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»... weit schärfer und gründlicher nachgedacht ... « Zur Musiktheorie Johann Philipp Kirnbergers



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Valerie Krupp Immanuel Ott Birger Petersen



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Timothy Dwight Edwards Kirnberger's Enigmatic Canons

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Kirnberger's Enigmatic Canons

Throughout Kirnberger's professional life his remarkable enigmatic canons appeared in print in didactic publications. The solutions to these puzzles provide clues to understand their creator, his theories, aspirations, values and humor. It is my aim here to unlock some of the details of these puzzles and shed light both on the person and his practice of enigmatic canon. The result will be to trace the development of Kirnberger's enigmatic canons and their pedagogical use in publications spanning the years 1753 to 1782.

Those canons to be examined here, taken from three points in the life of the composer and theorist, show both remarkable technical musical skill and a progressively sophisticated integration into the theoretical work in which they are included. At the apex of this development Kirnberger worked one canon in particular into the subject matter and organization of his final essay, *Gedanken über die verschiedenen Lehrarten in der Komposition als Vorbereitung zur Fugenkenntniss*¹ (heretofore *Gedanken über die verschiedenen Lehrarten*) brining together contrpuntal techniques advancing his program of promoting the pedagogic and compositional techniques of Johann Sebastian Bach, even while taking on an autobiographical connotation, reflecting upon the anticipation of Kirnberger's own death.

For the most part, the canons to be examined here are both brief and perpetual. They are part of a didactic musical form unto themselves, standing distinct from pieces that fulfill the role of a movement in a larger work, such as any of Telemann's *18 Canons Mélodieux* (TWV 40:118–123) or the canonic movement from C. P. E. Bach's palindromic *Menuet in C Major* (Wq 116:5). Didactic enigmatic canons such as Kirnberger's derive their value from their concision and symmetry but also by how well they exemplify the contrapuntal devices that are their focus. They tend to focus on a single motive or phrase.

Most often these canons are included in a treatise with a sepcific didactic purpose, but they can have a broader significance. Kirnberger's canons help to advance Bach's approach to counterpoint, harmony and dissonance. *Gedanken über die verschiedenen Lehrarten*, is as much concerned with conveying that updated sense of the treatment of dissonance as it is focused on the canon that functions as its gemstone.

¹ Johann Philipp Kirnberger, *Gedanken über die verschiedenen Lehrarten in der Komposition, als Vorbereitung zur Fugenkenntniß*, Berlin 1782 (Thoughts on the Different Methods of Teaching Composition as Preparation for Understanding Fugue).

The uses and purposes of an enigmatic canon in the context of a treatise can be multifarious. They can engage readers independently of the flow of the prose, to help in the instruction of singing, to enhance the reader's experience either visually or through a diversionary puzzle, to entice the reader into further study in order to solve a riddle or challenge implying a solution to be revealed within, to teach about the practice of canon, but above all, in the eighteenth century, to teach skills and contrapuntal devices applicable to the composition of fugue. Didactically, the problem of solving a puzzle invites direct exploration; finding the proper solution necessitates the ability to discern what works and what does not and calls for reflection on the contrapuntal issues involved. The puzzle offered an opportunity for writers of didactic and theoretical treatises to unite theory and practice, pulling readers into investigating for themselves the melodic and harmonic relationships that such puzzles lend themselves to testing.

For the composer, the precomposition necessary in creating an enigmatic canon proves its rigor through the potential of the enigma to generate its complete solution. Kirnberger sought to continue and advance the practice of didactic canon, investing thought, energy and innovation into the practice as one of the last generation of the orists to do so. His canons have a story to tell, a conjecture to offer or a pedagogical lesson to convey. As a result, the effort involved in unlocking the mysteries of a canon yields not only a musical byproduct but a further clue as to the intention of its creator. Which of the aims listed above are the *raison d'être* of a particular canon— the purpose of including it where it appears? The answer to this question is not only relevant but can overshadow the mere musical product of the canon's solution; it is central to the present study.

As we examine Kirnberger's canons published in his own treatises and in a treatise by Friedrich Wilhelm Marpurg, it will help to keep in mind that both authors treat the concept of canon in a strict sense, involving an enigma and a solution. These two forms of a single canon are sometimes referred to as >closed< (enigmatic, *geschlossen, clauses*) and >open< (resolved, *aufgelöst, apertus*). These are not necessarily different kinds or species of canon but may also be two ways to notate a single canon. A command of enigmatic canon involves a fluency of translation between these two forms.² The open form must be composed with a view to the method of its riddle foremost in the mind of the composer so that it can be expressed in closed form. This seems straightforward when the contrapuntal device employed is simple imitation but can become quite complex as the devices are varied and layered. Aside from direct imitation, the techniques by which these canons are composed and encoded involve various transformations, such as imitation in contrary motion, augmentation, and

² See Friedrich Wilhelm Marpurg, *Abhandlung von der Fuge, nach den Grundsätzen und Exempeln der besten deutchen und ausländischen Meister entworfen von Friedrich Wilhelm Marpurg*, Berlin 1753–1754, Das sechste Hauptstück, erster Abschnitt §4, 1753–1754.

diminution, and constitute examples of various types of canon such as stacked canon, canon per tonos or canon over or under a *cantus firmus*.³

13 Canones Kirnbergeri

The time between Kirnberger's study of composition and performance with J.S. Bach and the inclusion of his set of thirteen canons by Marpurg in the first volume of his treatise *Abhandlung von der Fuge* (1753)⁴ was only about eight years.⁵ Not only was Marpurg's treatise to be the first of its kind, focusing systematically on fugal composition, but it was highly influenced by J.S. Bach himself. In fact, according to Christoph Wolff, Marpurg consulted Bach on fugal composition around the year 1749, most likely with his treatise in mind.⁶ »No theoretical work on fugal composition existed before Marpurg's 1753 treatise *Abhandlung von der Fuge*, which is largely based on *The Art of Fugue*, Bach's practical treatise.«⁷

Roughly following Bach's method of teaching, which includes first thorough bass, then invertible counterpoint, followed by canon before beginning fugal composition, Abhandlung von der Fuge follows Bach in a much more fundamental way: It includes a great deal of music. The multitude of musical examples is the greatest acknowledgment of Bach's practical music. Sixty pages of densely packed musical examples accompany each of the two volumes of Marpurg's treatise, more than can be thoroughly discussed on its pages. It is in this context that one must see that Kirnbeger's thirteen canons are placed in the middle of a sea of music, included in the first volume before any explicit discussion of canon. With the insertion of these canons, Marpurg steps outside of the aforementioned plan of Bach's teaching perhaps as a foreshadowing and perhaps as relief to the two lengthy fugues which precede them in the musical examples.8 Kirnberger's thirteen canons are included as a set followed immediately by a similar set of fifteen diverse canons by Marpurg. Apart from the heading appearing above these densely packed enigmas, Kirnberger is not mentioned by name in the text referring to them; his name appears only above the set of canons in the musical plates (tables) appearing separately on sixty pages following the chapters' text.⁹ This

9 The businesslike reference to the examples without referring to their source in the text is Marpurg's practice generally. In the preface to volume II of *Abhandlung* (as N.B. 4), p.4, »Vorläufige Erinnerungen«, Marpurg does mention several well-known composers and theorists of earlier

³ Further recommendations for understanding the history of terminology associated with canons include Charles Turner, »Sub Obscuritate Quadam Ostendens: Latin Canon in the Early Renaissance Motet«, in: *Early Music* 30 (2002), pp. 165–187; and Denis Collins, »Musical terminology in the canonic works of Bach: an historical context.« in: *Bach* 26 (1995), pp. 91–101.

⁴ Marpurg, Abhandlung (as N.B. 4).

⁵ Siegfried Borris, *Kirnbergers Leben und Werk und seine Bedeutung im Berliner Musikkreis um 1750*, Kassel 1933, p. 7.

⁶ Christoph Wolff, Bach the Learned Musician, New York 2001, p. 307, & ch. 12 footnote 9.

⁷ Ibid., p. 308

⁸ Bach's D-minor fugue from the second book of the *Well-Tempered Clavier*, BWV 875, and a fugue in the same key by Jean-Marie Leclair from his trio sonata, Op. 4 No. 3.

is still praise, as Marpurg with his very title, »Treatise on the fugue based on the principles and examples of the best German and foreign masters,« implicitly honors those whose examples he includes. Yet Marpurg disparages their style in an offhand, perhaps self-consciously apologetic remark:

"One will immediately recognize from the sight of the former [Kirnberger's canons] that observers of this type of writing are probably wrong when they want to persuade the world that there could be good, unforced and natural singing in this type of work. Unforced! Sure! One need look no more in these examples for operatic arias than in fugues. In such matters one should not expect anything other than a masculine and sedate melody that is not based on any fashionable passages."¹⁰

Both Kirnberger's thirteen and Marpurg's own fifteen canons are presented in an exceptionally enigmatic way; aside from the musical notation for each numbered canon only the number of voices is indicated, without epigrams, extra clefs or signs to indicate the times of subsequent entries.¹¹ Despite the challenging puzzles these enigmatic canons elicit, help is sporadically offered in the second volume of the treatise, published the following year, where without exception each canon is eventually revisited either with a written solution or with some clues added. All but one of Kirnberger's thirteen enigmas are represented by a single melody; number 12 is a double canon whose enigma includes two voices (see Example 1).

Many years later, around 1780, Johann Georg Albrechtsberger assembled a set of twenty-six canons by various authors, »26 canoni aperti varii autorii«.¹² Five of Kirnberger's 13 canons are included in Albrechtsberger's collection as well as another, »Wir irren allesamt« to be discussed later.

generations whose work he has drawn upon by name, J.S. Bach, Froberger, Frescobaldi, Bononcini, Bernhard, Theil, Stölzel, »und viele andere«, but not his contemporary, Kirnberger.

- 10 Marpurg, *Abhandlung* (as N.B. 4), p. 92. »Man wird aus dem Anblicke der erstern sogleich erkennen, dass die Berächter dieser Schreibart wohl Unrecht haben, wenn sie die Welt bereden wollen, es könnte sein guter, ungezwungener und natürlicher Gesang in dergleichen Art von Arbeit vorhauden seyn. ungezwungener! So gleich! Opernarienmässige Wendungen muß man so wenig darin suchen, als in periodischen Fugen. Man darf in solchen Sachen nichts anders erwarten, als eine männliche und gesetzte Melodie, die sich auf keine Modepassagen gründet.«
- 11 Marpurg did add epigraphs to two of his own canons one of which is a polymorphous canon capable of innumerable solutions, labeled »canon polymorphus«, and the other »triades harmonicae«.
- 12 Robert N. Freeman, Johann Georg Albrechtsberger's 26 >canoni aperti del varii autori«: The Edition in: *Theoria* 8 (1994), pp 1–53.



Example 1. Canones Kirnbergeri, enigmas, from Marpurg (B.1.XLV-XLVI).¹³

That Kirnberger's thirteen and Marpurg's fifteen canons are presented in the same unusually enigmatic way invites one to speculate that Marpurg may have changed the presentation of Kirnberger's enigmas, at least insofar as to omit Kirnberger's clues. For example, either Kirnberger's or Marpurg's fluency in translating back and forth between enigma and solution allows for alternate versions of the enigmas of canons 2, 3 and 13 discussed below. This is because some highly symmetrical canons may be encrypted into an enigma in more than one way. Kirnberger's enigmas for Canons 6 and 11, published again a decade later, are include more clues than they did in *Abhandlung*. (See Examples 1, 8 and 15.)

