A dictionary of modular threefolds

Dissertation zur Erlangung des Grades
“Doktor der Naturwissenschaften”

am Fachbereich Mathematik und Informatik
der Johannes Gutenberg-Universität in Mainz,

vorgelegt von
Christian Meyer,
geboren in Mainz.

Mainz, den 22. Februar 2005

D77 – Mainzer Dissertation

Contents

Introduction

The proof of the Taniyama-Shimura Conjecture by A. Wiles et al. in the 1990s (cf. [15]), which implied a proof of Fermat's Last Theorem, has been met with approval from the mathematical community and has even aroused great interest in the public (cf. [1], [95]). It connects, in a very fascinating way, different mathematical subjects, such as algebraic geometry and number theory.

The two main mathematical theories involved are those of elliptic curves and of modular forms. The Taniyama-Shimura conjecture relates the numbers of points on elliptic curves over finite fields to Fourier coefficients of certain modular forms of weight two.

An elliptic curve is a special case of a so called *Calabi–Yau manifold*, namely a Calabi–Yau manifold of dimension one. Calabi–Yau manifolds are of great importance in string theory, a main branch of modern theoretical physics. It is a very natural task to try to extend the results for elliptic curves to Calabi–Yau manifolds of higher dimension. Calabi–Yau manifolds of dimension two are called *K3 surfaces*. Their arithmetic, i.e., their properties over finite fields, has also been studied but we will take one further step forward and concentrate on Calabi–Yau manifolds of dimension three, the so called *Calabi–Yau threefolds*.

The arithmetic of Calabi–Yau threefolds defined over $\mathbb{Q}$ is mainly determined by the L-series of their middle étale cohomology space. The dimension of this space is a positive even number and can be used to classify Calabi–Yau threefolds. If the dimension is two then the threefold allows no complex deformations and is therefore called *rigid* (and *non-rigid* otherwise). For a rigid Calabi–Yau threefold X which is defined over $\mathbb{Q}$ there is a precise conjecture about its connection with modular forms. There should exist a newform of weight four for some Hecke subgroup $\Gamma_0(N)$ the L-series of which agrees with the L-series of the middle cohomology of X. In this case X is called *modular*.

The conjecture has been checked in several examples before and there is also a partial general result by Dieulefait and Manoharmayum (a modularity proof under mild restrictions concerning the primes of bad reduction). It is rather difficult to construct rigid Calabi–Yau threefolds.

For non-rigid Calabi–Yau threefolds the situation becomes much more complicated. We expect that the L-series of their middle cohomology is also determined by modular or automorphic forms. There are some examples where the L-series splits into two-dimensional pieces which are easier to handle.

The main subject of this thesis is the presentation of known results concerning modularity of

Calabi–Yau threefolds and the construction of many new examples.

In chapter 1 we collect the notations and facts concerning Calabi–Yau manifolds and their arithmetic. We also present general modularity results and tools for modularity proofs.

In chapters 2, 3, 4 and 5 we investigate many different examples of Calabi–Yau threefolds and study their modularity. Note that the level of detail is very different for the single examples. A detailed study of all occurring examples would require much more time and space. Nevertheless, the large number of examples makes it possible for the first time to give conjectures about the levels of the occurring newforms. Altogether there are hundreds of new examples of rigid and non-rigid Calabi–Yau threefolds. I would like to accentuate some results:

- In 3.1 and 3.2 the “standard family of quintics” is discussed. We present an equation for the mirror family as a family of quintics. Inside the mirror family there is a rigid Calabi–Yau manifold which corresponds to the Schoen quintic.
- Double coverings of $\mathbb{P}^3$ branched along an octic surface (so called *double octics*) are investigated in chapter 4. These Calabi–Yau threefolds are easier to handle because their geometry is determined by the (lower-dimensional) branch locus. This leads to large tables of modular examples.
- In 3.2 and 5.1 we construct two rigid Calabi–Yau threefolds with Euler characteristics 32 and 202. To my knowledge these are the smallest resp. largest known values. Note that it seems to be possible to produce larger values (cf. 5.11) but this requires additional work.
- It is an interesting question which prime numbers can occur in the levels of weight four modular forms connected with Calabi–Yau threefolds. We present examples involving the “new” primes 13, 19, 31 and 37.

In chapter 6 we try to link those modular Calabi–Yau threefolds which have the *same* modular form in their L-series. According to the Tate conjecture there should be correspondences between them. We present tables of examples and correspondences for examples connected with weight four newforms of small level. Afterwards we discuss the effect of primes of bad reduction on the level and formulate conjectures.

Appendix A contains a table of arrangements of eight planes defined over $\mathbb{Q}$ and the numerical data of the double coverings of $\mathbb{P}^3$ branched along these arrangements.

Appendix B contains tables of modular double coverings of $\mathbb{P}^3$ branched along the union of six planes and a smooth quadric surface.

Appendix C contains a large and almost complete table of weight four newforms for $\Gamma_0(N)$ with level $N \leq 2000$ and rational coefficients.

Appendix D contains a complete table of weight two newforms for $\Gamma_0(N)$ with level $N \leq 228$ and rational coefficients.

To keep the text from further expansion I omitted details on the background in algebraic geometry and number theory. The reader is referred to the standard texbooks of Hartshorne ([47]) on algebraic geometry, Serre ([91]) on Galois representations and Knapp ([58]), Dolgachev ([37]) or Milne ([72]) on modular forms. Further references on specific topics are given in the text. The table of references should be rather complete as far as the subject of modularity of Calabi–Yau threefolds is concerned.

I thank everybody who has helped me in one way or another during the time I have been writing this thesis. This includes everybody working in algebraic geometry at the university of Mainz. The working conditions at the institute of mathematics have been excellent.

During the time of writing I have been supported by the Deutsche Forschungsgemeinschaft. This work is a part of the “DFG Schwerpunktprogramm: Globale Methoden in der komplexen Geometrie”.

Chapter 1

Arithmetic on Calabi–Yau threefolds

1.1 Calabi–Yau varieties

Let X be a smooth complex projective variety of dimension d. X is called a *Calabi–Yau* variety if

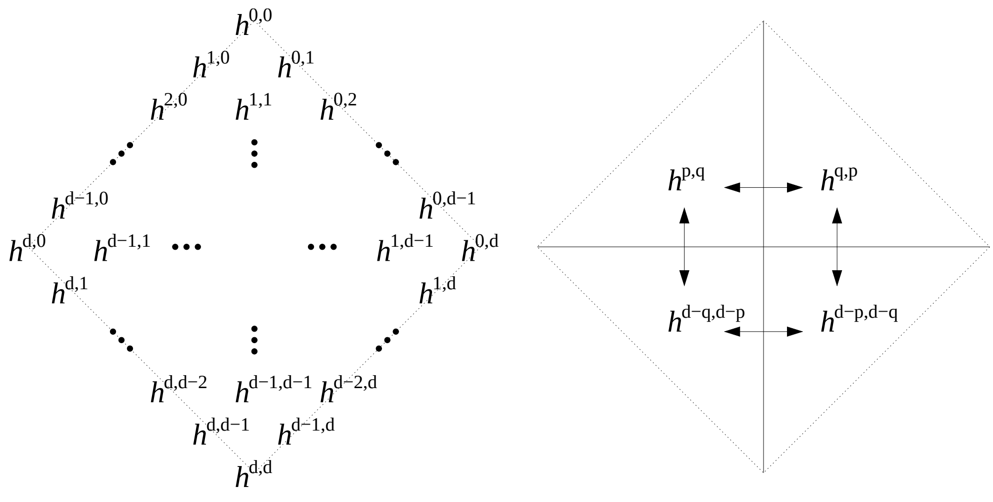
1. $H^i(X, \mathcal{O}_X) = 0$ for every i, $0 < i < d$, and
2. $K_X := \wedge^d \Omega^1_X \simeq \mathcal{O}_X$, i.e., the canonical bundle is trivial.

By the second condition and Serre duality we have

$$\dim H^0(X, K_X) = \dim H^d(X, \mathcal{O}_X) = 1,$$

i.e., the geometric genus of X is 1.

Let $\Omega^p_X := \wedge^p \Omega^1_X$ and let $H^q(\Omega^p_X)$ be the (p,q)-th *Hodge cohomology group* of X with *Hodge number* $h^{p,q}(X) := \dim_{\mathbb{C}} H^q(\Omega^p_X)$. The Hodge numbers are very important invariants of X. They are often displayed as a *Hodge diamond:*



All numbers not appearing in the diagram are zero. By complex conjugation we have $H^q(\Omega_X^p) = H^p(\Omega_X^q)$ and by Serre duality $H^q(\Omega_X^p) = H^{d-q}(\Omega_X^{d-p})$. The symmetries of the Hodge diamond are indicated in the second picture.

The $k-th$ *Betti number* of X is $h^k(X) := \dim_{\mathbb{C}} H^k(X, \mathbb{C})$. By the Hodge decomposition

$$H^k(X,\mathbb{C}) \cong \bigoplus_{p+q=k} H^q(\Omega_X^p)$$

we have

$$h^k(X) = \sum_{p+q=k} h^{p,q}(X) = \sum_{i=0}^{k} h^{i,k-i}(X).$$

Finally the *Euler characteristic* of X is

$$\chi(X) := \sum_{k=0}^{2d} (-1)^k h^k(X).$$

The conditions for X to be Calabi–Yau assert that $h^{i,0}(X) = 0$ for $0 < i < d$ and that $h^{0,0}(X) = h^{d,0}(X) = 1$.

A dimension $d = 1$ Calabi–Yau variety X (equipped with a rational point) is an elliptic curve with the following Hodge diamond:

$$\begin{array}{ccc} & 1 & \\ 1 & & 1 \\ & 1 & \end{array} \qquad \begin{array}{c} h^0(X) = 1 \\ h^1(X) = 1 + 1 = 2 \\ h^2(X) = 1 \\ \hline \chi(X) = 1 - 2 + 1 = 0 \end{array}$$

A dimension $d = 2$ Calabi–Yau variety X is called a $K3$ *surface*. It has the following Hodge diamond:

$$\begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 1 & & 20 & & 1 \\ & 0 & & 0 & \\ & & 1 & & \end{array} \qquad \begin{array}{c} h^0(X) = 1 \\ h^1(X) = 0 + 0 = 0 \\ h^2(X) = 1 + 20 + 1 = 22 \\ h^3(X) = 0 + 0 = 0 \\ h^4(X) = 1 \\ \hline \chi(X) = 1 - 0 + 10 - 0 + 1 = 22 \end{array}$$

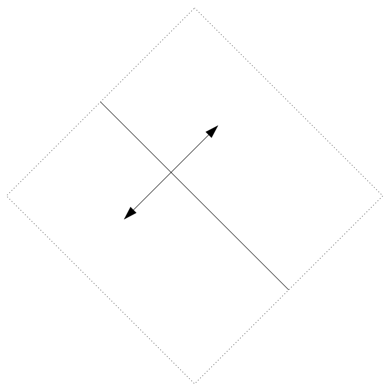
A dimension $d = 3$ Calabi–Yau variety X is simply called a *Calabi–Yau threefold*. It has the following Hodge diamond:

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1}(X) & & 0 & \\ 1 & & h^{2,1}(X) & & h^{1,2}(X) & & 1 \\ & 0 & & h^{2,2}(X) & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \qquad \begin{array}{c} h^0(X) = 1 \\ h^1(X) = 0 \\ h^2(X) = h^{1,1}(X) \\ h^3(X) = 2(1 + h^{2,1}(X)) \\ h^4(X) = h^{2,2}(X) = h^{1,1}(X) \\ h^5(X) = 0 \\ h^6(X) = 1 \\ \hline \chi(X) = 2(h^{1,1}(X) - h^{2,1}(X)) \end{array}$$

Calabi–Yau threefolds are Kähler manifolds, so $h^{1,1}(X) > 0$. Note also that for Calabi–Yau threefolds all 2-cycles are algebraic, i.e. $H^2(X, \mathbb{Z}) \simeq \mathrm{Pic}(X)$. In particular we have $h^{1,1}(X) = h^2(X) = \rho(X) := \mathrm{rk}\,\mathrm{Pic}(X)$. It is still an open problem if there is a constant bounding the absolute value of the Euler characteristics of Calabi–Yau threefolds (cf. [78]).

Physicists have discovered a phenomenon for Calabi–Yau threefolds, known as *mirror symmetry.* Given a Calabi–Yau threefold X there should exist (naively speaking) a *mirror* Calabi–Yau threefold $\hat{X}$ such that

$$
\begin{aligned}
h^{1,1}(X) &= h^{2,1}(\hat{X}), \\
h^{2,1}(X) &= h^{1,1}(\hat{X}), \\
\chi(X) &= -\chi(\hat{X}).
\end{aligned}
$$



The picture visualizes where mirror symmetry occurs in the Hodge diamond.

The mirror symmetry conjecture, as stated above, obviously fails for a certain type of Calabi–Yau threefolds, namely where $h^{2,1}(X) = 0$. In this case there is a generalized notion of mirror, cf. [9]. Since for a Calabi–Yau threefold X the Hodge number $h^{2,1}(X)$ equals the number of complex deformations, we call X *rigid* if $h^{2,1}(X) = 0$ and *non-rigid* if $h^{2,1}(X) > 0$.

1.2 Arithmetic on Calabi–Yau varieties

In the preceding section we introduced Calabi–Yau varieties over the complex numbers. Now we are going to deal with arithmetical questions, i.e., reduction of Calabi–Yau varieties over finite fields and number fields. All examples will be defined over $\mathbb{Q}$ so we are going to restrict the discussion to this case.

Let X be a Calabi–Yau variety of dimension d defined over $\mathbb{Q}$. Then X always has a model defined over $\mathbb{Z}$ (an *integral model*). We will use the following notation for the reduction of X over different fields:

notation	$\bar{X}$	X_q	$\bar{X}_p$
field	$\bar{\mathbb{Q}}$	$\mathbb{F}_q$, $q = p^r$, p prime	$\bar{\mathbb{F}}_p$, p prime

A prime p is called a *good* prime (or a *prime of good reduction*) if the reduction $\bar{X}_p$ is again a Calabi–Yau variety (in particular, it is smooth), otherwise a *bad* prime (or a *prime of bad reduction*). The set of bad primes is always finite.

Let p be a good prime and let $\mathrm{F}_p : \bar{X}_p \longrightarrow \bar{X}_p$ denote the *geometric Frobenius morphism* which takes coordinates to the p-th power. Let $\ell \neq p$ be a prime. The maps F_{p^r} induce endomorphisms $\mathrm{F}^*_{p^r} : H^i_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell) \longrightarrow H^i_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)$ on étale ℓ-adic cohomology (which is an ℓ-adic analogon of singular cohomology, $H^i_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell) \otimes_{\mathbb{Q}_\ell} \mathbb{C} \simeq H^i(X, \mathbb{C})$. By the smooth base change theorem we also have $H^i_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell) \simeq H^i_{\text{ét}}(\bar{X}_p, \mathbb{Q}_\ell)$. For details on étale cohomology, cf. [44], [71].). We have $\dim_{\mathbb{Q}_\ell} H^0_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell) = \dim_{\mathbb{Q}_\ell} H^{2d}_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell) = h^0(X) = 1$. The action of F^*_p on $H^0_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)$ resp. $H^{2d}_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)$ is the identity resp. multiplication with p^d.

There are ℓ-adic Galois representations

$$\rho^{(i)}_{X,\ell} : \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \mapsto \mathrm{GL}_{h^i(X)}(\mathbb{Q}_\ell), \qquad 0 \leq i \leq 2d,$$

unramified for all primes p of good reduction for X and compatible with the Poincaré perfect pairings

$$H^i_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell) \times H^{2d-i}_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell) \longrightarrow H^{2d}_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell) \simeq \mathbb{Q}_\ell.$$

Frobenius elements Frob_p at p are mapped to F^*_p.

Now let again p be a good prime and let

$$P_{i,p}(t) = \det(1 - t \cdot \mathrm{F}^*_p \,|H^i_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)).$$

By Weil and Deligne, the polynomials $P_{i,p}(t)$ have integer coefficients and

$$P_{i,p}(t) = \prod_{j=1}^{h^i(X)} (1 - t \cdot \omega_{ij})$$

where the ω_{ij} are algebraic integers, independent of ℓ, with $|\omega_{ij}| = p^{i/2}$. Let $\#X_q$ denote the number of points on X_q for a prime power q and let

$$Z_p(t) := \exp\left(\sum_{r=1}^{\infty} \#X_{p^r} \frac{t^r}{r}\right)$$

denote the *zeta function* of X. Then $Z_p(t)$ is a rational function of t and can be written as

$$Z_p(t) = \frac{P_{1,p}(t)P_{3,p}(t)\cdots P_{2d-1,p}(t)}{P_{0,p}(t)P_{2,p}(t)\cdots P_{2d,p}(t)}.$$

Now the *i-th (cohomological) L-series of X* is defined as the L-series of the (semi-simplification of the) Galois representation $\rho^{(i)}_{X,\ell}$. It is independent of ℓ and can be written as an Euler product

$$L(H^i_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell), s) = (*) \prod_p \frac{1}{P_{i,p}(p^{-s})}$$

where the product runs over the good primes and $(*)$ denotes possible Euler factors for the bad primes. In particular, the d-th L-series of X is called *the (cohomological) L-series of X* and denoted by

$$L(X, s) := L(H^d_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell), s).$$

We have a series expansion

$$L(X,s) = \sum_{k=1}^{\infty} \frac{a_k(X)}{k^s}$$

where $a_1(X) = 1$, $a_p(X) = \operatorname{tr}(\mathrm{F}_p^* \,|H^d_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell))$ and $a_k(X)$ is determined by the $a_p(X)$ for the prime divisors p of k.

The *Lefschetz fixed point formula* relates the number of points on X_p to the action of the Frobenius map:

$$\#X_{p^r} = \sum_{i=0}^{2d} (-1)^i \operatorname{tr}(\mathrm{F}_{p^r}^* \,|H^i_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell))$$

Note that, by Weil and Deligne, we have $\operatorname{tr}(\mathrm{F}_p^* \,|H^i_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)) \in \mathbb{Z}$ and

$$|\operatorname{tr}(\mathrm{F}_p^* \,|H^i_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell))| \leq h^i(X) \cdot p^{i/2}$$

for all good primes p.

1.3 Modular forms

By standard conjectures (cf. conjecture 1.2 and the subsequent remarks) the L-series of Calabi–Yau varieties should be determined by modular (or automorphic) forms. Modular forms for the congruence subgroups $\Gamma_0(N)$ play an important role so we will collect the basic facts. Modular forms for different congruence groups can similarly be defined. Good references on modular forms are [37], [58] and [72].

Let $\Gamma = \mathrm{SL}(2,\mathbb{Z})$ be the *full modular group*. The subgroups

$$\Gamma_0(N) := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \;\middle|\; c \equiv 0 \mod N \right\} \qquad \text{for } N \in \mathbb{N}$$

of finite index in Γ are called *Hecke subgroups* of Γ.

An *unrestricted modular form* of *weight* $k \in \mathbb{Z}$ and *level* $N \in \mathbb{N}$ is an analytic function f on the upper half plane $\mathbb{H}$ with

$$f\left(\frac{a\tau + b}{c\tau + d}\right) = (c\tau + d)^k f(\tau) \qquad \text{for all} \quad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(N), \quad \tau \in \mathbb{H}.$$

The function f always has a q-expansion

$$f(\tau) = \sum_{n=-\infty}^{\infty} c_n q^n \qquad \text{with } q = e^{2\pi i \tau}.$$

It is called a *modular form* if $c_n = 0$ for $n < 0$ and a *cusp form* if $c_n = 0$ for $n \leq 0$.

The set $M_k(\Gamma_0(N))$ of modular forms of weight k and level N is a finite-dimensional vector space. The subspace of cusp forms is denoted by $S_k(\Gamma_0(N))$.

The space $S_k(\Gamma_0(N))$ is the orthogonal sum of simultaneous eigenspaces for certain operators, the *Hecke operators*. A cusp form is called *eigenform* if it is an eigenvector for the Hecke operators; two eigenforms in the same eigenspace are called *equivalent*. If $r_1r_2|N$ and $f(\tau)$ is an eigenform for $\Gamma_0(N/r_1r_2)$ then $f(r_1\tau)$ is an eigenform for $\Gamma_0(N)$ with the same eigenvalues and is called an *oldform*. The oldforms span a subspace $S_k^{old}(\Gamma_0(N))$. The orthogonal complement is denoted by $S_k^{new}(\Gamma_0(N))$ and an eigenform in $S_k^{new}(\Gamma_0(N))$ is called a *newform*. The equivalence class of a newform is one-dimensional, and $S_k^{new}(\Gamma_0(N))$ is the orthogonal sum of these classes.

Let $f \in S_k^{new}(\Gamma_0(N))$, $k \geq 2$ be a newform and let the q-expansion

$$f(\tau) = \sum_{n=1}^{\infty} c_n q^n$$

have rational coefficients. We can normalize f to have $c_1 = 1$. By the theory of Hecke operators the coefficients c_n are in general algebraic integers and therefore integers in our case. Let ℓ be a prime. By Deligne ([31], cf. also [32]) there is a unique (up to isomorphism) ℓ-adic semi-simple Galois representation

$$\rho_{f,\ell} : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \longrightarrow \mathrm{GL}_2(\mathbb{Q}_\ell)$$

which is unramified for all primes p not dividing $N \cdot \ell$ with

$$\mathrm{tr}(\rho_{f,\ell}(\mathrm{Frob}_p)) = c_p \quad \text{and} \quad \det(\rho_{f,\ell}(\mathrm{Frob}_p)) = p^{k-1}.$$

The *L-series* of f is defined as the L-series of the Galois representation $\rho_{f,\ell}$ and is denoted by $L(f,s)$. It can be written as the Dirichlet series

$$L(f,s) = \sum_{n=1}^{\infty} \frac{c_n}{n^s}$$

and it has an Euler product expansion (which is convergent for the real part of s large enough) of the form

$$L(f,s) = \prod_{p\in\mathbb{P},p|N} \left(\frac{1}{1 - c_p p^{-s}}\right) \prod_{p\in\mathbb{P},p\nmid N} \left(\frac{1}{1 - c_p p^{-s} + p^{k-1-2s}}\right).$$

We will mainly be interested in newforms of weight two and four with rational coefficients for $\Gamma_0(N)$. Details on the computation of coefficients of such newforms can be found in 1.8.3 and large tables are presented in appendices C and D.

1.4 Dimension $\neq 3$

We give a short overview of the known connections between Calabi–Yau manifolds and modular forms in the dimension $d = 1$ and $d = 2$ cases. Our main interest will be in Calabi–Yau threefolds; this will be the subject of the rest of this thesis. For dimension > 3 there is almost nothing known. Ahlgren ([2]) connects the number of points on a certain fivefold with the coefficients of the unique normalized newform in $S_6(\Gamma_0(4))$. Note that this newform can be written as $\eta^{12}(2\tau)$ where η is the *Dedekind η function* (cf. 6.1).

1.4.1 Dimension 1: Elliptic curves

Let E be an elliptic curve (i.e., a Calabi–Yau variety of dimension 1) defined over $\mathbb{Q}$. The only non-trivial cohomology group of E is the middle cohomology group $H^1_{\text{ét}}(\bar{E}, \mathbb{Q}_\ell)$. The Lefschetz fixed point formula gives

$$a_p(E) = \operatorname{tr}(H^1_{\text{ét}}(\bar{E}, \mathbb{Q}_\ell)) = p + 1 - \#E_p.$$

Let N be the conductor of E (cf. 6.4.2). By the famous work of Wiles et al. (for an overview, cf. [15] or [79]) we know that there exists a cusp form

$$f(q) = \sum_{k=1}^{\infty} b_m q^m, \qquad q = e^{2\pi i z},$$

of weight 2 and level N such that for a prime ℓ the Galois representations $\rho^{(1)}_{E,\ell}$ and $\rho_{f,\ell}$ have the same semi-simplifications and

$$L(E, s) = L(f, s),$$

in particular $a_p(E) = b_p$ for all primes p of good reduction for E. Thus all elliptic curves defined over $\mathbb{Q}$ are *modular*.

1.4.2 Dimension 2: K3 surfaces

Let X be a K3 surface (i.e., a Calabi–Yau variety of dimension 2) defined over $\mathbb{Q}$. The only non-trivial cohomology group of X is again the middle cohomology group $H^2_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)$ (with dimension $h^2(X) = 22$).

Let $\mathrm{NS}(X)$ denote the *Néron–Severi group* of X, i.e., the group of divisors on X modulo algebraic equivalence. There is a natural embedding of $\mathrm{NS}(X)$ into $H^2(X, \mathbb{Z}) \simeq U_2^3 \perp (-E_8)^2$. This implies that $\mathrm{NS}(X)$ is a torsion free lattice of rank $\rho(X) = \operatorname{rk}\operatorname{Pic}(X) \leq 20$. The decomposition of lattices $H^2(X, \mathbb{Z}) = \mathrm{NS}(X) \otimes T(X)$, where $T(X)$ is the *transcendental part* of $H^2(X, \mathbb{Z})$, induces a decomposition of the L-series of X into

$$L(X, s) = L(\mathrm{NS}(X) \otimes \mathbb{Q}_l, s) \cdot L(T(X) \otimes \mathbb{Q}_l, s).$$

The K3 surface X is called *extremal* (or *singular*) if $\rho(X) = 20$.

