X$ differential cross sections. Note, however, that there will be also pions or rho-mesons produced in the nuclear part of the interaction, including the re-scattering of the produced nucleons, pions, etc. during their path exiting the nuclear environment. One should look at the $(E_d,\cos\theta_d)$-pattern provided by them, and focus the study in kinematical regions for which the number of nuclear events is much smaller than that due to pions or rho-mesons coming from the tau decay.

Another source of unwanted background is the high contamination of muon and/or electron (anti-)neutrinos that will be likely present in the available ν_τ and $\bar\nu_\tau$ beams. The nuclear $\nu_{\mu,e}$ and $\bar\nu_{\mu,e}$ interactions can give rise to an abundant production of pions and rho mesons. Though CC processes will produce an outgoing muon or electron which can be used to discard the event through transverse momentum balance, NC reactions represent a more challenging background to the $d^2\sigma_d^{(\nu_\tau,\bar\nu_\tau)}/(dE_d d\cos\theta_d)$ distribution, since the outgoing neutrino will not be detected. Given an experimental setup, these events, not coming from the τ decay, need to be taken into account for a meaningful analysis.

Note that this muon/electron neutrino NC background should no affect sequential decays initiated by an anti-neutrino $\bar\nu_\tau$ scattered off a bound-nucleon producing in the final state a τ^+, together with a hyperon $Y=\Lambda,\Sigma$. Though Cabibbo suppressed, the reaction $\bar\nu_\tau A_Z\to\tau^+(\pi^+\bar\nu_\tau,\rho^+\bar\nu_\tau)Y+X$ with the detection of the hyperon, and no kaons in the final state, should be considerably less affected by this unwanted NC background. However, in this case, the nuclear structure functions will be different to those probed in the Cabibbo allowed transitions, beginning from the fact that the outgoing hyperon will not be Pauli blocked [16,17].

In experimental analyses, it is common to use the longitudinal (p_{Ld}) and transverse (p_{Td}) pion/rho momentum components which are related to E_d and θ_d through the relations

$$p_{Ld}=\sqrt{E_d^2-m_d^2}\,\cos\theta_d,\qquad p_{Td}=\sqrt{E_d^2-m_d^2}\,\sin\theta_d \tag{15}$$

with $E_d^2=(m_d^2+p_{Ld}^2+p_{Td}^2)$. From the Jacobian of the transformation, we find for the corresponding double-differential cross section

$$\frac{d^2\sigma_d^{(\nu_\tau,\bar\nu_\tau)}}{dp_{Ld}\,dp_{Td}}=\frac{p_{Td}}{(p_{Ld}^2+p_{Td}^2)^{\frac12}}\frac{1}{(p_{Ld}^2+p_{Td}^2+m_d^2)^{\frac12}}\left[\frac{d^2\sigma_d^{(\nu_\tau,\bar\nu_\tau)}}{dE_d d\cos\theta_d}\right]=\frac{\sin\theta_d}{E_d}\left[\frac{d^2\sigma_d^{(\nu_\tau,\bar\nu_\tau)}}{dE_d d\cos\theta_d}\right] \tag{16}$$

4. Results

In this section and for illustrating purposes, we shall present results for the CCQE $d^2\sigma/(dp_{L\pi}dp_{T\pi})$ distributions for the sequential $\nu_\tau A_Z\to\tau^-(\pi^-\nu_\tau)X$ and $\bar\nu_\tau A_Z\to\tau^+(\pi^+\bar\nu_\tau)X$ processes evaluated in ^{16}O. We focus only on the pion production, since the polarization effects for the ρ particle are smaller (see Eq. (10), $a_{d=\rho}\approx 0.4$). The QE contribution has been computed using the LFG model of Ref. [11], and for simplicity, we will neither include RPA-type effects, nor account for the energy balance corrections. The latter are small for the relatively moderate neutrino energies which are considered in this work, while RPA correlations do not appreciably change the gross features of the tau polarization vector. The reason is that the polarization components are obtained as a ratio of linear combinations of nuclear structure functions and RPA changes similarly numerator and denominator [4,5]. RPA corrections might affect the nuclear response $F_{(\nu_\tau,\bar\nu_\tau)}$ that appears as a global factor in Eq. (10). However, we do not expect such effects to significantly change the qualitative characteristics of the distribution of pions from the sequential τ-decay that will be discussed below.