¹³ Abbreviations of musical examples from *Abhandlung von der Fuge* by Marpurg will take the following format: »B.2. XXXV.4« means Berlin edition of 1754, volume 2, table XXXV, figure 4.

| | voci | Method, Characteristics | Berlin Vol. II Tab. Fig. | in Vol. II | Albrechtsberger's 26 canoni aperti |
|----|------|-------------------------------------|-----------------------------|------------------------------------|---------------------------------------|
| 1 | a2 | retrograde | XXII.3 | solution | |
| 2 | a2 | augmentation, diminution | XXI.3-4 | solution | |
| 3 | a2 | parallel motion via contrary motion | XXII.4 (1XLIII) | solution | 13 (no.11) |
| 4 | a4 | interlocking half notes, inversion | XXIX.3 | solution | 14 (no.12) |
| 5 | a4 | invertible | XXX.8 | clues: segnos | 15 (no.13) |
| 6 | a4 | through the 5ths >per tonos< | XXXII.2 | solution | 16 (no.14) |
| 7 | a4 | invertible round - E minor | XXX.6 | clues: segnos | |
| 8 | a4 | augmentation 1:2:4:8 | XXX.3 | solution | |
| 9 | a6 | round (invertible) | XXXVII.5 | clues: segnos | |
| 10 | a6 | multiple intervals | XXXVI.11 | clues: rests, clefs, first note | |
| 11 | a4 | stacked, through the fifths | XXXIII.1 | solution | 17 (No.19) |
| 12 | a4 | retrograde, A major | XXIX.4 | solution | |
| 13 | a4 | 5th below, 8ve below, 12th below | XXXV.4 | clues: segnos | |

Table 1 – Canones Kirnbergeri (in Marpurg B1XLV–XLVI)

Canon 1: a2 in retrograde and contrary motion

Two-voice puzzle canons are generally the most difficult to solve, and this set begins with the most rare and unexpected of transformations: retrograde inversion (see Example 2). The only hint of this is that the melody's rhythm is nearly palindromic. In fact the solution given in volume 2 presents the *comes* (follower) in tenor clef so that the notes fall on the correct lines and spaces when turned upside down. Had Marpurg permitted more clues in the enigma, an inverted tenor clef could have been easily included as a hint at the end of the canon, making the enigma more intriguing.

Example 2. Canones Kirnbergeri No. 1.



b) Retrograde of solution. Notice the visual symmetry in both time and contour.

Canon 2: a2 in augmentation

As with the first canon, a rare form of canonic transformation is used, and this makes the second canon as difficult to solve as the first. Marpurg's notation of the solution to this canon illustrates the arbitrariness of the distinction between the labels >augmentation< and >diminution< as regards the canon as a whole. Marpurg's two solutions to the canon differ only in which note of the repeating melody is to be first, and because of that choice, the first solution is labeled as augmentation and the second as diminution (compare Example 3a and 3b). Example 3. Canones Kirnbergeri No. 2.

a) Canon 2 expressed as being. in augmentation, solution from Marpurg B.2.XXI.3



b) Canon 2. The same canon but expressed as being in diminution by starting at a different point, exchanging dux and comes, from Marpurg B.2.XXI.4



This is also a rare example of a >complete augmentation canon< in the sense that the diminished form of the melody appears twice against the augmented form, quite in contrary to the tradition of truncating the augmented version. Ordinarily an augmentation canon is understood to end when the most complete voice, the *dux*, reaches its end, and any augmented voices are truncated at that point. Rarely, a canon such as Canon 2 is devised where the *dux* may repeat as the augmented voice reaches its end and is not truncated.¹⁴

14 When a composer dies without giving the intended solution, a question can remain as to the correct solution. Thomas Op de Coul (in »The Augmentation Canon in J.S. Bach's >Musicalisches Opfer<« in: *Bach* 37 (2006), No. 1, pp. 50–77), explores a nineteenth-century idea that Bach's posthumous augmentation canon might take the form of a 'complete' augmentation canon. In response, Denis Collins (in »From Bull to Bach: In Search of Precedents for the >Complete< Version of the Canon by Augmentation and Contrary Motion in J.S. Bach's >Musical Offering<«, in: *Bach* 38 (2007), No. 2, pp. 39–63), gives some precedent and further discussion regarding Bach's canon, and Timothy D. Edwards (in »The Royal Theme's Hidden Symmetry: In Defense of the Concise Solution to the Augmentation Canon in J.S. Bach's >Musical Offering<«, in: *Bach* 41 (2010), No. 1, pp. 1–31, demonstrates that Bach's canon does not take this form.

Canon 3: parallel scales a2

The third canon illustrates of how a perpetual canon may be developed from a highly symmetrical musical phenomenon: a continuously oscillating major scale covering the span of a ninth (See Example 4.e.). Since a descending scale is equivalent to an ascending scale in contrary motion, a two-voice canon may be formed by such an oscillating scale with the *comes* in contrary motion, its descent moving in parallel with the descent of the *dux*. However, the melody is slightly more subtle than a simple scale in two ways: first, there is a rhythmic pause at the top and bottom of the scale, and second, for a short time some of the notes are shifted by an octave. The result is a fully parallel two-voice structure realized in thirds or sixths above, or in compound thirds or sixths.

In Examples 4a and 4b, the numerals between the staves indicate the continued parallel motion between the two voices, representing the imperfect consonances of thirds and sixths, often compound, formed in different possible solutions. Because the canon is presented in bass clef, the implication is that the missing voices will not be any lower than this voice. Yet it is possible to begin the *comes* on G so that it parallels first in thirds above the *dux*, but at the octave shift, briefly continuing in parallel sixths below the *dux*, as shown in Example 4c.

In the second volume of *Abhandlung*, Marpurg also illustrates that the entire perpetual duet can be performed in retrograde by introducing another canon formed from a retrograde of this canon but beginning on a more conveniently selected first note of the melody, shown in Example 4d.¹⁵

Example 4. Canones Kirnbergeri No. 3. Numerals denote diatonic interval class.

a) Canon 3 in contrary motion, solution from Marpurg (B.2.XXII.4), imitation at the octave.



b) Canon 3 in contrary motion, solution by Edwards: imitation at the twelfth.



c) Canon 3 in contrary motion, solution by Edwards: imitation at the fifth,

¹⁵ Marpurg, Abhandlung (as N.B. 4) vol. 2, Tab. XXII, Fig. 5 and inverted as well in Tab. XXII, Fig. 6.

d) Marpurg created another enigma (Marpurg B.2.XXII.5) from Canon 3 (bottom staff), formed from a retrograde of the solution. Compare the last notes of the upper melody with the second measure of the dux above.



e) The subject of Canon 3 consists of a scale, ascending and descending with rhythmic alterations and octave leaps. The octave displacements are removed in the upper staff reduced to a scale as compared with the subject in the lower staff.



Canon 4: interlocking tetrachords in contrary motion a4

What is most fascinating about this canon is the scarcity of possible solutions that may be properly obtained according to the principles of voice leading and harmony, given how simple the enigma appears. Although it is possible to swap the soprano and tenor voices, the configuration shown in Example 5 is essentially the same as the open-score solution presented in *Abhandlung* as well as Albrechtsberger's collection of 26 canons.¹⁶

Example 5. Canones Kirnbergeri No. 4. Canon in contrary motion, solution from Marpurg B.2.XXIX.3



16 Marpurg, Abhandlung, vol. 2, Tab 2. XXIX.3. Freeman, Johann Georg Albrechtsberger, p. 24.

Canon 5: in quadruple counterpoint

This arrangement is given in open score in Albrechtsberger (*26 canoni aperti varii autorii*) (see Example 6).¹⁷









The canon consists of a single melody overlapping with itself four times, effectively dividing its four measures into four melodic >strands<, labeled here A, B, C and D. When A, B, C and D are put together in order, they form the melody.

¹⁷ Freeman, *Johann Georg Albrechtsberger* (as N.B. 12), pp. 19–20. Other solutions to this enigma are possible that would create different vertical relationships between strands. For example the lowest voice could enter first followed by the tenor-range voice, and so on.

The solution to this canon, made clear by clues added in Marpurg's second volume, is found in Albrechtsberger's *26 canoni aperti* and shown in Example 6. The letters A, B, C and D appear above the measures marking where the strands occur.

| 1 | 2 | 3 | | 4 | 5 | 6 | 7 | |
|---|---|---|---|---|---|---|---|---|
| A | В | С | : | D | А | В | С | : |
| | А | В | : | С | D | А | В | : |
| | | А | : | В | С | D | А | : |
| | | | : | А | В | С | D | : |

In fact there is enough variety in this solution to demonstrate the quadruple counterpoint and to take full musical advantage of the independence of the parts.

There is one subtle but potentially controversial feature of this quadruple counterpoint: The meaning of the harmony changes depending upon the inversion at the cadence from beat 4 to the following downbeat. The neighbor note E that occurs on the fourth beat of strand C, is circled in Example 6 each time it occurs. Though seemingly innocuous as it recurs in various voices, it is a clever way to permit triadic harmony in quadruple counterpoint. Although the functioning harmony on the fourth beat of each measure should be a dominant G triad in the key of C, this would create trouble if the chord's fifth (the note D) were to occur in the bass. Luckily, there is a neighbor tone directly on beat 4, embellishing the note D with the note E instead. Thus, when the bass voice takes strand C (measure 6), the consonant harmony is an E minor triad, and the D which follows it is heard as a passing tone rather than the principal harmonic note. Only if the E is taken as the true bass note may the G's in the middle two staves be understood as consonant rather than poorly prepared fourths above D. The E as bass note changes the character of that fourth beat as compared with the fourth beat of other measures.

The trouble that a second-inversion triad can cause is perhaps best put in the words of Marpurg. In Howard Serwer's 1970 article »Marpurg versus Kirnberger: Theories of Fugal Composition«, the author relates the viewpoints of these two composers on matters relating to inversion and harmonic identity,¹⁸ citing as evidence a quotation from Marpurg's twenty-third *Kritischer Brief.* Although Marpurg is discussing another of Kirnberger's pieces, he clearly suggests that tricks such as Kirnberger's in Canon 5 would be prohibited: »The discussion here is only of correct harmony in contrapuntal contexts and here I maintain that all inversions or root position passages which produce incorrect harmony or even imply it are forbidden.« Soon after, he makes a further clarification:

"For example, in a triad the third is an invertible voice which, if it is used as the bass calls for a six-chord above it. However, the fifth, if not preceded by a tie, is, and

¹⁸ Howard Serwer, »Marpurg versus Kirnberger: Theories of Fugal Composition« in: *Journal of Music Theory* 14 No. 2 (1970), pp. 213–217.

remains, the middle voice in both, which calls for a six-four chord over itself, and therefore in the strict style of composition cannot form the proper bass of an unprepared chord."¹⁹

In other words, when a triad is inverted by moving its chordal third to the bass, a consonant first-inversion >six< chord is formed, but if a triad were to be inverted by moving its chordal fifth to the bass, a dissonant second-inversion >six-four< chord would be formed, and this is prohibited. This forbidden harmony would be the result of Kirnberger maintaining the D throughout beats 3 and 4 rather than employing the E has he has done. Yet by Marpurg's assertion that the implication of the harmony must be observed in an inversion, the foundational necessity of interpreting the E as a chordal root changes the implication of the harmony from a dominant-function G chord to an E-minor chord instead, contradicting the implication of a dominant function.

Marpurg's view on de facto reharmonization due to inversion in general is not favorable. This is evident from a dispute between Marpurg and Kirnberger prompted by the question: »In the composition of two-part invertible counterpoint, must the composer allow for the harmonic implication of the intervallic structure?«²⁰ According to Serwer's assessment,

"By thinking in terms of inverted chords and the fundamental bass, Marpurg judged Kirnberger's counterpoint according to its functional harmonic structure rather than as a mere succession of intervallic simultaneities. Kirnberger defended his work in intervallic rather than harmonic terms so that his position, while seemingly less rule-bound than Marpurg's, was in fact more old-fashioned."²¹

By extension the same question can be posed for invertible counterpoint of more voices. The same logic and the same disagreement between the two theorists can be extended to the case of our Canon 5, and Kirnberger would clearly defend his E-minor triad on beat 4.

Canon 6: stacked canon per tonos exhibiting counterpoint at the twelfth

This canon can be understood as a stacked canon that modulates as it recurs. A stacked canon is built when each new voice imitates the previous voice at the same time and pitch interval (other than the unison). In other words, as Example 7 shows, after the *dux* is heard to enter in the bass, each voice enters a perfect fifth above and two measures after the previous entry until all voices have entered. As each voice repeats, it does so a major third higher and five measures later. This transposition is indicated with the symbol at the end of the staff shown in the enigma in Example 1, known as a *direct* (German: >Wächter<; Latin: >custos<), indicating that the next note should be *E* rather than *C*.

Ibid., quoting and translating Marpurg's letter, pp. 216f.
Ibid., p. 213.
Ibid., p. 216.

