
Embodied Prediction

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Versions of the “predictive brain” hypothesis rank among the most promising and the most conceptually challenging visions ever to emerge from computational and cognitive neuroscience. In this paper, I briefly introduce (section 1) the most radical and comprehensive of these visions—the account of “active inference”, or “action-oriented predictive processing” (Clark 2013a), developed by Karl Friston and colleagues. In section 2, I isolate and discuss four of the framework’s most provocative claims: (i) that the core flow of information is top-down, not bottom-up, with the forward flow of sensory information replaced by the forward flow of prediction error; (ii) that motor control is just more top-down sensory prediction; (iii) that efference copies, and distinct “controllers”, can be replaced by top-down predictions; and (iv) that cost functions can fruitfully be replaced by predictions. Working together, these four claims offer a tantalizing glimpse of a new, integrated framework for understanding perception, action, embodiment, and the nature of human experience. I end (section 3) by sketching what may be the most important aspect of the emerging view: its ability to embed the use of fast and frugal solutions (as highlighted by much work in robotics and embodied cognition) within an over-arching scheme that includes more structured, knowledge-intensive strategies, combining these fluently and continuously as task and context dictate.

Keywords

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1 Mind turned upside down?

PP (Predictive processing; for this terminology, see Clark 2013a) turns a traditional picture of perception on its head. According to that once-standard picture (Marr 1982), perceptual processing is dominated by the forward flow of information transduced from various sensory receptors. As information flows forward, a progressively richer picture of the real-world scene is constructed. The process of construction would involve the use of stored knowledge of various kinds, and the forward flow of information was subject to modulation and nuancing by top-down (mostly attentional) effects. But the basic picture remained one in which perception was fundamentally a process of “bottom-up feature detection”. In Marr’s theory of vision, detected intensities (arising from surface discon-

tinuities and other factors) gave way to detected features such as blobs, edges, bars, “zero-crossings”, and lines, which in turn gave way to detected surface orientations leading ultimately (though this step was always going to be problematic) to a three-dimensional model of the visual scene. Early perception is here seen as building towards a complex world model by a feedforward process of evidence accumulation. Traditional perceptual neuroscience followed suit, with visual cortex (the most-studied example) being “traditionally viewed as a hierarchy of neural feature detectors, with neural population responses being driven by bottom-up stimulus features” (Egner et al. 2010, p. 16601). This was a view of the perceiving brain as passive and stimulus-driven, taking energetic inputs

from the senses and turning them into a coherent percept by a kind of step-wise build-up moving from the simplest features to the more complex: from simple intensities up to lines and edges and on to complex meaningful shapes, accumulating structure and complexity along the way in a kind of Lego-block fashion.

Such views may be contrasted with the increasingly active views that have been pursued over the past several decades of neuroscientific and computational research. These views (Ballard 1991; Churchland et al. 1994; Ballard et al. 1997) stress the active search for task-relevant information just-in-time for use. In addition, huge industries of work on intrinsic neural activity, the “resting state” and the “default mode” (for a review, see Raichle & Snyder 2007) have drawn our attention to the ceaseless buzz of neural activity that takes place even in the absence of ongoing task-specific stimulation, suggesting that much of the brain’s work and activity is in some way ongoing and endogenously generated.

Predictive processing plausibly represents the last and most radical step in this retreat from the passive, input-dominated view of the flow of neural processing. According to this emerging class of models, naturally intelligent systems (humans and other animals) do not passively await sensory stimulation. Instead, they are constantly active, trying to predict the streams of sensory stimulation before they arrive. Before an “input” arrives on the scene, these pro-active cognitive systems are already busy predicting its most probable shape and implications. Systems like this are already (and almost constantly) poised to act, and all they need to process are any sensed deviations from the predicted state. It is these calculated deviations from predicted states (known as *prediction errors*) that thus bear much of the information-processing burden, informing us of what is salient and newsworthy within the dense sensory barrage. The extensive use of top-down probabilistic prediction here provides an effective means of avoiding the kinds of “representational bottleneck” feared by early opponents (e.g., Brooks 1991) of representation-heavy—but feed-forward dominated—forms of pro-

cessing. Instead, the downward flow of prediction now does most of the computational “heavy-lifting”, allowing moment-by-moment processing to focus only on the newsworthy departures signified by salient (that is, high-precision—see section 3) prediction errors. Such economy and preparedness is biologically attractive, and neatly sidesteps the many processing bottlenecks associated with more passive models of the flow of information.

Action itself (more on this shortly) then needs to be reconceived. Action is not so much a response to an input as a neat and efficient way of selecting the next “input”, and thereby driving a rolling cycle. These hyperactive systems are constantly predicting their own upcoming states, and actively moving so as to bring some of them into being. We thus act so as to bring forth the evolving streams of sensory information that keep us viable (keeping us fed, warm, and watered) and that serve our increasingly recondite ends. PP thus implements a comprehensive reversal of the traditional (bottom-up, forward-flowing) schema. The largest contributor to ongoing neural response, if PP is correct, is the ceaseless anticipatory buzz of downwards-flowing neural prediction that drives both perception and action. Incoming sensory information is just one further factor perturbing those restless pro-active seas. Within those seas, percepts and actions emerge via a recurrent cascade of sub-personal predictions forged (see below) from unconscious expectations spanning multiple spatial and temporal scales.

Conceptually, this implies a striking reversal, in that the driving sensory signal is really just providing corrective feedback on the emerging top-down predictions.¹ As ever-active prediction engines, these kinds of minds are not, fundamentally, in the business of solving puzzles given to them as inputs. Rather, they are in the business of keeping us one step ahead of the game, poised to act and actively eliciting the sensory flows that keep us viable and fulfilled. If this is on track, then just about every aspect of the passive forward-flowing model is false. We are not passive cognitive couch potatoes so

¹ For this observation, see Friston (2005), p. 825, and the discussion in Hohwy (2013).

much as proactive predictors, forever trying to stay one step ahead of the incoming waves of sensory stimulation.

2 Radical predictive processing

Such models involve a number of quite radical claims. In the present treatment, I propose focusing upon just four:

1. The core flow of information is top-down, not bottom-up, and the forward flow of sensory information is replaced by the forward flow of prediction error.
2. Motor control is just more top-down sensory prediction.
3. Efference copies, and distinct “controllers” (inverse models) are replaced by top-down predictions.
4. Cost functions are absorbed into predictions.

One thing I shan’t try to do here is rehearse the empirical evidence for the framework. That evidence (which is substantial but importantly incomplete) is rehearsed in [Clark \(2013a\)](#) and [Hohwy \(2013, this collection\)](#). For a recent attempt to specify a neural implementation, see [Bastos et al. \(2012\)](#). I now look at each of these points in turn:

2.1 The core flow of information is top-down, not bottom-up, and the forward flow of sensory information is replaced by the forward flow of prediction error

This is the heart and soul of the radical vision. Incoming sensory information, if PP is correct, is constantly met with a cascade of top-down prediction, whose job is to predict the incoming signal across multiple temporal and spatial scales.