The minimum tau (anti-)neutrino energy considered in this work has been 3 GeV, for which the QE CC cross section in oxygen is about a factor one-hundred larger than the coherent pion production one, the latter estimated using the model of Ref. [18]. We have checked the coherent channel since its threshold is essentially m_τ, below that of the QE reaction-mechanism.

We start by showing in Fig. 2 the results for the sequential $\nu_\tau A_Z\to\tau^-(\pi^-\nu_\tau)X$ process at $E_\nu=3,4,6$ and 10 GeV (shown from top to bottom). The left panels show the full calculation of Eqs. (10) and (16), while in the middle ones we set $P_L^\tau=1$ and $P_T^\tau=0$, which corresponds in our case to pure negative-helicity taus. The latter would be similar to what is done in experimental analyses where Neutrino interaction Monte Carlo models generate the τ kinematics with models similar to the case of μ and e and let the TAUOLA [19] package to decay the τ normally assuming fully longitudinal or fixed $\tau^\pm$ polarizations. In the right panels we show the ratio of the difference of the two previous calculations over their sum. As it can be seen from the figure, at low neutrino energies the difference between the two calculations is substantial, while it decreases at higher neutrino energies. This behaviour can be understood since high energy neutrinos produce high energy taus and the latter tend to be in a negative-helicity state due to the $(1-\gamma_5)$ part of the weak production vertex (at high energies, compared to the mass of the lepton, helicity equals chirality). However, closer to the production threshold the taus are not so energetic and the helicity is not well defined in this case. Thus, for low neutrino energies accounting for the correct tau-polarization vector is essential.