Ordinarily the structure of a stacked canon does not require invertible counterpoint. This can be seen, again, by thinking of each one-measure strand of the melody as being represented by a letter, and seeing that the structure does not involve a vertical reordering of the strands from measure to measure. In a stacked canon that begins in the bass, newer strands always appear above older strands:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|-----|-----|-----|-----|
| | | | А | В | С | D | (E) |
| | | А | В | С | D | (E) | |
| | А | В | С | D | (E) | | |
| А | В | С | D | (E) | | | |

Examining this structure reveals that at no point are two strands inverted. However, when the first voice re-enters after the measure of rest (E), invertible counterpoint is used to ensure a sensible result. Compare strands A and C between the outer voices in measures 3 and 6, and notice that they are inverted:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | etc |
|---|---|---|---|-----|-----|-----|-----|---|-----|-----|
| | | | А | В | С | D | (E) | А | В | |
| | | Α | В | С | D | (E) | А | В | С | |
| | А | В | С | D | (E) | А | В | С | D | |
| А | В | С | D | (E) | Α | В | С | D | (E) | |

As shown by the brackets labeling the strands in Example 7a, the alto enters on the note d' in measure 4 (strand A), and the bass completes a 3–2 suspension on the notes c and B (strand C); as the voices are inverted on the bass's entrance in measure 6 on the note e (strand A), the soprano completes a 4–3 suspension. This is invertible counterpoint at the 12th. Consult the interval table in Table 2 to see the expected relationships between intervals when inverting melodies by a fifth or twelfth.

| Table 2 — Inversion Tables | | | | | | | | | |
|----------------------------|--------|--------|---------|----------------|------|--|--|--|--|
| At the | Octave | At the | Twelfth | At the Seventh | | | | | |
| 8 | unis | 12 | unis | 7 | unis | | | | |
| 7 | 2 | 11 | 2 | 6 | 2 | | | | |
| 6 | 3 | 10 | 3 | 5 | 3 | | | | |
| 5 | 4 | 9 | 4 | 4 | 4 | | | | |
| 4 | 5 | 8 | 5 | 3 | 5 | | | | |
| 3 | 6 | 7 | 6 | 2 | 6 | | | | |
| 2 | 7 | 6 | 7 | unis | 7 | | | | |
| unis | 8 | 5 | 8 | | | | | | |
| | | 4 | 9 | | | | | | |
| | | 3 | 10 | | | | | | |
| | | 2 | 11 | | | | | | |
| | | unis | 12 | | | | | | |



Example 7. Canones Kirnbergeri No. 6, canon a4 in contrary motion. Solution by Marpurg B.2.XXXII.2

It should be noted that the extended rest in strand E, marked (E), acts as a buffer, allowing some pairs of strands to avoid the necessity of requiring invertibility. While the pairs of strands (A, C), (A, D) and (B, D) are invertible, the pairs (A, B), (B, C) and (C, D) are not; strand A never appears below strand B; strand B never appears below strand C, and strand C never appears below strand D.

The enigma for this canon published in Kirnberger's *Kunst des reinen Satzes in der Musik* is different from that in Marpurg's *Abhandlung*. It is much longer because it traces the path of the *dux* through all twelve keys, (»Durch Quinten«) rather than just four, repeating at new pitch levels, beginning on *C* and ending on *C* again (compare Examples 7 and 8).



Example 8. Kirnberger's enigma of Canon a 4. durch Quinten (Canones Kirnbergeri No. 6), Courtesy University of North Carolina

Unlike the enigma included by Marpurg, which includes only one iteration of the >subject<, Kirnberger's enigma for this canon, seen in Example 8, is notated all the way through the circle of fifths including three iterations of the >subject< (i.e. the enigma published by Marpurg). The convenience of this longer enigma makes the journey through the fifths clear and accomplishes the enharmonic shift in its notation via a change in key signatures, rather than leaving the task of modulation and enharmonic spelling to the one solving the canon. It also accomplishes the octave correction that is necessary for the canon to avoid transposing a full octave upward with each repetition.

The numerical figures 6, 9 and 15 appearing above in the first two lines of Kirnberger's enigma (Example 8) signify intervals above the notes of the *dux*, interpreted in bass clef over which they appear, their metric positions showing the times of entry. Thus the second voice enters a sixth above the note *B* with the note *g*; The third entry begins a ninth above the bass's *c* on the note *d*', and the final follower begins on the note *a*' at the interval of a fifteenth (double octave) above the bass note *A*. These numerals resolve any ambiguity as to the correct order of the clefs and first notes of each entry.

Canon 7: invertible round a4

As with Canon 5, this four-part solution exhibits quadruple counterpoint. Since the voices are entirely invertible there is no reason that this particular firing order of parts needs to be followed. In Example 9 the four strands of which the *dux* is made are marked A through D.

There are two interesting contrapuntal situations, marked with asterisks in measures 3 and 4 where they occur in the bass. They are both subtle but worth mentioning since they allow full triads to occur in the canon's harmony. It is due to Kirnberger's deep understanding of triadic harmony that he is able to accomplish the use of this fuller harmony. As mentioned with Canon 5, a major or minor triad is consonant when its root or chordal third are in the bass, but the situation is more complex when the fifth appears in the bass. As David Beach points out,

"[...] the fact that Kirnberger identifies both a consonant and a dissonant six-four chord sets him apart from the other theorists of his time. The earlier figured-bass theorists like Heinichen and Mattheson, for example, considered all six-four chords to be dissonant because of the dissonant interval of a fourth. On the other hand, Rameau and his followers viewed all such chords as consonant because they are related to consonant triads by inversion. Kirnberger, however, stated that the six-four chord could be either consonant or dissonant, depending upon the context."²²

In Canon 5, Kirnberger avoided a six-four harmony where it would have occurred. Now in Canon 7 he introduces a six-four harmony on the second eighth note of measure 3 by arpeggiation. Both strands B and C include a chordal fifth by arpeggiation, and this allows the harmony to be fuller. This is in contrast to the very moment of the downbeat where, as is typical for quadruple counterpoint, only the root and third of a harmony are sounded. (Strands A, B and C begin with the root, *e*, of the tonic triad, and strand D begins with the tonic triad's third, *g*.) The arpeggiation is effective in any inversion just after the downbeat because it occurs after the harmony is established.



Example 9. Kirnberger's enigma of Canones Kirnbergeri No. 7, solution by Edwards

²² Johann Philipp Kirnberger, *The Art of Strict Musical Composition [Die Kunst des reinen Satzes in der Musik]* (Music Theory Translation Series, Number 4), London 1982, p. xiii.



As strand D occurs in the bass of measure 4, one can see two devices used to avoid the trouble caused by six-four chords. There, in Example 9, the asterisk marks a place where the bass voice has leapt to a higher register to sound the chordal fifth of the dominant B triad. There are two contrapuntal excuses for this note (F#) when it occurs in the bass. First, the leap to a higher register makes it much more likely that it appears by voice crossing above the triad's root or third sounding in another voice. Even if it were not heard in another voice, the leap suggests a polyphonic melody. Second, despite the leap, the overall motion in that measure is a passing motion from the chordal third, g, through f# to e. By this logic, the f# heard as the bass of a passing six-four. The higher register also makes it more likely that the chordal fifth is not heard as a bass note, and this helps to obviate an aural interpretation of the harmony as inverted.

Is Kirnberger cheating by having his melody cross voices with the one beyond it in order to avoid inverting certain troublesome notes? Perhaps it is wily, but it is creative. Although cumbersome to explain, the importance of this careful writing is that Kirnberger is able to create a highly invertible contrapuntal structure even while using full triads.

Canon 8: Augmentation canon with contrary motion in proportions 1:2:4:8

This augmentation canon follows the scheme of Bach's four part canon, number 14 >in augmentation and diminution< from BWV 1087. That is, the given voice mostly in sixteenth notes is augmented to the next follower in which it is predominantly eighth notes, with the next augmentation producing quarter notes in a third voice, and resulting in half notes in the ultimate level of augmentation.

Example 10a is a new solution similar to Marpurg's but more faithfully following the tonal implications of the enigma, adhering to the indicated key of C major, and ending on the final pitch indicated in the enigma rather than attempting to continue. These observances produce a more coherent conclusion than Marpurg's, shown in Example 10b.

As the dux begins and ends with confirmation of C major, Marpurg's decision to add B-flats in measures 3, 4 and 5 is clearly not Kirnberger's intention. After all, only one flat is used in the C-major dux. That no flat should be used in the tenor voice at the

end of measure 4 should be clear after considering the question as to why Kirnberger included the otherwise awkward eighth rest in the first measure of the enigma. It has no musical reason for being in the first measure but for the rest that its quadrupling augmentation will produce in the third measure, alto voice. This half rest in measure four is necessary to allow the tenor's *B*-natural to avoid a clash. The intention to employ *B*-natural and the avoidance of a false relation is the only logical reasons for the existence of the half rest in measure 4 and thus the reason for the eighth rest in measure 1.

Flats should not appear in measure 5 either. If flats are used in measure 5, then the *dux's* sudden *B*-naturals in the final measure give the impression of a half cadence in F rather than a full cadence in *C*. It is most likely that the B-flats in measure 5, and their weakening of the cadence are the cause of Marpurg's desire to carry the augmentation further, even extending the *dux* and adding >cet< (etcetera). However, the augmentation cannot be continued beyond the end of the *dux*. In the end, Marpurg's solution does not do justice to Kirnberger's conception.



Example 10. Canones Kirnbergeri No. 8 in augmentation in proportions 1:2:4:8.

a) Solution by Edwards based on Marpurg B.2.XXX.3



b) Solution by Marpurg (B.2. XXX 3)

Canon 9: six voices, sextuple counterpoint

Two solutions are presented to Canon 9: In Example 11a the enigma is resolved in six voices at the unison and presented in the style of a round, where each voice begins by singing the first staff, then sings the second staff as the second voice enters on the first, and so on, repeating until all voices have entered and all staves sung. This illustrates the fact that the melody is composed of six strands working in harmony with one another, but it does not reveal that these strands can be transposed by octaves to different registers without compromising the harmony of the canon. That is shown in Example 11b where the voices do enter in various registers. As each two-beat strand passes, the relationship between these six strands changes. By the time the repeat sign is reached, six different arrangements of these six strands have been heard, demonstrating that the tiny canon is capable of sextuple counterpoint (invertible counterpoint in six voices).

Example 11. Canones Kirnbergeri No. 9 a 6.



a) at the unison, solution by Edwards



b) in octaves exhibiting sextuple counterpoint, solution by Edwards

At the very least, invertible counterpoint in six voices requires that under all inversions of voices no prohibited parallel motion occur and that any voice may serve as the bass below the others without compromising the harmony. In short, any configuration of the six strands of which the melody is comprised must be viable. While no prohibited parallel motion occurs due to inversion in any combination, the treatment of the bass deserves comment.

Kirnberger finds uses for triads in such a way that when they are inverted to six-four chords they are used in two ways exemplified by the example of his teacher, Johann Sebastian Bach. We might refer to these two ways in contemporary terms as accented passing six-fours and pedal six-fours.

The first of these six-four scenarios arises from a dissonance known as an accented passing tone. In Kirnberger's words:

"Two notes against one are treated in two ways. First, the first note can be a consonance and the second a dissonance, which is known as a regular passing tone. Second, the first note can be dissonant and the second consonant, and this quality of the notes creates irregular passing tones."²³

These dissonances discussed in a two-voice setting can also occur in denser textures, and quite often in Bach's chorale harmonizations, whether in an upper voice or in the bass. Though not generally practiced in sixteenth-century counterpoint, these dissonances are ubiquitous in eighteenth century music. An accented passing six-four would be a second-inversion triad whose bass is passing by step, but that passing bass note is an accented passing tone. Example 12 shows three instances of accented passing tones in Bach's chorale harmonizations. Each of these the passing tone (circled) occurs in a more metrically accented position (on the beat) than its resolution (after the beat). In the first of these the chorale melody itself contains the passing tone; in the second, the dissonance occurs in the bass, and in the third, the bass note functions as an accented passing tone, producing an accented passing six-four chord.²⁴

Example 12. Excerpts from Bach chorale harmonizations, accented passing tones circled, resolutions marked »*«.



a) No. 237 »Was betrübst du dich, mein Herz«, BWV 423, meas. 3-4.