To see how this works in practice, consider a seminal proof-of-concept by [Rao & Ballard \(1999\)](#). In this work, prediction-based learning targets image patches drawn from natural scenes using a multi-layer artificial neural network. The network had no pre-set task apart from that of using the downwards connections

to match input samples with successful predictions. Instead, visual signals were processed via a hierarchical system in which each level tried (in the way just sketched) to predict activity at the level below it using recurrent (feedback) connections. If the feedback successfully predicted the lower-level activity, no further action was required. Failures to predict enabled tuning and revision of the model (initially, just a random set of connection weights) generating the predictions, thus slowly delivering knowledge of the regularities governing the domain. In this architecture, forward connections between levels carried only the “residual errors” ([Rao & Ballard 1999](#), p. 79) between top-down predictions and actual lower level activity, while backward or recurrent connections carried the predictions themselves.

After training, the network developed a nested structure of units with simple-cell-like receptive fields and captured a variety of important, empirically-observed effects. One such effect was “end-stopping”. This is a “non-classical receptive field” effect in which a neuron responds strongly to a short line falling within its classical receptive field but (surprisingly) shows diminishing response as the line gets longer. Such effects (and with them, a whole panoply of “context effects”) emerge naturally from the use of hierarchical predictive processing. The response tails off as the line gets longer, because longer lines and edges were the statistical norm in the natural scenes to which the network was exposed in training. After training, longer lines are thus what is first predicted (and fed back, as a hypothesis) by the level-two network. The strong firing of some level-one “edge cells”, when they are driven by shorter lines, thus reflects not successful feature detection by those cells but rather error or mismatch, since the short segment was not initially predicted by the higher-level network. This example neatly illustrates the dangers of thinking in terms of a simple cumulative flow of feature-detection, and also shows the advantages of re-thinking the flow of processing as a mixture of top-down prediction and bottom-up error correction.² In ad-

² This does not mean that there are no cells in v1 or elsewhere whose responses match the classical profile. PP claims that each neural area

dition it highlights the way these learning routines latch on to the world in a manner specified by the training data. End-stopped cells are simply a response to the structure of the natural scenes used in training, and reflect the typical length of the lines and edges in these natural scenes. In a very different world (such as the underwater world of some sea-creatures) such cells would learn very different responses.

These were early and relatively low-level results, but the predictive processing model itself has proven rich and (as we shall see) widely applicable. It assumes only that the environment generates sensory signals by means of nested interacting causes and that the task of the perceptual system is to invert this structure by learning and applying a structured internal model—so as to predict the unfolding sensory stream. Routines of this kind have recently been successfully applied in many domains, including speech perception, reading, and recognizing the actions of oneself and of other agents (see [Poepfel & Monahan 2011](#); [Price & Devlin 2011](#); [Friston et al. 2011](#)). This is not surprising, since the underlying rationale is quite general. If you want to predict the way some set of sensory signals will change and evolve over time, a good thing to do is to learn how those sensory signals are determined by interacting external causes. And a good way to learn about those interacting causes is to try to predict how the sensory signal will change and evolve over time.

Now try to imagine this this on a very grand scale. To predict the visually presented scene, the system must learn about edges, blobs, line segments, shapes, forms, and (ultimately) objects. To predict text, it must learn about interacting “hidden” causes in the linguistic domain: causes such as sentences, words, and letters. To predict all of our rich multimodal plays of sensory data, across many scales of space and time, it must learn about interacting hidden causes such as tables, chairs, cats, faces, people, hurricanes, football games, goals,

contains two kinds of cell, or at least supports two functionally distinct response profiles, such that one functionality encodes error and the other current best-guess content. This means that there can indeed be (as single cell recordings amply demonstrate) recognition cells in each area, along with the classical response profiles. For more on this important topic, see [Clark \(2013a\)](#).

meanings, and intentions. The structured world of human experience, if this is correct, comes into view only when all manner of top-down predictions meet (and “explain away”) the incoming waves of sensory information. What propagates forwards (through the brain, away from the sensory peripheries) is then only the mismatches, at every level, with predicted activity.

This makes functional sense. Given that the brain is ever-active, busily predicting its own states at many levels, all that matters (that is, all that is newsworthy, and thus ought to drive further processing) are the incoming surprises: unexpected deviations from what is predicted. Such deviations result in prediction errors reflecting residual differences, at every level and stage of processing, between the actual current signal and the predicted one. These error signals are used to refine the prediction until the sensory signal is best accommodated.

Prediction error thus “carries the news”, and is pretty much the hero (or anti-hero) of this whole family of models. So much so, that it is sometimes said that:

In predictive coding schemes, sensory data are replaced by prediction error, because that is the only sensory information that has yet to be explained. ([Feldman & Friston 2010](#), p. 2)

We can now savor the radicalism. Where traditional, feed-forward-based views see a progressive (though top-down modulated) flow of complex feature-detection, the new view depicts a progressive, complex flow of feature prediction. The top-down flow is not mere attentional modulation. It is the core flow of structured content itself. The forward-flowing signal, by contrast, has now morphed into a stream of residual error. I want to suggest, however, that we treat this apparently radical inversion with some caution. There are two reasons for this—one conceptual, and one empirical.

The first (conceptual) reason for caution is that the “error signal” in a trained-up predictive coding scheme is highly informative. Prediction error signals carry detailed information

about the mismatched content itself. Prediction errors are thus as structured and nuanced in their implications as the model-based predictions relative to which they are computed. This means that, in a very real sense, the prediction error signal is not a mere proxy for incoming sensory information – it *is* sensory information. Thus, suppose you and I play a game in which I (the “higher, predicting level”) try to describe to you (the “lower level”) the scene in front of your eyes. I can’t see the scene directly, but you can. I do, however, believe that you are in some specific room (the living room in my house, say) that I have seen in the past. Recalling that room as best I can, I say to you “there’s a vase of yellow flowers on a table in front of you”. The game then continues like this. If you are silent, I take that as your agreeing to my description. But if I get anything that matters wrong, you must tell me what I got wrong. You might say “the flowers are yellow”. You thus provide an error signal that invites me to try again in a rather specific fashion—that is, to try again with respect to the colour of the flowers in the vase. The next most probable colour, I conjecture, is red. I now describe the scene in the same way but with red flowers. Silence. We have settled into a mutually agreeable description.³

The point to note is that your “error signal” carried some quite specific information. In the pragmatic context of your silence regarding all other matters, the content might be glossed as “there is indeed a vase of flowers on the table in front of me but they are not yellow”. This is a pretty rich message. Indeed, it does not (content-wise) seem different in kind to the down-

ward-flowing predictions themselves. Prediction error signals are thus richly informative, and as such (I would argue) not radically different to sensory information itself. This is unsurprising, since mathematically (as Karl Friston has pointed out⁴) sensory information and prediction error are informationally identical, except that the latter are centred on the predictions. To see this, reflect on the fact that prediction error is just the original information minus the prediction. It follows that the original information is given by the prediction error plus the prediction. Prediction error is simply error relative to some specific prediction and as such it flags the sensory information that is as yet unexplained. The forward flow of prediction error thus constitutes a *forward flow of sensory information relative to specific predictions*.