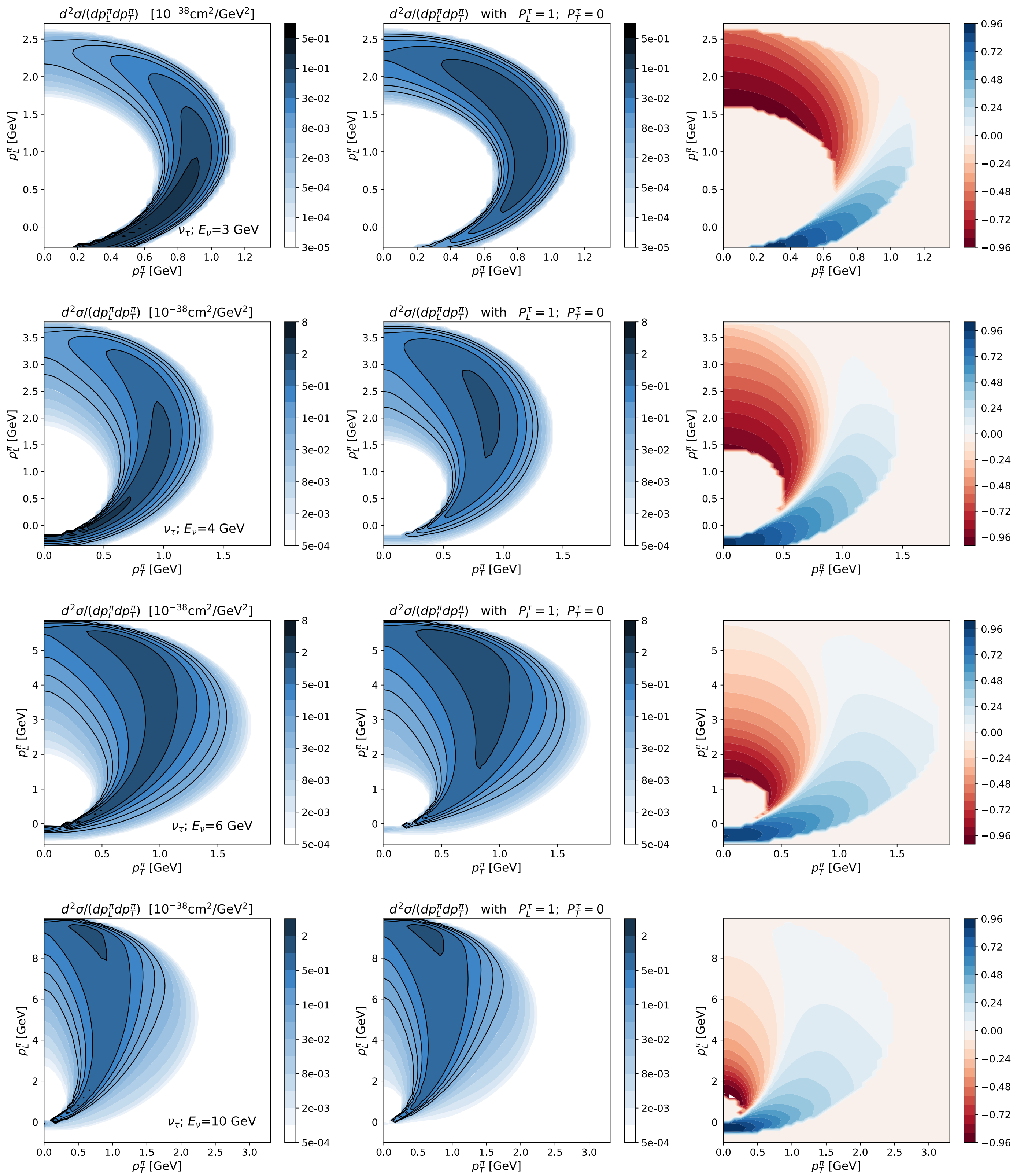

Fig. 2. Two-dimensional $d^2\sigma/(dp_{L\pi}dp_{T\pi})$ distributions, in units of 10^{-38} cm^2/GeV^2, for the sequential $\nu_\tau A_Z \to \tau^-(\pi^-\nu_\tau)X$ process evaluated in ^{16}O at $E_\nu = 3, 4, 6$ and 10 GeV (shown from top to bottom). The left panels show the full calculation of Eqs. (16). In the middle panels we set $P_L = 1$, $P_T = 0$, which corresponds in our case to pure negative-helicity taus. Finally, the right panels show the ratio of the difference of the two previous calculations over their sum. We consider only the QE contribution computed using the LFG model of Ref. [11], without including RPA and correct-energy balance corrections.