1.1 Theorem
Let X be an extremal K3 surface defined over $\mathbb{Q}$. Suppose that $\mathrm{NS}(X)$ is generated by algebraic cycles defined over some extension K of $\mathbb{Q}$. Then the L-series of X is given, up to finitely many Euler factors, by

$$L(X, s) = \zeta_K(s-1)^{20} L(f, s)$$

where $\zeta_K(s)$ is the Dedekind zeta function of K and $L(f, s)$ is the L-series of a cusp form f of weight 3 on a congruence subgroup of $\mathrm{PSL}_2(\mathbb{Z})$, e.g., $\Gamma_1(N)$ or $\Gamma_0(N)$ twisted by a character. The first factor corresponds to the algebraic part of $H^2_{\acute{e}t}(\bar{X}, \mathbb{Q}_\ell)$, the second to the (two-dimensional) transcendental part of $H^2_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)$. The level N depends on the discriminant of the lattice $\mathrm{NS}(X)$ and can be determined explicitly.

There are different proofs of the above theorem which can be found in [111]. They rely on the work of Inose and Shioda on the classification of extremal K3 surfaces ([53]) and on the work of Livné on modularity of motivic orthogonal two-dimensional Galois representations ([63]). Examples of modular K3 surfaces can be found in [4], [14], [59], [64], [65], [66], [77] and [98].

If the conditions of the above theorem are weakened (i.e., the K3 surface is not extremal) then there is not much known about modularity. In this case the Galois representation associated to the transcendental part of $H^2_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)$ is no longer two-dimensional.

1.5 Dimension 3: Calabi–Yau threefolds

Modularity of Calabi–Yau threefolds has been the subject of investigation of several authors. Several review articles have been written by N. Yui (cf. [108], [109], [110], [111]). Articles dealing with specific examples or questions include [3], [23], [28], [33], [34], [35], [49], [50], [51], [52], [64], [68], [69], [74], [75], [80], [81], [86], [87], [88], [89], [99], [100], [101], [104], [107].

Let X be a Calabi–Yau threefold defined over $\mathbb{Q}$, and let p be a prime of good reduction for X. We apply the Lefschetz fixed point formula:

$$\begin{aligned}\#X_p &= \sum_{i=0}^{6} (-1)^i \operatorname{tr}(\mathrm{F}_p^* \,|H^i_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)) \\ &= 1 + \operatorname{tr}(\mathrm{F}_p^* \,|H^2_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)) - \operatorname{tr}(\mathrm{F}_p^* \,|H^3_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)) + \operatorname{tr}(\mathrm{F}_p^* \,|H^4_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)) + p^3\end{aligned}$$

By Weil and Deligne, we have

$$\operatorname{tr}(\mathrm{F}_p^* \,|H^2_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)) = k_p(X) \cdot p$$

where $k_p(X) \in \mathbb{Z}$, $|k_p(X)| \le h^2(X) = h^{1,1}(X)$. The equality $k_p(X) = h^2(X)$ holds if $H^2_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)$ is generated by cycles defined over $\mathbb{Q}$ (in this case the action of F_p^* on $H^2_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)$ is just multiplication by p). By Poincaré duality we have

$$\operatorname{tr}(\mathrm{F}_p^* \,|H^4_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)) = k_p(X) \cdot p^2.$$

Remember also the notation

$$a_p(X) := \operatorname{tr}(\mathrm{F}_p^* \,|H^3_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)).$$

Altogether this gives the identity

$$a_p(X) = 1 + p^3 + (p^2 + p) \cdot k_p(X) - \#X_p,$$

so if we know $k_p(X)$ (which is not too difficult in many examples because the Picard group of X can be controlled) we can determine $a_p(X)$ by counting points on X_p.

1.5.1 Modularity of rigid Calabi–Yau threefolds

For arithmetical purposes the easiest Calabi–Yau threefolds are the rigid ones (i.e., $h^{2,1}(X) = 0$, $h^3(X) = 2$). For these there is a precise modularity conjecture:

1.2 Conjecture
Let X be a rigid Calabi–Yau threefold defined over $\mathbb{Q}$. Then X is modular, *i.e., there exists a newform*

$$f(q) = \sum_{k=1}^{\infty} b_m q^m, \qquad q = e^{2\pi i z},$$

of weight 4 for $\Gamma_0(N)$ such that for a prime ℓ the (two-dimensional) Galois representations $\rho^{(3)}_{X,\ell}$ and $\rho_{f,\ell}$ have the same semi-simplifications and

$$L(X, s) = L(f, s),$$

in particular $a_p(X) = b_p$ for all primes p of good reduction for X. The level N is only divisible by primes of bad reduction for X.

This conjecture was formulated in [81]. It is a concrete realization of a conjecture of Fontaine and Mazur in [43] that every irreducible ℓ-adic two-dimensional Galois representation arising from geometry should be modular. It is also a special case of Serre's conjectures in [94].

We are going to describe later how a modularity proof for a specific example can work. The most general result up to now is the following:

1.3 Theorem
(Dieulefait, Manoharmayum in [35], Dieulefait in [34]) Let X be a rigid Calabi–Yau threefold defined over $\mathbb{Q}$. Suppose that X satisfies one of the following conditions:

- *X has good reduction at 3 with $3 \nmid a_3(X)$, or*
- *X has good reduction at 3 and 7, or*
- *X has good reduction at 5 and some prime $p \equiv \pm 2 \mod 5$ with $5 \nmid a_p(X)$.*

Then X is modular.

The above theorem contains no information about the level of the modular form. The following theorem gives a bound for the powers of primes in the level:

1.4 Theorem
(Serre in [94], Dieulefait in [33]) Let X be a rigid Calabi–Yau threefold defined over $\mathbb{Q}$. Suppose that X is modular with modular form f of weight 4 for $\Gamma_0(N)$. Then the exponent e_p of a prime p dividing N is bounded by $e_p \leq 2$ if $p > 3$, $e_3 \leq 5$ and $e_2 \leq 8$.

To prove modularity of a specific Calabi–Yau threefold there is a powerful result based on work of Faltings ([42]) and Serre. The key part will be to check equality of finitely many coefficients of the two L-series and to conclude equality for almost all coefficients. We need some definitions first.

A subset T of a finite-dimensional vector space V is called *non-cubic* if every homogeneous polynomial of degree 3 on V which vanishes on T vanishes on V. For example, the set $V \setminus \{0\}$ is non-cubic for $V = (\mathbb{Z}/2\mathbb{Z})^3$ and there is no smaller non-cubic set for this vector space (for each nonzero vector in this space there is a cubic polynomial vanishing everywhere but at this vector). The set $W \setminus \{0, w\}$ is non-cubic for $W = (\mathbb{Z}/2\mathbb{Z})^4$, where $w \in W$ is an arbitrary vector.

Let $S = \{s_1, \dots, s_k\}$ be a finite set of primes and let $\mathbb{Q}_S = \mathbb{Q}(\sqrt{-1}, \sqrt{s_1}, \dots, \sqrt{s_k})$ be the compositum of all quadratic extensions of $\mathbb{Q}$ unramified outside S. For $s \in S \cup \{-1\}$, denote by $\chi_s : \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \longrightarrow \mathbb{Z}/2\mathbb{Z}$ the quadratic Galois character cutting out $\sqrt{s}$. Note that $\chi_s(\mathrm{Frob}_p) = \left(\frac{s}{p}\right)$. We can now interpret $\mathrm{Gal}(\mathbb{Q}_S/\mathbb{Q})$ as $\mathbb{Z}/2\mathbb{Z}$-vector space via the bijection

$$\mathrm{Gal}(\mathbb{Q}_S/\mathbb{Q}) \longrightarrow (\mathbb{Z}/2\mathbb{Z})^{k+1}, \qquad g \mapsto \left(\frac{1-\chi_{-1}(g)}{2}, \frac{1-\chi_{s_1}(g)}{2}, \dots, \frac{1-\chi_{s_k}(g)}{2} \right).$$

1.5 Theorem
(Theorem 4.3 in [62]) Let S be a finite set of primes and let $\rho_1, \rho_2 : \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \longrightarrow \mathrm{GL}_2(\mathbb{Q}_2)$ be continuous Galois representations, unramified outside S, and satisfying

1. $\operatorname{tr} \rho_1 \equiv \operatorname{tr} \rho_2 \equiv 0 \mod 2$ *and* $\det \rho_1 \equiv \det \rho_2 \mod 2$.
2. *There exists a finite set T of primes, disjoint from S, for which the image of the set $\{\mathrm{Frob}_t, t \in T\}$ in $\mathrm{Gal}(\mathbb{Q}_S/\mathbb{Q})$ (as $\mathbb{Z}/2\mathbb{Z}$-vector space as explained above) is non-cubic, and*

$$\operatorname{tr} \rho_1(\mathrm{Frob}_t) = \operatorname{tr} \rho_2(\mathrm{Frob}_t) \quad \textit{and} \quad \det \rho_1(\mathrm{Frob}_t) = \det \rho_2(\mathrm{Frob}_t)$$

for all $t \in T$.

Then ρ_1 and ρ_2 have isomorphic semi-simplifications. In particular, $\operatorname{tr} \rho_1(\mathrm{Frob}_p) = \operatorname{tr} \rho_2(\mathrm{Frob}_p)$ for all primes $p \notin S$.

1.6 Corollary
Let X be a rigid Calabi–Yau threefold defined over $\mathbb{Q}$ and let

$$f(q) = \sum_{k=1}^{\infty} b_m q^m, \qquad q = e^{2\pi i z}$$

be a newform of weight 4 for $\Gamma_0(N)$. Let S be a finite set of primes containing the primes of bad reduction for X and the prime divisors of N. Suppose that

$$a_p(X) \equiv b_p \equiv 0 \mod 2$$

for all primes $p \notin S$ and that there exists a finite set T of primes, disjoint from S, for which the image of the set $\{\mathrm{Frob}_t, t \in T\}$ in $\mathrm{Gal}(\mathbb{Q}_S/\mathbb{Q})$ is non-cubic, and

$$a_p(X) = b_p$$

for all $p \in T$. Then X is modular, i.e. $L(X,s) = L(f,s)$ except for possible Euler factors at the primes of bad reduction; in particular, $a_p(X) = b_p$ for all primes $p \notin S$.

Proof:
We apply theorem 1.5 to the two Galois representations $\rho_1 = \rho^{(3)}_{X,2}$ and $\rho_2 = \rho_{f,2}$. It is known (cf. [35]) that $\det \rho_1$ is the third power of the ℓ-adic cyclotomic character, so we have $\det \rho_1(\mathrm{Frob}_p) = \det \rho_2(\mathrm{Frob}_p) = p^3$ and the conditions for the determinants follow from the Tchebotarev density theorem. Since $a_p(X) = \mathrm{tr}\,\rho_1(\mathrm{Frob}_p)$ and $b_p = \mathrm{tr}\,\rho_2(\mathrm{Frob}_p)$ the same holds true for the traces. □

The above corollary still contains a condition on the traces for infinitely many primes but there is also a method to reduce this to finitely many conditions which was introduced in [62]. Let ρ be a continuous 2-adic Galois representation unramified outside the prime divisors of some number N and assume that the trace is not always even. Consider the kernel of the reduction $\bar{\rho}$ of ρ modulo 2. Since by assumption it contains an element of order 3, the Galois extension of $\mathbb{Q}$ cut out by $\ker \rho$ must have Galois group the symmetric group Σ_3 or the group with three elements C_3 while being unramified outside 2 and the prime divisors of N. The different possible extensions of $\mathbb{Q}$ have been classified in [54] by the cubic polynomials they are the splitting fields of. Now we choose for each such polynomial h a prime p such that h is irreducible over $\mathbb{F}_p$, which implies that the trace $\mathrm{tr}\,\rho(\mathrm{Frob}_p)$ is odd, since Frob_p has order 3 in $\mathrm{Gal}(\mathbb{Q}(h)/\mathbb{Q})$.

This way it is possible to find, for each set S of bad primes, a set U of primes, disjoint from S, such that if $\mathrm{tr}\,\rho(\mathrm{Frob}_p)$ is even for all $p \in U$ then it is even for all primes $p \notin S$.

We give a list of sets S of bad primes and corresponding sets T and U of good primes needed for a modularity proof. The list suffices for all modularity proofs in this thesis where corollary 1.6 can be applied but it could easily be extended (at least for small enough primes where the classification of the needed Galois extensions is accessible for computers).

S	T	U	References
2	3,5,7	3	[62], [104]
2,3	5,7,11,13,17,19,23	5,7,11,13	[104]
2,5	3,7,11,13,17,29,31	3	[62]
2,3,5	7,11,13,17,19,23,29,31,41,43,53,61,71,73	11,13,17,19, 23,29,31,37	[62]
2,7	3,5,11,17,23,29,31	5,11,13,19,23,31	[88]
2,3,7	5,11,13,17,19,23,29,31,37,43,47,59,73,79	5,11,13,19,23,31	[88]
2,3,5,7	11,13,17,19,23,29,31,37,41,43,47,53, 59,61,71,73,79,83,101,103,107,109,113, 127,173,193,211,241,281,283,311	11,13,17,19, 23,29,31,37	[51]

S	T	U	References
2,17	3,5,7,13,19,41,47	$\emptyset$	[88]
2,3,17	5,7,11,13,19,23,37,41,47,53,59,73,89,103	?	
2,73	3,5,7,11,17,23,37	3,13	[88]
2,3,73	5,7,11,13,17,19,23,37,41,43,47,79,149,193	?	

The main disadvantage of theorem 1.5 and corollary 1.6 is the condition that the traces of the two Galois representation must be even. This condition is not fulfilled in several examples.

There are some nice ideas by J.P. Serre to treat a more general situation. Apart from [93] they have only been communicated in letters and used once in [86]. Recently M. Schütt ([89]) explained the construction in detail and proved the following lemma:

1.7 Lemma
([89], Proposition 1) Let $\rho_1, \rho_2 : \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \longrightarrow \mathrm{GL}_2(\mathbb{Q}_2)$ *be continuous Galois representations, unramified outside* $\{2,3\}$*, with the same determinant and with* $\mathrm{tr}\,\mathrm{Frob}_{11}(\rho_1) \equiv 0 \mod 2$ *or* $\mathrm{tr}\,\mathrm{Frob}_{13}(\rho_1) \equiv 0 \mod 2$*. Then* ρ_1 *and* ρ_2 *have isomorphic semisimplifications if and only if* $\mathrm{tr}\,\mathrm{Frob}_p(\rho_1) = \mathrm{tr}\,\mathrm{Frob}_p(\rho_2)$ *for every* $p \in \{5,7,11,13,17,19,23,31,37\}$*.*

The lemma can be generalized to other sets of bad primes. This requires again knowledge about Galois extensions with Galois group Σ_3 or C_3 (as tabulated in [54]). For example, in [89] M. Schütt considered the set $\{2,5,11\}$.

1.5.2 Modularity of non-rigid Calabi–Yau threefolds

The modularity of non-rigid Calabi–Yau threefolds is much more difficult to handle. In this case the Galois representations $\rho^{(3)}_{X,\ell}$ induced by the Frobenius morphism are $h^3(X)$-dimensional with $h^3(X) > 2$. By standard conjectures they should agree with the Galois representations of certain automorphic forms but we do not know enough to make this conjecture more precise.

There are some examples in the literature (and we will present more) where the Galois representations $\rho^{(3)}_{X,\ell}$ split into two-dimensional pieces. The single pieces correspond to Galois representations of certain modular forms and the modularity can be proved. The most prominent examples are of the following type:

Assume that there exist elliptic curves E_i defined over $\mathbb{Q}$, $i = 1, \ldots, r$, and birational maps

$$\phi_i : E_i \times \mathbb{P}^1 \longrightarrow X$$

defined over $\mathbb{Q}$, i.e., there are certain elliptic surfaces $E_i \times \mathbb{P}^1$ inside X. We have induced endomorphisms

$$\phi_i^* : H_3(E_i \times \mathbb{P}^1, \mathbb{Z}) \longrightarrow H_3(X, \mathbb{Z}).$$

Assume further that the maps ϕ_i^* are non-zero and that the images $\phi_i^*(E_i \times \mathbb{P}^1)$ are linearly independent in $H_3(X, \mathbb{Z})$, so they span a subspace $V \subset H^3(X, \mathbb{Z})$ of dimension $2r$. In ℓ-adic cohomology we get the exact sequence

$$0 \longrightarrow U \longrightarrow H^3_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell) \longrightarrow \bigoplus_{i=1}^{r} H^3_{\text{ét}}(\overline{E \times \mathbb{P}^1}, \mathbb{Q}_\ell) \longrightarrow 0.$$

By the Künneth formula we have $H^3_{\text{ét}}(\overline{E \times \mathbb{P}^1}, \mathbb{Q}_\ell) = H^1_{\text{ét}}(\bar{E}, \mathbb{Q}_\ell) \otimes H^2_{\text{ét}}(\bar{\mathbb{P}}^1, \mathbb{Q}_\ell)$, such that

$$F_p^* | H^3_{\text{ét}}(\overline{E \times \mathbb{P}^1}, \mathbb{Q}_\ell) = p \cdot (F_p^* | H^1_{\text{ét}}(\bar{E}, \mathbb{Q}_\ell)).$$

and finally

$$a_p(X) = \text{tr}(\text{F}_p^* | U) + p \cdot a_p(E).$$

Since elliptic curves defined over $\mathbb{Q}$ are modular, the action of Frobenius on a $2r$-dimensional subspace of $H^3_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell)$ is determined by weight 2 modular forms for $\Gamma_0(N)$. If $h^3(X) - 2r = 2$, i.e. the remaining piece U is two-dimensional, then there is hope that the Galois representation associated to it is again determined by a weight 4 modular form for $\Gamma_0(N)$ (as in the rigid case).

This method has been used in [51] and [89] (and in a smiliar way in [75] although this is not so obvious, cf. 4.5) and we are going to apply it to further examples. Very recently ([52]) Hulek and Verrill explained the construction in detail, giving complete proofs. There are very few non-rigid Calabi–Yau threefolds that have been associated with different modular or automorphic forms than this "weight 4 plus weight 2" case. In [23] (cf. also 3.5) the arithmetic of a threefold X with $h^3(X) = 4$ is investigated, and there is numerical evidence that its L-series is that of a Hilbert modular form. Again the Galois representation $\rho^{(3)}_{X,\ell}$ splits, but over $\mathbb{Q}[\sqrt{5}]$ instead of $\mathbb{Q}$.

Further considerations about modularity for a certain class of Calabi–Yau threefolds are made in [64]. These threefolds are resolutions of $Y \times E$ divided by an involution, where Y is an extremal $K3$ surface and E is an elliptic curve. Consequently products of coefficients of weight 3 and weight 2 newforms occur in the L-series.

Note that we will meet many examples of non-rigid Calabi–Yau threefolds where the L-series seems to split (due to numerical observations) in a "weight 4 part" and a "p times something part", i.e.,

$$a_p(X) = b_p + p \cdot c_p$$

where b_p are the coefficients of a weight 4 newform. This can be relatively easily detected by comparing $a_p(X)$ modulo p with coefficients of suitable newforms. However, the quantity c_p is much more difficult to handle. It might be a sum of coefficients of weight 2 newforms as explained above but it is hard to detect the right newforms if there is no explicit geometrical explanation at hand.

1.6 Construction of Calabi–Yau threefolds

Now we want to see examples of Calabi–Yau threefolds. The easiest examples are complete intersections of k hypersurfaces in $\mathbb{P}^{3+k}$ in general position. Let $d_1, \ldots, d_k$ be the degrees of these hypersurfaces. Then the canonical bundle of their intersection is trivial if and only if

$$\sum_{i=1}^{k} d_i = k + 4.$$

This gives the following possibilities:

1. A quintic in $\mathbb{P}^4$, with Euler characteristic -200,
2. The intersection of a quartic and a quadric in $\mathbb{P}^5$, with Euler characteristic -176,
3. The intersection of two cubics in $\mathbb{P}^5$, with Euler characteristic -144,
4. The intersection of a cubic and two quadrics in $\mathbb{P}^6$, with Euler characteristic -144,
5. The intersection of four quadrics in $\mathbb{P}^7$, with Euler characteristic -128.

For these examples see [20] or [48]. We can generalize this to certain complete intersections of polynomials in products of (weighted) projective spaces, cf. for example [11], [17] and [80].

Other important examples are:

6. Double coverings of $\mathbb{P}^3$ branched along a smooth octic surface, with Euler characteristic -296.
7. Triple coverings of $\mathbb{P}^3$ branched along a smooth sextic surface, with Euler characteristic -204.

All these examples will be non-rigid (since a rigid Calabi–Yau threefold has positive Euler characteristic). The most important method to produce rigid Calabi–Yau threefolds (or those with small number $h^{2,1}$ of deformations) is to take a (highly) singular threefold X of one of the above types and resolve the singularities in such a way that the resolution $\tilde{X}$ is still Calabi–Yau.

1.6.1 Ordinary double points

Let us deal with isolated singularities first. The simplest and most common ones are *ordinary double points*, also called *(ordinary) nodes*. In suitable local analytic coordinates they are given by the equation

$$xy - zt = 0.$$

In general the tangent cone at a node is a smooth quadric surface, which is isomorphic to $\mathbb{P}^1 \times \mathbb{P}^1$. Blowing up the point in the ambient space replaces the node by its tangent cone and resolves the singularity. This is called a *big resolution*. Unfortunately this adds the exceptional surface to the canonical divisor so the result will not be Calabi–Yau.

There is another way to resolve a node, a so called *small resolution*. The idea is to replace the singularity by a set of codimension 2 which does not influence the canonical divisor. Consider the above local coordinates and the local meromorphic functions

$$\frac{x}{u} = \frac{v}{y} \qquad \text{and} \qquad \frac{x}{v} = \frac{u}{y}.$$

They have a point of indeterminancy at the critical point $x = y = z = t = 0$. Choosing one of them and taking the closure of its graph we add a $\mathbb{P}^1$ and resolve the node. Blowing up along this exceptional $\mathbb{P}^1$ we regain the big resolution of the node.

Now there is another problem. The above construction is an analytic one so in general the resolution will not be projective (or equivalently not kähler, in the Calabi–Yau case) anymore. There are a number of references on this problem, cf. [22], [25] and [106]. The following theorem characterizes projective algebraic small resolutions:

1.8 Theorem
([106], chapter XI) Let X be a singular projective threefold and assume that the singular locus of X consists of the nodes $P_1, \dots, P_s$. Let $\tilde{X}$ be a small resolution of X and L_i the exceptional $\mathbb{P}^1$ for the node P_i. Then $\tilde{X}$ is projective algebraic if and only if there is a divisor D on X with $D.L_i > 0$ for all $i = 1, \dots, s$.

This gives a very convenient way to prove the existence of a projective small resolution. Assume that for each node P_i of X there is a surface inside X which contains P_i and is smooth in this point. Then we obtain a projective small resolution of X by blowing up along these surfaces. This method has been used in many examples.

Now let X be a threefold containing only s nodes as singularities and let $\tilde{X}$ resp. $\hat{X}$ be a small resp. big resolution of (all the nodes of) X. Let X be a member of a family $\{X_t\}$ of smooth threefolds. Then we can compute the Euler characteristics

$$\begin{aligned}
\chi(X) &= \chi(X_t) + s, \\
\chi(\tilde{X}) &= \chi(X_t) + 2s, \\
\chi(\hat{X}) &= \chi(X_t) + 4s.
\end{aligned}$$

The first equality holds because removing a singular point from a threefold changes the Euler characteristic in the same way as removing the Milnor fibre from a smooth model, i.e., the Euler characteristic decreases by the Milnor number of the singularity, which is one for a node (cf. [36]). The second resp. third equality holds because each node is replaced by $\mathbb{P}^1$ resp. $\mathbb{P}^1 \times \mathbb{P}^1$.

The *defect* of X is defined as

$$d(X) := h^4(X) - h^2(X).$$

J. Werner ([106]) considers in particular the cases that X is a quintic in $\mathbb{P}^4$ or a double covering of $\mathbb{P}^3$ branched along an octic surface and proves that if there is a projective small resolution $\tilde{X}$ of all the nodes of X then $d(X) > 0$. Note also that in these cases we have

$$h^2(X) = 1, \qquad h^4(X) = h^4(\tilde{X}) = h^2(\tilde{X}) = 1 + d(X).$$

Werner also proves the following useful corollary:

1.9 Corollary
([106], chapter IV) Let X be a quintic in $\mathbb{P}^4$ or a double covering of $\mathbb{P}^3$ branched along an octic surface. Assume that X has only nodes as singularities. Let there be a group of automorphisms of X operating transitively on the set of nodes of X, and let $d(X) > 0$. Then there exist projective small resolutions (of all the nodes of X).

Equipped with the above theory we can investigate many examples of threefolds with nodes as only singularities. Note that the existence of projective small resolutions is only important for the construction of Calabi–Yau threefolds but not for arithmetical purposes since the arithmetic of big resolutions is almost the same.

Let $S_p \subset \mathbb{P}^3$ be a smooth quadric surface over $\mathbb{F}_p$, $p \neq 2$, with discriminant a. Then by [92], IV1.7, prop. 5, S_p is isomorphic over $\mathbb{F}_p$ to the quadric surface given by

$$\begin{cases} x^2+y^2+z^2+t^2=0, & \text{if } \left(\frac{a}{p}\right)=1, \\ x^2+y^2+z^2+at^2=0, & \text{if } \left(\frac{a}{p}\right)=-1. \end{cases}$$

This includes

$$\#S_p = \begin{cases} (p+1)^2, & \text{if } \left(\frac{a}{p}\right)=1, \\ p^2+1, & \text{if } \left(\frac{a}{p}\right)=-1. \end{cases}$$

In the first case all rulings of S_p are defined over $\mathbb{F}_p$. In the second case there is a pair of intersecting rulings on S_p which are not defined over $\mathbb{F}_p$ apart from the point of intersection.