There is more to the story at this point, since the (complex, non-linear) ways in which downward-flowing predictions interact are importantly different to the (simple, linear) effects of upward-flowing error signals. Non-linearities characterize the multi-level construction of the predictions, which do the “heavy lifting”, while the prediction error signals are free to behave additively (since all the complex webs of linkage are already in place). But the bottom line is that prediction error does not replace sensory information in any mysterious or conceptually challenging fashion, since prediction error is nothing other than that sensory information that has yet to be explained.

The second (empirical) reason for caution is that it is, in any case, only one specific implementation of the predictive brain story depicts the forward-flow as consisting solely of prediction error. An alternative implementation (due to [Spratling 2008](#) and [2010](#)—and see discussion in [Spratling 2013](#)) implements the same key principles using a different flow of prediction and error, and described by a variant mathematical framework. This illustrates the urgent need to explore multiple variant architectures for prediction error minimization. In fact, the PP schema occupies just one point in a large and complex space of probabilistic generative-

³ To complete the image using this parlour game, we’d need to add a little more structure to reflect the hierarchical nature of the message-passing scheme. We might thus imagine many even-higher-level “prediction agents” working together to predict which room (house, world, etc.) the layers below are currently responding to. Should sufficient prediction error signals accrue, this ensemble might abandon the hypothesis that signals are coming in from the living room, suggesting instead that they are from the boudoir, or the attic. In this grander version (which recalls the “mixtures of experts” model in machine learning—see [Jordan & Jacobs 1994](#))—there are teams (and teams of teams) of specialist prediction agents, all trying (guided top-down by the other prediction agents, and bottom-up by prediction errors from the level below) to decide which specialists should handle the current sensory barrage. Each higher-level “prediction agent”, in this multi-level version, treats activity at the level below as sensory information, to be explained by the discovery of apt top-down predictions.

⁴ Personal communication.

model-based approaches, and there are many possible architectures and possible ways of combining top-down predictions and bottom-up sensory information in this general vicinity. These include foundational work by Hinton and colleagues on deep belief networks (Hinton & Salakhutdinov 2006; Hinton et al. 2006), work that shares a core emphasis on the use of prediction and probabilistic multi-level generative models, as well as recent work combining connectionist principles with Bayesian angles (see McClelland 2013 and Zorzi et al. 2013). Meanwhile, roboticists such as Tani (2007), Saegusa et al. (2008), Park et al. (2012), Pezzulo (2008), and Mohan et al. (2010) explore the use of a variety of prediction-based learning routines as a means of grounding higher cognitive functions in the solid bedrock of sensorimotor engagements with the world. Only by considering the full space of possible prediction-and-generative-model-based architectures and strategies can we start to ask truly pointed experimental questions about the brain and about biological organisms; questions that might one day favor one of these models (or, more likely, one coherent sub-set of models⁵) over the rest, or else may reveal deep faults and failings among their substantial common foundations.

2.2 Motor control is just more top-down sensory prediction

I shall, however, continue to concentrate upon the specific explanatory schema implied by PP, as this represents (it seems to me) the most comprehensive and neuroscientifically well-grounded vision of the predictive mind currently available. What makes PP especially interesting—and conceptually challenging—is the seamless integration of perception and action achieved under the rubric of “active inference”.

To understand this, consider the motor system. The motor system (like the visual cortex) displays a complex hierarchical structure.⁶

⁵ One such subset is, of course, the set of hierarchical dynamic models (see Friston 2008).

⁶ The appeal to hierarchical structure in PP, it should be noted, is substantially different to that familiar from treatments such as Felleman & Van Essen (1991). Although I cannot argue for this here (for more on this see Clark 2013b; in press) the PP hier-

archical structure allows complex behaviors to be specified, at higher levels, in compact ways, the implications of which can be progressively unpacked at the lower levels. The traditional way of conceptualizing the difference, however, is that in the case of motor control we imagine a downwards flow of information, whereas in the case of the visual cortex we imagine an upwards flow. Descending pathways in the motor cortex, this traditional picture suggests, should correspond functionally to ascending pathways in the visual cortex. This is not, however, the case. Within the motor cortex the downwards connections (descending projections) are “anatomically and physiologically more like backwards connections in the visual cortex than the corresponding forward connections” (Adams et al. 2013, p. 1).

This is suggestive. Where we might have imagined the functional anatomy of a hierarchical motor system to be some kind of inverted image of that of the perceptual system, instead the two seem fundamentally alike.⁷ The explanation, PP suggests, is that the downwards connections, in both cases, take care of essentially the same kind of business—namely the business of predicting sensory stimulation. Predictive processing models subvert, we saw, the traditional picture with respect to perception. In PP, compact higher-level encodings are part of an apparatus that tries to predict the play of energy across the sensory surfaces. The same story applies, recent extensions (see below) of PP suggest, to the motor case. The difference is that motor control is, in a certain sense, subjunctive. It involves predicting the non-actual sensory trajectories that *would* ensue *were* we performing some desired action. Reducing prediction er-

archy is fluid in that the information-flows it supports are reconfigurable moment-by-moment (by, for example, changing β and theta band oscillations—see Bastos et al. 2015). In addition, PP dispenses entirely with the traditional idea (nicely reviewed, and roundly rejected, in Churchland et al. 1994) that earlier levels must complete their tasks before passing information “up” the hierarchy. The upshot is that the PP models are much closer to dynamical systems accounts than to traditional, feed forward, hierarchical ones.

⁷ For the full story, see Adams et al. (2013). In short: “[t]he descending projections from motor cortex share many features with top-down or backward connections in visual cortex; for example, corticospinal projections originate in infragranular layers, are highly divergent and (along with descending cortico-cortical projections) target cells expressing NMDA receptors” (Adams et al. 2013, p. 1).

rors calculated against these non-actual states then serves (in ways we are about to explore) to make them actual. We predict the sensory consequences of our own action and this brings the actions about.