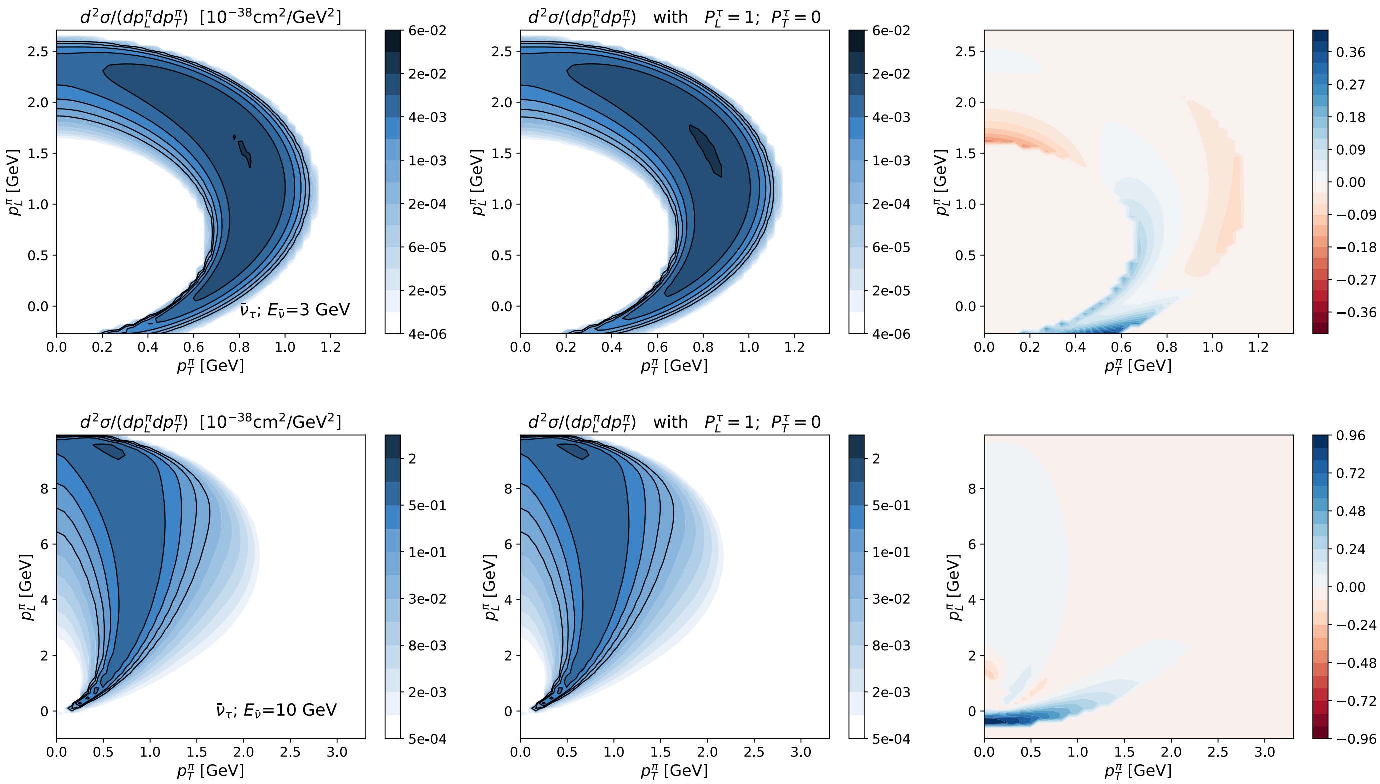

Fig. 3. Same as Fig. 2 but in this case for the anti-neutrino $\bar\nu_\tau A_Z \to \tau^+(\pi^+\nu_\tau)X$ reaction at only $E_{\bar\nu} = 3$ and 10 GeV.

The corresponding results for the anti-neutrino $\bar\nu_\tau A_Z \to \tau^+(\pi^+\bar\nu_\tau)X$ sequential process are shown in Fig. 3. In this case the full calculation and the one setting $P_L^\tau = 1$ and $P_T^\tau = 0$, that corresponds in our case to positive-helicity anti-taus, are very similar already at low anti-neutrino energies. This is in agreement with the findings in Ref. [6] and it is due to the fact that the $\bar\nu$ scattering is more forward peaked than the ν one for QE (and also for the 2p2h) mechanisms [20]. The latter effect is produced because of the different sign of W_3 term, which leads to a destructive interference for the anti-neutrino case that becomes more and more effective as $|q^2|$ increases (see Eq. D2 of Ref. [11]).

The results discussed in this manuscript also show the relevance of the transverse polarization in future oscillation experiments such as DUNE, which expects a sizeable amount of ν_τ CC interactions in the far detector [10] after oscillations. To show its relevance, we present in Fig. 4, the momentum and angular distributions of the π^- originating from τ^- decays for 3, 4 and 6 GeV neutrinos. The differences between the two models discussed in this work, one with full polarization and the other restricted to a pure longitudinal ($P_L = 1$) lepton polarisation, reveal the strong dependence of the π^- kinematics with the transverse polarisation. As expected, the difference is reduced for higher momentum τ^-'s. The majority of ν_τ flux in DUNE is expected below 7 GeV [10]. The case of 3 GeV is very conclusive. The transverse polarisation transforms a hard spectrum of π^-, typically above those produced in NC interactions, into a very soft one that overlaps the expected spectrum of π^- produced in NC.

It is also interesting to evaluate the distributions in the case where $W_4 = W_5 = 0$. As seen in Eqs. (3), (7) and (8), the contribution of these two structure functions is suppressed by powers of the charged lepton mass and thus they cannot be accessed in the corresponding reactions involving the first two lepton generations. The relevance of these structure functions is analyzed in Fig. 5 where, for the sequential $\nu_\tau A_Z \to \tau^-(\pi^-\nu_\tau)X$ process evaluated in ^{16}O at $E_\nu = 3$ GeV and 10 GeV, we compare the two calculations and we further show the ratio

$$\frac{\left(\frac{d^2\sigma}{dp_{L\pi}\,dp_{T\pi}}\Big|_{W_4=W_5=0}\right) - \frac{d^2\sigma}{dp_{L\pi}\,dp_{T\pi}}}{\frac{d^2\sigma}{dp_{L\pi}\,dp_{T\pi}}} \tag{17}$$

The results clearly show the significant impact of the W_4 and W_5 contributions for the case of τ production. We have checked that the effects are even more pronounced for anti-neutrinos. They produce a general reduction in the cross section. Their effect is present for all energies analyzed although it is more relevant at lower neutrino energies.