Let X again be a threefold containing only s nodes as singularities and let $\tilde{X}$ resp. $\hat{X}$ be a small resp. big resolution of (all the nodes of) X. Let p be a prime of good reduction for X. Consider the Leray spectral sequence for the blow–up:

$$0 \longrightarrow H^2_{\text{ét}}(\bar{X}_p, \mathbb{Q}_\ell) \longrightarrow H^2_{\text{ét}}(\bar{\hat{X}}_p, \mathbb{Q}_\ell) \longrightarrow \bigoplus H^2_{\text{ét}}(\bar{Q}_p, \mathbb{Q}_\ell)$$

Here the sum runs over the exceptional quadrics Q of the blow–up. In many examples (e.g., if X is singular of one of the types listed at the beginning of this section) we have

$$H^2_{\text{ét}}(\bar{X}_p, \mathbb{Q}_\ell) = \mathbb{Q}_l(-1),$$

i.e., it contains only multiples of the generic hyperplane section. In this case $H^2_{\text{ét}}(\bar{\hat{X}}_p, \mathbb{Q}_\ell)$ is determined by the action of F_p^* on $H^2_{\text{ét}}(\bar{Q}_p, \mathbb{Q}_\ell)$ for the exceptional quadrics Q. If all rulings of all the quadrics are defined over $\mathbb{F}_p$ then the action of F_p^* on $H^2_{\text{ét}}(\bar{\hat{X}}_p, \mathbb{Q}_\ell)$ (and also on $H^2_{\text{ét}}(\bar{\tilde{X}}_p, \mathbb{Q}_\ell)$) is just multiplication by p, and in general it depends only on the question if the discriminants of the quadrics are squares in $\mathbb{F}_p$. On the point counting side we have

$$\begin{aligned} \#\hat{X}_p &= \#X_p + s_p \cdot (p^2+p) + t_p \cdot p^2, \\ \#\tilde{X}_p &= \#X_p + s_p \cdot p - t_p \cdot p, \end{aligned}$$

where s_p denotes the number of nodes which are rational over $\mathbb{F}_p$ and whose rulings of the tangent cones are also rational over $\mathbb{F}_p$, and t_p denotes the number of nodes which are rational over $\mathbb{F}_p$ but not the rulings of their tangent cones.

For convenience we recapitulate formulas for some Legendre symbols that will occur in this thesis:

$$\left(\frac{-1}{p}\right) = \begin{cases} 1, & p \equiv 1 \mod 4 \\ -1, & p \equiv 3 \mod 4 \end{cases} \qquad \left(\frac{2}{p}\right) = \begin{cases} 1, & p \equiv 1,7 \mod 8 \\ -1, & p \equiv 3,5 \mod 8 \end{cases}$$

$$\left(\frac{-2}{p}\right) = \begin{cases} 1, & p \equiv 1,3 \mod 8 \\ -1, & p \equiv 5,7 \mod 8 \end{cases} \qquad \left(\frac{3}{p}\right) = \begin{cases} 1, & p \equiv 1,11 \mod 12 \\ -1, & p \equiv 5,7 \mod 12 \end{cases}$$

$$\left(\frac{-3}{p}\right) = \begin{cases} 1, & p \equiv 1 \mod 6 \\ -1, & p \equiv 5 \mod 6 \end{cases} \qquad \left(\frac{5}{p}\right) = \begin{cases} 1, & p \equiv 1,4 \mod 5 \\ -1, & p \equiv 2,3 \mod 5 \end{cases}$$

$$\left(\frac{-7}{p}\right) = \begin{cases} 1, & p \equiv 1,2,4 \mod 7 \\ -1, & p \equiv 3,5,6 \mod 7 \end{cases}$$

1.6.2 Threefolds with many nodes

It is an interesting task to construct threefolds of a certain type with many isolated singularities. Varchenko ([102]) gives bounds for the maximal number in the case of hypersurfaces. We compile a table of threefolds with many nodes leading to rigid Calabi–Yau threefolds. All examples will be discussed later.

type of threefold	# of nodes	reference
quintic in $\mathbb{P}^4$	125	3.1
quintic in $\mathbb{P}^4$	126	3.3
quintic in $\mathbb{P}^4$	130	3.6
intersection of quadric and quartic in $\mathbb{P}^6$	122	5.2
intersection of two cubics in $\mathbb{P}^6$	108	5.6
intersection of four quadrics in $\mathbb{P}^7$	96	5.4
double covering of $\mathbb{P}^3$ branched along octic surface	168	4.6

1.6.3 Higher singularities

We can also allow higher isolated singularities. The following examples have been discussed in [48] and [99], building on the results of [16] about small resolutions.

We say that a singularity is of type (a, b, c, d) (with $a < b < c < d$) if it can be given in local coordinates by $x^a + y^b + z^c + t^d = 0$. Let again X be a singular member of a smooth family $\{X_t\}$ of threefolds.

A singularity of type $(3, 3, 3, 3)$ can be resolved by a big resolution (blow–up of the point) without changing the canonical divisor. The resolving surface is $\mathbb{P}^2$ blown up in six points with Euler characteristic equal to 9. The Milnor number of the singularity is 16, so every singularity and its resolution enlarges the Euler characteristic by 24, compared with a smooth member X_t.

A singularity of type $(2,4,4,4)$ can also be resolved by a big resolution without changing the canonical divisor. The resolving surface is $\mathbb{P}^2$ blown up in seven points with Euler characteristic equal to 10. The Milnor number of the singularity is 27, so every singularity and its resolution enlarges the Euler characteristic by 36, compared with a smooth member X_t.

A singularity of type $(2,2,n+1,h(n+1))$ can be resolved small by a configuration of n curves isomorphic to $\mathbb{P}^1$. The Milnor number is $n(hn+h-1)$, the resolving curve has Euler number $2n-(n-1)=n+1$, so every resolved singularity enlarges the Euler characteristic by $nh(n+1)$, compared with a smooth member X_t. If all local divisors $\{x=\sqrt{-1}y, z=\sqrt[n+1]{-1}t^h\}$ of all singularities can be extended to global smooth divisors then there exist projective small resolutions. An ordinary node is of type $(2,2,2,2)$ and so a special case of this class of singularities.

A singularity given by the local equations

$$x^2+y^2+z^2+t^2+w^2 \quad = \quad ax^2+by^2+cz^2+dt^2+ew^2 \quad = \quad 0$$

with a,b,c,d,e pairwise distinct can be resolved by a big resolution without changing the canonical divisor. The resolving surface is $\mathbb{P}^2$ blown up in five points with Euler characteristic equal to 8. The Milnor number of the singularity is 9, so every singularity and its resolution enlarges the Euler characteristic by 16, compared with a smooth member X_t.

We can also allow non-isolated singularities, e.g. singular lines. This will be disussed in examples.

1.7 Correspondences and twists

1.7.1 Correspondences and relatives

There are many examples of pairs of Calabi–Yau threefolds with an isomorphism between some pieces of their middle étale cohomologies and the appropriate Galois representations. In particular, if we can attach modular forms to these pieces then these modular forms will be the same. If on the other hand we detect the same modular forms in the middle étale cohomologies of two Calabi–Yau threefolds then this should have a geometrical reason:

1.10 Conjecture
(The Tate conjecture, as formulated in [111, Conj. 5.8]) If two isomorphic two-dimensional Galois representations ρ_1, ρ_2 occur in the étale cohomology of varieties X_1, X_2 defined over $\mathbb{Q}$, then there should be a correspondence between the two varieties (i.e., an algebraic cycle on the product of the two varieties) defined over $\mathbb{Q}$, which induces an isomorphism between ρ_1 and ρ_2.

Following [50] we will call two Calabi–Yau threefolds defined over $\mathbb{Q}$ *relatives* if the same (weight four) modular form occurs in their L-series. Finding a correspondence between two relatives is a highly non-trivial task. It can be induced by a birational map defined over $\mathbb{Q}$ or more generally by a finite map between the two threefolds but this does not have to be the case. If a correspondence is induced by a birational map then by a result of Batyrev ([8]) the two Calabi–Yau threefolds must have the same Betti (and Hodge) numbers (and so the same Euler characteristic). It is very interesting to find Calabi–Yau threefolds with the same L-series but different Hodge numbers. Many explicit correspondences can be found in 6.1.

1.7.2 Relatives by construction

The Calabi–Yau threefolds constructed in 1.6 (and others) may be closely related to each other (also in the above sense). We will illustrate this with an example.

Let X be a double covering of $\mathbb{P}^3$ branched along the union of eight planes. Under certain conditions on the intersection of the planes the threefold X will have a Calabi–Yau resolution. This is the subject of 4.1 and 4.2. The variety X is given by an equation of the form

$$\{u^2 = \prod_{i=1}^{8} f_i(x, y, z, t)\} \subset \mathbb{P}^4(4, 1, 1, 1, 1)$$

where the f_i are linear homogeneous polynomials in x, y, z, t. Now consider the following three-folds:

Let the complete intersection threefold $X_{2,2,2,2} \subset \mathbb{P}^7$ be given by the equations

$$\begin{aligned} u_1^2 &= f_1(x, y, z, t) \cdot f_2(x, y, z, t), \\ u_2^2 &= f_3(x, y, z, t) \cdot f_4(x, y, z, t), \\ u_3^2 &= f_5(x, y, z, t) \cdot f_6(x, y, z, t), \\ u_4^2 &= f_7(x, y, z, t) \cdot f_8(x, y, z, t). \end{aligned}$$

There is a $8 : 1$ map $X_{2,2,2,2} \longrightarrow X$ induced by the map

$$\mathbb{P}^7 \dashrightarrow \mathbb{P}^4(4, 1, 1, 1, 1), \quad (u_1 : u_2 : u_3 : u_4 : x : y : z : t) \mapsto (u_1 u_2 u_3 u_4 : x : y : z : t).$$

Let the quintic threefold $X_5 \subset \mathbb{P}^4$ be given by the equation

$$u^2 \cdot \prod_{i=1}^{3} f_i(x, y, z, t) = \prod_{i=4}^{8} f_i(x, y, z, t)$$

There is a $1 : 1$ map $X_5 \longrightarrow X$ induced by the map

$$\mathbb{P}^4 \dashrightarrow \mathbb{P}^4(4, 1, 1, 1, 1), \quad (u : x : y : z : t) \mapsto (u \cdot \prod_{i=1}^{3} f_i(x, y, z, t) : x : y : z : t).$$

Let the complete intersection threefold $X_{3,3} \subset \mathbb{P}^5$ be given by the equations

$$\begin{aligned} u^2 f_1(x, y, z, t) &= \prod_{i=2}^{4} f_i(x, y, z, t), \\ v^2 f_5(x, y, z, t) &= \prod_{i=6}^{8} f_i(x, y, z, t). \end{aligned}$$

There is a $2 : 1$ map $X_{3,3} \longrightarrow X$ induced by the map

$$\mathbb{P}^5 \dashrightarrow \mathbb{P}^4(4, 1, 1, 1, 1), \quad (u : v : x : y : z : t) \mapsto (uv \cdot f_1(x, y, z, t) \cdot f_5(x, y, z, t) : x : y : z : t).$$

Let the complete intersection threefold $X_{2,4} \subset \mathbb{P}^5$ be given by the equations

$$\begin{aligned} u^2 &= f_1(x,y,z,t) \cdot f_2(x,y,z,t), \\ v^2 \cdot f_3(x,y,z,t) \cdot f_4(x,y,z,t) &= \prod_{i=5}^{8} f_i(x,y,z,t). \end{aligned}$$

There is a $2:1$ map $X_{2,4} \longrightarrow X$ induced by the map

$$\mathbb{P}^5 \dashrightarrow \mathbb{P}^4(4,1,1,1,1), \quad (u:v:x:y:z:t) \mapsto (uv \cdot f_3(x,y,z,t) \cdot f_4(x,y,z,t) : x : y : z : t).$$

Let the complete intersection threefold $X_{2,2,3} \subset \mathbb{P}^6$ be given by the equations

$$\begin{aligned} u^2 &= f_1(x,y,z,t) \cdot f_2(x,y,z,t), \\ v^2 &= f_3(x,y,z,t) \cdot f_4(x,y,z,t), \\ w^2 \cdot f_5(x,y,z,t) &= \prod_{i=6}^{8} f_i(x,y,z,t). \end{aligned}$$

There is a $4:1$ map $X_{2,2,3} \longrightarrow X$ induced by the map

$$\mathbb{P}^6 \dashrightarrow \mathbb{P}^4(4,1,1,1,1), \quad (u:v:w:x:y:z:t) \mapsto (uvw \cdot f_5(x,y,z,t) : x : y : z : t).$$

It is not a priori clear if the singularities of the threefolds $X_{2,2,2,2}$, X_5, $X_{3,3}$, $X_{2,4}$ and $X_{2,2,3}$ admit Calabi–Yau resolutions. But if they do then the above maps will induce non-zero maps on étale cohomology, and some pieces of the L-series will agree with some pieces of the L-series of X. Note also that in the above construction we may construct examples with different geometry by permutation of the f_i. This way and with generalizations it is possible to construct many examples of Calabi–Yau threefolds and correspondences between them. Examples will be given in the following chapters.

1.7.3 Twists

The previous section about correspondences was concerned with maps defined over $\mathbb{Q}$ giving rise to isomorphisms on étale cohomology and leading to the same modular forms. If such maps are not defined over $\mathbb{Q}$ but over some finite extension this may cause twists of the modular forms involved. We will have a closer look at a special case. The reference is the appendix of [111] by H. Verrill.

Let $d \in \mathbb{Z}$ be a square free number and let X, X_d be Calabi–Yau threefolds defined over $\mathbb{Q}$ and isomorphic over $\mathbb{Q}[\sqrt{d}]$. In this case the induced maps $H^3_{\text{ét}}(\bar{X}, \mathbb{Q}_\ell) \longrightarrow H^3_{\text{ét}}(\bar{X}_d, \mathbb{Q}_\ell)$ commute with the action of $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}[\sqrt{d}])$, and so restricted to this group the Galois representations are equal. Hence (up to conjugation) they are equal up to tensoring by a character $\chi_d : \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}[\sqrt{d}]) \longrightarrow \mathbb{Q}_l^\times$. This character is either trivial or given by $\mathrm{Frob}_p \mapsto \left(\frac{d}{p}\right)$. If the weight four newform $f = \sum a_n q^n$ occurs in the L-series of X then in the first case also f and in the second case the twisted modular form $f_d = \sum \left(\frac{d}{n}\right) a_n q^n$ occurs in the L-series of X_d. If d is odd then f_d has level $8d^2$ if $d \equiv 1 \mod 4$, and level $16d^2$ otherwise (cf. [111], Lemma 9.4).

For some classes of Calabi–Yau threefolds it is very easy to write down equations of pairs of examples X, X_d isomorphic not over $\mathbb{Q}$ but over some $\mathbb{Q}[\sqrt{d}]$. For example, let X be a double covering of $\mathbb{P}^3$ branched along an octic surface, i.e., X is given by an equation of the form

$$u^2 = f(x, y, z, t)$$

where f is a homogeneous polynomial of degree 8 in x, y, z, t (cf. chapter 4). We define X_d by the equation

$$u^2 = d \cdot f(x, y, z, t).$$

Now assume that X is modular. Choosing d it is possible to write down examples of modular Calabi–Yau threefolds with arbitrary primes appearing in the level of the corresponding modular form (but it is a kind of "cheating").

In what follows, if we detect a modular Calabi–Yau threefold X then we will always give the twist of the modular form involved that has minimal level. This does not always mean that we are able to write down equations for a Calabi–Yau threefold isomorphic with X over some finite extension of $\mathbb{Q}$ with exactly this modular form in its L-series (but I conjecture the existence of such a variety).

1.8 Computational matters

1.8.1 Computation of Hodge and Betti numbers

Computing the Hodge numbers $h^{1,1}$ and $h^{2,1}$ of a Calabi–Yau threefold (or equivalently the Betti numbers h^2 and h^3) is a highly nontrivial task. But there is a method invented by van Geemen (cf. [75], [99], [100]) that works in a special situation and reduces the problem to the counting of points.

Let X be a Calabi–Yau threefold defined over $\mathbb{Q}$ and assume that the Frobenius map F_p^* acts by multiplication with p on $H^2_{\text{ét}}(X, \mathbb{Q}_\ell)$ for a prime p. This can quite often be checked by explicitly determining generators of the Picard group $\mathrm{Pic}(X)$. Combining the Weil conjectures and the Lefschetz fixed point formula we find the estimate

$$|a_p(X)| = |1 + p^3 + (p^2 + p) \cdot h^2(X) - \#X_p| \le h^3(X) \cdot p^{3/2} = (2 + 2h^2(X) - \chi(X)) \cdot p^{3/2}.$$

Consequently if we know the Euler characteristic $\chi(X)$ (which is not too difficult to compute in most examples) and if there is a prime p as above with

$$\sqrt{p} + \frac{1}{\sqrt{p}} > 2 \cdot h^3(X)$$

we can determine $h^2(X)$ and so all Hodge and Betti numbers of X by counting points on X_p. We can rewrite the above condition as

$$p \ge 4 \cdot \left(h^3(X)\right)^2 - 2.$$

If X has a small number of deformations (i.e., $h^{2,1}(X)$ and so $h^3(X)$ is small) then also a very small p will suffice (if for example X is rigid, i.e. $h^3(X) = 2$, then we get the condition $p \geq 17$). If on the other hand $h^3(X)$ is large then also the required prime p is large and we need fast algorithms for counting points.

1.8.2 Algorithms for counting points

Let X be a threefold defined over $\mathbb{Q}$. If we want to count points on X_p then we will have to do this with a computer. Basically we can take every point of the ambient space and check if it belongs to X_p. This way we get running times (with respect to the number of evaluations of the defining equations) depending on the dimension of the ambient space. In all examples appearing in this thesis I could reduce the running time to $O(p^3)$ (or sometimes only to $O(p^4)$). This is of course only possible by close inspection of the single examples.

To illustrate this consider a double octic X which is given by an equation of the form $u^2 = f(x, y, z, t)$ where $(x : y : z : t)$ are coordinates on $\mathbb{P}^3$. We first create a table of Legendre symbols $\left(\frac{u}{p}\right)$. Then we insert all possible values $(x : y : z : t) \in \mathbb{P}^3(\mathbb{F}_p)$ into f and check if the result is a square in $\mathbb{F}_p$ (and we add 0, 1 or 2 points to the computed number). This algorithm already has running time $O(p^3)$.

It also makes sense to consider symmetry. This will of course only reduce the running time by a constant (but this constant can be rather large; there are examples where the symmetric group Σ_6 operates on X).

I used the C++ programming language for all point counting algorithms. This is much faster than any computer algebra system for this purpose since a typical program uses only loops and elementary integer arithmetic, and a computer algebra system would only interpret but not compile the source.

Typical running times for counting points with an $O(p^3)$ algorithm on a 3 Gigahertz machine (cf. 1.8.4) would be less than a second for a prime $p \sim 200$ and 6 hours for a prime $p \sim 20000$.

Note that there are attempts to efficiently count points on varieties over finite fields using p-adic methods. A good starting point is [60]. Using these methods there should be an algorithm with running time $O(p^{2+\varepsilon})$ that counts points on a threefold but as far as I know at the moment this is not implemented (there are implementations for curves).

1.8.3 Computation of coefficients of modular forms

To find candidates of modular forms connected with Calabi–Yau threefolds it is desirable to have large tables of newforms of weight 4 and weight 2 for $\Gamma_0(N)$. W. Stein has set up a web page ([97]) containing tables. He also wrote the computation package HECKE which is now part of the computer algebra system MAGMA ([112]). I used HECKE to compute coefficients for the first 25 primes (between 2 and 97) of all newforms with rational coefficients of weight 4 for $\Gamma_0(N)$ with $N \leq 2000$ and all newforms with rational coefficients of weight 2 for $\Gamma_0(N)$ with $N \leq 3000$. The computations afford quite a lot of memory and time, around 2 Gigabytes and 6 hours on a 3 Gigahertz machine for one level ~ 2000 in the weight 4 case (weight 2 is easier). For some levels HECKE could not complete the computations due to lack of memory. These are

$$\begin{array}{llll}
1849 = 43 \cdot 43, & 1853 = 17 \cdot 109, & 1883 = 7 \cdot 269, & 1897 = 7 \cdot 271, \\
1903 = 11 \cdot 173, & 1909 = 23 \cdot 83, & 1919 = 19 \cdot 101, & 1921 = 17 \cdot 113, \\
1927 = 41 \cdot 47, & 1937 = 13 \cdot 149, & 1939 = 7 \cdot 277, & 1943 = 29 \cdot 67, \\
1957 = 19 \cdot 103, & 1961 = 37 \cdot 53, & 1963 = 13 \cdot 151, & 1967 = 7 \cdot 281, \\
1969 = 11 \cdot 179, & 1981 = 7 \cdot 283, & 1985 = 5 \cdot 397, & 1991 = 11 \cdot 181.
\end{array}$$

The complete table of computed weight four newforms can be found in appendix C. I also included a small table of weight two newforms in appendix D. It contains all weight two newforms occurring in this thesis.

Throughout the text we will use the notation N/m for the m-th newform of weight four for $\Gamma_0(N)$ with rational coefficients in the table in appendix C. Stein ([97]) uses a different notation; whenever a newform also occurs in his tables (this is only the case for levels $\sim< 300$) we will give both notations. We will also use his notation for weight two newforms for $\Gamma_0(N)$.

1.8.4 Hard- and software

This thesis was written between March 2001 and January 2005. In the beginning I could use an AMD Duron PC running at 800 MHz, with 256 megabytes of RAM. In 2003 this machine was replaced by an INTEL Pentium IV PC running at 2.6 GHz, with 512 megabytes of RAM. Whenever I am referring to a “3 Gigahertz machine” in the text, I am thinking of a PC with approximately this capacity (which is still state of the art in the year 2005). I am indebted to W. Stein who gave me access to the multiprocessor machine MECCAH at Harvard university which I used to compute coefficients of modular forms.

All algorithms counting points on threefolds, searching for modular examples or classifying threefolds I have implemented in the C++ programming language. I used the computer algebra system MAGMA ([112]) and in particular W. Stein’s computation package HECKE to compute coefficients of modular forms. For all other computer algebra purposes I used SINGULAR ([45]).

I estimate the total computing time used to produce the results of this thesis to one year on a 3 Gigahertz machine. This makes it obvious that it could not have been written ten years ago.

Chapter 2

Fibre products of elliptic surfaces

2.1 Examples of Schoen and Schütt

The construction method for the threefolds appearing in this section is due to Schoen ([84]). The modularity proofs for the different examples can be found in [81] and [111] and recently in [87], [88] and [89].

Let (Y, r), (Y', r') be relatively minimal, regular elliptic surfaces with r, r' surjecting onto $\mathbb{P}^1$. Let $W := (Y, r) \times_{\mathbb{P}^1} (Y', r')$ denote their fibre product. In general W will not be smooth; the singularities are the points (x, x') where x and x' are singular points of the fibres of (Y, r) and (Y', r') over a common cusp $s \in S'' = S \cap S'$, where S and S' denote the images of the singular fibres of (Y, r) and (Y', r') in $\mathbb{P}^1$.

In order to avoid singularities worse than ordinary double points, we are going to assume that all fibres over S'' are either irreducible nodal rational curves or cycles of smooth rational curves. In Kodaira's notation these are of type I_b, where $b > 0$ denotes the number of irreducible components. Such fibres are also called *semi-stable*. If both Y and Y' are rational and have sections then the fibre product W has trivial canonical bundle.

Now consider a small resolution $\tilde{W}$ of the nodes of W. There are projective small resolutions (i.e., $\tilde{W}$ is a Calabi–Yau threefold) if $r = r'$ or if for all $s \in S''$, neither $r^{-1}(s)$ nor r'^{-1} is irreducible.

Now assume that $(Y, r) = (Y', r')$ and that Y has exactly four singular fibres and that they are of type I_{b_1}, I_{b_2}, I_{b_3}, I_{b_4} for some integers $b_i > 0$. Then $\tilde{W}$ is a rigid Calabi–Yau threefold.

In order to find suitable elliptic surfaces Y we consider a torsion free congruence subgroup $\Gamma \subset \mathrm{PSL}_2(\mathbb{Z})$ (of level > 2) and the respective modular curve $C_\Gamma = (\mathbb{H}/\Gamma)^*$. Then there is a universal family of elliptic curves $\pi : S_\Gamma \longrightarrow C_\Gamma$, called the *elliptic modular surface* associated to Γ (see [96]). In fact these surfaces have exactly four singular fibres of type I_b, and all rational elliptic surfaces with this property are of this type. They have been classified by Beauville ([12]). We give a list of the six possible cases, including the congruence subgroup Γ and equations of surfaces $Y_\Gamma \subset \mathbb{P}^2 \times \mathbb{P}^1$ such that S_Γ is a resolution of Y_Γ. The fibration in each case is given by projecting to $\mathbb{P}^1$. The elliptic surface $Y_{\Gamma(3)}$ is also known as *Hesse pencil.*

	Γ	sing. fibres	equation for Y_Γ
I	$\Gamma(3)$	I_3, I_3, I_3, I_3	$(x^3 + y^3 + z^3)\mu = \lambda xyz$
II	$\Gamma_1(4) \cap \Gamma(2)$	I_4, I_4, I_2, I_2	$(x + y)(x^2 + y^2 + 2xy + 4z^2 + 4yz - 4xz)\mu = \lambda xyz$
III	$\Gamma_1(5)$	I_5, I_5, I_1, I_1	$(x + y)(x + y - z)(y - z)\mu = \lambda xyz$
IV	$\Gamma_1(6)$	I_6, I_3, I_2, I_1	$(x + y)(y + z)(z + x)\mu = \lambda xyz$
V	$\Gamma_0(8) \cap \Gamma_1(4)$	I_8, I_2, I_1, I_1	$(x + y)(xy - z^2)\mu = \lambda xyz$
VI	$\Gamma_0(9) \cap \Gamma_1(3)$	I_9, I_1, I_1, I_1	$(x^2y + y^2z + z^2x)\mu = \lambda xyz$

Now let W_Γ be a projective small resolution of $Y_\Gamma \times_{\mathbb{P}^1} Y_\Gamma$. Then W_Γ is a rigid Calabi–Yau threefold defined over $\mathbb{Q}$. Saito and Yui ([81]) and Verrill (appendix of [111]) have given different modularity proofs for all six cases. We list the Hodge numbers $h^{1,1}(W_\Gamma)$ and Euler numbers $\chi(W_\Gamma) = 2 \cdot h^{1,1}(W_\Gamma)$ and the weight four newforms involved. We also give different notations for W_Γ occuring in the literature (corresponding to the congruence subgroups).