The upshot is that the downwards connections, in both the motor and the sensory cortex, carry complex predictions, and the upwards connections carry prediction errors. This explains the otherwise “paradoxical” (Shipp et al. 2013, p. 1) fact that the functional circuitry of the motor cortex does not seem to be inverted with respect to that of the sensory cortex. Instead, the very distinction between the motor and the sensory cortex is now eroded—both are in the business of top-down prediction, though the kind of thing they predict is (of course) different. The motor cortex here emerges, ultimately, as a multimodal sensorimotor area issuing predictions in both proprioceptive and other modalities.

In this way, PP models have been extended (under the umbrella of “active inference”—see Friston 2009; Friston et al. 2011) to include the control of action. This is accomplished by predicting the flow of sensation (especially that of proprioception) that would occur were some target action to be performed. The resulting cascade of prediction error is then quashed by moving the bodily plant so as to bring the action about. Action thus results from our own predictions concerning the flow of sensation—a version of the “ideomotor” theory of James (1890) and Lotze (1852), according to which the very idea of moving, when unimpeded by other factors, is what brings the moving about. The resulting schema is one in which:

The perceptual and motor systems should not be regarded as separate but instead as a single active inference machine that tries to predict its sensory input in all domains: visual, auditory, somatosensory, interoceptive and, in the case of the motor system, proprioceptive. (Adams et al. 2013, p. 4)

In the case of motor behaviors, the key driving predictions, Friston and colleagues suggest, are

predictions of the proprioceptive patterns⁸ that would ensue were the action to be performed (see Friston et al. 2010). To make an action come about, the motor plant responds so as to cancel out proprioceptive prediction errors. In this way, predictions of the unfolding proprioceptive patterns that would be associated with the performance of some action serve to bring that action about. Proprioceptive predictions directly elicit motor actions (so traditional motor commands are simply replaced by those proprioceptive predictions).

This erases any fundamental computational line between perception and the control of action. There remains, to be sure, an obvious (and important) difference in direction of fit. Perception here matches neural hypotheses to sensory inputs, and involves “predicting the present”; while action brings unfolding proprioceptive inputs into line with neural predictions. The difference, as Elizabeth Anscombe (1957) famously remarked,⁹ is akin to that between consulting a shopping list to select which items to purchase (thus letting the list determine the contents of the shopping basket) and listing some actually purchased items (thus letting the contents of the shopping basket determine the list). But despite this difference in direction of fit, the underlying form of the neural computations is now revealed to be the same. Indeed, the main difference between the motor and the visual cortex, on this account, lies more in what kind of thing (for example, the proprioceptive consequences of a trajectory of motion) is predicted, rather than in how it is predicted. The upshot is that:

The primary motor cortex is no more or less a motor cortical area than striate (visual) cortex. The only difference

⁸ Proprioception is the “inner” sense that informs us about the relative locations of our bodily parts and the forces and efforts that are being applied. It is to be distinguished from exteroceptive (i.e., standard perceptual) channels such as vision and audition, and from interoceptive channels informing us of hunger, thirst, and states of the viscera. Predictions concerning the latter may (see Seth 2013 and Pezzulo 2014) play a large role in the construction of feelings and emotions.

⁹ Anscombe’s target was the distinction between desire and belief, but her observations about direction of fit generalize (as Shea 2013 notes) to the case of actions, here conceived as the motoric outcomes of certain forms of desire.

between the motor cortex and visual cortex is that one predicts retinotopic input while the other predicts proprioceptive input from the motor plant. (Friston et al. 2011, p. 138)

Perception and action here follow the same basic logic and are implemented using the same computational strategy. In each case, the systemic imperative remains the same: the reduction of ongoing prediction error. This view has two rather radical consequences, to which we shall now turn.

2.3 Efference copies and distinct “controllers” are replaced by top-down predictions

A long tradition in the study of motor control invokes a “forward model” of the likely sensory consequences of our own motor commands. In this work, a copy of the motor command (known as the “efference copy”; Von Holst 1954) is processed using the forward model. This model captures (or “emulates”—see Grush 2004) the relevant biodynamics of the motor plant, enabling (for example) a rapid prediction of the likely feedback from the sensory peripheries. It does this by encoding the relationship between motor commands and predicted sensory outcomes. The motor command is thus captured using the efference copy which, fed to the forward model, yields a prediction of the sensory outcome (this is sometimes called the “corollary discharge”). Comparisons between the actual and the predicted sensory input are thus enabled.

But motor control, in the leading versions of this kind of account, requires in addition the development and use of a so-called “inverse model” (see e.g., Kawato 1999; Franklin & Wolpert 2011). Where the forward model maps current motor commands in order to predicted sensory effects, the inverse model (also known as a controller) “performs the opposite transformation [...] determining the motor command required to achieve some desired outcome” (Wolpert et al. 2003, p. 595). Learning and deploying an inverse model appropriate to some

task is, however, generally much more demanding than learning the forward model, and requires solving a complex mapping problem (linking the desired end-state to a nested cascade of non-linearly interacting motor commands) while effecting transformations between varying co-ordinate schemes (e.g., visual to muscular or proprioceptive—see e.g., Wolpert et al. 2003, pp. 594–596).

PP (the full “action-inclusive” version just described) shares many key insights with this work. They have common a core emphasis on the prediction-based learning of a forward (generative) model, which is able to anticipate the sensory consequences of action. But active inference, as defended by Friston and others—see e.g., Friston (2011); Friston et al. (2012)—dispenses with the inverse model or controller, and along with it the need for efference copy of the motor command. To see how this works, consider that action is here reconceived as a direct consequence of predictions (spanning multiple temporal and spatial scales) about trajectories of motion. Of special importance here are predictions about proprioceptive consequences that implicitly minimize various energetic costs. Subject to the full cascade of hierarchical top-down processing, a simple motor command now unfolds into a complex set of predictions concerning both proprioceptive and exteroceptive effects. The proprioceptive predictions then drive behavior, causing us to sample the world in the ways that the current winning hypothesis dictates.¹⁰

Such predictions can be couched, at the higher levels, in terms of desired states or trajectories specified using extrinsic (world-centered, limb-centered) co-ordinates. This is possible because the required translation into intrinsic (muscle-based) co-ordinates is then devolved to what are essentially classical reflex arcs set up to quash proprioceptive prediction errors. Thus:

if motor neurons are wired to suppress proprioceptive prediction errors in the dorsal horn of the spinal cord, they effect-

¹⁰ For a simulation-based demonstration of the overall shape of the PP account, see Friston et al. (2012). These simulations, as the authors note, turn out to implement the kind of “active vision” account put forward in Wurtz et al. (2011).

ively implement an inverse model, mapping from desired sensory consequences to causes in intrinsic (muscle-based) coordinates. In this simplification of conventional schemes, descending motor commands become topdown predictions of proprioceptive sensations conveyed by primary and secondary sensory afferents. (Friston 2011, p. 491)

The need (prominent in approaches such as Kawato 1999; Wolpert et al. 2003; and Franklin & Wolpert 2011) for a distinct inverse model/optimal control calculation has now disappeared. In its place we find a more complex forward model mapping prior beliefs about desired trajectories to sensory consequences, some of which (the “bottom level” proprioceptive ones) are automatically fulfilled.