5. Summary

We have analysed the sequential $\nu_\tau A_Z \to \tau^-(\pi^-\nu_\tau, \rho^-\nu_\tau)X$ and $\bar\nu_\tau A_Z \to \tau^+(\pi^+\bar\nu_\tau, \rho^+\bar\nu_\tau)X$ reactions. For the first time we give the general expression [Eq. (10)] for the outgoing hadron (pion or rho meson) energy and angular differential cross section. Within this context, we have investigated the role of the (anti-)tau polarization vector in the analysis of these processes. We have shown that such distributions depend on the tau inclusive nuclear CC differential cross section and the longitudinal and transverse polarization observables integrated, with certain dynamical weights, over the outgoing τ available phase space. Since the τ momentum can not be easily reconstructed, the

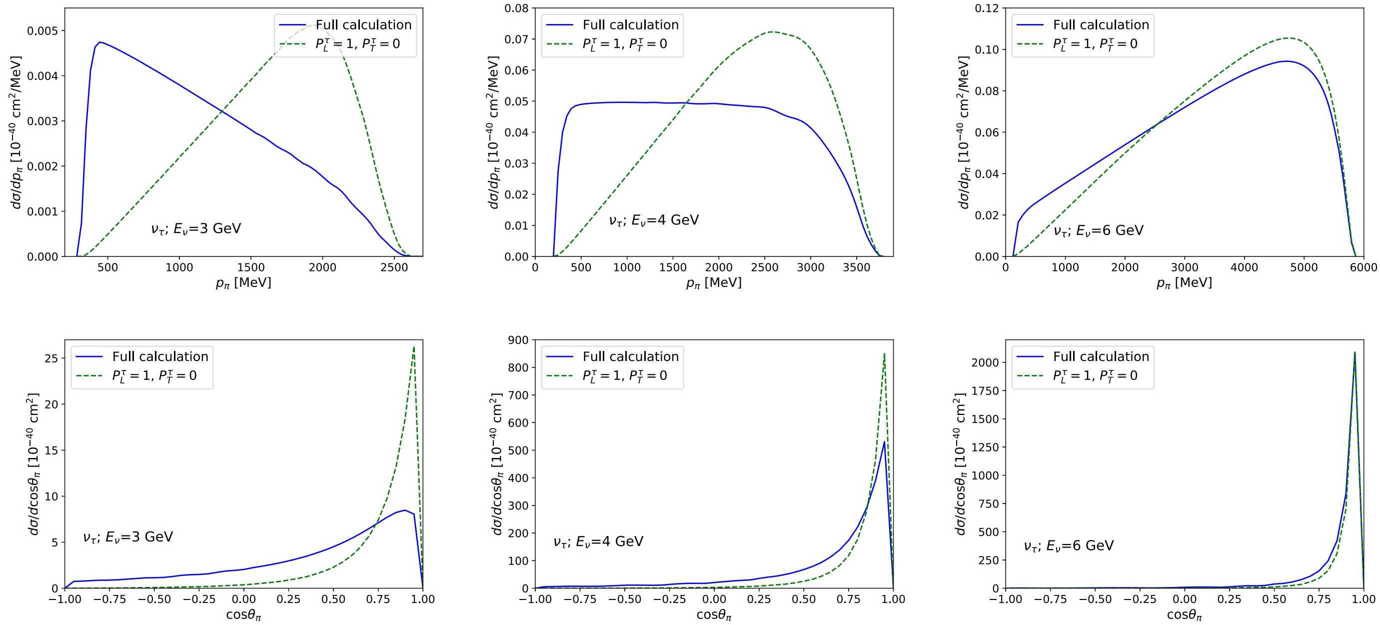

Fig. 4. $d\sigma/dp_\pi$ (upper row) and $d\sigma/d\cos\theta_\pi$ (lower row) differential cross section for the π^- from the $\nu_\tau A_Z \to \tau^-(\pi^-\nu_\tau)X$ sequential reaction at three different energies $E_\nu = 3, 4,$ and 6 GeV. Results are shown for full polarization and longitudinal polarization ($P_L = 1, P_T = 0$), with the areas under both types of distributions equal, as follows from the discussion of Eq. (14).