- 23 Johann Philipp Kirnberger, Richard B. Nelson, and Donald R. Boomgaarden, »Kirnberger's >Thoughts on the Different Methods of Teaching Composition as Preparation for Understanding Fugue« in: *Journal of Music Theory* 30 (1986), p. 71–94: p. 78.
- 24 Johann Sebastian Bach, *371 vierstimmige Choräle für ein Tasteninstrument* (Orgel, Klavier, Cembalo) *nach der Ausgabe von 1784-1787 (J. Ph. Kirnberger, C. Ph. E. Bach), herausgegeben von Klaus Schubert.* Breitkopf & Härtel, Wiesbaden, 1990. Nos. 237, 190 and 356, respectively. In Example 8c, it should be noted that the six-four appearing on beat 2 resolves within the subdominant function chord built on *g*, so that the tenor note *d'* remains structurally consonant, preparing the suspension on beat 3. Understanding the six-four sonority on the first eighth-note of beat 2 to be passing between inversions of a subdominant sonority, rather than anticipating a cadential suspension, helps to support the view that the tenor suspension is handled in the traditional, effective manner.



b) No. 190 »Herr, nun laß in Friede«, BWV 337, meas. 3-4.



c) No. 356 »Jesu, meine Freude«, BWV 358, meas. 5-6.

In Canon 9 the two six-four harmonies occurring on beats 1 and 3 of the second repeating measure, marked with asterisks in Example 11b, are explicable by regarding their bass notes as accented passing tones. When harmonized, these accented passing tones can serve as the dissonant bass of a >passing six-four< but rhythmically offset in contradiction to traditional practice.

The second of these six-four scenarios challenging a traditional view of invertible counterpoint is seen in the form of a >pedal six-four<, in other words, a second-inversion triad whose existence can be explained by the fact that the bass is functioning as a pedal tone. As stated in *Die wahren Grundsätze zum Gebrauch der Harmonie*:

"A pedal point, which usually appears near the end of a fugal composition, is formed by a succession of harmonies sounding against a sustained note in the bass, or in the soprano voice, or even (albeit infrequently) in a middle voice. In determining the fundamental harmony, the sustained note is disregarded when the harmonies are not directly related to it; rather the fundamental harmony is derived from the moving parts, which must always be composed as if the sustained note were not there."²⁵

²⁵ Johann Philipp Kirnberger, »The True Principles for the Practice of Harmony.« Translated with introduction by David W. Beach and Jurgen Thym in: *Journal of Music Theory*, 23, no. 2 (1979), p. 203, §21.

As with Canon 5 which also employs quadruple counterpoint, we see edge cases for how rules can be applied in the most contemporary manner. In Canon 9 a challenge is made to the traditional view, and even to Marpurg's contemporary view, as stated above.

Critically comparing the two solutions shown in Examples 11a and 11b reveals the advantages of invertible counterpoint. While both solutions rely on the use of pedal tone *D*, that reliance is not constant in the longer version since that note sometimes occurs in a higher register. Three chief advantages of the second solution are 1) that it provides textural variety, 2) that it provides a wider range in pitch, and 3) that it avoids the obfuscation of each melody caused by voice crossing. In contrast, the unisono solution creates incessant repetition, each two beats being equivalent, the individual lines unencumbered by too many voices in the same register, each voice's middle-register notes obscured, and the poignant resemblance of the salient highest notes with each repetition drawing the ear's attention away from hearing any one part.

Since sextuple counterpoint is exceptionally rare, its occurrence deserves scrutiny. But the argument that the pedal point is an invalid rationale for accepting the invertibility must also be applied to the unisono solution, since both of these solutions rely on that same pedal point. If one accepts the unisono canon, one must by the same logic accept a solution in octaves.

Canon 9 along with Canons 5 and 7 explore triadic multiple counterpoint. Though their mutual inclusion in the collection may at first seem redundant, they do explore different methods for accomplishing invertibility in so many voices. Canon 5 avoids the second inversion of the dominant triad through shades of harmonic meaning; Canon 7 introduces the triadic fifth through metrically careful arpeggiation and registral leaps, and Canon 9 employs the passing and pedal six-four chords. Between them these three canons address the three non-cadential contexts of six-four chord: passing, arpeggiating, pedal.

Vir Türnberger. 2. Vox. 3. tia. 4. ta. 5.ta. 6.ta.

Example 13. Canones Kirnbergeri No. 10, clues in Marpurg (B.2. XXXVI.11), Courtesy of the Newberry Library.

Canon 10: six voices, different intervals

In Canon 10 Kirnberger created an unusual chord progression that seems two hundred years ahead of its time. The harmonies seem to be drawn from American popular music of the 1970's, not German music of the 1750's. The effect is caused by a saturation of seventh chords, many of them minor-seventh sonorities. This is possible in the eighteenth century because each seventh is prepared and resolves, not simply struck in full as it would be in jazz or soul music.

It is interesting to note that seventh chords are formed on nearly every scale degree in this canon and that Kirnberger's theory accepts the idea of essential seventh chords on each scale degree other than the seventh. Whether or not they are essential, in each of these chords the seventh is prepared. That is, they are rhythmically-displaced dissonances by means of suspension. This is also true of the ninth which resolves in harmony with the seventh in the fourth measure.

Example 14. Canones Kirnbergeri No. 10, a6 at multiple intervals



c) Theoretical harmonic relationship of sounding bass-clef notes to missing partials in measure 5



d) Example 74 from *Die wahren Grundsätze zum Gebrauch der Harmonie…* (Kirnberger/Schulz 1773) Among the »most natural progressions of the fundamenal bass.«



This canon's voicing and thickness of texture and harmony, saturated with seventh chords in six voices. is not unique. There is a strong resemblance to a similar voicing and harmony in J.S. Bach's *Ricercare a6* from the *Musical Offering*, BWV 1079 (see Example 14; asterisks denote suspensions), and it is possible that the remarkable sequence from Bach's *Ricercare* was an inspiration to Kirnberger to explore this richer six-voice harmony in his canon.



Example 15. Bach, »Ricercare a6« from *Musicalisches Opfer*, excerpt m.29–31.

More remarkable in Canon 10 are the six-four sonorities which call to mind echoes of Marpurg's words. These are harmonies that in contrast to cadential six-fours do represent the root-position triads of which they are inversions. One might say that Canon 10 provides empirical evidence for the consonant six-four (measures 5 and 6, Example 14). It is significant that in a case where Kirnberger chose to include a six-four sonority whose fourth is not prepared, and whose root is the expected resolution of the previous harmony in such a clear fifths progression, that the sonority is compact, consisting of notes with a single shared but absent fundamental note.

The six-four sonorities in the bottom three voices in measures 5 and 6 have a special acoustic relationship: They represent consecutive partials of a single (absent) fundamental note,²⁶ as shown in Example 13c. Organists will recognize that this major-triad six-four voicing is the same that sounds when a three-rank cornet organ stop is played without an 8-foot stop, these pitches representing the immediate overtones of the note produced by that 8-foot organ stop. There could be no stronger voicing of three notes in a six-four harmony.

To say that the chords occur over bass pedal tones on the downbeats of measures 5 and 6 is hardly tenable, since the harmony is in constant motion. In fact, the hypothesis that the harmonies on the downbeats of these measures constitute Kirnberger's

26 The cornet organ stops include the third, fourth and fifth partials of the note played on the keyboard. Due to the inverse relationship between wavelength (pipe length) and frequency, the pipe lengths of the three components of a three-rank cornet stop are not, at a glance, within this relationship, but the overtone relationship is the source of the genesis of this organ pipe construction. The three (non-breaking) ranks of a three-rank cornet stop include the tierce 1-3/5-foot, block flute 2-foot and na-sard 2-2/3-foot stops, a four-rank cornet would add the 4-foot flute stop, and a five-rank would add the 8-foot fundamental tone. These numbers, if inverted, show an overtone frequency series, 1, 2, 3, 4, 5 calculated thus: 8' ÷ 1.6' = 5; 8' ÷ 2' = 4; 8' ÷ 2.666' = 3; 8' ÷ 4' = 2; 8' ÷ 8' = 1. It is this series that the listener's ear combines into a single tone, even when only partials 3, 4 and 5 are heard.

>consonant six-fours<, formed from the fundamental bass line depicted in Example 13b, is that Kirnberger himself (or his ghostwriter Schulz) used almost the same progression as the first example of one of the most natural progressions of the fundamental bass (Example 13d).

The fourth is not prepared in measures 5, 6 or 7 of Canon 10. Marpurg's quotation reciting rules of harmony and declaring such fourths to be dissonant was from 1759 in response to a fugue by Kirnberger, but Kirnberger's canon was published by Marpurg in 1752. Marpurg's harsh tone is clearly in response to a long-held disagreement between the two men regarding what is acceptable in deliberate, traditional harmony. Canon 10 is precisely the type of musical example that might elicit this kind of response from Marpurg. In every other respect, Canon 10 follows the rules of traditional, style antico harmony in six voices, and this makes the unusual six-fours stand out all the more.

To allay any suspicion in the reader that the enigma given for Canon 10 with clefs, ensuring this voicing and order of entries, might have been devised by Marpurg rather than Kirnberger, one will find on examination that no matter how the voices are arranged through inversion, there is no way to avoid the fourths that result from chordal fifths in the bass.²⁷

Given the rarity of such a sonority used in this way, it is surprising that this canon would appear in Marpurg's treatise without discussion or explanation. The controversial six-four sonority in this canon lends one to doubt that Kirnberger would want it to be published without further discussion. Whatever the debate elicited by this canon, it is not mentioned or even realized in open form in Marpurg's treatise, and this omission constitutes a denial of Marpurg to engage Kirnberger's musico-philosophical inquiry into the limits of strict harmony. In light of this unexplained inversion, it is interesting that 20 years later in *Die Kunst des reinen Satzes*, Kirnberger points out to his readers passages from Bach's Sinfonia in F minor, BWV 795, where unprepared fourths occur.

"Bach, who has often written for learned ears, can of course not be understood by the aspiring contrapuntal students, who in this case cannot feel and judge the missing basic voice clearly enough. An imitation of this is not to be recommended to them until they are due more mature insights, as the six-four Accord leads to frequent mistakes."²⁸

- 27 This is easily demonstrated by examining in which voices the chordal fifths arise, numbered descending from 1 to 6, soprano to bass. Beginning at the repeat sign, measure 3, the chordal fifth *a*' appears in voice 3, then the chordal fifth *d* appears in voices 2 and 4. In measure 4 the chordal fifth *E* appears in voice 1, then the chordal fifth *A* appears in voice 2. In measure 5 the chordal fifth *D* appears in the bass, voice 6, then the chordal fifth *G* appears in voice 5. In these three measures each of the six voices contains a chordal fifth. Therefore, no matter which of the six entries is performed in the bass it is doomed to undergird a second-inversion chord.
- 28 Kirnberger, Kunst des reinen Satzes, vol. 2, part 2, pp. 39–41. See also Nicholas Stoia, »Triple Counterpoint and Six-Four Chords in Bach's Sinfonia in F Minor«, in: Music Analysis 34, no. 3 (2015), pp. 305– 334.

Kirnberger is making two important assertions here: first, that the proper hearing of otherwise forbidden six-four chords by a master composer involve the skill of being able to hear a missing voice–one would assume this to be the fundamental bass–and, second, that an explanation is not forthcoming. This stance protects the integrity of the mysterious six-four chords occurring occasionally in music by learned masters including those in Kirnberger's Canon 10.

Canon 11: stacked canon progressing through the circle of fifths

After Canon 6, this canon is the second of two canons which Kirnberger referred to as canons >Durch Quinten< (through the fifths) in *Die Kunst des reinen Satzes* where their enigmas were again included.²⁹ These two canons as Kirnberger presented them are unique among modulating canons insofar as all of the modulation occurs within the canon's subject rather than by means of imitation or transposition on repetition. Thus the canon is not properly a canon *per tonos*, as characterized by Marpurg in his solution,³⁰ or at any other interval but a perpetual stacked canon with imitation at the fifth but repetition at the original pitch.³¹ It just so happens that the *dux* and *comes* contain modulations that carry them through the circle of fifths before they repeat. For this reason, Kirnberger's characterization >Durch Quinten< is more apt. Rather than reiterating a single canonic subject at continuous intervals of modulation as did Canon 6, this canon traverses the twelve keys through a continuously varying *dux*. The enigma thus matches the one published in *Die Kunst des reinen Satzes in der Musik*, except of course with the clues absent in the Marpurg.