The need for efference copy has also disappeared. This is because descending signals are already (just as in the perceptual case) in the business of predicting sensory (both proprioceptive and exteroceptive) consequences. By contrast, so-called “corollary discharge” (encoding predicted sensory outcomes) is now endemic and pervades the downwards cascade, since:

[...] every backward connection in the brain (that conveys topdown predictions) can be regarded as corollary discharge, reporting the predictions of some sensorimotor construct. (Friston 2011, p. 492)

This proposal may, on first encounter, strike the reader as quite implausible and indeed too radical. Isn't an account of the functional significance and neurophysiological reality of efference copy one of the major success stories of contemporary cognitive and computational neuroscience? In fact, most (perhaps all) of the evidence often assumed to favour that account is, on closer examination, simply evidence of the pervasive and crucial role of forward models and corollary discharge—it is evidence, that is to say, for just those parts of the traditional story that are preserved (and made even more central) by PP. For example, Sommer & (Wurtz's influential (2008) review paper makes very little

mention of efference copy as such, but makes widespread use of the more general concept of corollary discharge—though as those authors note, the two terms are often used interchangeably in the literature. A more recent paper, Wurtz et al. (2011), mentions efference copy only once, and does so only to merge it with discussions of corollary discharge (which then occur 114 times in the text). Similarly, there is ample reason to believe that the cerebellum plays a special role here, and that that role involves making or optimizing perceptual predictions about upcoming sensory events (Bastian 2006; Roth et al. 2013). But such a role is, of course, entirely consistent with the PP picture. This shows, I suggest, that it is the general concept of forward models (as used by e.g., Miall & Wolpert 1996) and corollary discharge, rather than the more specific concept of efference copy as we defined it above, that enjoys the clearest support from both experimental and cognitive neuroscience.

Efference copy figures prominently, of course, in one particular set of computational proposals. These proposals concern (in essence) the positioning of forward models and corollary discharges within a putative larger cognitive architecture involving multiple paired forward and inverse models. In these “paired forward inverse model” architectures (see e.g., Wolpert & Kawato 1998; Haruno et al. 2003) motor commands are copied to a stack of separate forward models that are used to predict the sensory consequences of actions. But acquiring and deploying such an architecture, as even its strongest advocates concede, poses a variety of extremely hard computational challenges (see Franklin & Wolpert 2011). The PP alternative neatly sidesteps many of these problems—as we shall see in section 2.4. The heavy lifting that is usually done by traditional efference copy, inverse models, and optimal controllers is now shifted to the acquisition and use of the predictive (generative) model—i.e., the right set of prior probabilistic “beliefs”. This is potentially advantageous if (but only if) we can reasonably assume that these beliefs “emerge naturally as top-down or empirical priors during hierarchical perceptual inference” (Friston 2011, p. 492).

The deeper reason that efference copy may be said to have disappeared in PP is thus that the whole (problematic) structure of paired forward and inverse models is absent. It is not needed, because some of the predicted sensory consequences (the predicted proprioceptive trajectories) act as motor commands already. As a result, there are no distinct motor commands to copy, and (obviously) no efference copies as such. But one could equally well describe the forward-model-based predictions of proprioceptive trajectories as “minimal motor commands”: motor commands that operate (in essence) by specifying results rather than by exerting fine-grained limb and joint control. These minimal motor commands (proprioceptive predictions) clearly influence the even wider range of predictions concerning the exteroceptive sensory consequences of upcoming actions. The core functionality that is normally attributed to the action of efference copy is thus preserved in PP, as is the forward-model-based explanation of core phenomena, such as the finessing of time-delays (Bastian 2006) and the stability of the visual world despite eye-movements (Sommer & Wurtz 2006; 2008).

2.4 Cost functions are absorbed by predictions.

Active inference also sidesteps the need for explicit cost or value functions as a means of selecting and sculpting motor response. It does this (Friston 2011; Friston et al. 2012) by, in essence, building these in to the generative model whose probabilistic predictions combine with sensory inputs in order to yield behaviors. Simple examples of cost or value functions (that might be applied to sculpt and select motor behaviors) include minimizing “jerk” (the rate of change of acceleration of a limb during some behavior) and minimizing rate of change of torque (for these examples see Flash & Hogan 1985 and Uno et al. 1989 respectively). Recent work on “optimal feedback control” minimizes more complex “mixed cost functions” that address not just bodily dynamics but also systemic noise and the required accuracy of outcomes (see Todorov 2004; Todorov & Jordan 2002).

Such cost functions (as Friston 2011, p. 496 observes) resolve the many-one mapping problem that afflicts classical approaches to motor control. There are many ways of using one’s body to achieve a certain goal, but the action system has to choose one way from the many available. Such devices are not, however, needed within the framework on offer, since:

In active inference, these problems are resolved by prior beliefs about the trajectory (that may include minimal jerk) that uniquely determine the (intrinsic) consequences of (extrinsic) movements. (Friston 2011, p. 496)

Simple cost functions are thus folded into the expectations that determine trajectories of motion. But the story does not stop there. For the very same strategy applies to the notion of desired consequences and rewards at all levels. Thus we read that:

Crucially, active inference does not invoke any “desired consequences”. It rests only on experience-dependent learning and inference: experience induces prior expectations, which guide perceptual inference and action. (Friston et al. 2011, p. 157)

Notice that there is no *overall* computational advantage to be gained by this reallocation of duties. Indeed, Friston himself is clear that:

[...] there is no free lunch when replacing cost functions with prior beliefs [since] it is well-known [Littman et al. (2001)] that the computational complexity of a problem is not reduced when formulating it as an inference problem. (2011, p. 492)

Nonetheless, it may well be that this reallocation (in which cost functions are treated as priors) has conceptually and strategically important consequences. It is easy, for example, to specify whole paths or trajectories using prior beliefs about (you guessed it) paths and trajectories! Scalar reward functions, by contrast, specify points or peaks. The upshot is that everything

that can be specified by a cost function can be specified by some prior over trajectories, but not vice versa.