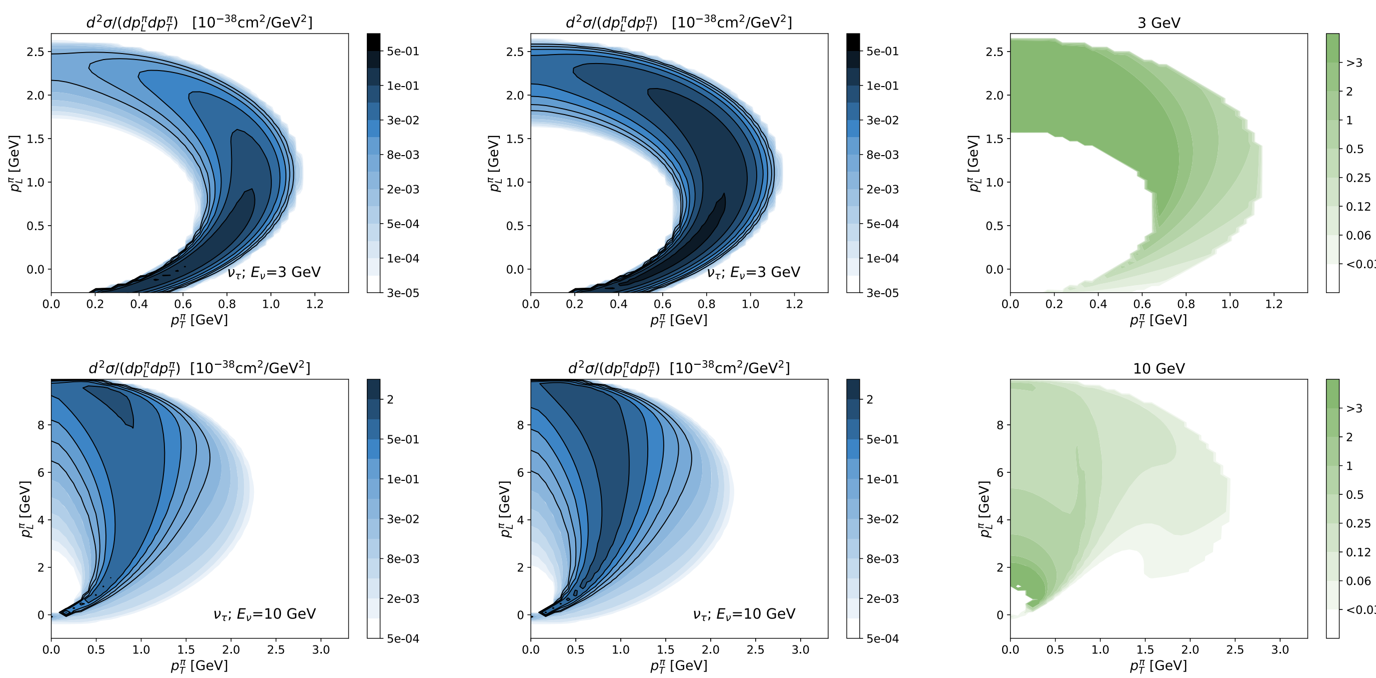

Fig. 5. Left panels: two-dimensional $d^2\sigma/(dp_{L\pi}dp_{T\pi})$ distribution, in units of 10^{-38} cm^2/GeV^2, for the sequential $\nu_\tau A_Z \to \tau^-(\pi^-\nu_\tau)X$ reaction evaluated in ^{16}O at $E_{\bar\nu} = 3$ GeV (top) and 10 GeV (bottom) and obtained within the complete QE model. Middle panels: same as before but with the contributions from W_4 and W_5 switched-off. Right panels: the corresponding ratios as defined in Eq. (17).

study of the visible kinematics of its sequential decays is the best that can be done to extract the information on the nuclear response and reaction mechanism encoded in the polarization state of the produced tau. This information goes beyond that obtained from the inclusive nuclear weak CC differential cross section and it can be used to further constrain nuclear models.

Though all possible neutrino-nucleus reaction mechanisms contribute to the visible distribution, $d^2\sigma_d/(dE_d d\cos\theta_d)$, depending on the neutrino energy and implemented cuts, it may be possible to isolate/enhance the contribution of different (anti-)neutrino-nucleus reaction channels. Our results for the pion decay mode in oxygen at $E_\nu \le 6-10$ GeV and for the QE reaction show the relevance of considering the correct τ polarization versus some simplifications ($P_L \sim 1$ and $P_T \sim 0$) usually assumed in experimental neutrino oscillation analyses [10]. One exception known to us is the SuperKamiokande atmospheric ν_τ analysis based on the polarization calculations given in [3], though some approximations are still adopted. This is more significant for ν_τ-induced reactions that for the ones initiated by $\bar\nu_\tau$. In the latter case, the more forward character of the QE reaction enhances the P_L component of the τ^+ polarization vector, which renders the full calculation in a much better agreement with the approximate one. We have also explored the relevance of the contributions of the W_4 and W_5 nuclear structure functions. The contributions associated to these two structure functions are proportional to the charged lepton mass and, thus, they play a minor role in reactions involving the two light lepton families. However, they give a significant contribution for CC processes initiated by tau (anti-)neutrinos. Again the fact that more structure functions play a role for these sequential reactions helps in constraining nuclear models and possible reaction mechanisms.