In Kirnberger's enigma, shown in Example 16, the numeral 2 above the bass's A indicates that the second entry begins on B at the interval of a second above that A, answering the leader's E up a fifth. The third entry begins on F#, as dictated both by its being a fifth above the previous entry and by the numeral #6 above the bass note A. Similarly, the fourth voice begins as expected on C#, two octaves above the bass's C# as indicated by the numeral 15.

²⁹ Kirnberger, *Kunst des reinen Satzes*, vol. 2, part 3. The enigma for Canon 5 is on pages 60f., and the enigma for Canon 11 is on pages 61f.

³⁰ Marpurg, *Abhandlung*, vol.2, tab XXXIII, fig. 1.

³¹ A stacked canon is one where each successive entry occurs at the same pitch and time interval as the previous entry. Alan Gosman's article »Stacked Canon and Renaissance Compositional Procedure« explores this topic thoroughly (*Journal of Music Theory* 41 No. 2 (1997), pp. 289–317).

Example 16. Canon a 4. »durch Quinten« enigma appearing in *Die Kunst des reinen Satzes*, volume 2 part 3, pages 61–62. Also: Canones Kirnbergeri No. 11.



Being another canon through the fifths, Canon 11 follows the same plan as Canon 6. In Kirnberger's enigmas for both the canons through the fifths (numbers 6 and 11), the clefs ascend by fifth as well, so that the answers can be read by means of these clefs and the key signatures given. However, in solving the canon some judgment must be made as to the interpretation of accidentals, remembering that each voice represents a chromatic rather than a diatonic transposition. Even as the *dux* makes its way through the successive keys over the course of its thirteen measures, it is imitated in successive fifths, each new entry participating in the changing keys. Rather than strictly spending one measure per key, the canon includes an extra, thirteenth measure, gracefully pausing before the succession begins again. Both Canon 6 and this canon appear in all three collections: Kirnberger, Marpurg, and Albrechtsberger's *26 canoni aperti dei varii autori* (ca. 1780). The solution reproduced in a condensed score in Example 17 is given in open score in Marpurg and Albrechtsberger.



Example 17. Canones Kirnbergeri No. 11, »durch Quinten.« Solution: Marpurg (B.2.XXXII.2).

Canon 12: double retrograde canon in four voices

The two-voice enigma of Canon 12 is solved in four voices by duplicating each of the two parts in retrograde, as shown in Marpurg's solution in Example 18. This transcription includes Marpurg's asterisk marking the axis of temporal symmetry but omits the repeat sign he added to his solution.

Like the enigma of Canon 1, the relatively palindromic rhythm and contour of Canon 12's enigma give a hint of a retrograde solution, but because Canon 12 retrogrades at the unison, the overall effect is that of a true palindrome.

Example 18. Canones Kirnbergeri No. 12, Marpurg's solution (B.2.XXIX.4), repeat sign removed.

Canon 13: at the fifth below, octave below and twelfth below

This four-in-one canon involves imitation at the fifth below, the octave below and the compound fifth below. Another way to understand the pitch relationship between the voices in this canon is as two pairs of voices, each pair consisting of two entries an octave apart, and each pair being a fifth apart from the other (see Example 19).³² This intervallic relationship perfectly represents the relative ranges of the standard voice designations in a choir: soprano, alto, tenor and bass.

Example 19. *Canones Kirnbergeri* No. 13, Canon at the fifth below, octave below, twelfth below, author's solution, strands marked A through G.



The fact that invertible counterpoint plays an important role in this canon is best understood by breaking down the seven-measure enigmatic melody into seven onemeasure strands as shown above the soprano voice in Example 19. Once the canon is repeating, all of the voices continue in the same unchanging relationship with respect to the other voices as each repeats the eleven-note, seven-measure melody.

This configuration of entries is reminiscent of that in a stacked canon, but since the pitch and time intervals between successive entries is not consistent in Canon 13, it is not a stacked canon. However, a comparison of this canon with Canon 6 will be useful. In both Canons 6 and 13, double counterpoint is utilized to allow for new overlaps of strands to occur as original entries enter with the repeat of the canon.

Waiting a measure before introducing the second pair of entries creates new difficulties for the composer, even as it avoids others. It has the effect of extending the overlap of the last strands (D, E, F and G) with the first strands potentially increasing the need for more double counterpoint, but it also ensures that when particular consecutive strands (for example, strand A and strand B) occur simultaneously, they do so only in pairs that employ imitation at the fifth. This careful coordination of staggered pairs of entries helps to manage the need for double counterpoint, but with such a close and concentrated succession of entries, some inversion is inevitable.

³² There is some indication that Marpurg went back and forth between open and closed form of this canon. In its reappearance (solution) in Marpurg's second volume it has a different starting point than in its introduction in volume one. Since it is a repeating (perpetual) canon this only affects the beginning, but the version in volume 2 with clues begins on the third measure of the version in volume 1.

The heptagonal figure in Example 20 shows how every strand of the canon melody is combined contrapuntally with every other strand at some point throughout the repeating seven measures, but only those strands that are maximally distant in the repeating seven-measure unit are susceptible to inversion. In Example 20, the vertices of the heptagonal figure represent strands, and the line segments that connect them represent contrapuntal relationships between pairs of strands, some non-invertible, and others invertible. Each of these segments is labeled according to the type of contrapuntal relationship that occurs between the strands. For example, the pair A, B is connected with a symbol denoting that strand A always occurs below strand B, and that this pair is never inverted (see measures 2, 5 and 9). Strands E and A, however, are inverted, and this relationship is represented in the heptagonal figure with an equal sign connecting vertex A with vertex E. The fact that all vertices are interconnected demonstrates that every strand overlaps with every other strand at some point in the canon.



Example 20. Inversional relationships between strands in Canon 13.

Example 21 shows the inversion in more detail. The top system shows the entire canon beginning at measure 5 with the repeat written out; the second system includes the soprano and bass voices from measure 5 through measure 11, while the bottom system shows those same strands inverted at the (compound) twelfth from measure 9 through measure 15, now in the bass and alto voices.

For example, in measure 5, strand E occurs in the soprano above strand A in the bass, but in measure 9, strand A in the alto is above strand E in the bass. As the canon is

constructed without melodic embellishment, every interval occurring between these pairs of voices sounds as a consonance above the bass, either a third, fifth, octave or compound instance of these, all inverting perfectly at the twelfth as if in a textbook example.



Example 21. Canones Kirnbergeri No. 13, measures 5–15, invertible counterpoint: Soprano/Bass & Alto/Bass. Some compound intervals' numerals are expressed as simple intervals for clarity.

The clever device that allows this canon to rejoin itself in seven-measure iterations lies in the invertible counterpoint at the 12th designed between the second and fourth entries occurring at an interval of three measures an octave below. This invertibility allows the first voice to occur (again) three measures after the fourth voice but now a twelfth higher rather than an octave lower. The fact that seven is a prime number prevents unintended collisions of the strands in different combinations, because any time interval of imitation is necessarily coprime with the canon's overall repeating length.

The 13 Canones Kirnbergeri

In summary, we see in these 13 canons not a randomly meandering set of dry, encoded curiosities, but deliberate contrapuntal studies that explore possibilities and demonstrate skills, principles and symmetries. The stories that these canons tell, though technical, show a mismatch between their composition and the forum in which they are first shared: as a diversion within Marpurg's first volume. In other words, when asking why these canons are included in the first volume independently of the flow of the topics, there is no explicit answer. In the first volume, Marpurg gives a practical reason for the absence of the customary clues:

"If we omitted the usual heading in these canons, we did it so as not to take away from those who have already practiced this type of writing the difficulties of solving them.

For as soon as one notices the entrances with certain signs, and also puts the clefs in order on the staff, the canon is as good as solved, and from a lucky guess one cannot do too much for oneself." 33

It is undoubtedly also the case that Marpurg included these canons in such a bare and puzzling way as a matter of intrigue, to entice the reader's interest. Perhaps in leaving out the clues, Marpurg might sell more copies of his second volume to enthusiasts eager to learn the answers. Those first readers who did find themselves intrigued and waited in anticipation for Marpurg's second volume would find considerable difficulty, searching through five dozen pages of new musical examples with no clues as to where the solutions may be found. Kirnberger's name is absent from Mapurg's index,³⁴ and the thirteen canons are never again acknowledged as a set in the second volume of *Abhandlung*. Marpurg offered little or no discussion of their construction or the method of their composition. It would have been interesting to know what Kirnberger would have to say about his canons.

Kirnberger's Canon a 4 with basso continuo Wir irren allesamt

The title page of the first volume of Johann Philipp Kirnberger's treatise *Die Kunst des reinen Satzes in der Musik* (1771) bears on its title page a humorously notated puzzle canon packed with subtleties and esoteric knowledge (Example 22). Among those details are to be found both a profound statement about a conundrum of intonation and a practical way to resolve it.

^{33 »}Wenn wir bey diesen Canons die sonst gewönliche Ueberschrift wegließen, thaten wir es darum, um denjenigen, die sich in dieser Schreibart schon geübt haben, nicht daß Vergnügen der Auflösung zu nehmen. Denn sobald man die Eintritte mit gewissen Charaktern bemerkt, und die Schlüssel noch dazu in ihrer Ordnung aufs System hinsetzt, so ist der Canon schon so gut als aufgelöst, und man kann sich alsdann auf das glückliche Errathen desselben wohl nichts zu gute thun.« Marpurg, *Abhandlung* (N.B. 4) volume 1, 1806 edition, p. 92f.

³⁴ Names which do appear in Marpurg's index to both volumes, after page 147, include the following: Bach, Battiferri, Lebègue, Berardi, Bernhardi, Boivin, Bombardo, Bononcini, Dandrieu, Danglebert, Eberlin, Fasch, Frescobaldi, Froberger, Fux, Gebel, Graupner, Gregorius, Händel, Heinichen, Keirleber, Kirchoff, Kreising, Kuhnau, Mattheson, Michael Romanus, Muffat, Pepusch, Rameau, Scheibe, Scacchi, Sellius, Steffani, Stölzel, Telemann, Theil, Valentinus, and Werckmeister.

Example 22. Title page from the 1771 printing of *Die Kunst des reinen Satzes in der Musik*, v.1 by Kirnberger, showing the enigmatic canon »Wir irren allsamt, nur jeder irret anderst.«, Courtesy of the Newberry Library



Depicted in Kirnberger's illustration is an enigmatic canon whose two seemingly incompatible melodies are engraved on a monolith planted firmly in the ground. Kirnberger's image is reminiscent of iconography employed in illustrations of dozens of canons in Giovanni Battista Martini's *Storia della Musica*, the first volume of which was published in Bologna in 1757. See Example 23 showing a canon illustrating the end of Martini's fifth chapter.³⁵

Padre Martini, a Conventual Franciscan friar, composer and music historian, used puzzle canons to help his students reflect upon and teach biblical passages, further nurturing their religious education while reassuring their parents of the piety of the instructor as well. Each canon in Martini's *Storia della musica* appears at the beginning or end of a chapter as an illustration and includes a biblical reference which might easily be misinterpreted as a citation for the lyrics. The dozens of canons in Martini's treatise are often depicted in illustrations as on monoliths, obelisks, banners and plaques often surrounded by cherubs seen to be engraving or painting the musical notation.

35 Giovanni Battista Martini, Storia Della Musica: Volume 1, Bologna 1757, p. 41.

Example 23. Illustration from Giovanni Battista Martini's treatise Storia della Musica, vol. 1, published a decade before Kirnberger's *Kunst des reinen Satzes in der Musik*, Courtesy Bayerische Staatsbibliothek



Although there is an iconography of canon that dates back to the renaissance,³⁶ there is a reason that this newer iconography of permanence emerged when it did in the middle of the eighteenth century. As the value of canon seemed threatened, theorists sought to venerate it with images suggesting glory, endurance and permanence. The act of enshrining canon perhaps marked a reaction to a fear that the art of canon would perish. Ironically, a monolith may also inadvertently suggest a tombstone. For both Martini and Kirnberger, these illustrations might be either dismissed by the reader as decoration or celebrated as dedications or treated as epigraphs, but because the canons are illustrations and positioned as chapter epigraphs, both authors can safely distance the canons from the subject matter of their treatises and refrain from any commentary whatsoever upon them. Both authors do refrain, but as we will see, Kirnberger's canon locks much in its cipher, including a joke.