Related concerns have led many working roboticists to argue that explicit cost-function-based solutions are inflexible and biologically unrealistic, and should be replaced by approaches that entrain actions in ways that implicitly exploit the complex attractor dynamics of embodied agents (see e.g., [Thelen & Smith 1994](#); [Mohan & Morasso 2011](#); [Feldman 2009](#)). One way to imagine this broad class of solutions (for a longer discussion, see [Clark 2008](#), Ch. 1) is by thinking of the way you might control a wooden marionette simply by moving the strings attached to specific body parts. In such cases:

The distribution of motion among the joints is the “passive” consequence of the [...] forces applied to the end-effectors and the “compliance” of different joints. ([Mohan & Morasso 2011](#), p. 5)

Solutions such as these, which make maximal use of learnt or inbuilt “synergies” and the complex bio-mechanics of the bodily plant, can be very fluently implemented (see [Friston 2011](#); [Yamashita & Tani 2008](#)) using the resources of active inference and (attractor-based) generative models. For example, [Namikawa et al. \(2011\)](#) show how a generative model with multi-timescale dynamics enables a fluent and decomposable (see also [Namikawa & Tani 2010](#)) set of motor behaviors. In these simulations:

Action per se, was a result of movements that conformed to the proprioceptive predictions of [...] joint angles [and] perception and action were both trying to minimize prediction errors throughout the hierarchy, where movement minimized the prediction errors at the level of proprioceptive sensations. ([Namikawa et al. 2011](#), p. 4)

Another example (which we briefly encountered in the previous section) is the use of downward-flowing prediction to side-step the need to transform desired movement trajectories from

extrinsic (task-centered) to intrinsic (e.g., muscle-centered) co-ordinates: an “inverse problem” that is said to be both complex and ill-posed ([Feldman 2009](#); [Adams et al. 2013](#), p. 8). In active inference the prior beliefs that guide motor action already map predictions couched (at high levels) in extrinsic frames of reference onto proprioceptive effects defined over muscles and effectors, simply as part and parcel of ordinary online control.

By re-conceiving cost functions as implicit in bodies of expectations concerning trajectories of motion, PP-style solutions sidestep the need to solve difficult (often intractable) optimality equations during online processing (see [Friston 2011](#); [Mohan & Morasso 2011](#)) and—courtesy of the complex generative model—fluidly accommodate signaling delays, sensory noise, and the many-one mapping between goals and motor programs. Alternatives requiring the distinct and explicit computation of costs and values thus arguably make unrealistic demands on online processing, fail to exploit the helpful characteristics of the physical system, and lack biologically plausible means of implementation.

These various advantages come, however, at a price. For the full PP story now shifts much of the burden onto the acquisition of those prior “beliefs”—the multi-level, multi-modal webs of probabilistic expectation that together drive perception and action. This may turn out to be a better trade than it at first appears, since (see [Clark in in press](#)) PP describes a biologically plausible architecture that is just about maximally well-suited to installing the requisite suites of prediction, through embodied interactions with the training environments that we encounter, perturb, and—at several slower timescales—actively construct.

3 Putting predictive processing, body, and world together again

An important feature of the full PP account (see [Friston 2009](#); [Hohwy 2013](#); [Clark in press](#)) is that the impact of specific prediction error signals can be systematically varied according to their estimated certainty or “precision”. The precision of a specific prediction error is

its inverse variance—the size (if you like) of its error bars. Precision estimation thus has a kind of meta-representational feel, since we are, in effect, estimating the uncertainty of our own representations of the world. These ongoing (task and context-varying) estimates alter the weighting (the gain or volume, to use the standard auditory analogy) on select prediction error units, so as to increase the impact of task-relevant, reliable information. One key effect of this is to allow the brain to vary the balance between sensory inputs and prior expectations at different levels (see [Friston 2009](#), p. 299) in ways sensitive to task and context.¹¹ High-precision prediction errors have greater gain, and thus play a larger role in driving processing and response. More generally, variable precision-weighting may be seen as the PP mechanism for implementing a wide range of attentional effects (see [Feldman & Friston 2010](#)).

Subtle applications of this strategy, as we shall shortly see, allow PP to nest simple (“quick and dirty”) solutions within the larger context of a fluid, re-configurable inner economy; an economy in which rich, knowledge-based strategies and fast, frugal solutions are now merely different expressions of a unified underlying web of processing. Within that web, changing ensembles of inner resources are repeatedly recruited, forming and dissolving in ways determined by external context, current needs, and (importantly) by flexible precision-weighting reflecting ongoing estimations of our own uncertainty. This process of inner recruitment is itself constantly modulated, courtesy of the complex circular causal dance of sensorimotor engagement, by the evolving state of the external environment. In this way (as I shall now argue) many key insights from work on embodiment and situated, world-exploiting action may be comfortably accommodated within the emerging PP framework.

¹¹ Malfunctions of this precision-weighting apparatus have recently been implicated in a number of fascinating proposals concerning the origins and persistence of various forms of mental disturbance, including the emergence of delusions and hallucinations in schizophrenia, “functional motor and sensory symptoms”, Parkinson’s disease, and autism—see [Fletcher & Frith \(2009\)](#), [Frith & Friston \(2012\)](#), [Adams et al. \(2012\)](#), [Brown et al. \(2013\)](#), [Edwards et al. \(2012\)](#), and [Pellicano & Burr \(2012\)](#).

3.1 Nesting simplicity within complexity

Consider the well-known “outfielder’s problem”: running to catch a fly ball in baseball. Giving perception its standard role, we might assume that the job of the visual system is to transduce information about the current position of the ball so as to allow a distinct “reasoning system” to project its future trajectory. Nature, however, seems to have found a more elegant and efficient solution. The solution, a version of which was first proposed in [Chapman \(1968\)](#), involves running in a way that seems to keep the ball moving at a constant speed through the visual field. As long as the fielder’s own movements cancel any apparent changes in the ball’s optical acceleration, she will end up in the location where the ball hits the ground. This solution, OAC (Optical Acceleration Cancellation), explains why fielders, when asked to stand still and simply predict where the ball will land, typically do rather badly. They are unable to predict the landing spot because OAC is a strategy that works by means of moment-by-moment self-corrections that, crucially, involve the agent’s own movements. The suggestion that we rely on such a strategy is also confirmed by some interesting virtual reality experiments in which the ball’s trajectory is suddenly altered in flight, in ways that could not happen in the real world—see [Fink et al. 2009](#)). OAC is a succinct case of fast, economical problem-solving. The canny use of data available in the optic flow enables the catcher to sidestep the need to deploy a rich inner model to calculate the forward trajectory of the ball.¹²

Such strategies are suggestive (see also [Maturana & Varela 1980](#)) of a very different role of the perceptual coupling itself. Instead of using sensing to get enough information inside, past the visual bottleneck, so as to allow the reasoning system to “throw away the world” and solve the problem wholly internally, such strategies use the sensor as *an open conduit allowing environmental magnitudes to exert a constant influence on behavior*. Sensing is here

¹² There are related accounts of how dogs catch Frisbees—a rather more demanding task due to occasional dramatic fluctuations in the flight path (see [Shaffer et al. 2004](#)).