	W_Γ	$h^{1,1}(W_\Gamma)$	$\chi(W_\Gamma)$	weight four newform
I	$W(3)$	36	72	9/1 (9k4A1)
II	$W_1(4)$	40	80	8/1 (8k4A1)
III	$W_1(5)$	52	104	5/1 (5k4A1)
IV	$W_1(6)$	50	100	6/1 (6k4A1)
V	$W_0(8)$	70	140	16/1 (16k4A1, twist of 8/1)
VI	$W_0(9)$	84	168	9/1 (9k4A1)

Note that for example no. V we can also get the weight four newform 8/1 in the L-series by using the equation $(x + y)(xy + z^2)\mu = \lambda xyz$ instead (cf. [111]). The resulting threefolds are isomorphic over $\mathbb{Q}[\sqrt{-1}]$. This perfectly agrees with the fact that the newform 16/1 is a twist of the newform 8/1 by $\left(\frac{-1}{p}\right)$.

The L-series of $W(3)$ and $W_0(9)$ are the same so there should be a correspondence defined over $\mathbb{Q}$ between them. C. Schoen found such a correspondence (cf. 6.1.5).

Recently ([88], [87]) M. Schütt modified the above construction by twisting, i.e., he considered small resolutions of

$$(Y_\Gamma, \mathrm{pr}) \times_{\mathbb{P}^1} (Y_\Gamma, \pi \circ \mathrm{pr})$$

where π is an automorphism of $\mathbb{P}^1$ chosen in such a way that some small resolutions are still projective. Most possibilities do not lead to different modular forms (cf. [87]), but for $\Gamma = \Gamma_1(6)$ there are very interesting results. Let W_i denote a small resolution of $(Y_\Gamma, \mathrm{pr}) \times_{\mathbb{P}^1} (Y_\Gamma, \pi_i \circ \mathrm{pr})$. Note that Schütt uses an equation for Y_Γ which is slightly different from that in the above table. With that equation the cusps are 0, 1, ∞ and -8, and the automorphisms π_i below permute the first three and do not fix the fourth.

i	π_i	$h^{1,1}(W_i)$	$\chi(W_i)$	weight four newform
1	$t \mapsto 1-t$	48	96	17/1 (17k4A1)
2	$t \mapsto \frac{1}{t}$	40	80	21/2 (21k4A1)
3	$t \mapsto \frac{t}{t-1}$	33	66	10/1 (10k4A1)
4	$t \mapsto \frac{1}{1-t}$	36	72	73/1 (73k4A1)
5	$t \mapsto \frac{t-1}{t}$	36	72	73/1 (73k4A1)

Very recently ([89]) Schütt generalized this even further by also looking at non-rigid examples and at examples that do not allow a projective small resolution of all nodes (which is not important from an arithmetical standpoint). He gives the following examples:

Consider the group $\Gamma = \Gamma_1(5)$ (number III in the above table) and the twisted self-fibre product $(Y_\Gamma, \mathrm{pr}) \times_{\mathbb{P}^1} (Y_\Gamma, \pi \circ \mathrm{pr})$ where $\pi : t \mapsto -11 - t$ is an automorphism of $\mathbb{P}^1$ (Schütt uses a slightly different equation for Y_Γ so his automorphism is also slightly different). The variety has 27 nodes but only 25 of them allow a projective small resolution. Let $\tilde{W}_1$ denote a *mixed* resolution of $(Y_\Gamma, \mathrm{pr}) \times_{\mathbb{P}^1} (Y_\Gamma, \pi \circ \mathrm{pr})$ where the remaining 2 nodes are resolved by a big resolution. We have $h^{1,1}(\tilde{W}_1) = 37$ and $h^{2,1}(\tilde{W}_1) = 8$. Schütt proves that (up to Euler factors at the primes of bad reduction) the L-series of $\tilde{W}_1$ splits into

$$L(\tilde{W}_1, s) = L(f, s) \cdot L(g, s-1)^8$$

where f is the weight four newform 55/1 (55k4A1) and g is the weight two newform 11A1. In particular we have

$$a_p(\tilde{W}_1) = b_p + 8 \cdot p \cdot c_p$$

where b_p resp. c_p are the coefficicents of f resp. g. The "weight two part" comes from the fibre of Y_Γ above 11 which is an elliptic curve with conductor 11 and so associated to the newform g.

Now consider the elliptic surface Y' arising from the following pencil of cubics:

$$(x+y+z)(\frac{11}{8}xy + \frac{11}{8}yz + \frac{125}{88}xz) - (t + \frac{125}{88})xyz.$$

It has singular fibres of type I_6, I_2, I_2, I_1 and I_1. The fibre product $Y_\Gamma \times_{\mathbb{P}^1} Y'$ where $\Gamma = \Gamma_1(5)$ has again only nodes as singularities two of which do not allow a projective small resolution. Let $\tilde{W}_2$ denote a mixed resolution of $Y_\Gamma \times_{\mathbb{P}^1} Y'$ as above. We have $h^{1,1}(\tilde{W}_2) = 45$ and $h^{2,1}(\tilde{W}_2) = 1$. Schütt conjectures that (up to Euler factors at the primes of bad reduction) the L-series of $\tilde{W}_2$ splits into

$$L(\tilde{W}_2, s) = L(f, s) \cdot L(g', s-1)^8$$

where f is again the weight four newform 55/1 (55k4A1) and g' is a weight two newform for $\Gamma_0(39490)$. In particular we have

$$a_p(\tilde{W}_2) = b_p + p \cdot c'_p$$

where b_p resp. c'_p are the coefficicents of f resp. g'. The "weight two part" comes from the fibre of Y_Γ above $-\frac{125}{88}$ which is an elliptic curve with conductor 39490 and so associated to the newform g'. Note that with more computer power than it is available today a proof of this conjecture would be possible (the problem is that $\tilde{W}_2$ has bad reduction at 359).

Now consider the elliptic surface Y'' arising from the Weierstrass equation

$$y^2 = x(x-1)(x + (t^2 - 11t - 1)).$$

It has singular fibres of type I_4 and four times I_2. The fibre product $Y_\Gamma \times_{\mathbb{P}^1} Y''$ where $\Gamma = \Gamma_1(5)$ has again only nodes as singularities some of which do not allow a projective small resolution. Let $\tilde{W}_3$ denote a mixed resolution of $Y_\Gamma \times_{\mathbb{P}^1} Y''$ as above. We have $h^{1,1}(\tilde{W}_3) = 39$ and $h^{2,1}(\tilde{W}_3) = 1$. Schütt proves that (up to Euler factors at the primes of bad reduction) the L-series of $\tilde{W}_3$ splits into

$$L(\tilde{W}_3, s) = L(f', s) \cdot L(g, s-1)$$

where f' is the weight four newform 22/2 ($22k4C1$) and g is again the weight two newform $11A1$. In particular we have

$$a_p(\tilde{W}_3) = b'_p + p \cdot c_p$$

where b'_p resp. c_p are the coefficicents of f' resp. g. The "weight two part" comes again from the fibre of Y_Γ above 11.

Finally consider the group $\Gamma = \Gamma(3)$ (number I in the above table) and the twisted self-fibre product $(Y_\Gamma, \mathrm{pr}) \times_{\mathbb{P}^1} (Y_\Gamma, \pi \circ \mathrm{pr})$ where $\pi : t \mapsto 3 - t$ is an automorphism of $\mathbb{P}^1$. All nodes of this threefold allow a projective small resolution. Let $\tilde{W}_4$ denote such a resolution. We have $h^{1,1}(\tilde{W}_4) = 31$ and $h^{2,1}(\tilde{W}_4) = 4$. Schütt proves that (up to Euler factors at the primes of bad reduction) the L-series of $\tilde{W}_4$ splits into

$$L(\tilde{W}_4, s) = L(f_{27}, s) \cdot L(g_{27}, s-1)^4$$

where f_{27} is the weight four newform 27/2 ($27k4B1$) and g is the weight two newform $27A1$. In particular we have

$$a_p(\tilde{W}_1) = d_p + 4 \cdot p \cdot e_p$$

where d_p resp. e_p are the coefficicents of f_{27} resp. g_{27}. The "weight two part" comes from the fibre of Y_Γ above 6 which is an elliptic curve with conductor 27 and so associated to the newform g_{27}. The proof makes use of lemma 1.7.

2.2 Experiments

Inspired by Schütt's constructions I performed some numerical experiments. I counted points on twisted self-fibre products $W_{\Gamma,\pi} = (Y_\Gamma, \mathrm{pr}) \times_{\mathbb{P}^1} (Y_\Gamma, \pi \circ \mathrm{pr})$ where Y_Γ is one of the six Beauville surfaces and π is an automorphism of $\mathbb{P}^1$ defined over $\mathbb{Q}$,

$$\pi(t) = \frac{at + b}{ct + d}, \quad a, b, c, d \in \mathbb{Z}, \quad ad - bc \neq 0.$$

For certain automorphisms π we find

$$\#W_{\Gamma,\pi,p} \equiv b_p \mod p$$

for all checked good primes p where b_p are the coefficients of a certain weight four newform, suggesting that this newform occurs in the L-series of (a resolution of) $W_{\Gamma,\pi}$. The "rest" of the L-series could again be determined by weight two newforms but I did not check this.

The following tables list the results for the parameter set $|a|, |b|, |c|, |d| \leq 32$ (the computations took several days). Some examples in it have been discussed in [87], [88] and [89].

Example I: $\Gamma = \Gamma(3)$**:**

Here I used the equation

$$x^3 + y^3 + z^3 = txyz$$

for the elliptic surface Y_Γ.

a, b, c, d	weight four newform	references
$0, 9, 1, 0$	27/2 (27k4B1, twist of 27/1)	5.5
$0, 18, -1, 0$	54/3 (54k4A1, twist of 54/1)	
$1, -6, 0, -1$	54/2 (54k4D1)	
$1, 0, 0, 1$	9/1 (9k4A1)	[81], [84], [87], [111], 5.6
$1, 3, 0, -1$	27/2 (27k4B1, twist of 27/1)	[89]
$3, -9, 1, 6$	27/1 (27k4A1)	
$3, 0, -1, -3$	54/2 (54k4D1)	
$3, 18, -2, -3$	54/4 (54k4C1, twist of 54/2)	
$3, 18, 1, -3$	9/1 (9k4A1)	[87]
$6, -18, -1, -6$	54/1 (54k4B1)	

Note that the surface given by

$$x^3 + y^3 \pm d \cdot z^3 = txyz,$$

with $d \in \mathbb{N}$ a cubefree number, is isomorphic to $Y_{\Gamma(3)}$ over $\mathbb{Q}[\sqrt[3]{d}]$. Consequently self-fibre products of such surfaces lead to closely related newforms. M. Schütt suggested that they can be constructed from the newform 9/1 by tensoring with the cubic reciprocity character $\mathrm{Frob}_p \mapsto \left(\frac{d}{p}\right)_3$ or its square. For $d = 2, 3, 4, 5, 6, 7, 9, 10, 12, 18, 25, 28, 36, 49, 98$ we get the newforms 108/3 (108k4A1), 243/4 (243k4B1), 108/1 (108k4B1), 675/1, 972/2, 1323/2, 243/3 (243k4A1), 900/18, 972/1, 972/4, 675/4, 1764/3, 972/3, 1323/3, 1764/2.

Example II: $\Gamma = \Gamma_1(4) \cap \Gamma(2)$**:**

Here I used the equation

$$(x + y)(x^2 + y^2 + 2xy + 4z^2 + 4yz - 4xz) = txyz$$

for the elliptic surface Y_Γ. It is different from that in [87] so that the cusps are in different positions and also the automorphisms are different from those in [87].

The automorphism $t \mapsto -t$ corresponds to the coordinate change $z \mapsto -z$ so to keep the table short I only display the results modulo this automorphism.

a,b,c,d	weight four newform	references
$0,32,1,-4$	$48/2$ ($48k4C1$, twist of $12/1$)	
$0,32,1,4$	$48/2$ ($48k4C1$, twist of $12/1$)	
$0,64,1,-16$	$6/1$ ($6k4A1$)	
$0,64,1,-8$	$12/1$ ($6k4A1$)	
$0,64,1,8$	$12/1$ ($6k4A1$)	
$0,64,1,16$	$6/1$ ($6k4A1$)	
$0,64,1,0$	$8/1$ ($8k4A1$)	[87]
$1,-16,0,-1$	$6/1$ ($6k4A1$)	
$1,-16,0,1$	$6/1$ ($6k4A1$)	
$1,-8,0,-1$	$12/1$ ($6k4A1$)	
$1,0,0,1$	$8/1$ ($8k4A1$)	[81], [84], [87], [111]
$1,8,0,-2$	$48/2$ ($48k4C1$, twist of $12/1$)	
$1,8,0,-1$	$12/1$ ($6k4A1$)	
$1,8,0,2$	$48/2$ ($48k4C1$, twist of $12/1$)	
$4,0,-1,4$	$6/1$ ($6k4A1$)	
$4,0,1,4$	$6/1$ ($6k4A1$)	
$8,-64,-1,-8$	$16/1$ ($16k4A1$, twist of $8/1$)	[87]
$8,64,1,-8$	$16/1$ ($16k4A1$, twist of $8/1$)	[87]
$8,0,-1,8$	$12/1$ ($6k4A1$)	
$8,0,-1,16$	$48/2$ ($48k4C1$, twist of $12/1$)	
$8,0,1,8$	$12/1$ ($6k4A1$)	
$8,0,1,16$	$48/2$ ($48k4C1$, twist of $12/1$)	

Example III: $\Gamma = \Gamma_1(5)$**:**

Here I used the equation

$$(x+y)(x+y-z)(y-z) = txyz$$

for the elliptic surface Y_Γ. It is different from that in [87] and [89] so that the cusps are in different positions and also the automorphisms are different from those in [89].

a,b,c,d	weight four newform	references
$0,1,-1,0$	$5/1$ ($5k4A1$)	[87]
$0,1,1,0$	$22/2$ ($22k4C1$)	
$0,1,1,11$	$55/1$ ($55k4A1$)	
$1,-1,1,1$	$110/5$ ($110k4E1$)	
$1,0,0,-1$	$22/2$ ($22k4C1$)	
$1,0,0,1$	$5/1$ ($5k4A1$)	[81], [84], [87], [111]
$1,0,11,-1$	$55/1$ ($55k4A1$)	
$1,11,0,-1$	$55/1$ ($55k4A1$)	[89]
$2,11,11,-2$	$550/5$ (twist of $22/2$)	
$11,-2,-2,-11$	$550/5$ (twist of $22/2$)	

Example IV: $\Gamma = \Gamma_1(6)$**:**

Here I used the equation

$$(x+y)(y+z)(z+x) = txyz$$

for the elliptic surface Y_Γ. Note that some examples appear as relatives of some of Hulek's and Verrill's modular threefolds (cf. 5.8). To see this we can rewrite the above equation for Y_Γ:

$$(x+y+z)(xy+xz+yz) = (x+y)(y+z)(z+x) + xyz = (t+1)xyz$$

a,b,c,d	weight four newform		references
$0,1,-1,-1$	73/1	($73k4A1$)	[87], [89]
$0,1,1,0$	21/2	($21k4A1$)	[87], [89]
$0,8,-9,-8$	90/2	($90k4A1$, twist of 10/1)	
$0,8,-1,0$	6/1	($6k4A1$)	[87]
$0,8,1,-16$	102/3	($102k4D1$)	
$0,8,1,-8$	10/1	($10k4A1$)	
$0,8,1,-7$	21/2	($21k4A1$)	
$0,8,1,0$	14/2	($14k4A1$)	
$0,8,1,1$	17/1	($17k4A1$)	
$0,8,1,2$	60/1	($60k4A1$)	
$0,8,9,1$	657/1	(twist of 73/1)	
$0,9,-1,-1$	153/2	($153k4D1$, twist of 17/1)	
$0,9,1,-8$	657/1	(twist of 73/1)	
$1,-16,0,-1$	102/3	($102k4D1$)	
$1,-8,-10,-1$	10/1	($10k4A1$)	
$1,-8,-1,-1$	18/1	($18k4A1$, twist of 6/1)	[87]
$1,-8,-1,17$	306/8	($306k4F1$, twist of 102/3)	
$1,-8,0,-1$	10/1	($10k4A1$)	
$1,-8,0,8$	73/1	($73k4A1$)	
$1,-8,0,9$	90/2	($90k4A1$, twist of 10/1)	[51], 5.8
$1,-8,1,1$	126/4	($126k4D1$, twist of 14/2)	
$1,-8,8,8$	63/3	($63k4B1$, twist of 21/2)	
$1,-7,0,-1$	21/2	($21k4A1$)	
$1,0,-2,-1$	102/3	($102k4D1$)	
$1,0,-1,-1$	10/1	($10k4A1$)	[87], [89]
$1,0,-1,8$	73/1	($73k4A1$)	
$1,0,-1,9$	90/2	($90k4A1$, twist of 10/1)	
$1,0,0,-8$	21/2	($21k4A1$)	
$1,0,0,-1$	14/2	($14k4A1$)	
$1,0,0,1$	6/1	($6k4A1$)	[81], [84], [87], [111]
$1,1,-1,8$	63/3	($63k4B1$, twist of 21/2)	
$1,1,0,-9$	657/1	(twist of 73/1)	
$1,1,0,-1$	17/1	($17k4A1$)	[87], [89]

a,b,c,d	weight four newform	references
$1, 2, 0, -1$	60/1 (60k4A1)	[51], 5.8
$1, 10, -1, -1$	180/5 (180k4D1, twist of 60/1)	
$4, -32, 5, -4$	180/5 (180k4D1, twist of 60/1)	
$4, 0, 1, -4$	60/1 (60k4A1)	
$7, 16, 2, -7$	14/2 (14k4A1)	
$8, -8, 1, 8$	126/4 (126k4D1, twist of 14/2)	
$8, 0, -8, -9$	657/1 (twist of 73/1)	
$8, 0, -7, -8$	21/2 (21k4A1)	
$8, 0, 1, -8$	17/1 (17k4A1)	
$8, 0, 1, 9$	153/2 (153k4D1, twist of 17/1)	
$8, 8, -17, -8$	306/8 (306k4F1, twist of 102/3)	
$8, 8, 0, 9$	153/2 (153k4D1, twist of 17/1)	
$8, 8, 1, -8$	18/1 (18k4A1, twist of 6/1)	[87]
$8, 8, 1, 10$	180/5 (180k4D1, twist of 60/1)	
$8, 17, 1, -8$	17/1 (17k4A1)	

Example V: $\Gamma = \Gamma_0(8) \cap \Gamma_1(4)$**:**

Here I used the equation

$$(x + y)(xy + z^2) = txyz$$

for the elliptic surface Y_Γ. The advantage is that all cusps are then defined over $\mathbb{Q}$ and there are more possibilities to permute some of them by an automorphism defined over $\mathbb{Q}$ (which may lead to interesting arithmetical results). In fact, using the equation $(x + y)(xy - z^2) = txyz$ I have not detected any modular forms in the L-series of the twisted fibre products, except for the trivial cases $t \mapsto \pm t$.

The automorphism $t \mapsto -t$ corresponds to the coordinate change $z \mapsto -z$ so to keep the table short I only display the results modulo this automorphism.

a,b,c,d	weight four newform	references
$0, 8, 1, -2$	48/2 (48k4C1, twist of 12/1)	
$0, 8, 1, 2$	48/2 (48k4C1, twist of 12/1)	
$0, 16, 1, -8$	6/1 (6k4A1)	
$0, 16, 1, -4$	12/1 (12k4A1)	
$0, 16, 1, 0$	8/1 (8k4A1)	[87]
$0, 16, 1, 4$	12/1 (12k4A1)	
$0, 16, 1, 8$	6/1 (6k4A1)	
$0, 32, 1, -4$	48/2 (48k4C1, twist of 12/1)	
$0, 32, 1, 4$	48/2 (48k4C1, twist of 12/1)	
$1, -8, 0, -1$	6/1 (6k4A1)	
$1, -4, 0, -1$	12/1 (12k4A1)	
$1, 0, 0, 1$	8/1 (8k4A1)	[81], [84], [87], [111]
$1, 4, 0, -2$	48/2 (48k4C1, twist of 12/1)	

a,b,c,d	weight four newform	references
$1,4,0,-1$	12/1 $(12k4A1)$	
$1,4,0,2$	48/2 $(48k4C1$, twist of 12/1)	
$1,8,0,-1$	6/1 $(6k4A1)$	
$2,0,-1,2$	6/1 $(6k4A1)$	
$2,0,1,2$	6/1 $(6k4A1)$	
$4,-16,-3,-4$	12/1 $(12k4A1)$	
$4,-16,-1,-4$	16/1 $(16k4A1$, twist of 8/1)	
$4,-16,3,-4$	48/3 $(48k4A1$, twist of 6/1)	
$4,0,-1,4$	12/1 $(12k4A1)$	
$4,0,-1,8$	48/2 $(48k4C1$, twist of 12/1)	
$4,0,1,4$	12/1 $(12k4A1)$	
$4,0,1,8$	48/2 $(48k4C1$, twist of 12/1)	
$4,16,-3,-4$	48/3 $(48k4A1$, twist of 6/1)	
$4,16,-1,-12$	48/3 $(48k4A1$, twist of 6/1)	
$4,16,-1,12$	12/1 $(12k4A1)$	
$4,16,1,-12$	12/1 $(12k4A1)$	
$4,16,1,-4$	16/1 $(16k4A1$, twist of 8/1)	
$4,16,1,12$	48/3 $(48k4A1$, twist of 6/1)	
$4,16,3,-4$	12/1 $(12k4A1)$	

Example VI: $\Gamma = \Gamma_0(9) \cap \Gamma_1(3)$:

Here I used the equation

$$x^2y + y^2z + z^2x = txyz$$

for the elliptic surface Y_Γ. The results are exactly the same as for example I ($\Gamma = \Gamma(3)$). This is not unexpected because of the correspondence between them (cf. 6.1.5).

Other examples:

Let (Y, pr) be given by the equation

$$(x + y + z)(xy + xz + 4yz) = txyz.$$

(Twisted) self-fibre products of this elliptic surface are also rather interesting because they appear as relatives of some of Hulek's and Verrill's modular threefolds (cf. 5.8).

a,b,c,d	weight four newform	references
$1,0,0,-1$	40/2 $(40k4B1)$	5.8
$1,0,0,1$	12/1 $(12k4A1)$	[51], 5.8
$1,0,0,4$	30/2 $(30k4A1)$	
$2,0,-1,6$	240/11 $(240k4H1$, twist of 120/4)	
$2,0,1,-2$	336/4 (twist of 168/1)	

a,b,c,d	weight four newform		references
$3, 0, 1, -4$	78/2	(78k4D1)	
$4, 0, -1, -4$	480/7	(twist of 480/2)	
$4, 0, 1, -16$	78/2	(78k4D1)	
$4, 0, 1, -12$	96/3	(96k4F1, twist of 96/2)	
$4, 0, 1, -4$	96/3	(96k4F1, twist of 96/2)	
$8, 0, 1, -8$	384/8	(twist of 384/1)	
$12, 0, -1, 16$	168/2	(168k4E1)	
$16, 0, 1, -16$	1344/9	(twist of 168/2)	
$16, 0, 5, -16$	960/1	(twist of 30/2)	

I performed some more numerical experiments with (twisted) fibre products of two elliptic surfaces. This way I found threefolds connected with the weight four newforms 28/1 (28k4B1), 68/1 (68k4A1) and 88/2 (88k4A1). Very recently ([52]) Hulek and Verrill continued the study of non-rigid fibre products. Among others they give an example connected with the weight four newform 35/1 (35k4A1).

2.3 Relatives

All of the above elliptic surfaces Y_Γ are given in $\mathbb{P}^2 \times \mathbb{P}^1$ by an equation of the form

$$F(x, y, z) = t \cdot G(x, y, z)$$

with homogeneous polynomials F and G of degree 3. There is a model in $\mathbb{P}^2 \times \mathbb{P}^2$ of their self-fibre products $(Y_\Gamma, \mathrm{pr}) \times_{\mathbb{P}^1} (Y_\Gamma, \mathrm{pr})$, with equation

$$F(x, y, z) \cdot G(r, s, t) = F(r, s, t) \cdot G(x, y, z).$$

A birational relative of such a variety is the complete intersection of two cubics in $\mathbb{P}^5$ given by

$$F(x, y, z) = F(r, s, t),$$
$$G(x, y, z) = G(r, s, t).$$

If we consider twisted self-fibre products instead, i.e., $(Y_\Gamma, \mathrm{pr}) \times_{\mathbb{P}^1} (Y_\Gamma, \pi \circ \mathrm{pr})$ where π is an automorphism of $\mathbb{P}^1$ then we may get different complete intersections of two cubics as relatives. If π is of the form $t \mapsto a \cdot t$ with $a \in \mathbb{P}^1$ then a birational relative is given by the equations

$$F(x, y, z) = \lambda \cdot F(r, s, t),$$
$$G(x, y, z) = \mu \cdot G(r, s, t).$$

with $\lambda, \mu \in \mathbb{P}^1$, $\mu/\lambda = a$.