³⁶ See for example a study of iconography of musical notation in Renaissance painting including many canons. Volker Scherliess, *Musikalische Noten auf Kunstwerken der italienischen Renaissance bis zum Anfang des 17. Jahrhunderts*, Hamburg 1972. Also, Laurence Wuidar, *Canons énigmes et hiéroglyphes musicaux dans l'Italie du 17e siècle. Études de Musicologie 1*, Brussels 2008.

Example 24. Detail from the title page of the 1771 (Berlin) printing of *Die Kunst des reinen Satzes in der Musik*, vol. 1 by Kirnberger, showing the enigmatic canon »Wir irren allsamt, nur jeder irret anderst.«



The enigma, shown in Example 24, consists of two seemingly different melodies, the first with a key signature of six flats in quadruple meter (4/4) as indicated by the »C«, and the second melody, marked »basso continuo«, with a key signature of six sharps in duple meter (2/2), indicated with Ȣ«.

Both the keys and the meters of the two melodies seem to contradict one another, but despite these paradoxes, the two melodies do belong together. The metrical paradox, four measures of quadruple meter against eight measures of duple meter is easily reconciled: either the continuo must be diminuted (twice as fast), or the canon must be augmented (half as fast). The solution in Example 25 employs the former remedy, reconciling the two melodies to a 4/4 time signature.

Example 25. Canon a4 con Basso Continuo, »Wir irren allesamt, nur jeder irret anderst«, author's solution, corroborated in Albrechstberger's 26 canoni aperti varii autori but in the key of F. Bass figures added.



The key signatures, of course, are enharmonic, and to make matters easy, the dux of this canon can be transposed from G-flat major to F-sharp major to match the continuo part. This transposition, however, glosses over an important assumption which will be dealt with below.

The mysterious numbers (60, 45, 48, 64) appearing below the music refer to the four canonic voices. They represent the relative frequencies of the first notes of the leader and the three canonic voices in the order that they enter. The last two of these numbers are upside down, indicating that the last two voices are to follow in contrary motion.

One thing that every eighteenth-century reader of a musical treatise would come to understand is that intervals are represented by ratios. Multiplying the frequency of one pitch by an interval's ratio produces the note at the interval represented by that ratio. Until musicians learn about musical ratios, they are apt to imagine that intervals combined by addition, but in terms of acoustics, intervals represent ratios. If the leader begins on a#',³⁷ arbitrarily represented by >60<, then the first follower must begin on a lower note bearing the ratio 45:60 with the entrance note of the *dux*. Since this ratio reduces to 3:4, that of the perfect fourth below, the answer must fall on *e*#', the note whose frequency is a perfect fourth below *a*#'. Likewise, the inverted >48< indicates the first note of the next voice is to begin with the note given by the ratio 48:60 with respect to the entrance note of the *dux*. This fraction reduces to 4:5, and indicates that the third voice should enter on *f*#', a major third below *a*#', continuing in contrary motion because the number 48 is inverted on the page. Lastly, the fourth voice's ratio with respect to the *dux* is 60:64, which reduces to the numerals again suggests that the fourth voice continue in contrary motion.

These numbers 60, 45, 48, 64, have no absolute significance with respect to the notes they represent. Their significance lies in their proportions relative to one another. They are the lowest such numbers that can be used to represent this configuration of ratios. These proportions are represented in Example 26.³⁸

Example 26. Pitch numerals and proportions in the enigmatic canon a4 with basso continuo »Wir irren allesamt, nur jeder irret anderst.«



While these enigmatic devices might seem to indicate an intentional obfuscation that is not musically justified, one must remember that the principles, relations and concepts behind the solutions to the mysteries of this puzzle are to be learned in Kirnberger's treatise, including the less common baritone and mezzo-soprano clefs, the

³⁷ In the enigma, the first note is bb', but I will discuss the canon in terms of the key chosen for the solution.

³⁸ The numbers may also be understood to represent the 60th, 45th, 48th and 64th partials of the fundamental note six octaves below *b*'.

symbols for time signatures, the missing basso continuo figures (added to the solution in Example 25) and the justly-intoned harmonic proportions governing the relative pitches of the entries. This enigmatic frontispiece to Kirnberger's treatise therefore presents some intrigue, both jocular and perplexing, perhaps even intimidating to the would-be student or to Princess Amalia herself.

The text of the canon is »Wir irren allesamt, nur jeder irret anderst«. At first glance the text's theme does not seem to address the contradictions or cryptic numerals, but it is nonetheless keenly apt. The familiar phrase, »We are all err, only we all err differently« if not coined by Albrecht von Haller is at least remembered from his 1729 poem »Gedanken« from the collection *Versuch Schweizerischer Gedichte (An attempt at Swiss poems*). Here is a brief excerpt of von Haller's poem of 388 lines.

"Unseliges Geschlecht, das nichts aus Gründen thut! Dein wissen ist Betrug und Tand dein höchstes Gut. Du fehlst, so bald du glaubst, und fällst, so bald du wanderst, Wir irren allesammt, nur jeder irret anderst."³⁹

The saying has also proved even greater longevity as Beethoven used the line for his canon (WoO 198) in 1826.

Kirnberger's enigma presents two melodies apparently contrasting both in meter and key. However, the disagreement and the joke are much more profound than to be a matter of notation or even of enharmonic compatibility. There is not only a contradiction between keys, but a confrontation between the foundations of intonation. There is only one system of intonation that allows for the equivalence of the keys of F sharp and G-flat, and that system is equal temperament. Yet the numbers that represent the entries of this canon precisely represent justly-intoned intervals and are therefore incompatible with equal temperament. In the same canon we not only have two different tuning systems, but two incompatible tuning systems! In other words, for harmony to be made, everyone needs to err a little bit, to approximate. This is the sentiment of the lyric and the crux of the joke.

The incompatibility is not merely a matter of intellectual rumination but a practical matter which we today relegate to our piano tuners. If it were agreed among performers who had established two perfectly harmonious pitches an octave apart, one on c and the other on c', then how could they find their way to the same pitch, one rising a certain interval to f# and the other descending by the same interval to g. The only way to bisect the octave is to find the frequency ratio R that satisfies the equation R • R = 2. Multiplying R times the frequency of c would establish the note f# enharmonically equivalent to g. Again, multiplying that pitch's frequency by R would exactly equal the note c', because only the number R multiplied twice produces an octave. Solving this equation for R yields the square root of 2, and that is an irrational number, one that is not expressible as a ratio. The decimal equivalent of the square root of 2

³⁹ Albrecht von Haller, Dr. Albrecht Hallers Versuch Von Schweizerischen Gedichten, 2. verm. und veränderte Aufl., Bern 1734. http://www.zeno.org/Literatur/M/Haller,+Albrecht+von/Gedichte/Versuch+Schweizerischer+Gedichte/5.+Gedanken [23. Juli 2021].

continues forever without repeating: 1.141213562373095... There was no physical or acoustical way to establish this ratio precisely in Kirnberger's day as could be done for those traditional consonances such as the perfect fifth (3:2), octave (2:1) or any interval in the various forms of just intonation represented as ratios.⁴⁰

As it happens, the set of four numbers in Kirnberger's canon span a just-intoned diminished fifth! The ratio of e# to b by these numbers forms the ratio 64:45, which of course does not equal the square root of 2 (see Example 27). The decimal equivalent of 64:45 is 1.422222 ... (the 2's repeating forever), and the complementary interval that would be needed to rise from b to e#', an augmented fourth, would be 45:32 whose decimal equivalent is precisely 1.40625. (This ratio when multiplied by 64:45 gives the number 2 which is an octave.) The untempered gap between these two sizes of justly-intoned tritones measures 20 cents or one fifth of a semitone, and that is far too large for musical harmony.⁴¹

Example 27. Ratios and radicals from »Wir irren allesamt«. For any three notes, the long side of the triangle connecting the notes represents the product of the values on the shorter two sides.



a) Just intonation: Kirnberger's four tones (»Wir irren allesamt«)

b) Equal temperament: Kirnberger's four tones, hypothetically

40 See Jessulat, in this publication.

^{41 1200} x log(1.422222/1.40625)/log(2).

Understanding the logarithmic nature of equal intervals is one of the benefits of the enlightenment in Kirnberger's time, but with it comes the understanding that compromise is necessary. In his canon *Wir irren allesamt …*, Kirnberger has humorously acknowledged this incompatibility whose atonement is made through its text. In the end, the music sounds fine because everyone is a little bit wrong.

The Gedanken Quodlibet⁴²

While the 1771 canon *Wir irren allesamt* represents a unique intellectual meditation on intonation, Kirnberger's equally impressive canon combining familiar Lutheran chorale melodies serves as the focus of his 1782 publication *Gedanken über die verschiedenen Lehrarten in der Komposition, als Vorbereitung zur Fugenkenntniß*,⁴³ (henceforth *Gedanken über die verschiedenen Lehrarten*) where it is the fifty-fourth and final brief musical example, shown in Example 28. It is difficult to imagine a canon being more intimately bound up in the essay to which it is the culmination, but much of the relationship between the essay and the canon lies below the surface.

Example 28. Canonic Quodlibet, enigma: »Aus tiefer Noth« from Kirnberger's *Gedanken über die verschiedenen Lehrarten in der Komposition*, Berlin, 1782, p.31.

- 42 Although the term quodlibet, even in its name, suggests good natured fun and less serious matters, the term has no equivalent for simultaneous contrapuntal combinations of a grave character. For want of a better term, I will use it.
- 43 Gedanken über die verschiedenen Lehrarten in der Komposition, als Vorbereitung zur Fugenkenntniß von Johann Philipp Kirnberger, Ihrer Königl. Hoheit der Prinzeßin Amalia von Preußen Hof. Musicus (Thoughts on the different methods of teaching composition as preparation for the study of fugue by Johann Philipp Kirnberger, court musician to Her Royal Highness Princess Amalia of Prussia), p. 31, published by George Jacob Decker, 1782 in Berlin.

Just as Bach on his deathbed famously invoked the hymn *Wenn wir in höchsten Nothen seyn* calling upon its association with death and dying, dictating the chorale that would be included in the publication of what was to be named *Die Kunst der Fuge*, Kirnberger in his illness, near the end of his life draws upon the same tradition in preparing for death.

Concluding his demonstration of invertible counterpoint, Kirnberger concludes his tract with this canon as the last of his musical examples using these melodic phrases together in quodlibet, each line quoting a different hymn.

Aus tiefer Noth ruf ich zu dir, Ach Gott vom Himmel sieh darein, Wenn wir in höchsten Nöthen seyn.

From the depths I call to you, Ah, God, look down from heaven, When we are in dire straits.

This is not a single hymn, but a combination of the first lines of three hymns.

Example 29. Sixteenth century sources for »Aus tiefer Noth«, Pages 29 and 24 from Enchiridion Geistliche Gesänge by Johann Walter and Martin Luther 1524. Red brackets indicate the first phrase of each melody as quoted by Kirnberger. Source: Wikimedia Foundation.

Darumb fpzicht Bot ich muß auff feyn-oie armen

The canon's enigmatic melody consists of three phrases, each quoting its familiar early Lutheran hymn tune. The first two of these strands, *Aus tiefer Noth* and *Ach Gott vom Himmel sieh darein* date back to the the very first Lutheran hymnbook, the *Acht-liederbuch* of 1523.⁴⁴ See Example 26, showing these melodies as they appear in a subsequent hymnal the following year. Kirnberger faithfully employs the traditional chorale melodies associated with these first lines.

The third of the canon's phrases has a more complex relation between text and melody. In this case, the first line of the hymn, *Wenn wir in höchsten Nöthen seyn*, borrows with slight modification the second melodic line from its familiar associated melody. In other words, Kirnberger combines the text of the first line with the melody of the second line, though slightly modified. This can be seen Example 30 through comparison with J.S. Bach's verbatim quotation of the traditional melody's second melodic line as it appears in the *Art of Fugue* chorale.⁴⁵

Example 30. Comparison of contours between the third line of Kirnberger's canonic quodlibet, text »Wenn wir in höchsten Nöthen seyn« and the second melodic strand of the Lutheran chorale by the same name as faithfully quoted in Bach's *Kunst der Fuge*.