depicted as the opening of a channel, with successful whole-system behavior emerging when activity in this channel is kept within a certain range. In such cases:

[T]he focus shifts from accurately representing an environment to continuously engaging that environment with a body so as to stabilize appropriate co-ordinated patterns of behaviour. (Beer 2000, p. 97)

These focal shifts may be fluidly accommodated within the PP framework. To see how, recall that “precision weighting” alters the gain on specific prediction error units, and thus provides a means of systematically varying the relative influence of different neural populations. The most familiar role of such manipulations is to vary the balance of influence between bottom-up sensory information and top-down model-based expectation. But another important role is the implementation of fluid and flexible forms of large-scale “gating” among neural populations. This works because very low-precision prediction errors will have little or no influence upon ongoing processing, and will fail to recruit or nuance higher-level representations. Altering the distribution of precision weightings thus amounts, as we saw above, to altering the “simplest circuit diagram” (Aertsen & Preißl 1991) for current processing. When combined with the complex, cascading forms of influence made available by the apparatus of top-down prediction, the result is an inner processing economy that is (see Clark in press) “maximally context-sensitive”.

This suggests a new angle upon the outfielder’s problem. Here too, already-active neural predictions and simple, rapidly-processed perceptual cues must work together (if PP is correct) to determine a pattern of precision-weightings for different prediction-error signals. This creates a pattern of effective connectivity (a temporary distributed circuit) and, within that circuit, it sets the balance between top-down and bottom-up modes of influence. In the case at hand, however, efficiency demands selecting a circuit in which visual sensing is used to cancel the optical acceleration of the fly ball.

This means giving high weighting to the prediction errors associated with cancelling the vertical acceleration of the ball’s optical projection, and (to put it bluntly) not caring very much about anything else. Apt precision weightings here function to select *what to predict* at any given moment. They may thus select a pre-learned, fast, low-cost strategy for solving a problem, as task and context dictate. Contextually-recruited patterns of precision weighting thus accomplish a form of set-selection or strategy switching—an effect already demonstrated in some simple simulations of cued reaching under the influence of changing tonic levels of dopamine firing—see Friston et al. (2012).

Fast, efficient solutions have also been proposed in the context of reasoning and choice. In an extensive literature concerning choice and decision-making, it has been common to distinguish between “model-based” and “model-free” approaches (see e.g., Dayan & Daw 2008; Dayan 2012; Wolpert et al. 2003). Model-based strategies rely, as their name suggests, on a model of the domain that includes information about how various states (worldly situations) are connected, thus allowing a kind of principled estimation (given some cost function) of the value of a putative action. Such approaches involve the acquisition and the (computationally challenging) deployment of fairly rich bodies of information concerning the structure of the task-domain. Model-free strategies, by contrast, are said to “learn action values directly, by trial and error, without building an explicit model of the environment, and thus retain no explicit estimate of the probabilities that govern state transitions” (Gläscher et al. 2010, p. 585). Such approaches implement “policies” that typically exploit simple cues and regularities while nonetheless delivering fluent, often rapid, response.

The model-based/model-free distinction is intuitive, and resonates with old (but increasingly discredited) dichotomies between reason and habit, and between analytic evaluation and emotion. But it seems likely that the image of parallel, functionally independent, neural subsystems will not stand the test of time. For example, a recent functional Magnetic Resonance Imaging (fMRI) study (Daw et al. 2011) sug-

gests that rather than thinking in terms of distinct (functionally isolated) model-based and model-free learning systems, we may need to posit a single “more integrated computational architecture” (Daw et al. 2011, p. 1204), in which the different brain areas most commonly associated with model-based and model-free learning (pre-frontal cortex and dorsolateral striatum, respectively) *each* trade in both model-free and model-based modes of evaluations and do so “in proportions matching those that determine choice behavior” (Daw et al. 2011, p. 1209). Top-down information, (Daw et al. (2011) suggest, might then control the way different strategies are combined in differing contexts for action and choice. Within the PP framework, this would follow from the embedding of shallow “model-free” responses within a deeper hierarchical generative model. By thus combining the two modes within an overarching model-based economy, inferential machinery can, by and large, identify the appropriate contexts in which to deploy the model-free (“habitual”) schemes. “Model-based” and “model-free” modes of valuation and response, if this is correct, name extremes along a single continuum, and may appear in many mixtures and combinations determined by the task at hand.

This suggests a possible reworking of the popular suggestion (Kahneman 2011) that human reasoning involves the operation of two functionally distinct systems: one for fast, automatic, “habitual” response, and the other dedicated to slow, effortful, deliberative reasoning. Instead of a truly dichotomous inner organization, we may benefit from a richer form of organization in which fast, habitual, or heuristically-based modes of response are often the default, but within which a large variety of possible strategies may be available. Humans and other animals would thus deploy multiple—rich, frugal and all points in between—strategies defined across a fundamentally unified web of neural resources (for some preliminary exploration of this kind of more integrated space, see Pezzulo et al. 2013). Some of those strategies will involve the canny use of environmental structure – efficient embodied prediction machines, that is to say, will often deploy minimal

neural models that benefit from repeated calls to world-altering action (as when we use a few taps of the smartphone to carry out a complex calculation).

Nor, finally, is there any fixed limit to the complexities of the possible strategic embeddings that might occur even within a single more integrated system. We might, for example, use some quick-and-dirty heuristic strategy to identify a context in which to use a richer one, or use intensive model-exploring strategies to identify a context in which a simpler one will do. From this emerging vantage point the very distinction between model-based and model-free response (and indeed between System 1 and System 2) looks increasingly shallow. These are now just convenient labels for different admixtures of resource and influence, each of which is recruited in the same general way as circumstances dictate.¹³

3.2 Being human

There is nothing specifically human, however, about the suite of mechanisms explored above. The basic elements of the predictive processing story, as Roepstorff (2013, p. 45) correctly notes, may be found in many types of organism and model-system. The neocortex (the layered structure housing cortical columns that provides the most compelling neural implementation for predictive processing machinery) displays some dramatic variations in size but is common to all mammals. What, then, makes us (superficially at least) so very different? What is it that allows us—unlike dogs, chimps, or dolphins—to latch on to distal hidden causes that include not just food, mates, and relative social rankings, but also neurons, predictive processing, Higgs bosons, and black holes?

One possibility (Conway & Christiansen 2001) is that adaptations of the human neural apparatus have somehow conspired to create, in us, an even more complex and context-flexible

¹³ Current thinking about switching between model-free and model-based strategies places them squarely in the context of hierarchical inference, through the use of “Bayesian parameter averaging”. This essentially associates model-free schemes with simpler (less complex) lower levels of the hierarchy that may, at times, need to be contextualized by (more complex) higher levels.

hierarchical learning system than is found in other animals. Insofar as the predictive processing framework allows for rampant context-dependent influence within the distributed hierarchy, the same basic operating principles might (given a few new opportunities for routing and influence) result in the emergence of qualitatively novel forms of behavior and control. Such changes might explain why human agents display what Spivey (2007, p. 169) describes as an “exceptional sensitivity to hierarchical structure in *any* time-dependent signal”.