If π is of the form $t \mapsto a/t$ with $a \in \mathbb{P}^1$ then a birational relative is given by the equations

$$F(x, y, z) = \lambda \cdot G(r, s, t),$$
$$G(r, s, t) = \mu \cdot G(x, y, z),$$

with $\lambda, \mu \in \mathbb{P}^1$, $\lambda \cdot \mu = a$.

Sometimes it may be interesting to study these models. We will do this in 5.5 and 5.6 for two examples. The first one corresponds to $(Y_{\Gamma(3)}, \mathrm{pr}) \times_{\mathbb{P}^1} (Y_{\Gamma(3)}, \pi \circ \mathrm{pr})$ where $\pi(t) = 9/t$. It is a special member of a family of smooth Calabi–Yau threefolds which has been studied by several authors. The second one corresponds to $(Y_{\Gamma(3)}, \mathrm{pr}) \times_{\mathbb{P}^1} (Y_{\Gamma(3)}, \mathrm{pr})$. It has 108 ordinary nodes as only singularities which seems to be the highest known value for a complete intersection of two cubics in $\mathbb{P}^5$.

Chapter 3

Quintics in $\mathbb{P}^4$

3.1 Schoen's quintic and the standard family of quintics

Let $X_\mu \subset \mathbb{P}^4$ be the quintic threefold defined by the equation

$$x_0^5 + x_1^5 + x_2^5 + x_3^5 + x_4^5 - 5\mu x_0 x_1 x_2 x_3 x_4 = 0.$$

If μ is general (i.e., no 5-th root of unity and not 0 or ∞) then X_μ is smooth and so a Calabi–Yau threefold. On X_μ there is an action of the group $G \simeq (\mathbb{Z}/5\mathbb{Z})^3$ generated by the coordinate transformations

$$(x_0 : x_1 : x_2 : x_3 : x_4 : x_5) \mapsto (x_0 : x_1 \cdot \xi_5^{\lambda_1} : x_2 \cdot \xi_5^{\lambda_2} : x_3 \cdot \xi_5^{\lambda_3} : x_4 \cdot \xi_5^{\lambda_4})$$

with $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathbb{Z}/5\mathbb{Z}$, $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \equiv 0 \mod 5$ and ξ_5 a fixed primitive 5-th root of unity. The mirror partner of X_μ can be described as a resolution of the quotient X_μ/G (we will come back to that in 3.2 below). There is a lot of literature on the varieties X_μ; good starting points are [73] for the mirror and [18] and [19] where the zeta function of X_μ is discussed.

If μ is a 5-th root of unity then X_μ has 125 nodes as only singularities, namely the points on the orbit of $(1 : \mu : \mu : \mu : \mu)$ under the action of the group G. We are especially interested in $X := X_1$, which is defined over $\mathbb{Q}$. We will give an account of the modularity question for X which was discussed in [86].

Let $\tilde{X}$ be a small resolution of X. Then $\tilde{X}$ has Euler characteristic

$$\chi(\tilde{X}) = -200 + 2 \cdot 125 = 50.$$

The defect of X is $d(X) = h^2(\tilde{X}) - 1 = 24 \neq 0$ (see the computation of $h^2(\tilde{X})$ below). Since the group G acts transitively on the set of nodes of X there exist projective small resolutions (cf. corollary 1.9). Equivalently this may be deduced from the existence of smooth quadric surfaces on X through all the nodes (cf. [86]).

If 5-th roots of unity exist in $\mathbb{F}_p$ (i.e., $p \equiv 1 \mod 5$) then all the nodes and the rulings of their tangent cones are rational over $\mathbb{F}_p$. In this case the Lefschetz fixed point formula gives

$$\begin{aligned}|\#\tilde{X}_p - 1 - p^3 - h^2(\tilde{X}) \cdot (p^2+p)| &= |\#X_p + 125p - 1 - p^3 - h^2(\tilde{X}) \cdot (p^2+p)| \\ &\leq p^{3/2} h^3(\tilde{X}) = p^{3/2}(2 + 2h^2(\tilde{X}) - 50).\end{aligned}$$

Counting points over $\mathbb{F}_{31}$ we find

$$h^2(\tilde{X}) = 25, \quad h^3(\tilde{X}) = 2,$$

and so $\tilde{X}$ is rigid.

Note that $p = 5$ is the only prime of bad reduction for X. If $p \not\equiv 1 \mod 5$ then only the node $(1:1:1:1:1)$ is rational over $\mathbb{F}_p$. The tangent cone at this node is given by the smooth quadric surface with equation

$$2(x^2+y^2+z^2+w^2) - xy - xz - xw - yz - yw - zw = 0.$$

The discriminant of the corresponding quadratic form is 125. Consequently if 5 is a square in $\mathbb{F}_p$ (i.e., in our case $p \equiv 4 \mod 5$) we find

$$|\#\tilde{X}_p - 1 - p^3 - k \cdot (p^2+p)| = |\#X_p + p - 1 - p^3 - k \cdot (p^2+p)| \leq 2p^{3/2}$$

with $k \in \mathbb{Z}$, $|k| \leq 25 = h^2(\tilde{X})$. Counting points over $\mathbb{F}_{19}$ gives $k = 1$.

If $p \equiv 2, 3 \mod 5$ then we have the estimate

$$|\#\tilde{X}_p - 1 - p^3 - l(p^2+p)| = |\#X_p - p - 1 - p^3 - l(p^2+p)| \leq 2p^{3/2}$$

with $l \in \mathbb{Z}$, $|l| \leq 25 = h^2(\tilde{X})$. Counting points over $\mathbb{F}_7$ gives $l = 1$.

We end up with the formula

$$a_p(\tilde{X}) = \begin{cases} p^3 + 25p^2 - 100p + 1 - \#X_p, & p \equiv 1 \mod 5, \\ p^3 + p^2 + 1 - \#X_p, & p \equiv 4 \mod 5, \\ p^3 + p^2 + 2p + 1 - \#X_p, & p \equiv 2,3 \mod 5. \end{cases}$$

Not all of the $a_p(\tilde{X})$ are even so theorem 1.5 can not be applied to prove the modularity of $\tilde{X}$ but the principle still works. In [86] it is checked that the $a_p(\tilde{X})$ agree with the coefficients of the weight four newform 25/1 ($25k4A1$) for the primes $p \in \{3, 7, 11, 13\}$, and a proof is given that they agree for all good primes.

3.2 Equations for the mirror

Let $Y_\mu \subset \mathbb{P}^4$ be the quintic threefold defined by the equation

$$(x_0 + x_1 + x_2 + x_3 + x_4)^5 - (5\mu)^5 x_0 x_1 x_2 x_3 x_4 = 0.$$

For general μ (i.e., μ is no 5-th root of unity and not 0 or ∞) the singular locus of Y_μ consists of the $\binom{5}{2} = 10$ lines given by

$$x_i = x_j = x_k + x_l + x_m = 0$$

where $\{i, j, k, l, m\} = \{0, 1, 2, 3, 4\}$. If μ is a 5-th of unity then there is an additional singularity at the point $(1 : 1 : 1 : 1 : 1)$. This is an ordinary node.

There is a rational dominant map $X_\mu \longrightarrow Y_\mu$ induced by

$$\phi : \mathbb{P}^4 \longrightarrow \mathbb{P}^4, \quad (x_0 : x_1 : x_2 : x_3 : x_4) \mapsto (x_0^5 : x_1^5 : x_2^5 : x_3^5 : x_4^5).$$

The map is generically $125 : 1$. The degree reduces to $25 : 1$ on the singular lines and to $5 : 1$ on the 10 intersection points of three lines (i.e., the points on the orbit of $(0 : 0 : 0 : 1 : -1)$ under permutation of coordinates). Let A denote the union of the 10 singular lines and let B denote the union of the 10 intersection points. We can now relate the Euler characteristic of a general Y_μ to that of X_μ:

$$\begin{aligned} -200 = \chi(X_\mu) &= 125 \cdot \chi(Y_\mu \setminus A) + 25 \cdot \chi(A \setminus B) + 5 \cdot \chi(B) \\ &= 125 \cdot \chi(Y_\mu \setminus A) + 25 \cdot 10 \cdot (2 - 3) + 5 \cdot 10 \\ &= 125 \cdot \chi(Y_\mu \setminus A) - 200, \end{aligned}$$

thus $\chi(Y_\mu \setminus A) = 0$ and

$$\chi(Y_\mu) = \chi(Y_\mu \setminus A) + \chi(A) = 0 + 10 \cdot 2 - 2 \cdot 10 = 0.$$

If μ is a 5-th root of unity then we set $Y := Y_\mu = Y_1$ and we find

$$\begin{aligned} -75 = \chi(X) &= 125 \cdot \chi(Y \setminus A) + 25 \cdot \chi(A \setminus B) + 5 \cdot \chi(B) \\ &= 125 \cdot \chi(Y \setminus A) + 25 \cdot 10 \cdot (2 - 3) + 5 \cdot 10 \\ &= 125 \cdot \chi(Y \setminus A) - 200, \end{aligned}$$

thus $\chi(Y \setminus A) = 1$ and

$$\chi(Y) = \chi(Y \setminus A) + \chi(A) = 1 + 10 \cdot 2 - 2 \cdot 10 = 1.$$

The map ϕ exactly divides out the action of the group G on X_μ. Thus the quintic Y_μ is a model for the mirror X_μ/G of X_μ. The resolution of singularities of X_μ/G has been discussed in [73]. The singular lines are lines of A_4 singularities. The 10 intersection points of singular lines look locally like the quotient $\mathbb{C}^3/H$ where the group $H \cong \{(\xi_1, \xi_2, \xi_3),\ \xi_1^5 = \xi_2^5 = \xi_3^5 = \xi_1\xi_2\xi_3 = 1\}$ acts diagonally on $\mathbb{C}^3$. One choice of resolution is the following:

Blow up the 10 intersection points of the singular lines. This produces three exceptional divisors for each point and 30 lines of A_1 singularities where two of these divisors intersect.

Blow up the 10 lines of A_4 singularities and the 30 lines of A_1 singularities. This produces $50 = 2 \cdot 10 + 1 \cdot 30$ new exceptional divisors.

Blow up the remaining singular curves (intersection of divisors coming from the blowup of the lines of A_4 singularities). This produces $20 = 2 \cdot 10$ new exceptional divisors.

Now the singular locus consists of $60 = 6 \cdot 10$ nodes (2 nodes on each exceptional divisor from the first step) which can be resolved by a projective small resolution.

Altogether the resolution of singularities replaces 30 points by $\mathbb{P}^2$ and 70 copies of $\mathbb{P}^1$ by $\mathbb{P}^1 \times \mathbb{P}^1$. Denote by $\tilde{Y}_\mu$ such a resolution. The Euler characteristic and Hodge numbers of $\tilde{Y}_\mu$ are

$$\chi(\tilde{Y}_\mu) = \chi(Y_\mu) + 200 = 200, \quad h^{2,1}(\tilde{Y}_\mu) = 1, \quad h^{1,1}(\tilde{Y}_\mu) = 100,$$

and over the finite field $\mathbb{F}_p$ we have

$$\#\tilde{Y}_{\mu,p} = \#Y_{\mu,p} + 100 \cdot (p^2 + p).$$

On $Y = Y_1$ there is the additional node $(1 : 1 : 1 : 1 : 1)$. It is the image of the 125 nodes of the Schoen quintic X under the map ϕ. On X the node $(1 : 1 : 1 : 1 : 1)$ is contained in the smooth quadric surface Q given by the equations

$$\begin{aligned}&x_0 + \xi_5 x_1 + \xi_5^2 x_2 + \xi_5^3 x_3 + \xi_5^4 x_4 = \\ &x_0x_1 + \xi_5 x_0x_2 + \xi_5^2 x_0x_3 + \xi_5^3 x_0x_4 + \xi_5^2 x_1x_2 + \xi_5^3 x_1x_3 + \xi_5^4 x_1x_4 + \xi_5^4 x_2x_3 + x_2x_4 + \xi_5 x_3x_4 = 0\end{aligned}$$

where ξ_5 is a primitive 5-th root of unity. Thus on Y the node $(1 : 1 : 1 : 1 : 1)$ is contained in the smooth surface $\phi(Q)$ (which does not meet the singular lines) so there exist projective small resolutions. Let $\tilde{Y}$ denote such a small resolution of (all the singularities of) Y. The Euler characteristic of $\tilde{Y}$ is

$$\chi(\tilde{Y}) = \chi(Y) + 200 + 1 = 202.$$

The tangent cone at the node $(1 : 1 : 1 : 1 : 1)$ is given by the smooth quadric surface

$$2(x^2 + y^2 + z^2 + t^2) - (xy + xz + xt + yz + yt + zt) = 0$$

with discriminant 125. Thus if $p \equiv 1, 4 \mod 5$ we have

$$\#\tilde{Y}_{\mu,p} = \#Y_{\mu,p} + 100 \cdot (p^2 + p) + p$$

and the Lefschetz fixed point formula gives

$$\begin{aligned}|\#\tilde{Y}_p - 1 - p^3 - h^2(\tilde{Y}) \cdot (p^2 + p)| &= |\#Y_p + 100(p^2 + p) + p - 1 - p^3 - h^2(\tilde{Y}) \cdot (p^2 + p)| \\ &\le p^{3/2} h^3(\tilde{Y}) = p^{3/2}(2 + 2h^2(\tilde{Y}) - 202).\end{aligned}$$

Counting points over $\mathbb{F}_{31}$ we find

$$h^2(\tilde{Y}) = h^{1,1}(\tilde{Y}) = 101, \quad h^3(\tilde{Y}) = 2, \quad h^{2,1}(\tilde{Y}) = 0,$$

and so $\tilde{Y}$ is rigid.

If $p \equiv 2, 3 \mod 5$ then we have the estimate

$$|\#\tilde{Y}_p - 1 - p^3 - k \cdot (p^2 + p)| = |\#Y_p + 100(p^2 + p) - p - 1 - p^3 - k \cdot (p^2 + p)| \le 2p^{3/2}$$

with $k \in \mathbb{Z}$, $|k| \leq 101 = h^2(\tilde{Y})$. Counting points over $\mathbb{F}_{23}$ gives $k = 101$. We end up with the formula

$$a_p(\tilde{Y}) = \begin{cases} p^3 + p^2 \qquad\quad + 1 - \#Y_p, & p \equiv 1, 4 \mod 5, \\ p^3 + p^2 + 2p + 1 - \#Y_p, & p \equiv 2, 3 \mod 5. \end{cases}$$

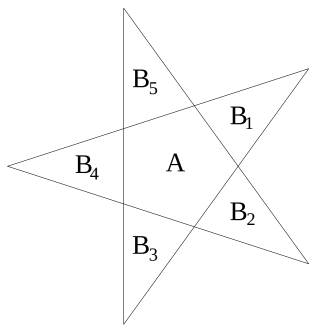
Counting points we find that the $a_p(\tilde{Y})$ agree with the coefficients of the weight four newform 25/1 ($25k4A1$) for all primes $p \in \{3, 7, 11, 13\}$, so they agree for all good primes. Note that this was clear from the start because of the correspondence between X and Y.

It would also be interesting to explicitly compute the zeta functions of the mirror families and compare with the recent results of Wan ([105]) and Haessig ([46]).

3.3 Hirzebruch's quintic

The manifold in this section was constructed in [48]. It was further discussed in [99] where also its modularity was proven.

Let $\{f(x, y) := \prod_{i=1}^{5} f_i(x, y) = 0\}$ be the quintic curve in the real (x, y)-plane which is given by the product of the five lines of a regular pentagon.



As a function of two real variables x and y, f has relative extrema in the center A of the pentagon and in one point b_i of each triangle B_i. So both partial derivatives of f vanish at these six points and at the ten intersection points of the five lines. By symmetry, $f(b_i) = f(b_j)$ for all i and j. Thus the function f can be normalized so that $f(b_i) = b < 0$ for all $i = 1, \ldots, 5$. Now we consider the threefold $V \subset \mathbb{P}^4$ given by the homogenisation of the equation

$$f(x, y) - f(z, w) = 0.$$

There are no singularities at infinity. If (x, y, z, w) is a singular point in the affine part of V, then (x, y) and (z, w) are critical points of f. There are three possibilities:

$$\begin{array}{lclcll} f(x,y) & = & 0 & = & f(z,w) & \text{(100 points)}, \\ f(x,y) & = & b & = & f(z,w) & \text{(25 points)}, \\ f(x,y) & = & f(A) & = & f(z,w) & \text{(1 point}, f(A) > 0). \end{array}$$

So the threefold V has 126 isolated singularities (which are all nodes) and Euler characteristic $\chi(V) = -200 + 126 = -74$.

Now we choose the coordinates of the vertices of the pentagon to be

$$\left(-\frac{1}{2}, \pm\frac{u\sqrt{2-u}}{2}\right), \quad \left(\frac{1-u}{2}, \pm\frac{\sqrt{2-u}}{2}\right), \quad (u, 0),$$

with

$$u = \frac{\sqrt{5}-1}{2}, \quad \text{so} \quad u^2 = 1-u.$$

This gives:

$$\begin{aligned} f(x,y) &= \left(x+\frac{1}{2}\right)\left(y^2 - \left(\frac{-4u+3}{5}\right)(x+u+1)^2\right)\left(y^2 - \left(\frac{4u+7}{5}\right)(x-u)^2\right) \\ &= (x+\frac{1}{2})(y^4 - y^2(2x^2-2x+1) + \frac{1}{5}(x^2+x-1)^2). \end{aligned}$$

The critical points of f are the 5 vertices of the pentagon, the other 5 intersection points with coordinates

$$\left(\frac{u+1}{2u}, \pm\frac{(u+1)\sqrt{2-u}}{2}\right), \quad \left(-\frac{1}{2}, \pm\frac{(u+2)\sqrt{2-u}}{2}\right), \quad (-(u+1), 0),$$

the point $(0,0)$ and the 5 points in the orbit of $(-1,0)$ under the symmetry group of the pentagon, with coordinates

$$\left(\frac{1}{2u}, \pm\frac{\sqrt{2-u}}{2}\right), \quad \left(-\frac{1}{2(u+1)}, \pm\frac{(1+u)\sqrt{2-u}}{2}\right), \quad (-1, 0).$$

On V there are the planes

$$f_i(x,y) = f_j(z,w) = 0, \quad i,j \in \{1,\ldots,5\}$$

containing the 100 nodes arising from the intersection points of the lines and the planes

$$z = \left(\frac{\cos 2\pi k}{5}\right)x - \left(\frac{\sin 2\pi k}{5}\right)y, \quad w = \left(\frac{\sin 2\pi k}{5}\right)x + \left(\frac{\cos 2\pi k}{5}\right)y, \quad k \in \{1,\ldots,4\}$$

containing the other 26 nodes so there exist projective small resolutions. Let $\tilde{V}$ be a small resolution of V. Then $\tilde{V}$ has Euler characteristic $\chi(\tilde{V}) = -74 + 126 = 52$.

All critical points appear over finite fields $\mathbb{F}_p$ where 5 is a square (i.e., $p \equiv 1, 4 \mod 5$) and where $2-u$ is a square. This condition is equivalent to the equation

$$v^4 = 5v^2 - 5$$

being solvable in $\mathbb{F}_p$, which is the case exactly for $p \equiv -1, 1 \mod 20$.

The rulings of the tangent cones at the nodes are also defined over $\mathbb{F}_p$ in this case so the Lefschetz fixed point formula gives

$$\begin{aligned}|\#\tilde{V}_p - 1 - p^3 - h^2(\tilde{V})(p+p^2)| &= |\#V_p + 126p - 1 - p^3 - h^2(\tilde{V})(p+p^2)| \\ &\le p^{3/2}h^3(\tilde{V}) = p^{3/2}(2 + 2h^2(\tilde{V}) - 52).\end{aligned}$$

Counting points over $\mathbb{F}_{41}$ we find

$$h^2(\tilde{V}) = 26, \quad h^3(\tilde{V}) = 2,$$

so $\tilde{V}$ is rigid.

If $u^2 = 1 - u$ has solutions in $\mathbb{F}_p$, but $v^4 = 5v^2 - 5$ has not, then the only critical points of f over $\mathbb{F}_p$ are

$$(u,0), \quad (-(u+1),0), \quad (-1,0), \quad (0,0),$$

which means that over $\mathbb{F}_p$ the variety V has only $2\cdot 2 + 1\cdot 1 + 1\cdot 1 = 6$ nodes. Again the rulings of the tangent cones are defined over $\mathbb{F}_p$. The Lefschetz fixed point formula gives

$$|\#\tilde{V}_p - 1 - p^3 - k(p+p^2)| = |\#V_p + 6p - 1 - p^3 - k(p+p^2)| \le 2p^{3/2}$$

with $k \in \mathbb{Z}, |k| \le h^2(\tilde{V})$. Counting points over $\mathbb{F}_{11}$ we find $k = 2$.

If $u^2 = 1 - u$ has no solutions in $\mathbb{F}_p$ (i.e., $p \not\equiv 1, 4 \mod 5$) then the only critical points of f over $\mathbb{F}_p$ are

$$(-1,0), \quad (0,0),$$

which means that over $\mathbb{F}_p$ the variety V has only 2 nodes. Again the rulings of the tangent cones are defined over $\mathbb{F}_p$. The Lefschetz fixed point formula gives

$$|\#\tilde{V}_p - 1 - p^3 - l(p+p^2)| = |\#V_p + 2p - 1 - p^3 - l(p+p^2)| \le 2p^{3/2}$$

with $l \in \mathbb{Z}, |l| \le h^2(\tilde{V})$. Counting points over $\mathbb{F}_{13}$ we find $l = 2$.

We end up with the formula

$$a_p(\tilde{V}) = \begin{cases} p^3 + 26p^2 - 100p + 1 - \#V_p, & p \equiv 1, 19 \mod 20, \\ p^3 + 2p^2 - 4p + 1 - \#V_p, & p \equiv 9, 11 \mod 20, \\ p^3 + 2p^2 + 1 - \#V_p, & p \equiv 3, 7 \mod 10, \end{cases}$$

and 2 and 5 are the primes of bad reduction. Counting points gives the following table:

p	3	7	11	13	17	29	31
$a_p(\tilde{V})$	-2	-26	-28	-12	64	90	-128

As far as calculated the $a_p(\tilde{V})$ agree with the coefficients of the weight 4 newform 50/3 ($50k4B1$) and by corollary 1.6 they agree for all $p \ne 2, 5$.

A similar construction

If we consider the family of threefolds $W_\lambda \subset \mathbb{P}^4$ given by the homogenisation of the equation

$$f(x,y) - \lambda \cdot f(z,w) = 0$$

then the general member of this family has $10 \cdot 10 = 100$ nodes arising from the intersection points of the lines as only singularities. All the nodes are contained in some of the planes

$$f_i(x,y) = f_j(z,w) = 0, \quad i,j \in \{1,\dots,5\}$$

so there exist projective small resolutions. The Euler characteristic of a small resolution is $\chi(\tilde{W}_\lambda) = 0$. I have not detected any weight 4 modular form in the L-series of W_λ.

The special member W_1 is Hirzebruch's quintic discussed above. Another very interesting special member is $W := W_{-1}$.

The function f as chosen above has critical values $\frac{1}{10}$ resp. $-\frac{1}{10}$ at $(0,0)$ resp. at the five points on the orbit of $(-1,0)$. Thus the threefold W has $10 \cdot 10 + 5 \cdot 1 + 5 \cdot 1 = 110$ nodes.

It is not clear if there exist projective small resolutions. The 100 nodes arising from the intersection points of the lines are again contained in the planes

$$f_i(x,y) = f_j(z,w) = 0, \quad i,j \in \{1,\dots,5\}.$$

Since

$$f(x,0) = \frac{1}{5}\left(x+\frac{1}{2}\right)\left(\left(x+\frac{1}{2}\right)^2 - \frac{5}{4}\right)^2,$$

is odd around $x = -\frac{1}{2}$ the other 10 nodes are contained in the lines

$$f_i(x,y) = -f_i(z,w), \quad \tilde{f}_i(x,y) = \tilde{f}_i(z,w) = 0, \quad i \in \{1,\dots,5\}$$

where $\tilde{f}_i$ is the line in the real plane through $(0,0)$ and perpendicular to f_i. But so far I have not been able to find a smooth divisor on W containing these nodes. The defect of W is $d(W) = h^2(\tilde{W}) - 1 = 17$ (see the computation of $h^2(\tilde{W})$ below). The part coming from the above planes is 16-dimensional (cf. [99]) so there is still some space left.