Bach's quotation in Kunst der Fuge the canto voice in the second line of the chorale melody

The canon's three melodic phrases are roughly four measures each, and these are followed by extended rest to complete the sixteen-measure canonic *dux*. The result is a dynamic texture as the rests allow alternating combinations of three-parts in different voices.

Although the three hymn texts paraphrase different Psalms, Kirnberger has unified them in subtle ways, for example, the juxtaposition of »Noth« and »Nöthen« to equate depths to anguish, and the substitution of the verb »schreien« (to cry) in the traditional hymn (»Aus tiefer Not schrei ich zu dir«) with »rufen« (to call).⁴⁶ The word painting that is already built into the contour of the first notes of *Aus tiefer Not* is put

⁴⁴ Martin Luther, Paul Speratus, Etlich Cristlich Lider / Lobgesang und Psalm, Nürnberg 1524.

⁴⁵ The hymn text by Paul Eber from the year 1566 is for use in the time of trouble, related to death and dying, and is based upon »In tenebris nostrae« (in our darkness) by his teacher Joachim Camerarius from about 1546. The melody originally comes from »Leve de cœur« by Louis Bourgeois (1547). — Bach Cantatas Website: Chorale Melodies used in Bach's Vocal Works - *Wenn wir in höchsten Nöten sein* (https://www.bach-cantatas.com/CM/Z394.htm) [6. Juli 2021].

⁴⁶ This is also to attenuate the emotion and better align character with music.

to good use in Kirberger's canon, but »aus« (out of [the depths]) is highlighted as well in the sense that the canon rises slowly in pitch with repetition.

This canon represents a unique combination of canon types. As well as having the three strands serve as cantus firmi, it is a canon per tonos in four voices, similar to Canons 6 and 11 from Canones Kirnbergeri (in Marpurg) discussed above, but in this canon, instead of each successive voice beginning in a new key, two voices enter in each key. Since it is a 4-in-1 canon, and each strand must serve as the bass, invertible counterpoint at the twelfth must be employed, as in Canon 13 of Canones Kirnbergeri. For example, the two strands in measures 5 through 8, Ach Gott vom Himmel in the soprano and Aus tiefer Noth in the bass may be compared with their inversion at the twelfth in measures 9–12 in the bass and alto (See Appendix). Similarly, Wenn wir in... and Ach Gott vom Himmel are invertible at the twelfth as exemplified in measures 9-12 in soprano and bass as compared with 13–16 in alto and bass. Remarkably, however, the remaining two strands, Wenn wir in... and Aus tiefer Noth must be invertible at the octave. To see this, compare soprano and alto in measures 9–12 with bass and tenor in measures 13–16. Invertibility at both the octave and twelfth allows for the interval of imitation of consecutive entries to change from an octave to a twelfth even as the same strands of melody serve now as bass and now as upper voice.

Thus in one canon, Kirnberger has combined five techniques: (1) two-part canon upon a *cantus firmus*, (2) quodlibet, or the contrapuntal combination of two or perhaps three *cantus firmi*, (3) invertible counterpoint at the octave and (4) at the fifth, and (5) canon *per tonos*, all while venerating the tradition of Lutheran hymns and managing a rhyme between the last two. The canon is a perfect example of Kirnberger's dedication to the purest understanding and practice of the techniques of and traditions of counterpoint.

Given its contrapuntal focus on hymn tunes associated with death, *Gedanken über die verschiedenen Lehrarten* can in part be understood to follow in the tradition of contrapuntal music associated with death and dying.⁴⁷ As Yearsley relates, the musical tradition comes from a cultural ritual of preparing for death. However, the existence of such music does not lessen the shock that Kirnberger seems to be suggesting that he may not live much longer.

The second and third hymn tunes relate to death as well. *Ach Gott vom Himmel sieh darein* paraphrases Psalm 12, a lament, and of course, *Wenn wir in höchsten Nöten sein* is known as Bach's deathbed chorale.

The hint of the author's impending death is only one of several revelations designed to align at the essay's close. In order to understand how Kirnberger used this canon to focus moments of revelation in the reader, it will be important to understand how the essay is structured.

⁴⁷ Johann Pachelbel's Musicalische Sterbens-Gedancken, Christian Flor's Todesgedanken in dem Liede: >Auf meinen lieben Gott, mit umgekehrtem Contrapuncte fürs Clavier sehr künstlich gesetzt und gedruckt zu Hamburg 1692, and Dieterich Buxtehude's Mit Fried und Freud ich fahr dahin (BuxWV 76), as well as Bach's final chorale, dictated from his deathbed and appearing in the Art of Fugue. Cf. David Yearsley, Bach and The Meanings Of Counterpoint, Cambridge 2008, pp.1–13.

As in Marpurg's *Abhandlung*, all the musical examples in Kirnberger's *Gedanken über die verschiedenen Lehrarten* are separate from the text, but unlike Marpurg's musical examples, Kirnberger's are brief and discussed in the order of their numerically labeled sequence. These examples consist of a series of brief counterpoints that resemble Fux's species counterpoint exercises, progressing as Fux's do from note-against-note to more complex. These are bookended by two important canons. First, a canon by Agostino Bendinelli serves as the frontispiece of the essay, and lastly the aforementioned canon by Kirnberger is the final musical example.

The inclusion of Bendinelli's canon is an honor to that composer but is at the same time an honor to Bendinelli's student to whom it is dedicated through its text, composer and theorist Giovanni Maria Bononcini, whose treatise *Musico prattico* (1666) Kirnberger describes as »more in keeping with the good taste of our time than is Berardi«. The canon's text is »Voi che di ben compor brama tenete / il Bononcin leggete.«⁴⁸.

Kirnberger's essay begins by discussing what makes a good education toward the study of fugue, attempting to steer his readers on an aesthetic course between certain strict masters and a newer style. Bononcini, Berardi, Fux and Cima are all honored in a sense but at the same time >cannot be recommended<. Although it is Kirnberger's aim to update the student's sophistication with regard to the treatment of dissonance, he praises Bononcini, Berardi and Fux even while finding reason to criticize each of their work, instead holding up Bach both as the best composer and teacher, and regretting that since Bach left no theoretical works, it was left to his students to do so. Still, the fault that Kirnberger finds with these other composers consists almost entirely in whether their music and teaching is modern enough and not too overly strict to recommend as an aide in teaching fugue.

Kirnberger honors Fux's compositional achievement and even his pedagogical method albeit with a few emphatic caveats, going as far as to emulate it in many of his examples even as he seeks to expand Fux's rules to accommodate contemporary practice. He praises Berardi, and Bononcini,⁴⁹ but finds them both lacking in the qualities that are Bach's exemplary strengths: a strong knowledge of contemporary harmony and his confidence in and stress on maintaining the correct character throughout a composition. Of course, these are precisely the compositional traits for which Bach is famous.

Diving into the treatment of dissonance, Kirnberger progresses through two-part *cantus firmus* exercises strongly resembling those of Fux, but introducing irregular passing tones, one more modern type of dissonance used by Bach though not by Palestrina, Fux's model. He then moves on to discuss invertible counterpoint at the octave, tenth and twelfth, and quite remarkably to illustrate these by combining the same two Lutheran chorale tunes in his canon at various degrees of invertibility,

^{48 »}You who care to compose well, read Bononcini.«

⁴⁹ Kirnberger apparently praises the compositions of the younger Giovanni Bononcini (1670–1747) rather than the elder Giovanni Battista Bononcini (1642–1678) author of the 1673 treatise, *Musico Prattico*, published in Bologna by Giacomo Monti.

whether with each other or with additional lines. Dissonances that appear in these contrapuntal demonstrations include the irregular (accented) passing tone. Although some of these contrapuntal demonstrations as well as his final canon include the *anticipation*, he failed to mention this type of dissonance in his program of updating Fux. After being so bold in his criticism at the beginning of the essay, Kirnberger now mentions only in the most casual way the remarkable fact that either voice may serve as the *cantus firmus*! (See Example 31) In Fux this never happens. Kirnberger was so modest about discussing the combination of the two chorale tunes that it is natural for a reader to assume that he must be mistaken. His translators seem to have come to the same conclusion as I did on my first reading, that the author must have made an error in this statement, but Kirnberger is subtly introducing an unusual surprise:

"23) In the thirty-fourth example, which is composed in a different manner in double counterpoint, one can make either the upper or the lower voice into a *Cantus firmus* [sic], whereby the added second voice is then called the counterpoint."

In other words, since both upper and lower voices are sacred melodies, either can be called a *cantus firmus*, and the other can be treated as the counterpoint. (See Example 31.) Furthermore, by combining the chorale tunes Kirnberger has both composed a Fuxian counterpoint and exempted the lines from criticism.

"24) After the thirty-fourth example, in which one can make the uppermost or lowest voice into a *Cantus firmus* [sic], inversions of counterpoint at the twelfth and transpositions at the fifth follow in examples thirty-five, thirty-six, thirty-seven, and thirty-eight."⁵⁰

Example 31. Kirnberger's Examples 34 & 35 from *Gedanken über die verschiedenen Lehrarten*, p. 24. These two chorale tunes are the same that appear in Kirnberger's canon (see Example 28, measures 5–9 and 1–4.

50 Kirnberger, Gedanken (as N.B. 1), trans. Nelson et al. p. 83.

After demonstrating the counterpoint that would be used in his canon and finding new contrapuntal inversions and transpositions including settings in three and four voices, Kirnberger turns our attention back to the canon by Bendinelli from the first page (Example 32). The student is now invited to revisit Bendinelli's canon, and armed with the experience of working through and studying the contrapuntal potential of these melodies, to reflect upon it, investigate it, learn from it and hopefully discover that it is a canon *per tonos*, just as Kirnberger's final canon is.

Example 32. Bendinelli's enigmatic canon on the title page of Kirnberger's *Gedanken über die verschiedenen Lehrarten*.

Example 33. Bendinelli, Canone a quattro voci, solution by Edwards

A careful reading of Kirnberger's essay will show that his return to the topic of Bendinelli's canon is relevant to the essay's conclusion. It is perhaps not simply a coincidence that Bendinelli's canon and Kirnberger's introduction praise Bononcini, since Bononcini was famous among German theorists for his theoretical work on the modes. The transpositions that Kirnberger has been including among his counterpoints are modal combinations. Kirnberger goes further in promoting his own abilities in his penultimate pair of examples by not only transposing Bendinelli's canon melody into the major mode but presenting a chromatically precise contrary-motion inversion of it, also resulting in its transformation into the major mode as well.

Example 34. Kirnberger's examples 52 & 53 showing his transformations of Bendinelli's canon melody.

Kirnberger once again calls attention to Bendinelli's canon by presenting a transformation of its enigma into the major mode (Example 34), which he favors over the original:

"If one puts it into the major mode (G major), as has been done in the fifty-second example, it not only sounds more pleasant, but is also capable of being inverted in all

voices, as has been shown in the fifty-third example, where the bass becomes the soprano and the tenor becomes the alto." 51

This remark leaves no doubt that Kirnberger intends that the transposed canon rendered in contrary motion in his example be solved and studied in four voices. Kirnberger's inversion of the the inverted Bendinelli solution (Example 35) is significant precisely because it embodies the spirit of Kirnberger's generous pedagogical thrust. Despite his criticism, he values the work of the composer-theorists whom he mentions in his first pages. Indeed the precedents of Fux and Bendinelli are intertwined with the method of Kirnberger's essay. He has not mentioned these men for the purpose of criticizing them; he seeks to both connect to their traditions and stress the necessity to update their methods in order to teach a more contemporary aesthetic, one bound up in the contrapuntal materials and compositional practice with which he is working. By giving his first musical examples in the style of Fux but changing the rules to account for Bach's treatment of dissonance, he is attempting to put Bach on equal footing with Fux's models such as Palestrina. By being conscious of the use of modes, Kirnberger has transformed Bendinelli's Dorian melody, recasting it in a major mode. Furthermore, he has demonstrated that the invertibility of the counterpoint has been preserved and has made the harmony more functional and less antiquated to the ear. Through these integrations Kirnberger asserts a connection with the venerated music and learned methods of the past. This must be understood to be directly related to his mission to teach and defend the contrapuntal purity of Bach's musical practice.

Example 35. Kirnberger's inversion in contrary motion of Bendinelli's canon.