Finally, as we have already mentioned, unless one is able to select a certain type of events by imposing a certain additional signature, all possible mechanisms contribute to $d^2\sigma_d/(dE_d d\cos\theta_d)$. Therefore, it becomes essential to obtain the $W_{1,2,3,4,5}$ structure functions, which determine the hadron tensor $W^{\mu\nu}$, for other nuclear inclusive reaction channels (2p2h, pion production, DIS, etc.) to correctly compute P_L, P_T and consequently the visible distributions of the tau-decay products.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] C.H. Llewellyn Smith, Phys. Rep. 3 (1972) 261.
[2] K.S. Kuzmin, V.V. Lyubushkin, V.A. Naumov, Nucl. Phys. B, Proc. Suppl. 139 (2005) 154, arXiv:hep-ph/0408107.
[3] K. Hagiwara, K. Mawatari, H. Yokoya, Nucl. Phys. B 668 (2003) 364, Erratum: Nucl. Phys. B 701 (2004) 405–406, arXiv:hep-ph/0305324.
[4] M. Valverde, J.E. Amaro, J. Nieves, C. Maieron, Phys. Lett. B 642 (2006) 218, arXiv:nucl-th/0606042.
[5] K.M. Graczyk, Nucl. Phys. A 748 (2005) 313, arXiv:hep-ph/0407275.
[6] J.E. Sobczyk, N. Rocco, J. Nieves, Phys. Rev. C 100 (2019) 035501, arXiv:1906.05656 [nucl-th].
[7] J. Nieves, J.E. Sobczyk, Ann. Phys. 383 (2017) 455, arXiv:1701.03628 [nucl-th].
[8] A. Fatima, M. Sajjad Athar, S.K. Singh, Phys. Rev. D 102 (2020) 113009, arXiv:2010.10311 [hep-ph].
[9] F. Zaidi, V. Ansari, M.S. Athar, H. Haider, I.R. Simo, S.K. Singh, arXiv:2111.07609 [nucl-th], 2021.
[10] P. Machado, H. Schulz, J. Turner, Phys. Rev. D 102 (2020) 053010, arXiv:2007.00015 [hep-ph].
[11] J. Nieves, J.E. Amaro, M. Valverde, Phys. Rev. C 70 (2004) 055503, Erratum: Phys. Rev. C 72 (2005) 019902, arXiv:nucl-th/0408005.
[12] B. Bourguille, J. Nieves, F. Sánchez, J. High Energy Phys. 04 (2021) 004, arXiv:2012.12653 [hep-ph].
[13] N. Penalva, E. Hernández, J. Nieves, J. High Energy Phys. 06 (2021) 118, arXiv:2103.01857 [hep-ph].
[14] N. Penalva, E. Hernández, J. Nieves, J. High Energy Phys. 10 (2021) 122, arXiv:2107.13406 [hep-ph].
[15] N. Penalva, E. Hernández, J. Nieves, J. High Energy Phys. 04 (2022) 026.
[16] S.K. Singh, M.J. Vicente Vacas, Phys. Rev. D 74 (2006) 053009, arXiv:hep-ph/0606235.
[17] J.E. Sobczyk, N. Rocco, A. Lovato, J. Nieves, Phys. Rev. C 99 (2019) 065503, arXiv:1901.10192 [nucl-th].
[18] J.E. Amaro, E. Hernandez, J. Nieves, M. Valverde, Phys. Rev. D 79 (2009) 013002, arXiv:0811.1421 [hep-ph].
[19] Z. Was, P. Golonka, Nucl. Phys. B, Proc. Suppl. 144 (2005) 88, arXiv:hep-ph/0411377.
[20] J. Nieves, I. Ruiz Simo, M.J. Vicente Vacas, Phys. Lett. B 721 (2013) 90, arXiv:1302.0703 [hep-ph].