Now let $\tilde{W}$ be a small (maybe not projective) resolution of W. It has Euler characteristic $\chi(\tilde{W}) = -200 + 2 \cdot 110 = 20$. If $p \equiv -1, 1 \mod 20$ then all the nodes and the rulings of their tangent cones are defined over $\mathbb{F}_p$ and the Lefschetz fixed point formula gives

$$\begin{aligned}|\#\tilde{W}_p - 1 - p^3 - h^2(\tilde{W})(p+p^2)| &= |\#W_p + 110p - 1 - p^3 - h^2(\tilde{W})(p+p^2)| \\ &\le p^{3/2}h^3(\tilde{W}) = p^{3/2}(2 + 2h^2(\tilde{W}) - 20).\end{aligned}$$

Counting points over $\mathbb{F}_{179}$ and $\mathbb{F}_{199}$ we find

$$h^2(\tilde{W}) = 18, \quad h^3(\tilde{W}) = 18.$$

If $p \equiv 9, 11 \mod 20$ then only 6 nodes (and the rulings of their tangent cones) are rational over $\mathbb{F}_p$. In this case we have the estimate

$$|\#W_p + 6p - 1 - p^3 - k \cdot p(p+1)| \leq 18p^{3/2}$$

with $k \in \mathbb{Z}$, $|k| \leq 18$. Counting points over $\mathbb{F}_{349}$ we find $k = 2$.

If $p \equiv 3, 7 \mod 10$ then only 2 nodes are rational over $\mathbb{F}_p$. The discriminant of the corresponding quadratic form is 5 times a square so on the tangent cone there is a pair of rulings not defined over $\mathbb{F}_p$. In this case we have the estimate

$$|\#W_p - 2p - 1 - p^3 - l \cdot p(p+1)| \leq 18p^{3/2}$$

with $l \in \mathbb{Z}$, $|l| \leq 18$. Counting points over $\mathbb{F}_{337}$ we find $l = 0$.

We end up with the formula

$$a_p(\tilde{W}) = \begin{cases} p^3 + 18p^2 - 92p + 1 - \#W_p, & p \equiv 1, 19 \mod 20, \\ p^3 + \ \ 2p^2 - \ \ 4p + 1 - \#W_p, & p \equiv 9, 11 \mod 20, \\ p^3 \qquad\qquad + \ \ 2p + 1 - \#W_p, & p \equiv 3, 7 \quad \mod 10, \end{cases}$$

and 2 and 5 are the primes of bad reduction.

Now let b_p be the coefficients of the weight 4 newform 50/4 (50k4A1) which is a twist by $\left(\frac{5}{p}\right)$ of the weight 4 newform 50/3 (50k4B1) connected with Hirzebruch's quintic. For all good primes $p < 1000$ we find by counting points

$$\begin{cases} b_p \equiv a_p(\tilde{W}) \mod 8p, & p \equiv 1, 19 \mod 20, \\ b_p = a_p(\tilde{W}), & p \not\equiv 1, 19 \mod 20. \end{cases}$$

The following table lists the numbers $(a_p(\tilde{W}) - b_p)/p$ for $p \equiv 1, 19 \mod 20$:

p	$(a_p(\tilde{W}) - b_p)/p$	p	$(a_p(\tilde{W}) - b_p)/p$	p	$(a_p(\tilde{W}) - b_p)/p$
19	40	379	−200	641	336
41	−24	401	−24	659	120
61	16	419	−120	661	256
79	−80	421	−224	701	96
101	−144	439	−320	719	−240
139	40	461	96	739	160
179	−120	479	240	761	−24
181	16	499	160	821	336
199	160	521	−24	839	0
239	0	541	256	859	40
241	136	599	240	881	−144
281	−144	601	−104	919	−80
359	240	619	160	941	96

Let c_p be the coefficients of the weight two newform $50B1$ and d_p be the coefficients of the weight two newform $50A1$ (which is a twist of $50B1$ by $\left(\frac{5}{p}\right)$). For all good primes $p < 1000$ we find

$$a_p(\tilde{W}) \;=\; b_p + 4p \cdot (1+\chi_p) \cdot c_p \;=\; b_p + 4p \cdot (1+\chi_p) \cdot d_p$$

where χ_p is the character defined by

$$\chi_p = \begin{cases} 1, & v^4 - 5v^2 + 5 \equiv 0 \mod p \text{ has solutions} \\ -1, & \text{otherwise} \end{cases} = \begin{cases} 1, & p \equiv 1, 19 \mod 20 \\ -1, & p \not\equiv 1, 19 \mod 20 \end{cases}.$$

An explanation for this formula has still to be found.

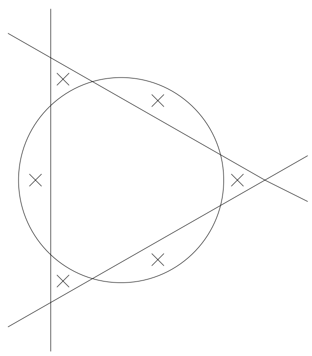
3.4 Van Geemen's and Werner's quintics

In [99] and [100] van Geemen and Werner generalized the construction from 3.3 to produce quintic hypersurfaces in $\mathbb{P}^4$ with many nodes and quintics which have Calabi–Yau resolutions with different Euler numbers. All quintics are projectivisations of affine varieties given by an equation of the form

$$F(x,y) - G(z,w) = 0$$

with F, G of degree 5.

We start with the examples constructed in [100]. Let us consider a symmetric configuration of a circle and an equilateral triangle where the radius of the circle is chosen in such a way that the critical values at the six critical points lying in the marked areas are all the same.



The equation of such a configuration can be written as

$$G(x,y) = (x+1)\left(y^2 - \frac{1}{3}(x-2)^2\right)\left(x^2 + y^2 - \frac{8}{5}\right) = 0$$

where $(0,0)$ is the center of the circle. Now let the threefold $Y \subset \mathbb{P}^4$ be defined by the homogenisation of the equation

$$G(x,y) - G(z,w) = 0.$$

Then Y has 118 ($= 9 \cdot 9 + 6 \cdot 6 + 1 \cdot 1$) nodes as only singularities. Let $\hat{Y}$ denote a big resolution of Y. Van Geemen and Werner compute

$$\chi(\hat{Y}) = 272, \quad h^2(\hat{Y}) = 137, \quad h^3(\hat{Y}) = 4.$$

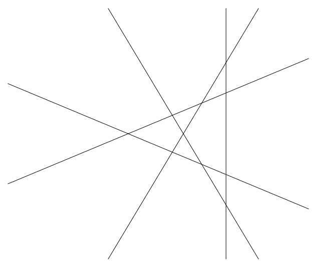
The most interesting thing is that they prove that there does not exist a triple $(\mathbb{K}, C, \phi)$ where $\mathbb{K}$ is a number field, C is a curve defined over $\mathbb{K}$ and ϕ is a non-trivial map of $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{K})$ representations

$$\phi : H^1_{\text{ét}}(\bar{C}, \mathbb{Q}_\ell(-1)) \longrightarrow H^3_{\text{ét}}(\bar{\hat{Y}}, \mathbb{Q}_\ell).$$

If there was such a triple then at each prime of $\mathbb{K}$ some eigenvalues of (every) Frobenius map on $H^3_{\text{ét}}(\bar{\hat{Y}}, \mathbb{Q}_\ell)$ would have to be equal to q times an algebraic integer, with q the norm of the prime. Van Geemen and Werner show that this is not the case for the prime 59.

This means that if the 4-dimensional Galois representation associated to $H^3_{\text{ét}}(\bar{\hat{Y}}, \mathbb{Q}_\ell)$ splits into 2-dimensional pieces over some number field then this splitting can not be caused by elliptic surfaces on $\hat{Y}$ (see 1.5.2).

Next we consider configurations of 5 lines which only meet in pairs and which are stable under $(x, y) \mapsto (x, -y)$. Van Geemen and Werner call such a configuration a *skew pentagon*.



The defining equation of a skew pentagon has 10 critical points at the intersection points of the lines and 2 critical points on the x-axis. There are additional four critical points, and we claim that their critical values are the same. Such skew pentagons are given by an equation of the form

$$H_t(x, y) = \left(x + \frac{t(t+5)}{t^2 - 5}\right)(y^2 - x^2)\left(y^2 - \frac{t^2}{5}(x+1)^2\right)$$

Now let the threefold $Z_t \subset \mathbb{P}^4$ be defined by the homogenisation of the equation

$$H_t(x, y) - H_t(z, w) = 0.$$

Then for general t the quintic Z_t has 118 ($= 10 \cdot 10 + 4 \cdot 4 + 1 \cdot 1 + 1 \cdot 1$) nodes as only singularities. Let $\hat{Z}_t$ denote a big resolution of Z_t. Van Geemen and Werner compute

$$\chi(\hat{Z}_t) = 272, \quad h^2(\hat{Z}_t) = 138, \quad h^3(\hat{Z}_t) = 6.$$

For $\tilde{t} = -3 \pm 2\sqrt{5}$ the critical values at the two critical points on the x-axis are the same. The corresponding skew pentagon can also be given by the equation

$$(x-2)(y^4 - y^2(2x^2 - 2x + 1) + \frac{1}{5}(x^2 + x - 1)^2).$$

The threefold $Z_{\tilde{t}}$ has 120 ($= 10 \cdot 10 + 4 \cdot 4 + 2 \cdot 2$) nodes as only singularities. Van Geemen and Werner compute

$$\chi(\hat{Z}_{\tilde{t}}) = 280, \quad h^2(\hat{Z}_{\tilde{t}}) = 141, \quad h^3(\hat{Z}_{\tilde{t}}) = 4.$$

Again they prove that there does not exist a triple $(\mathbb{K}, C, \phi)$ where $\mathbb{K}$ is a number field, C is a curve defined over $\mathbb{K}$ and ϕ is a non-trivial map of $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{K})$ representations

$$\phi : H^1_{\text{ét}}(\bar{C}, \mathbb{Q}_\ell(-1)) \longrightarrow H^3_{\text{ét}}(\bar{\hat{Z}}_{\tilde{t}} \mathbb{Q}_\ell).$$

Now let the threefold X_{gh} be defined by the homogenisation of the equation

$$G(x, y) - cH_t(z, w)$$

where c is a constant chosen such that the critical values agree at the six critical points of G and the four critical points of H_t mentioned above. Then X_{gh} has 114 ($= 9 \cdot 10 + 6 \cdot 4$) nodes as only singularities. Let $\hat{X}_{gh}$ denote a big resolution of X_{gh}.Van Geemen and Werner compute

$$\chi(\hat{X}_{gh}) = 256, \quad h^2(\hat{X}_{gh}) = 131, \quad h^3(\hat{X}_{gh}) = 8.$$

Now let the threefold V_t be defined by the homogenisation of the equation

$$f(x, y) - c(t)H_t(z, w)$$

where $f(x, y)$ is the equation for the regular pentagon from 3.3 and $c(t)$ is chosen so that V_t has in general 120 ($= 10 \cdot 10 + 5 \cdot 4$) nodes as only singularities. Let $\hat{V}_t$ denote a big resolution of V_t. Van Geemen and Werner compute

$$\chi(\hat{V}_t) = 280, \quad h^2(\hat{V}_t) = 141, \quad h^3(\hat{V}_t) = 4.$$

For $t = -5 - 2\sqrt{5}$ we obtain again the Hirzebruch quintic with 126 nodes. So this quintic is a special element of a family $\{V_t\}$, such that V_t has in general 120 nodes.

Finally let the threefold W be defined by the homogenisation of the equation

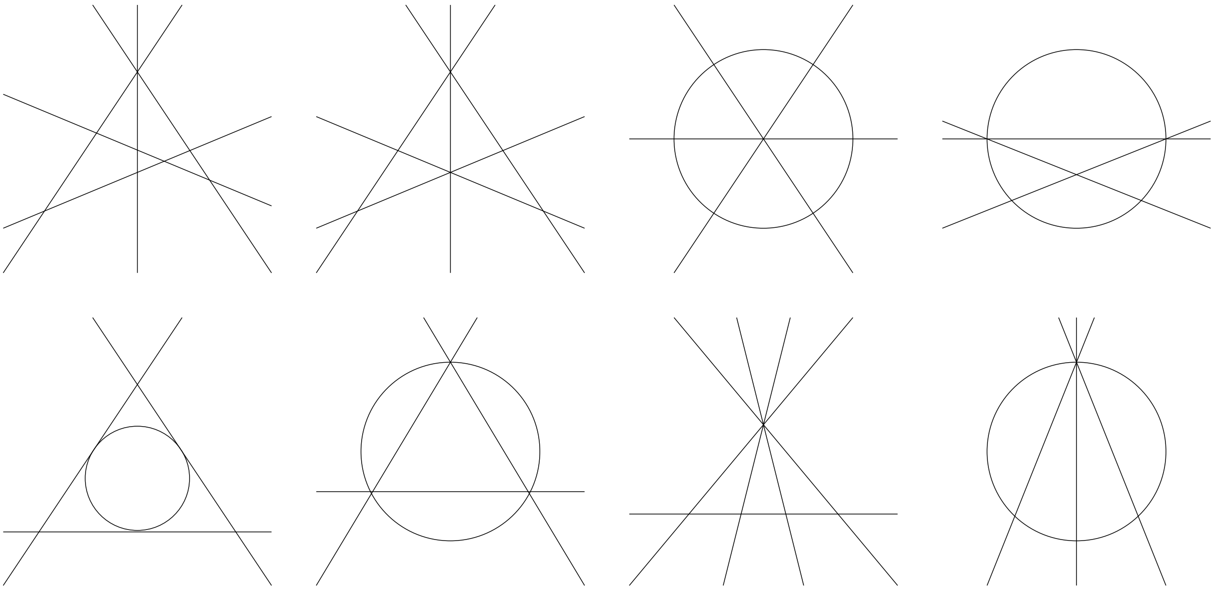
$$f(x, y) - \frac{625}{1296} G(z, w).$$

The factor $\frac{625}{1296} = (\frac{5}{6})^4$ is chosen so that W has also $120 = (9 \cdot 10 + 5 \cdot 6)$ nodes as only singularities (but this example is not mentioned in [100]). Let $\hat{W}$ denote a big resolution of W. We can compute

$$\chi(\hat{W}) = 280, \quad h^2(\hat{W}) = 141, \quad h^3(\hat{W}) = 4.$$

Note that in all of the above examples it is not clear if there exist projective small resolutions. The nodes coming from intersection points of lines are always contained in certain planes on the quintic but it is difficult to identify smooth divisors through the other nodes.

In [99], section 2, different configurations of lines and conics are allowed, as sketched below.



The configurations containing intersection points of three or more lines or tangency of a line and a conic lead to quintic threefolds with higher isolated singularities.

I have made some numerical experiments with the quintics constructed in this section but I have not detected any weight four modular form connected with their middle cohomology. The results of section 3.5 suggest that the situation may indeed be more complicated.

Note that the examples listed in [99] have a high number of deformations. By closer examination of critical points not coming from intersection of lines and conics we could add some nodes and so reduce the number of deformations.

3.5 Consani's and Scholten's quintic

In [23] Consani and Scholten investigated the L-series of a quintic threefold closely related to one of van Geemen's and Werner's examples. Consider the polynomial

$$P_5(x, y) = (x^5 + y^5) - 5xy(x^2 + y^2) + 5xy(x + y) + 5(x^2 + y^2) - 5(x + y)$$

and the quintic X defined by the homogenisation of the equation

$$P_5(x, y) - P_5(z, w).$$

Then X has 120 nodes as only singularities. It is isomorphic over $\mathbb{Q}[i]$ to van Geemen's and Werner's threefold $Z_{\tilde{t}}$ from section 3.4 (see [23] for the isomorphism). Let $\hat{X}$ denote a big resolution of X. We have

$$\chi(\hat{X}) = 280, \quad h^2(\hat{X}) = 141, \quad h^3(\hat{X}) = 4.$$

The threefold $\hat{X}$ has good reduction outside the set $\{2,3,5\}$. Consani and Scholten prove that the Galois representation ρ associated to $H^3_{\text{ét}}(\bar{\hat{X}}, \mathbb{Q}_\ell(\sqrt{5}))$ splits into two two-dimensional pieces. Furthermore they construct a Hilbert modular newform f of weight $(2,4)$ and conductor 30 on a finite extension E_λ of $\mathbb{Q}_l$ with ring of integers $\mathcal{O}_\lambda$ and an associated 2-dimensional λ-adic Galois representation $\sigma_{f,\lambda} \longrightarrow \mathrm{GL}_2(\mathcal{O}_\lambda)$. They give numerical evidence for the fact that the Galois representation $\sigma_{f,\lambda}$ appears as a two-dimensional piece of ρ (a complete proof would be possible with more computer power). This perfectly agrees with the results of van Geemen and Werner on $Z_{\tilde{t}}$ from section 3.4.

3.6 Van Straten's Σ_6-symmetric quintics

Let

$$S_i \quad := \quad S_i(x_0, x_1, \ldots, x_5) \quad := \sum_{0 \le j_1 < \cdots < j_i \le 5} x_{j_1} x_{j_2} \cdots x_{j_i}$$

be the i-th *elementary-symmetric function* in six variables x_j. The equations

$$\begin{aligned} S_1 &= 0, \\ \alpha S_5 + \beta S_2 S_3 &= 0 \quad \text{with } (\alpha : \beta) \in \mathbb{P}^1 \end{aligned}$$

define the pencil $\{\mathcal{M}_{(\alpha:\beta)}\}$ of quintics in $\mathbb{P}^4$ that are invariant under the operation of the symmetric group Σ_6 by permutation of coordinates.

These varieties were first investigated over $\mathbb{C}$ by van Straten in [101]. The general member of the pencil has exactly 100 nodes as its only singularities, namely the points on the Σ_6-orbit of the point $(1:1:1:-1:-1:-1)$ (the 10 *Segre nodes*; they are also the singularities of the *Segre cubic* $S_1 = S_3 = 0$) and the points on the Σ_6-orbit of the point $(1:1:-1:-1:z:-z)$ where z is a solution of $\beta z^2 + \alpha + 2\beta = 0$ (the 90 *moving nodes*).

For 6 choices of $(\alpha : \beta) \in \mathbb{P}^1$ the singular locus of $\mathcal{M}_{(\alpha:\beta)}$ is different:

$(\alpha : \beta)$	singular locus
(1:1)	100 nodes of the general variety and 30 *extra nodes* on the Σ_6-orbit of $(1:1:1:1:\sqrt{-3}-2:-\sqrt{-3}-2)$. $\mathcal{M}_{(1:1)}$ is the quintic with the highest number of nodes that is known.
(25:1)	100 nodes of the general variety and 6 additional nodes on the Σ_6-orbit of $(1:1:1:1:1:-5)$.
(-3:1)	10 singularities of type $(3,3,3,3)$ (*Del Pezzo nodes*).
(-2:1)	10 nodes (the *Segre nodes*) and 15 lines given by the Σ_6-orbit of $\{(x:x:y:y:z:z), x+y+z=0\}$.
(1:0)	10 nodes (the *Segre nodes*) and 20 lines given by the Σ_6-orbit of $\{(0:0:0:x:y:z), x+y+z=0\}$. $\mathcal{M}_{(1:0)}$ is known as *Barth-Nieto quintic* and was investigated in [6] and [50]; see also 3.7.
(0:1)	The surface $S_2 = S_3 = 0$.

Each $\mathcal{M}_{(\alpha:\beta)}$ contains the 15 so called *Segre planes* given by the Σ_6-orbit of

$$x_0 + x_1 = x_2 + x_3 = x_4 + x_5 = 0.$$

The *Segre nodes* (and the singularities of $\mathcal{M}_{(-3:1)}$) and the *moving nodes* are contained in these planes, so for general $(a : b)$ there exist projective small resolutions $\tilde{\mathcal{M}}_{(a:b)}$ of $\mathcal{M}_{(a:b)}$. Furthermore we can prove (with the help of point counting arguments)

$$\chi(\tilde{\mathcal{M}}_{(a:b)}) = 0, \quad h^2(\tilde{\mathcal{M}}_{(a:b)}) = 15, \quad h^3(\tilde{\mathcal{M}}_{(a:b)}) = 32.$$

The modularity of most of the special members has been previously discussed in [69] (cf. also [68]).

The L-series of $\mathcal{M}_{(1:1)}$

The coordinates of the singularities of $\mathcal{M}_{(1:1)}$ over $\mathbb{C}$ are defined over $\mathbb{Q}[\sqrt{-3}]$, so it is reasonable to assume that the situation over $\mathbb{F}_p$ depends on the existence of $\sqrt{-3}$. In fact, all nodes and the rulings of their tangent cones are rational over $\mathbb{F}_p$ for $p \geq 5$ if $\sqrt{-3}$ exists which means that $p \equiv 1 \mod 6$. For $p \equiv 5 \mod 6$ only the 10 *Segre nodes* (and the rulings of their tangent cones) are rational over $\mathbb{F}_p$.

On $\mathcal{M}_{(1:1)}$ there are 40 *extra planes* given by the Σ_6-orbit of

$$\begin{aligned} x_0 + x_1 + x_2 + x_3 + x_4 + x_5 = 0, \\ x_0 + \omega x_1 + \omega^2 x_2 = 0, \\ x_3 + \omega x_4 + \omega^2 x_5 = 0, \end{aligned}$$

where $\omega = \frac{-1+\sqrt{-3}}{2}$. The *extra nodes* are contained in these planes so there exist projective small resolutions.

Let $\tilde{\mathcal{M}}_{(1:1)}$ be a small resolution of $\mathcal{M}_{(1:1)}$. Then $\tilde{\mathcal{M}}_{(1:1)}$ has Euler characteristic

$$\chi(\tilde{\mathcal{M}}_{(1:1)}) = -200 + 2 \cdot 130 = 60.$$

The primes of bad reduction are 2 and 3. In the case of $p \equiv 1 \mod 6$ all singularities of $\mathcal{M}_{(1:1)}$ and the rulings of their tangent cones are rational over $\mathbb{F}_p$. The Lefschetz fixed point formula gives

$$\begin{aligned} &|\#\tilde{\mathcal{M}}_{(1:1),p} - 1 - p^3 - h^2(\tilde{\mathcal{M}}_{(1:1)}) \cdot p(p+1)| \\ &\quad = |\#\mathcal{M}_{(1:1),p} + 130p - 1 - p^3 - h^2(\tilde{\mathcal{M}}_{(1:1)}) \cdot p(p+1)| \\ &\quad \leq p^{3/2} h^3(\tilde{\mathcal{M}}_{(1:1)}) = p^{3/2}(2 + 2h^2(\tilde{\mathcal{M}}_{(1:1)}) - 60). \end{aligned}$$

Counting points over $\mathbb{F}_{13}$ we find

$$h^2(\tilde{\mathcal{M}}_{(1:1)}) = 30, \quad h^3(\tilde{\mathcal{M}}_{(1:1)}) = 2,$$

so $\tilde{\mathcal{M}}_{(1:1)}$ is rigid.

If $p \equiv 5 \mod 6$ then only the 10 Segre nodes (and the rulings of their tangent cones) are rational over $\mathbb{F}_p$ and we have the estimate

$$|\#\tilde{\mathcal{M}}_{(1:1),p} - 1 - p^3 - k \cdot p(p+1)| = |\#\mathcal{M}_{(1:1),p} + 10p - 1 - p^3 - k \cdot p(p+1)| \leq 2p^{3/2}$$

for some $k \in \mathbb{Z}, |k| \leq h^2(\tilde{\mathcal{M}}_{(1:1)}) = 30$. Counting points over $\mathbb{F}_{11}$ gives $k = 10$.

We end up with the formula

$$a_p(\tilde{\mathcal{M}}_{(1:1)}) = \begin{cases} p^3 + 30p^2 - 100p + 1 - \#\mathcal{M}_{(1:1),p}, & p \equiv 1 \mod 6, \\ p^3 + 10p^2 \qquad\qquad + 1 - \#\mathcal{M}_{(1:1),p}, & p \equiv 5 \mod 6. \end{cases}$$

Counting points we detect that for all primes $5 \leq p \leq 97$ the $a_p(\tilde{\mathcal{M}}_{(1:1)})$ agree with the coefficients of the weight four newform 6/1 ($6k4A1$), and by corollary 1.6 they agree for all $p \geq 5$.

The L-series of $\mathcal{M}_{(25:1)}$

To get an idea what the primes of bad reduction are we can look at the parameter $(25 : 1)$. Modulo 2, 3, 5 and 7 it becomes $(1 : 1)$, $(1 : 1)$, $(0 : 1)$ and $(-3 : 1)$ which is in any case the parameter of a special member of the pencil of quintics.

Let $\tilde{\mathcal{M}}_{(25:1)}$ be a small resolution of $\mathcal{M}_{(25:1)}$. Then $\tilde{\mathcal{M}}_{(25:1)}$ has Euler characteristic

$$\chi(\tilde{\mathcal{M}}_{(25:1)}) = -200 + 2 \cdot 106 = 12.$$

It is not clear if there exist projective small resolutions.