One effect of putting Beninelli's transposed canon on the same page⁵² of the essay as his own canon is that it shows another celebrated canon which ends with a copious amount of rest. Clearly a three-voice structure is easier to manage than a four-voice structure, and both Bendinelli and Kirnberger take advantage of that freedom to different degrees by silencing a fourth voice. The varying textures of each of their resultant vocal canons are a benefit of that modesty. As we shall see, the contrapuntal discipline required to avoid that rest is formidable. Kirnberger has avoided having to write counterpoint at the seventh as Bendinelli has done (see Table 2 on page 57). Comparison shows that Kirnberger's and Bendinelli's canons have a great deal in common, sharing the same complex structure. In both canons, double counterpoint at the 12th is utilized in two different ways: first to permit the structure of imitation at both the twelfth and octave in the exposition (as with canon 13 of Canones Kirnbergeri), and second to allow the overlap of strands that is necessary for the canon to advance and repeat *per tonos*. These relationships are described in Example 36. Example 36. Similar structures between Bendinelli's and Kirnberger's canons.

a) Bendinelli's original canon. Each iteration of the canon melody is symbolized by »ABCD«, each letter representing one of the four strands that together form the melody. Lower-case letters represent the minor keys of the entrances. Curved shapes encircle related pairs of strands involved in invertible counterpoint. These relationships occur between all other pairs of strands, though not circled here. For example, B & C are invertible at the twelfth; strands A & D are invertible at the 7th and 12th.

b) Kirnberger's *Gedanken Quodlibet* has the same basic structure despite a different order of entries. Strand D is in parentheses because it consists almost entirely of rest.

If one has made careful study of Bendinelli's canon and the inversion of the canon in contrary motion as Kirnberger has suggested, one may be surprised by three things: 1) The inverted version sounds more contemporary, and this is of course one of the themes running through Kirnberger's discussion. 2) The structures of Bendinelli's original canon and Kirnberger's *Gedanken Quodlibet* are essentially the same: Both canons are canons per tonos, and both alternate imitation at the octave with imitation at the fifth; the fluency of this pattern across high and low voice ranges is facilitated by invertible counterpoint at the octave and twelfth, respectively, as demonstrated earlier in his essay. Not only has double counterpoint been used by Kirnberger but it was used in the same way by Bendinelli. Before Kirnberger's readers have been shown his ultimate canon, they have already been introduced to the melodies of

which it is formed and have studied the invertibility of their combination. The canon itself, however, is a surprise. 3) A reader may discover for the first time the principle that when an entire composition is both inverted and in contrary motion, invertible counterpoint still holds.

Scholars of music will be further impressed by Kirnberger's flipped version of Bendinelli's canon, in the sense that the canon's transmission through Bononcini touches on the skill that Bononcini is best known for, since Bononcini was famous among German theorists for his theoretical work on the modes.⁵³ Kirnberger's inverted version of Bendinelli's canon gains its modal meaning through its first transformation into the major mode in Kirnberger's example fifty-two. (See Example 34) Kirnberger's chromatically precise contrary-motion inversion results in its transformation into the major mode as well. Although the transformed version of the melody in Kirnberger's example fifty-two does not yield an exciting canon, it is necessary that Kirnberger include this intermediate step so that he may bring the topic of mode to into his discussion and he can show an intrinsic, mode-based derivation of the inverted Bendinelli canon. In short, Kirnberger's command of both invertibility and mode yielded for him a more contemporary rendition of Bendinelli's canon based on Kirnberger's example fifty-three. (my Example 34)

In the end, counterpoint and composition by way of canon became so intertwined in Kirnberger's essay, that they became inseparable even as his words unwaveringly urged they must. In part the essay became a playground and a forum, a place to show both the interesting things he found that fit together, such as the building blocks and associations that point toward and support his final canon and those that he found along the way that are simply interesting such as the invertible counterpoint at the tenth and other relations between the chorale melody phrases that do not happen to be featured in the final canon but still hold between its subjects. Still, the many short examples weave connections with the canon at the end, preparing us for it and even giving us the means of understanding and recognizing the counterpoint. The groundwork laid by these earlier examples provides the means of evaluating the rigor of the counterpoint used in the final canon.

The flow of the essay toward this canon is more sophisticated than to simply be described as a cumulatively progressive curriculum. It seems that the structure of Kirnberger's essay is engineered to concentrate surprise and revelation around its conclusion, the canon. In the final paragraph of the essay Kirnberger hints that he may die soon, partly in words, referring to »finishing my work« and using the phrase »if heaven gives me enough power of body and soul«⁵⁴ and partly in his choice of its chorale tunes. In case the reader did not realize the melodies being used in Kirnberger's examples thirty-four through fifty-one were chorale melodies associated with death and suffering, the revelation occurs with the study of Kirnberger's canon which includes the words. The connotations of the melodies could be overlooked until this

⁵³ Gregory Barnett, »Giovanni Maria Bononcini and the Uses of the Modes« in: *The Journal of Musicology*, 25 (2008) No. 3, p. 231–232.

⁵⁴ Kirnberger, Gedanken (as N.B. 1), p.15.

point if a student were too focused on examining intervals and voice leading when approaching these lessons rather than the contour, sound or significance of the melodies. Therefore, it may be only at the end of the treatise, after seeing and working with the canon, that the meaning of Kirnberger's earlier statement in his thirty-fourth example is understood, when he says about the counterpoint that either of the two voices may serve as *cantus firmus*. A knowledge of the invertibility of the canon's themes allows the further realization that this relationship between those themes allows for the complex structure of a canon such as his or Bendinelli's. The reader may also at this point come to the realization that two chorale tunes were combined with one another in counterpoint, in a quodlibet, if you will. In examining the canon's text, the substitution of the word >rufen< for >schreien<, discussed above may become apparent, and this may also register in the reader with the urgency that the composition's character match the meaning of the text, arousing a concern for the well-being of the author.

Although the condition of Kirnberger's health is easily dismissed, it is never the focus of his discussion. His most direct hints arise as he discusses the context of this study of counterpoint and canon within the overall study of fugue. This is also where the differences between his approach and Marpurg's become clear with respect to copious musical examples. Both authors agree that many musical examples are necessary for the study of fugue. Marpurg's method includes large amounts of musical examples throughout the two volumes of his treatise, approximately sixty pages in each volume, but Kirnberger emphasizes that there are two more steps in the study of fugue and the next following Gedanken über die verschiedenen Lehrarten will be the study of various styles (Nationaltänze). There a great many examples are promised, and their study is necessary in order that the student may understand how the present contrapuntal principles are implemented in multifarious ways that composition in so many styles enables. While both authors presumably influenced by Bach in their value of the study of a great number of varied examples, we may intelligently speculate that Kirnberger's interpretation of the proper place of musical analysis within the curriculum of the study of fugue is more faithful to the ordering in Bach's own teaching. Focusing first on thorough bass, then contrapuntal techniques before attempting to analyze the music of other composers makes possible a more sophisticated understanding of the application of counterpoint in those examples.

Why did Kirnberger refer to his essay as »Thoughts«? Such a title may seem at first glance to be at odds with a superficial assessment of the essay's structure as critical commentary followed by applied theory, but that is not the deep reading. I would hold that core of these »thoughts« do not lie only in the early question of which composers a student should follow and why, but that the exercises themselves, the contrapuntal examples and even the canons are part of the argument Kirnberger makes that purity of writing does not necessitate Fux's strict rules as applied in his *Gradus ad Parnassum* but that J.S.Bach follows a purity of style in his compositions that is more deserving of study.⁵⁵ Kirnberger remains focused on purity of composition, offering Bach as a

55 Kirnberger, Gedanken (as N.B. 1), p. 4.

better example: »Johann Sebastian Bach führt in allen seinen Stücken einen durchgängig reinen Satz [...]«, and he aims to make explicit in the only way possible how that style may remain pure despite its defiance of Fux's overly strict rules, through demonstration. First by offering up Fuxian counterpoints that allow for metrically displaced dissonances, and then by offering Lutheran hymns as examples thereof, showing how they might be combined in counterpoint. Kirnberger's thoughts are shared with his readers in a musical way, inviting them to weigh the treatment of dissonance between short exercises and a more complex canon, and above all inviting the reader to think, to make connections and to investigate. The thoughts referred to in Kirnberger's title are not criticism or polemics but musical thoughts. They are about the many associations between the methods and tools of teaching counterpoint and composition, and many of these thoughts are not stated in words. The greatest benefit of the essay is gained by these deeper musical reflections, but without those reflections Kirnberger's treatment of the composers with whom he is concerned might seem petty.

Over the years Kirnberger's success at integrating his canons into pedagogy advanced. This integration benefited from decades of reflection about how canons could be more effectively and prominently shared and utilized after the disassociated presentation of his *Canones Kirnbergeri* in Marpurg's *Abhandlung*. In the first volume of Marpurg's treatise, the brevity of the canons is underscored through their juxtaposition with two fully notated fugues. Their inclusion enigmatically stands out of step with the pedagogical flow of the treatise. As a collection within the flow of the treatise their solutions are given little focus. In *Die Kunst des reinen Satzes*, canons are given more prominence. The frontispiece canon has a broader significance and scope even while standing aloof from the treatise's broad pedagogical program. In *Gedanken über die verschiedenen Lehrarten*, the art of canon is fully integrated into the essay. The canons are longer than the musical examples and are placed only at the very beginning and the very end of the essay. Hints are multifarious and multifaceted. Most importantly, the canon argues for the main thesis of the essay. The canon celebrates the essay, and the essay celebrates the canon.

It is interesting that unlike Marpurg, Kirnberger adhered to the practice of including only the enigmatic canons and not the solutions, implying that it is part of the learning process for the student to seek and find the combinations that work and for the teacher to verify them. That Marpurg regarded enigmatic canons as mere puzzles is suggested first by his effort to present the enigmas in the most puzzling way possible by removing clues, and second by revealing the solutions in his second volume. While Marpurg's initial presentation of Kirnberger's canons categorically omitted clues, implying that bafflement and battling wits were the province of enigmatic canon, Kirnberger in *Die Kunst des reinen Satzes*, judiciously included clues where they directed readers to concentrate their efforts on appropriate mental tasks, while in *Gedanken über die verschiedenen Lehrarten*, he planted subtle clues to his final canon throughout the essay, beginning with the abbreviations for Sopran, Alto, Tenor, and Bass in the frontispiece canon whose structure would be reflected in his own capstone canon.

In a way, Kirnberger's final essay on counterpoint freed him from the predicament in which he found himself as a theorist. He had no help from Bach or others in providing a theoretical publication to explain and advance what he perceived as Bach's rigorous treatment of dissonance. *Gedanken über die verschiedenen Lehrarten* establishes an opportunity to advance Bach's treatment of dissonance, specifically accented passing tones and anticipations, discussed here in relation to Canon 9 of Canones Kirnbergeri (Example 11) and Bach's chorales (Example 12), Enigmatic canon would be the indispensable vehicle for testing and demonstrating that deliberate rigor. The study and enjoyment of a canon that features accented passing tones and anticipations, carefully included in the contrapuntal syntax would argue for the rigor of that syntax. The implication of Fux's rules is that anything beyond that violated them would not be pure but blemished. It is in direct oppositon to these implications that I understand Kirnberger's adjective >reinen< to mean both pure and clean.

Whether as tombstones or monoliths, Kirnberger's canons trace a development from a focus on melodic transformation and contrapuntal combinations in the *Canones Kirnbergeri* published in 1753, to deeper, more philosophical and perhaps irreconcilable issues regarding intonation in his canon *Wir irren allesamt* published in 1771, bearing the fruits of his collaboration with Sulzer, to the complex and devotional quodlibet published in his penultimate year. Through these three glimpses into Kirnberger's career we can see the enigmatic canon's function to lend permanence and memorialization to Bach's legacy and Kirnberger's understanding of a pure musical style, even as the use of canon becomes more careful, less investigative, and more purposeful.

Kirnberger extolled the music of J.S. Bach, but in his earlier theoretical canons understood in their theoretical contexts, he can be seen to test and demonstrate some of Bach's practices and investigate and demonstrate his own understanding of consonance and dissonance, presumably in agreement with Bach's practice. Further investigation of Kirnberger's canons will undoubtedly reveal more about the person, composer, theorist and teacher. As canon becomes more recognized as a tool of theoretical practice it should be understood that further research would certainly reveal more relationships between Kirnberger's novel theories and their implementation in his canons.

APPENDIX – The Gedanken Quodlibet

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Aus tief

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