Another (possibly linked, and certainly highly complementary) possibility involves a potent complex of features of human life, in particular our ability to engage in temporally coordinated social interaction (see Roepstorff et al. 2010) and our ability to construct artifacts and design environments. Some of these ingredients have emerged in other species too. But in the human case the whole mosaic comes together under the influence of flexible and structured symbolic language (this was the target of the Conway and Christiansen paper mentioned above) and an almost obsessive drive (Tommasello et al. 2005) to engage in shared cultural practices. We are thus able to redeploy our core cognitive skills in the transformative context of exposure to what Roepstorff et al. (2010) call “patterned sociocultural practices”. These include the use of symbolic codes (encountered as “material symbols” (Clark 2006) and complex social routines (Hutchins 1995, 2014)—and more general, all the various ploys and strategies known as “cognitive niche construction” (see Clark 2008).

A simple example is the way that learning to perform mental arithmetic has been scaffolded, in some cultures, by the deliberate use of an abacus. Experience with patterns thus made available helps to install appreciation of many complex arithmetical operations and relations (for discussion of this, see Stigler 1984). The specific example does not matter very much, to be sure, but the general strategy does. In such cases, we structure (and repeatedly re-structure) our physical and social environments in ways that make available new knowledge and skills—see Landy & Goldstone (2005). Prediction-hungry brains, ex-

posed in the course of embodied action to novel patterns of sensory stimulation, may thus acquire forms of knowledge that were genuinely out-of-reach prior to such physical-manipulation-based re-tuning of the generative model. Action and perception thus work together to reduce prediction error against the more slowly evolving backdrop of a culturally distributed process that spawns a succession of designed environments whose impact on the development (e.g., Smith & Gasser 2005) and unfolding (Hutchins 2014) of human thought and reason can hardly be overestimated.

To further appreciate the power and scope of such re-shaping, recall that the predictive brain is not doomed to deploy high-cost, model-rich strategies moment-by-moment in a demanding and time-pressured world. Instead, that very same apparatus supports the learning and contextually-determined deployment of low-cost strategies that make the most of body, world, and action. A maximally simple example is painting white lines along the edges of a winding cliff-top road. Such environmental alterations allow the driver to solve the complex problem of keeping the car on the road by (in part) predicting the ebb and flow of various simpler optical features and cues (see e.g., Land 2001). In such cases, we are building a better world in which to predict, while simultaneously structuring the world to cue the low-cost strategy at the right time.

3.3 Extending the predictive mind

All this suggests a very natural model of “extended cognition” (Clark & Chalmers 1998; Clark 2008), where this is simply the idea that bio-external structures and operations may sometimes form integral parts of an agent’s cognitive routines. Nothing in the PP framework materially alters, as far as I can tell, the arguments previously presented, both pro and con, regarding the possibility and actuality of genuinely extended cognitive systems.¹⁴ What PP

¹⁴ For a thorough rehearsal of the positive arguments, see Clark (2008). For critiques, see Rupert (2004, 2009), Adams & Aizawa (2001), and Adams & Aizawa (2008). For a rich sampling of the ongoing debate, see the essays in Menary (2010) and Estany & Sturm (2014).

does offer, however, is a specific and highly “extension-friendly” proposal concerning the shape of the specifically neural contribution to cognitive success. To see this, reflect on the fact that known external (e.g., environmental) operations provide—by partly constituting—additional strategies apt for the kind of “meta-model-based” selection described above. This is because actions that engage and exploit specific external resources will now be selected in just the same manner as the inner coalitions of neural resources themselves. Minimal internal models that involve calls to world-recruiting actions may thus be selected in the same way as a purely internal model. The availability of such strategies (of trading inner complexity against real-world action) is the hallmark of embodied prediction machines.

As a simple illustration, consider the work undertaken by Pezzulo et al. (2013). Here, a so-called “Mixed Instrumental Controller” determines whether to choose an action based upon a set of simple, pre-computed (“cached”) values, or by running a mental simulation enabling a more flexible, model-based assessment of the desirability, or otherwise, of actually performing the action. The mixed controller computes the “value of information”, selecting the more informative (but costly) model-based option only when that value is sufficiently high. Mental simulation, in such cases, then produces new reward expectancies that can determine current action by updating the values used to determine choice. We can think of this as a mechanism that, moment-by-moment, determines (as discussed in previous sections) whether to exploit simple, already-cached routines or to explore a richer set of possibilities using some form of mental simulation. It is easy to imagine a version of the mixed controller that determines (on the basis of past experience) the value of the information that it believes would be made available by some kind of cognitive extension, such as the manipulation of an abacus, an iPhone, or a physical model. Deciding when to rest, content with a simple cached strategy, when to deploy a more costly mental simulation, and when to exploit the environment itself as a cognitive resource are thus all options apt for the same

kind of “meta-Bayesian” model-based resolution.

Seen from this perspective, the selection of task-specific inner *neural* coalitions within an interaction-dominated PP economy is entirely on a par with the selection of task-specific *neural–bodily–worldly* ensembles. The recruitment and use of extended (brain–body–world) problem-solving ensembles now turns out to obey many of the same basic rules, and reflects many of the same basic normative principles (balancing efficacy and efficiency, and reflecting complex precision estimations) as does the recruitment of temporary inner coalitions bound by effective connectivity. In each case, what is selected is a temporary problem-solving ensemble (a “temporary task-specific device”—see Anderson et al. 2012) recruited as a function of context-varying estimations of uncertainty.

4 Conclusion: Towards a mature science of the embodied mind

By self-organizing around prediction error, and by learning a generative rather than a merely discriminative (i.e., pattern-classifying) model, these approaches realize many of the goals of previous work in artificial neural networks, robotics, dynamical systems theory, and classical cognitive science. They self-organize around prediction error signals, perform unsupervised learning using a multi-level architecture, and acquire a satisfying grip—courtesy of the problem decompositions enabled by their hierarchical form—upon structural relations within a domain. They do this, moreover, in ways that are firmly grounded in the patterns of sensorimotor experience that structure learning, using continuous, non-linguaform, inner encodings (probability density functions and probabilistic inference). Precision-based restructuring of patterns of effective connectivity then allow us to nest simplicity within complexity, and to make as much (or as little) use of body and world as task and context dictate.

This is encouraging. It might even be that models in this broad ballpark offer us a first glimpse of the shape of a fundamental and unified science of the embodied mind.

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