The *Segre nodes* and the rulings of their tangent cones are always rational over $\mathbb{F}_p$, the *moving nodes* and the rulings of their tangent cones only for $p \equiv 1 \mod 6$. The six additional nodes are always rational over $\mathbb{F}_p$ but the rulings of their tangent cones only if $\sqrt{5}$ exists, i.e., $p \equiv 1, 4 \mod 5$. Thus for $p \equiv 1, 4 \mod 15$ the Lefschetz fixed point formula gives

$$\begin{aligned} &|\#\tilde{\mathcal{M}}_{(25:1),p} - 1 - p^3 - h^2(\tilde{\mathcal{M}}_{(25:1)}) \cdot p(p+1)| \\ &\quad = |\#\mathcal{M}_{(25:1),p} + 106p - 1 - p^3 - h^2(\tilde{\mathcal{M}}_{(25:1)}) \cdot p(p+1)| \\ &\quad \leq p^{3/2} h^3(\tilde{\mathcal{M}}_{(25:1)}) = p^{3/2}(2 + 2h^2(\tilde{\mathcal{M}}_{(25:1)}) - 12). \end{aligned}$$

Counting points over $\mathbb{F}_{31}$ and $\mathbb{F}_{139}$ gives

$$h^2(\tilde{\mathcal{M}}_{(25:1)}) = 15, \quad h^3(\tilde{\mathcal{M}}_{(25:1)}) = 20.$$

For $p \not\equiv 1, 4 \mod 15$ we have the estimates

$$\begin{aligned} |\#\mathcal{M}_{(25:1),p} + 94p - 1 - p^3 - k \cdot p(p+1)| &\leq 20p^{3/2}, \quad p \equiv 7, 13 \mod 15, \\ |\#\mathcal{M}_{(25:1),p} + 16p - 1 - p^3 - l \cdot p(p+1)| &\leq 20p^{3/2}, \quad p \equiv 11, 14 \mod 15, \\ |\#\mathcal{M}_{(25:1),p} + 4p - 1 - p^3 - m \cdot p(p+1)| &\leq 20p^{3/2}, \quad p \equiv 2, 8 \mod 15, \end{aligned}$$

with $k, l, m \in \mathbb{Z}$, $|k|, |l|, |m| \leq 15$. Counting points over $\mathbb{F}_{43}$, $\mathbb{F}_{149}$ and $\mathbb{F}_{107}$ gives $k = l = m = 15$. We end up with the formula

$$a_p(\tilde{\mathcal{M}}_{(25:1)}) = \begin{cases} p^3 + 15p^2 - 91p + 1 - \#\mathcal{M}_{(25:1),p}, & p \equiv 1,\ 4 \mod 15, \\ p^3 + 15p^2 - 79p + 1 - \#\mathcal{M}_{(25:1),p}, & p \equiv 7, 13 \mod 15, \\ p^3 + 15p^2 - \quad p + 1 - \#\mathcal{M}_{(25:1),p}, & p \equiv 11, 14 \mod 15, \\ p^3 + 15p^2 + 11p + 1 - \#\mathcal{M}_{(25:1),p}, & p \equiv 2,\ 8 \mod 15. \end{cases}$$

For all primes $11 \leq p \leq 149$ we find

$$a_p(\tilde{\mathcal{M}}_{(25:1)}) = b_p + 9 \cdot p \cdot c_p$$

where b_p are the coefficients of the weight four newform 210/9 ($210k4F1$) and c_p the coefficients of the weight two newform $210C1$.

The L-series of $\mathcal{M}_{(-3:1)}$

To get an idea what the primes of bad reduction are we can look at the parameter $(-3 : 1)$. Modulo 2, 3 and 7 it becomes $(1 : 1)$, $(0 : 1)$ and $(25 : 1)$ which is in any case the parameter of a special member of the pencil of quintics.

Let $\tilde{\mathcal{M}}_{(-3:1)}$ be a big resolution of $\mathcal{M}_{(-3:1)}$ (which is Calabi–Yau, cf. 1.6.3). Then $\tilde{\mathcal{M}}_{(-3:1)}$ has Euler characteristic

$$\chi(\tilde{\mathcal{M}}_{(-3:1)}) = -200 + 10 \cdot 16 + 10 \cdot (9 - 1) = 40.$$

The tangent cone at the singularities is locally isomorphic to the cone over the smooth cubic surface given by

$$\begin{aligned} 0 = {} & x^2y + xy^2 + x^2z + xz^2 \\ & + y^2z + yz^2 + y^2w + yw^2 \\ & + z^2w + zw^2 + 2xyz + 2yzw \\ & + 3x^2w + 3xw^2 + 4xyw + 4xzw. \end{aligned}$$

Over $\mathbb{F}_p$ for all primes p that I checked this surface contains

$$\begin{cases} p^2 + 7p + 1, & p \equiv 1 \mod 3, \\ p^2 + 5p + 1, & p \equiv 2 \mod 3 \end{cases}$$

points. It is isomorphic to $\mathbb{P}^2$ blown up in six points, and if someone identifies a suitable configuration of six points then four of them should be defined over $\mathbb{Q}$ and two over $\mathbb{Q}[\xi_3]$ for a third root of unity ξ_3. Now for $p \equiv 1 \mod 3$ the Lefschetz fixed point formula gives

$$\begin{aligned} & |\#\tilde{\mathcal{M}}_{(-3:1),p} - 1 - p^3 - h^2(\tilde{\mathcal{M}}_{(-3:1)}) \cdot p(p+1)| \\ & \quad = |\#\mathcal{M}_{(-3:1),p} + 10(p^2 + 7p) - 1 - p^3 - h^2(\tilde{\mathcal{M}}_{(-3:1)}) \cdot p(p+1)| \\ & \quad \leq p^{3/2} h^3(\tilde{\mathcal{M}}_{(-3:1)}) = p^{3/2}(2 + 2h^2(\tilde{\mathcal{M}}_{(-3:1)}) - 40). \end{aligned}$$

Counting points over $\mathbb{F}_{73}$ and $\mathbb{F}_{97}$ gives

$$h^2(\tilde{\mathcal{M}}_{(-3:1)}) = 25, \quad h^3(\tilde{\mathcal{M}}_{(-3:1)}) = 12.$$

For $p \equiv 2 \mod 3$ we have the estimate

$$\begin{aligned} &|\#\tilde{\mathcal{M}}_{(-3:1),p} - 1 - p^3 - k \cdot p(p+1)| \\ &\quad = |\#\mathcal{M}_{(-3:1),p} + 10(p^2+5p) - 1 - p^3 - k \cdot p(p+1)| \leq 12p^{3/2} \end{aligned}$$

for some $k \in \mathbb{Z}$, $|k| \leq 25$. Counting points over $\mathbb{F}_{83}$ gives $k = 25$. We end up with the formula

$$a_p(\tilde{\mathcal{M}}_{(-3:1)}) = \begin{cases} p^3 + 15p^2 - 45p + 1 - \#\mathcal{M}_{(-3:1),p}, & p \equiv 1 \mod 3, \\ p^3 + 15p^2 - 25p + 1 - \#\mathcal{M}_{(-3:1),p}, & p \equiv 2 \mod 3. \end{cases}$$

For $p = 5$ and all primes $11 \leq p \leq 149$ we find

$$a_p(\tilde{\mathcal{M}}_{(-3:1)}) = b_p + 5 \cdot p \cdot c_p$$

where b_p are the coefficients of the weight four newform 21/1 ($21k4B1$) and c_p the coefficients of the weight two newform $21A1$.

The L-series of $\mathcal{M}_{(-2:1)}$

I have not studied the L-series of $\mathcal{M}_{(-2:1)}$ but there is numerical evidence that it is also connected to the weight four newform 6/1 ($6k4A1$). Let b_p the coefficients of this newform. For all primes $5 \leq p \leq 151$ we have the formula

$$b_p = p^3 + 15p^2 - 40p + 1 - \#\mathcal{M}_{(-2:1),p}$$

(cf. the tables in [68]), suggesting that $\mathcal{M}_{(-2:1)}$ has a rigid Calabi–Yau desingularization $\tilde{\mathcal{M}}_{(-2:1)}$ and that $a_p(\tilde{\mathcal{M}}_{(-2:1)}) = b_p$. A proof would require a closer look at the resolution of the singularities of $\mathcal{M}_{(-2:1)}$.

3.7 The Barth-Nieto quintic and its double cover

The Barth-Nieto quintic is the variety given by

$$N = \left\{ \sum_{i=0}^{5} x_i = \sum_{i=0}^{5} \frac{1}{x_i} = 0 \right\} \subset \mathbb{P}^5,$$

so we have $N = \mathcal{M}_{1:0}$ (cf. 3.6). It was studied by Barth and Nieto in [6]. We will also consider the inverse image $\tilde{N}$ of N under the double covering of $\mathbb{P}^5$ branched along the union of the 6 hyperplanes $\{x_k = 0\}$.

In [50] Hulek, Spandaw, van Geemen and van Straten proved that the varieties N and $\tilde{N}$ have smooth Calabi–Yau models, denoted by Y and Z respectively. They also determined their L-series. The L-series of Y was also determined independently in [68]. We will sketch the results.

There are smooth Calabi–Yau models Y resp. Z of N resp. $\tilde{N}$. Note that there exist projective small resolutions of the nodes (cf. 3.6). We have

$$\chi(Y) = 100, \quad h^{1,1}(Y) = 50, \quad h^{2,1}(Y) = 0$$

and

$$\chi(Z) = 80, \quad h^{1,1}(Z) = 40, \quad h^{2,1}(Z) = 0,$$

so both Y and Z are rigid. Using theorem 1.5 we can prove

$$a_p(Y) = a_p(Z) = b_p$$

for all primes $p \geq 5$, where b_p are the coefficients of the weight four newform 6/1 ($6k4A1$). This confirms the Tate conjecture for Y and Z which predicts that since there is a $2:1$ map $Z \dashrightarrow Y$ the L-series of the two varieties should be the same. In 6.1.2 we will give correspondences between Y and other threefolds with the same L-series.

Chapter 4

Double octics

4.1 Cynk's octic arrangements

Let $X \xrightarrow{\pi} \mathbb{P}^3$ be a double covering of $\mathbb{P}^3$ branched along an octic surface D. We will regard X as a hypersurface in the weighted projective space $\mathbb{P}^4(1,1,1,1,4)$. If D is smooth then X is a (smooth) Calabi–Yau threefold, if D is singular then X is also singular, and the singularities of X are in one–to–one correspondence with the singularities of D. The singularities of X can be resolved by a sequence of blow–ups of $\mathbb{P}^3$, more precisely there is a sequence of blow–ups with smooth centers $\sigma : Y \longrightarrow \mathbb{P}^3$, and a smooth, reduced divisor D^* such that $\sigma(D^*) = D$ and D^* is an even element of the Picard group $\mathrm{Pic}(Y)$ of Y. Then the double covering $\tilde{X}$ of Y branched along D^* is a smooth model of X (for details see, f.i., [41]). If X has only certain types of singularities then $\tilde{X}$ is a smooth Calabi–Yau threefold.

The study of such double coverings was initiated by C.H. Clemens in [22]. He investigated branch loci with ordinary nodes as only singularities. Other authors continued these investigations (cf. [25], [40], [106]). S. Cynk and his co-authors (cf. [26], [27], [29], [30]) extended the class of surfaces and studied new aspects. We are going to report about the results. Modularity of some examples was investigated in [28] and we will extend this work.

4.1 Definition
Let $D \subset \mathbb{P}^3$ be a surface. We call D an arrangement *if it is a sum of irreducible surfaces $D_1, \ldots, D_r$ with only isolated singular points satisfying the following conditions:*

1. *For any $i \neq j$ the surfaces D_i and D_j intersect transversally along a smooth irreducible curve $C_{i,j}$ or they are disjoint,*
2. *The curves $C_{i,j}$ and $C_{k,l}$ either coincide, are disjoint or intersect transversally.*

A singular point of D_i we call an isolated singular point of the arrangement. *A point $P \in D$ which belongs to p of the surfaces $D_1, \ldots, D_r$ we call an* arrangement p–fold point. *We say that* *an irreducible curve $C \subset D$ is a q–*fold curve *if exactly q of the surfaces $D_1, \dots, D_r$ pass through it.*

We will use the following numerical data for an arrangement:

- d_i *The degree of D_i,*
- p_q^i *The number of arrangement q–fold points lying on exactly i triple curves,*
- l_3 *The number of triple lines,*
- m_q *The number of isolated q–fold points.*

If D has degree 8 then we call it an octic arrangement.

Away from the isolated singularities an arrangement looks locally like a sum of planes. Note also that for an octic arrangement with triple curves there are only two possibilities: Either there is one triple elliptic curve and no other triple curves or there are only triple lines. The octic arrangements with triple elliptic curves were classified in [29] (there are four cases).

4.2 Theorem ([27])
If an octic arrangement D contains only

- *double and triple curves,*
- *arrangement q–fold points, $q = 2, 3, 4, 5$,*
- *isolated q–fold points, $q = 2, 4, 5$,*

then the double covering of $\mathbb{P}^3$ branched along D has a non–singular model $\tilde{X}$ which is a Calabi–Yau threefold. Moreover if D contains no triple elliptic curves then

$$\begin{aligned}\chi(\tilde{X}) = 8 &- \sum_i (d_i^3 - 4d_i^2 + 6d_i) \\ &+ 2\sum_{i<j}(4 - d_i - d_j)d_i d_j - \sum_{i<j<k} d_i d_j d_k \\ &+ 4p_4^0 + 3p_4^1 + 16p_5^0 + 18p_5^1 + 20p_5^2 + l_3 + 2m_2 + 36m_4 + 56m_5.\end{aligned}$$

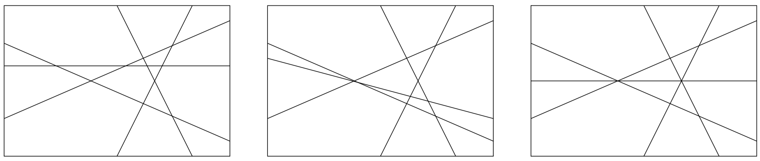
The ordinary double points (nodes) play a special role in the above theorem. They are resolved by a small resolution (on the double covering). As a consequence $\tilde{X}$ can not be in general realized as a double covering, and it is even non–projective (or equivalently non–kähler). In this case it is easier to study a large resolution of X which is a blow–up of the small resolution at the exceptional lines. These matters have already been discussed in 1.6.

For an octic arrangement there can be up to two triple curves going through a 5–fold point and up to one triple curve going through a 4–fold point.

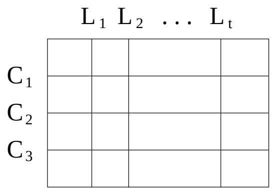
The resolution of singularities (and with that the proof of the above theorem) is done in the following way:

1. **Blow–up of isolated singular points:** For points of even multiplicity we take the strict transform of the branch divisor as the new branch divisor, for points of odd multiplicity we take the strict transform of the branch divisor plus the exceptional divisor as the new branch divisor. In the latter case we get a new double curve (projectivisation of the normal cone).

2. **Blow–up of arrangement 5–fold points:** We take the strict transform of the branch divisor plus the exceptional divisor as the new branch divisor. This introduces five double lines (lying on the exceptional divisor) and a p_4^1 point for each triple curve going through the point. Here is a picture of the exceptional divisor $\mathbb{P}^2$ in the three cases of 0, 1 or 2 triple curves:



3. **Blow–up of triple curves:** We take the strict transform of the branch divisor plus the exceptional divisor as the new branch divisor. We get three copies C_1, C_2, C_3 of the blown–up curve C as double curves. Moreover every 4–fold point lying on that curve gives rise to a double line. Here is a picture of the exceptional divisor $C \times \mathbb{P}^1$ for t 4–fold points on the blown–up curve:



4. **Blow–up of arrangement 4–fold points:** We take the strict transform of the branch divisor as the new branch divisor (no new singularities).

5. **Blow–up of double curves:** We take the strict transform of the branch divisor as the new branch divisor (no other singularities). Observe that arrangement triple points disappear.

The next important thing to compute are the Hodge numbers of $\tilde{X}$. In principle this can be done by counting points, using van Geemen's method. In the present case it is also possible to compute $h^{1,2}(\tilde{X})$ (and so $h^{1,1}(\tilde{X})$) with computer algebra methods. The advantage is that we do not need additional information on the action of Frobenius on $H^2_{\text{ét}}(\tilde{X})$. The algorithm has been implemented in SINGULAR ([45]) by S. Cynk.

4.3 Lemma
Let D be an octic arrangement as in theorem 4.2 without triple curves.

1. $h^2(Y) = \operatorname{rk}\operatorname{Pic}(Y) = 1 + \binom{r}{2} + p_4^0 + p_4^1 + 6p_5^0 + 7p_5^1 + 8p_5^2 + l_3 + m_4 + 2m_5$,
2. $h^{1,2}(Y) = 6m_5 + \frac{1}{2}\sum_{i<j} d_i d_j (d_i + d_j - 4) + \binom{r}{2}$,
3. $h^1(\mathcal{T}_Y(\log D^*)) = \dim_{\mathbb{C}}(I_{\text{eq}}/Jf)_8$,
4. $h^{1,2}(\tilde{X}) = h^{1,2}(Y) + h^1(\mathcal{T}_Y(\log D^*))$,

where I_{eq} is the equisingular ideal of D defined by

$$I_{\text{eq}} = \bigcap_C \left(I_C^{mult_C D} + Jf \right),$$

the intersection being taken over all multiple curves and points of the arrangement D, and

$$Jf := \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \frac{\partial f}{\partial t} \right)$$

is the Jacobian ideal of D.

4.2 Arrangements of eight planes

Now we are ready to look for nice examples. First we are going to investigate sums of eight planes. These are always octic arrangements in the above sense. In [28] we already gave 88 examples. The aim now is to give an at least heuristically complete list. In [28] we also noticed that the numbers p_4^0, p_4^1, p_5^0, p_5^1, p_5^2 and l_3 are not sufficient to determine the geometry of the arrangement; they even do not determine the Hodge numbers. To refine the classification we can look at all subarrangements of six planes:

4.4 Lemma
Up to projective equivalence there are exactly 10 possible arrangements of six planes (in arbitrary characteristic $\neq 2, 3$) containing no 6–fold points and no fourfold lines. We list the numerical data and a sample equation:

no.	l_3	p_5^2	p_5^1	p_5^0	p_4^1	p_4^0	p_3	*equation:* $0 = \cdots$
1	2	1	0	0	2	0	4	$xyzt(x+y)(x+z)$
2	2	0	0	0	6	0	0	$xyzt(x+y)(z+t)$
3	1	0	1	0	1	0	7	$xyzt(x+y)(x-y+z)$
4	1	0	0	0	3	0	10	$xyzt(x+y)(x+z+t)$
5	1	0	0	0	3	1	6	$xyzt(x+y)(x+y+z+t)$
6	0	0	0	1	0	0	10	$xyzt(x+y+z)(x-y+2z)$
7	0	0	0	0	0	0	20	$xyzt(x+y+z+t)(x-y+2z-2t)$
8	0	0	0	0	0	1	16	$xyzt(x+y+z)(x+2y-z+t)$
9	0	0	0	0	0	2	12	$xyzt(x+y+z+t)(x+y-z-t)$
0	0	0	0	0	0	3	8	$xyzt(x+y+z)(x+y+t)$

Proof:
There can not be three triple lines since this would result in a 6–fold point.

Let there be two triple lines. If they meet in a p_5^2 point then the plane that does not contain a triple line intersects the triple lines in two p_4^1 points (and there are no other 4–fold or 5–fold points). If the triple lines do not meet then each plane intersects the triple line which is not contained in that plane in a p_4^1 point.

Let there be only one triple line. If there is a p_5^1 point then the plane which does not contain that point intersects the triple line in a p_4^1 point (and there are no other 4–fold or 5–fold points). If there is no p_5^1 point then the three planes which do not contain the triple line intersect that line in three p_4^1 points. These three planes also intersect in a point which either is contained in one of the other planes or not.

Let there be no triple lines. If there is a p_5^0 point then there are no other 4–fold or 5–fold points. If there is no p_5^0 point then there can be up to three p_0^4 points (if there are two p_0^4 points then they lie on a double line; if there are three p_0^4 points then each two lie on a double line, the arrangement is a cube). □

The table in appendix A lists 450 examples of arrangements of eight planes defined over $\mathbb{Q}$ that have been found with a computer search. We give the numerical data of the arrangements, the Hodge numbers $h^{1,1} = h^{1,1}(\tilde{X})$ and $h^{1,2} = h^{1,2}(\tilde{X})$ and the Euler number $\chi = \chi(\tilde{X})$ of the Calabi–Yau resolution $\tilde{X}$ of the double covering and the list of types of subarrangements of six planes (in lexicographical order with respect to some numbering of the planes, i.e., from $D_0 \cup \cdots \cup D_5$ to $D_2 \cup \cdots \cup D_7$).

The computer search was organized in the following way: One by one, I fixed one of the ten possible subarrangements of six planes (with equations as in the table in lemma 4.4) and added all sets of two planes with bounded absolute value of integral coefficients. I determined the numerical data of the arrangement and all subarrangements of six planes and compared the result with the existing list (this includes of course considering all possible permutations of the eight planes).

At first I bounded the absolute value of the coefficients of the additional planes by two. This way I already found 447 examples. The remaining 3 examples had some coefficients ± 3 (no. 275, 276, 385). I ran the program again, this time bounding the absolute value of the coefficients by six, and did not find any new examples (this took more than a week).

Note that it is not clear that two arrangements with the same numerical data and the same (ordered) set of subarrangements have the same geometry (but it seems plausible). I checked it for fun in two simple cases:

There are exactly 3 arrangements with two p_4^0 points and no other multiple points: the two points can lie on two, one or zero common planes of the arrangement. These arrangements correspond to no. 446, no. 447 and no. 448 in the list. Note that to distinguish no. 447 and no. 448 we really need to look at the *ordered* set of subarrangements.

There are exactly 7 arrangements with three p_4^0 points and no other multiple points: Denote the three p_4^0 points of such an arrangement with A, B and C and the planes with $D_0, \ldots, D_8$. Note that it is impossible that no two of the points lie in a common plane of the arrangement.

Now assume that the three points do not lie in a common plane. Then without loss of generality $A \in D_0$ and $B \in D_0$. Up to permutation there are four cases (note that there must be two points lying on a double line):

- $A = D_0 \cap D_1 \cap D_2 \cap D_3$, $B = D_0 \cap D_1 \cap D_4 \cap D_5$, $C = D_2 \cap D_4 \cap D_6 \cap D_7$ (A and B lie on a double line, A and C lie on a common plane, B and C lie on a common plane; no. 443)
- $A = D_0 \cap D_1 \cap D_2 \cap D_3$, $B = D_0 \cap D_1 \cap D_4 \cap D_5$, $C = D_2 \cap D_3 \cap D_6 \cap D_7$ (A and B lie on a double line, A and C lie on a double line; no. 440)
- $A = D_0 \cap D_1 \cap D_2 \cap D_3$, $B = D_0 \cap D_1 \cap D_4 \cap D_5$, $C = D_2 \cap D_3 \cap D_4 \cap D_7$ (A and B lie on a double line, A and C lie on a double line, B and C lie on a common plane; no. 441)
- $A = D_0 \cap D_1 \cap D_2 \cap D_3$, $B = D_0 \cap D_1 \cap D_4 \cap D_5$, $C = D_2 \cap D_3 \cap D_4 \cap D_5$ (A and B lie on a double line, A and C lie on a double line, B and C lie on a double line; no. 444)

Now assume that the three points lie in a common plane, say D_0. This time there are three cases up to permutation:

- $A = D_0 \cap D_1 \cap D_2 \cap D_3$, $B = D_0 \cap D_1 \cap D_4 \cap D_5$, $C = D_0 \cap D_2 \cap D_6 \cap D_7$ (A and B lie on a double line, A and C lie on a double line; no. 442)
- $A = D_0 \cap D_1 \cap D_2 \cap D_3$, $B = D_0 \cap D_1 \cap D_4 \cap D_5$, $C = D_0 \cap D_2 \cap D_4 \cap D_6$ (A and B lie on a double line, A and C lie on a double line, B and C lie on a double line; no. 439)
- $A = D_0 \cap D_1 \cap D_2 \cap D_3$, $B = D_0 \cap D_1 \cap D_4 \cap D_5$, $C = D_0 \cap D_1 \cap D_6 \cap D_7$ (A, B and C lie on a double line; no. 445)

Here are schematic pictures of the seven arrangements:

Now we are going to discuss several aspects of the list of examples.

Forgotten arrangements of eight planes

There are some configurations of multiple points and lines that did not appear in the list in [28]:

no.	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	$h^{1,2}$	$h^{1,1}$	χ
30	14	1	6	1	0	1	2	2	52	100
52	15	0	4	0	3	0	2	2	56	108
64	12	3	10	0	0	0	2	2	44	84
144	14	4	5	1	0	0	1	1	45	88
145	18	3	5	1	0	0	1	2	44	84
146, 147, 148	22	2	5	1	0	0	1	3	43	80
149	26	1	5	1	0	0	1	4	42	76
150, 151	30	0	5	1	0	0	1	5	41	72
197, 198, 199, 200	22	6	0	1	0	0	0	1	41	80

Hodge numbers are not determined by numerical data

As mentioned before, the Hodge numbers are not determined by the numbers p_4^0, p_4^1, p_5^0, p_5^1, p_5^2 and l_3. This can be observed for the arrangements with only p_4^0 points where in some cases the Hodge number $h^{1,2}$ is lower than expected (the idea is that adding a p_4^0 point should decrease $h^{1,2}$ by one):

• no. 384 with $p_4^0 = 6$, • no. 287 with $p_4^0 = 7$, • no. 260, no. 263, no. 269, no. 271, no. 272 with $p_4^0 = 8$, • no. 243, no. 244, no. 246 with $p_4^0 = 9$, • no. 242 with $p_4^0 = 10$.

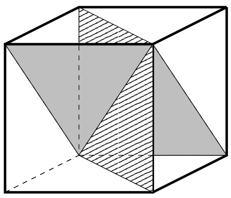
Rigid arrangements of eight planes

In the table of arrangements of eight planes in appendix A there are exactly 11 rigid arrangements. All of them but no. 241 were already discussed in [28] (where they were numbered in a different way). We list the numbers, the old numbers from [28], the Hodge number $h^{1,1} = h^{1,1}(\tilde{X})$, the rank $\rho = \operatorname{rk}\operatorname{Pic}(Y)$ of the Picard group of Y and a sample equation.

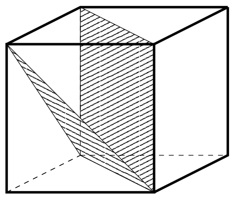
no.	old	$h^{1,1}$	ρ	sample equation: $0 = \cdots$
1	2	70	70	$xyzt(x+y)(y+z)(z+t)(t+x)$
3	6	62	62	$xyzt(x+y)(y+z)(y-t)(x-y-z+t)$
19	23	54	54	$xyzt(x+y)(y+z)(x-z-t)(x+y+z-t)$
32	29	50	50	$xyzt(x+y)(y+z)(x-y-z-t)(x+y-z+t)$
69	44	50	50	$xyzt(x+y)(x-y+z)(x-y-t)(x+y-z-t)$
93	62	46	46	$xyzt(x+y)(x-y+z)(y-z-t)(x+z-t)$
238	87	44	41	$xyzt(x+y+z-t)(x+y-z+t)(x-y+z+t)(-x+y+z+t)$
239	86^a	40	39	$xyzt(x+y+z)(x+y+t)(x+z+t)(y+z+t)$
240	86	40	39	$xyzt(x+y+z)(x+y-z+t)(x-y+z+t)(x-y-z-t)$
241		40	39	$xyzt(x+y+z+t)(x+y-z-t)(y-z+t)(x+z-t)$
245	84	38	38	$xyzt(x+y+z)(y+z+t)(x-y-t)(x-y+z+t)$

All of the above arrangements can be realized as a cube with two additional planes since they contain subarrangements of six planes of type zero. Note that in all cases except no. 32 there is more than one subcube and we could draw different pictures of the same arrangement. E.g., for arrangements no. 239 and no. 240 different subcubes were chosen in [28].

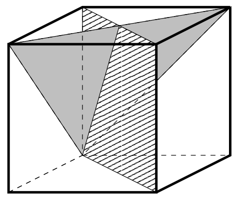
Now we will present pictures and geometrical descriptions of all rigid arrangements.



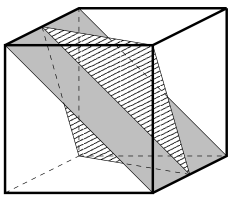
Arr. no. 1



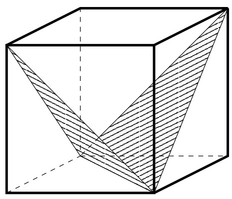
Arr. no. 3



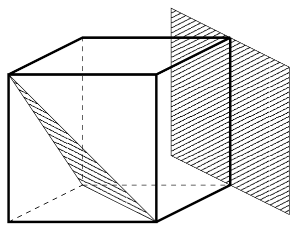
Arr. no. 19



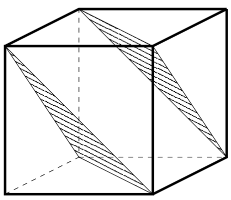
Arr. no. 32



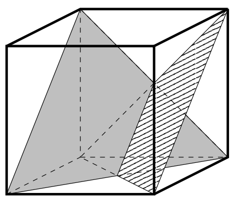
Arr. no. 69



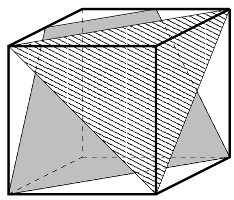
Arr. no. 93



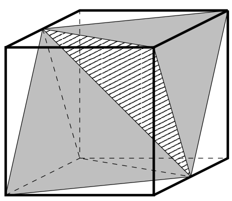
Arr. no. 238



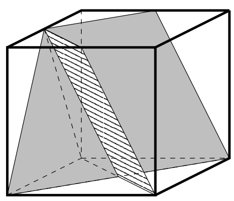
Arr. no. 239



Arr. no. 240



Arr. no. 241



Arr. no. 245

Arrangement no. 1: the additional two planes pass through four vertices of the cube each and intersect along a diagonal of the cube.

Equivalently this arrangement may be described as a tetrahedron and additional four planes going through four edges of the tetrahedron and intersecting in one point.

Arrangement no. 3: one additional plane goes through three vertices and the other through four vertices of the cube; they intersect along the diagonal of a face.

Arrangement no. 19: one additional plane goes through three vertices and the other through four vertices of the cube; they have only one of the vertices of the cube in common.

Arrangement no. 32: one additional plane goes through four vertices of the cube, the other through two opposite vertices not belonging to the first plane and two midpoints of edges belonging to the first plane.

Arrangement no. 69: the additional planes pass through three vertices of the cube each and intersect along the diagonal of a face.

Arrangement no. 93: one additional plane goes through an edge of the cube and is parallel to a diagonal of the cube, the other plane goes through three vertices of the cube not belonging to the first plane.

Arrangement no. 238: the additional planes pass through three vertices of the cube each and are parallel.

Equivalently this arrangement may be described as a symmetric octahedron. The 4–fold points are then: six vertices of the octahedron and six points at infinity of intersections of parallel edges. Note that this arrangement was already described in [48] and [75].

Arrangement no. 239: the additional planes pass through three vertices of the cube each; they intersect in a line going through two midpoints of faces of the cube.

Arrangement no. 240: one additional plane goes through three vertices of the cube, the other goes through two vertices and two midpoints of edges such that the planes are parallel.

Arrangement no. 241: the additional planes pass through two opposite vertices of the cube each and intersect in a line through the midpoints of two opposite edges of the cube.

Arrangement no. 245: the additional planes pass through two vertices and two midpoints of edges of the cube each and intersect in a line through a midpoint of an edge and a midpoint of a face of the cube.

Modularity of the rigid arrangements

Now we are going to verify the modularity conjecture for the Calabi–Yau threefolds constructed from the eleven rigid arrangements above.

4.5 Lemma
The Calabi–Yau manifolds $\tilde{X}_p$ *associated to arrangements no.* $1, 3, 19, 32, 69, 93, 238, 241$ *are smooth for all primes* $p \geq 3$*, the Calabi–Yau manifolds* $\tilde{X}_p$ *associated to arrangements no.* $239, 240, 245$ *are smooth for all primes* $p \geq 5$*.*

Proof:
Since the singularities of arrangements of planes are defined by ranks of some minors of 8×4 matrices of coefficients, it is enough to verify the lemma for the primes dividing any minor of the matrices. This is easily done with a computer. □

The coefficients of the L–series can now be computed from the Lefschetz fixed point formula

$$a_p(\tilde{X}) = 1 + p^3 + k_p(\tilde{X})(p + p^2) - \#\tilde{X}_p$$

where $k_p(\tilde{X}) \in \mathbb{Z}$, $|k_p(\tilde{X})| \leq h^{1,1}(\tilde{X})$, $k_p(\tilde{X}) \cdot p = \operatorname{tr}(\operatorname{Frob}_p^* | H^2_{\text{ét}}(\tilde{X}))$.

Now the Picard group $\operatorname{Pic}(\tilde{X})$ of $\tilde{X}$ splits into a sum of symmetric part and skew–symmetric part. The symmetric part is naturally isomorphic to $\operatorname{Pic}(Y)$. By Lemma 4.3

$$\operatorname{rk}\operatorname{Pic}(Y) = 29 + p_4^0 + p_4^1 + 6p_5^0 + 7p_5^1 + 8p_5^2 + l_3.$$

Consequently for arrangements no. $1, 3, 19, 32, 69, 93, 245$ we get $\operatorname{Pic}(\tilde{X}) \cong \operatorname{Pic}(Y)$, i.e., all the divisors are even and defined over $\mathbb{Q}$. Thus Frob_p^* acts on $H^2_{\text{ét}}(\tilde{X})$ by multiplication with p, and $k_p(\tilde{X}) = h^{1,1}(\tilde{X})$ for all good primes p.

For arrangements no. 239, 240 and 241 the rank of the skew–symmetric part of $\operatorname{Pic}(\tilde{X})$ is one. For arrangement no. 239 it is generated by the divisor associated to the contact hyperplane $x - t = 0$, for arrangement no. 240 it is generated by the divisor associated to the contact hyperplane $x + y - z + t$; so also in these cases we have $k_p(\tilde{X}) = h^{1,1}(\tilde{X})$ for all good primes p.

For arrangement no. 241 there seems to be no contact hyperplane (there is at least no plane through four double lines as for no. 239 and no. 240). But since $h^{1,1}(\tilde{X}) - \operatorname{rk}\operatorname{Pic}(Y) = 1$ and $k_p(\tilde{X}) \in \mathbb{Z}$, the "missing eigenvalue" of Frob_p^* on $H^2_{\text{ét}}(\tilde{X})$ can only be $\pm p$, so $k_p(\tilde{X}) = h^{1,1}(\tilde{X})$ or $k_p(\tilde{X}) = h^{1,1}(\tilde{X}) - 2$. Once we know $\#\tilde{X}_p$ we can thus determine $k_p(\tilde{X})$ for all needed primes since $|a_p(\tilde{X})| \leq 2p^{3/2}$ and $p^2 + p > 2p^{3/2}$.

For arrangement no. 238 the rank of the skew–symmetric part of $\operatorname{Pic}(\tilde{X})$ is three. On $\tilde{X}$ there are the skew–symmetric divisors associated to the contact hyperplanes $x + y + z - t = 0$, $x + y + t - z = 0$, $x + z + t - y = 0$ and $y + z + t - x = 0$. It is not easy to check if they generate all of the skew–symmetric part of $\operatorname{Pic}(\tilde{X})$, but anyway we have $h^{1,1}(\tilde{X}) - k_p(\tilde{X}) \in \{0, 1, 2, 3, 4\}$ and we can determine $k_p(\tilde{X})$ as above.

To compute $\#\tilde{X}_p$ we first count points on the singular double covering X_p of $\mathbb{P}^3(\mathbb{F}_p)$, i.e., the number of points in $\mathbb{P}^3(\mathbb{F}_p)$ for which the value of the branch divisor equation is a square (in $\mathbb{F}_p$). Note that the number does not only depend on the branch divisor, but actually on its equation. Multiplying the equation of the branch divisor by squarefree integers we get new (non–isomorphic over $\mathbb{Q}$) Calabi–Yau manifolds. Then we have to take into account the resolution of singularities.

Blowing up a 5–fold point replaces a point on the double covering by a plane (since the exceptional divisor is contained in the branch locus), but we add five double lines and 0, 1 or two p_4^1 points (depending on the number of triple lines through this point).

Blowing up a triple line replaces a line on the double covering by $\mathbb{P}^1 \times \mathbb{P}^1$. This introduces new double lines, altogether 3 plus the number of 4–fold points on the triple line.

Blowing up a double line replaces a line on the double covering by a double covering of $\mathbb{P}^1 \times \mathbb{P}^1$ which is also $\mathbb{P}^1 \times \mathbb{P}^1$, so we add $p^2 + 2p + 1 - (p + 1) = p^2 + p$ points.

Altogether blowing up double and triple lines and 5–fold points adds

$$(p_4^1 + 6p_5^0 + 7p_5^1 + 8p_5^2 + l_3 + 28)(p + p^2)$$

points to the double covering.

We can not write down a similarly simple formula for blowing up a 4-fold point. The reason is that the blow–up of a 4–fold point replaces a point on the double covering by a double covering of a projective plane branched along four lines (projectivisation of the normal cone).

Let P be a p_4^0 point with coordinates $(p_x : p_y : p_z : 1)$ and let D_i, $i = 1, \dots, 8$ be the equations for the eight planes of the arrangement. The projectivisation of the normal cone at P is then given by

$$\left\{ u^2 = \prod_{D_i(P)\neq 0} D_i(P) \prod_{D_i(P)=0} D_i(x : y : z : 0) \right\} \subset \mathbb{P}^3(1, 1, 1, 2)$$

so it is of the form

$$\left\{ u^2 = \prod_{i=1}^{4} (a_{i1}x + a_{i2}y + a_{i3}z) \right\} \subset \mathbb{P}^3(1, 1, 1, 2).$$

Let

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{pmatrix}, \qquad M = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

Then $|M| \neq 0$ and

$$|M| \cdot A \cdot M^{-1} = \begin{pmatrix} |M| & 0 & 0 \\ 0 & |M| & 0 \\ 0 & 0 & |M| \\ c_1 & c_2 & c_3 \end{pmatrix}$$

with

$$\begin{aligned} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} &= |M|(a_{41}, a_{42}, a_{43})M^{-1} \\ &= \begin{pmatrix} (a_{22}a_{33} - a_{23}a_{32})a_{41} + (a_{23}a_{31} - a_{21}a_{33})a_{42} + (a_{21}a_{32} - a_{22}a_{31})a_{43} \\ (a_{13}a_{32} - a_{12}a_{33})a_{41} + (a_{11}a_{33} - a_{13}a_{31})a_{42} + (a_{12}a_{31} - a_{11}a_{32})a_{43} \\ (a_{12}a_{23} - a_{13}a_{22})a_{41} + (a_{13}a_{21} - a_{11}a_{23})a_{42} + (a_{11}a_{22} - a_{12}a_{21})a_{43} \end{pmatrix} \end{aligned}$$

and $c_1c_2c_3 \neq 0$. This means that our surface is birationally equivalent over $\mathbb{Q}$ with the surface

$$\{u^2 = |M|^3 \cdot xyz(c_1x + c_2y + c_3z)\} \subset \mathbb{P}^3(1, 1, 1, 2)$$

and so also to the surface

$$S_\alpha = \{u^2 = \alpha \cdot xyz(x+y+z)\} \subset \mathbb{P}^3(1,1,1,2)$$

where α is the squarefree part of $|M|c_1c_2c_3$. Now consider the smooth quadric surface

$$Q_\alpha = \{t^2 = \alpha(ab+ac+bc)\} \subset \mathbb{P}^3(\mathbb{K})$$

with discriminant $-4\alpha^3$. The map

$$\mathbb{P}^3(1,1,1,2) \longrightarrow \mathbb{P}^3(\mathbb{K}), \qquad (x:y:z:u) \mapsto (u:yz:xz:xy)$$

maps $S_\alpha \setminus \{xyz=0\}$ birationally to $Q_\alpha \setminus \{abc=0\}$ (the inverse map is given by $(t:a:b:c) \mapsto (bc:ac:ab:tabc)$). The set $S_\alpha \cap \{xyz=0\}$ is the union of three lines in a plane, the set $Q_\alpha \cap \{abc=0\}$ is the union of three plane conics where each two meet in one point. Thus over the finite field $\mathbb{F}_p$ we have

$$\#S_{\alpha,p} = \#Q_{\alpha,p} = p^2 + p + \left(\frac{-\alpha}{p}\right)p + 1.$$

Going back to our Calabi–Yau manifolds $\tilde{X}$ we can now compute

$$\#\tilde{X}_p = \#X_p + (p_4^0 + p_4^1 + 6p_5^0 + 7p_5^1 + 8p_5^2 + l_3 + 28)(p+p^2) + \sum \left(\frac{-\alpha}{p}\right)p$$

where the sum runs over the p_4^0 points and α is defined as above. Counting points and comparing we find that the $a_p(\tilde{X})$ agree with the coefficients of certain weight four newforms (where I multiplied the equation of the branch divisor with λ to twist the modular form to reach a minimal level). To prove the modularity we can use corollary 1.6.

no.	λ	newform
1	1	8/1 (8k4A1)
3	1	32/2 (32k4B1)
19	2	32/1 (32k4A1)
32	−1	8/1 (8k4A1)
69	−1	8/1 (8k4A1)
93	2	8/1 (8k4A1)
238	1	8/1 (8k4A1)
239	1	12/1 (12k4A1)
240	−2	6/1 (6k4A1)
241	1	8/1 (8k4A1)
245	−2	6/1 (6k4A1)

One-parameter families

In the table of arrangements of eight planes in appendix A there are exactly 63 arrangements with $h^{2,1}(\tilde{X}) = 1$. Some of them were already discussed in [28] (where they were numbered in a different way). We list the numbers, the old numbers from [28] and equations of one-parameter families containing these arrangements. I have not been able to find a linear parametrization for families no. 275 and no. 276. But of course this does not mean that there is no such parametrization.

no.	old	equation: $0 = xyzt\cdots$
2	1	$(x+y)(y+z)(z+t)(Ax+Bt)$
4		$(x+y)(y+z)(Ax+By+Bz-At)(Ax+Ay+Bz-At)$
5	5	$(x+y)(y+z)(x+y+z-t)(Ax+By+Az-At)$
8	11	$(x+y)(y+z)(z-t)(Ax-By-Bz+Bt)$
10	10	$(x+y)(y+z)(z-t)(Ax-By-Bz-At)$
13	14	$(x+y)(y+z)(x-z-t)(Ax-Az+Bt)$
16	18	$(x+y)(y+z)(Ay-Bz-At)(Bx-Ay+At)$
20	22	$(x+y)(y+z)(x-z+t)(Ay-Bz-At)$
21		$(x+y)(y+z)(Ax-By-(A+B)t)(Ax+Bz-At)$
33		$(x+y)(y+z)(x-z+t)(Ax-Ay-Az+Bt)$
34	28	$(x+y)(x+z)(x+y+z+t)(Ay-Az+Bt)$
35		$(x+y)(x+y+t)(Ax-Ay+Bz+At)(Ay-Bz+At)$
36		$(x+y)(y-z+t)(Ax-By+Bz+At)(Ax+Ay+Bz+At)$
53	32	$(x+y)(z+t)(Ax-By-Az-At)(Bx+By-Bz+At)$
70		$(x-y+z)(y-z-t)(x-y-t)(Ax+By)$
71	43	$(x+y)(x+y+z+t)(Ax-By+Az)(By-Az-At)$
72		$(x+y+z)(y+z+t)(x-y-t)(Ay+Bz+Bt)$
73		$(x+y-z-t)(y-z-t)(Ax+Ay+Bz+Bt)(Ax-By+Bt)$
94		$(x+y)(x+y+z-t)(Ax-By+Az)(By-Az-Bt)$
95		$(x+y)(x+y-z+t)(Ax-By+Bz)(Ax-By-Az-Bt)$
96		$(x+y)(x+y-z+t)(Ax-By+Bz+At)(Ay+Bz+At)$
97		$(x+y)(x+y+z+t)(y-z-t)(Ax-Bz+At)$
98	61	$(x+y+z)(y+z+t)(x+z-t)(Ay+Bz+Bt)$
99		$(x+y+z)(x+z-t)(Ax+(A+B)y-Bz+Bt)(Ax-By-Bz)$
100		$(x+y-z+t)(Ax+Ay+Bz)(Ay+Bz+At)(By-Bz-At)$
144		$(x-y+z+t)(Ax+By+Az)(By+Az+At)(Bx-By-Az+Bt)$
152		$(x+y+z+t)(y+t)(x-y-z+t)(Ax-Ay+Bz-Bt)$
153		$(x+y+z)(y+z+t)(Ax-By+At)(Ax-By+Az+At)$
154	55	$(x+y+z)(x+y+z-t)(Ax+(A+B)y-Bz+Bt)(Ax-Bz-At)$
155		$(Ax+By+Az)(Ax+(A+B)y-Bz+At)\cdot$ $\cdot(Ax-Bz-Bt)(Ax+By+Az+At)$
197		$(x-y-z+t)(Ax+By+Bz)(By+Bz+At)(Ax+Bz+At)$
198		$(x+y+z)(y+z+t)(x-y-t)(Ax-Ay-Az+Bt)$

no.	old	equation: $0 = xyzt\cdots$
199		$(x+y+z)(y+z+t)(Ax+By+(A-B)z)(Ax+By+Az+Bt)$
200		$(x+y+z+t)(Ax+Ay-Bz-Bt)(Ay-Bz+At)(Ax-By-Bt)$
242	85	$(x+y+z)(x+z-t)\cdot$ $\cdot(Ax+(A+B)y-Bz+Bt)((A+B)x+(A+B)y+Bt)$
243		$(x+y+z)(y+z+t)(x+y+t)(Ax+By+Az+At)$
244	83	$(x+y+z+t)(Ax+Ay+Bz+Bt)(Ay+Bz+At)(Ax+Bz+At)$
246		$(x+y+z)(Ax+(A+B)y-Bz+Bt)\cdot$ $\cdot(Ax-Bz-At)(Ax+(A+B)y+Az-At)$
247		$(x+y+z)(y+z+t)(x-y-t)(Ax-Bz+Bt)$
248		$(x+y+z)(y-z-t)(x+z+t)(Ax+(A+B)y-Bz+At)$
249		$(x+y+z)(x+z+t)(Ax+(A+B)y-Bz+At)(By-Bz+At)$
250		$(x+y+z)(y+z-t)(x+z+t)(Ax+By-Az+At)$
251		$(x+y+z)(x+z-t)(Ax+(A+B)y-Bz+Bt)(Ax-By-Bz-At)$
252		$(x+y+z)(x+y+t)(Ax+2Ay-Bz+At)(Ax-Bz-At)$
253		$(x+y+z)(x+z-t)(Ax+(A+B)y-Bz+Bt)(Ax+Ay-Bz-At)$
254		$(x+y+z+t)(Ax+Ay-Bz-Bt)(Ay-Bz+At)(Ax-By-Bz)$
255		$(Ax+Ay+Bz+Bt)(x+y-2z-2t)\cdot$ $\cdot(Ay+(B-2A)z+Bt)(Bx+(B-2A)y+(4A-2B)z-2Bt)$
256		$(x+y+2z)(Ay-Bz+Bt)\cdot$ $\cdot(Ax+Ay+(2A-B)z+Bt)(Bx+(B-2A)y+2Bz-2Bt)$
257		$(x+y+2z+2t)(Ax+Ay+Bz+Bt)\cdot$ $\cdot(Ay+(B-2A)z+Bt)((2A-B)x-By+(4A-2B)z-2Bt)$
258		$(x-y+2z-2t)(y-z+2t)(x-y+z-t)(Ax+By+Az+Bt)$
259		$(x+y+z+t)(x-y-z+t)(Ax-Ay+Bz-Bt)(Ax-By+Az-Bt)$
261		$(x+y+z+t)(x-y-z+t)(Ax-Ay+Bz-Bt)(Ax+Ay+Bz+Bt)$
262		$(x-z-t)(Ax+Ay+Bz)(Ax+(A+B)y-Az+Bt)(By-(A+B)z-At)$
264		$(y-2z+2t)(Ax+Ay+Bz)\cdot$ $\cdot(Ax+2Ay+(B-2A)z+(2A-B)t)(Ax+Ay-2Az+(2A-B)t)$
265		$(x+y-z+2t)(Ax+2Ay-Az+Bt)\cdot$ $\cdot(By-2Az+2Bt)(Bx+By+(2A-B)z)$
266		$(y-2z+2t)(2x+y+2t)(Ax+By+Az)(Ax+(A+B)y-Az+At)$
267		$(Ax+Ay+(B-A)z)(Ax+By-Az+At)\cdot$ $\cdot((B-A)y-Bz+Bt)(Bx+By-Az+Bt)$
268		$(x+y+z)(Ay-2Bz+2Bt)(2Bx+2By+At)((2B-A)x+2By-Az+At)$
270		$(x+y+z)(y+z+t)(Ax+2Ay-Bz+At)(Bx-2Ay+Bz+Bt)$
273		$(x+y+z)(2y+2z+t)(2x-2z-t)(Ax+2By-Az+Bt)$
274		$(x+y+z)(x+z-t)(Ax+(A+B)y-Bz+Bt)(Ax+Ay-Bz+(A+B)t)$
275		$xyzt(x+y+z)((A+B-C)x+2Bz+t)(2Cy+(B+C-A)z+t)\cdot$ $\cdot(2Ax+(A+C-B)y+t), \qquad A^2+B^2+C^2=2(AB+AC+BC)$
276		$xyzt(Ax+By+Cz)(Cy+Bz+At)\cdot$ $\cdot(Bx+Bz+Ct)(Cx+By+Bt), \qquad B^2=C^2-AC$

I ran a computer search to find modular examples. For some families and certain parameters we find

$$a_p(\tilde{X}) = b_p + p \cdot c_p$$

for all primes $5 \leq p \leq 97$, where b_p are the coefficients of a weight four newform and c_p are the coefficients of a weight two newform. Below there is a list of the results. Again I multiplied the equations of the branch divisors with λ to obtain a weight four newform of minimal level.

no.	$(A : B)$	λ	weight 4	weight 2
4	$(1 : -1)$	1	32/1 $(32k4A1)$	$32A1$
4	$(1 : 2)$	1	32/1 $(32k4A1)$	$32A1$
4	$(2 : 1)$	2	32/1 $(32k4A1)$	$32A1$
8	$(3 : 1)$	-1	24/1 $(24k4A1)$	$24A1$
13	$(1 : -2)$	1	32/1 $(32k4A1)$	$32A1$
13	$(1 : 1)$	1	32/1 $(32k4A1)$	$32A1$
13	$(2 : -1)$	1	32/1 $(32k4A1)$	$32A1$
21	$(2 : -1)$	1	32/2 $(32k4B1)$	$32A1$
53	$(1 : 1)$	1	32/2 $(32k4B1)$	$32A1$
154	$(2 : -3)$	-1	8/1 $(8k4A1)$	$72A1$
244	$(1 : -1)$	1	12/1 $(12k4A1)$	$48A1$
249	$(2 : 1)$	1	24/1 $(24k4A1)$	$24A1$
249	$(2 : -3)$	1	24/1 $(24k4A1)$	$24A1$
267	$(1 : -1)$	-1	96/4 $(96k4B1)$	$96B1$
267	$(1 : 2)$	-1	96/4 $(96k4B1)$	$96B1$
267	$(2 : 1)$	-1	96/4 $(96k4B1)$	$96B1$
274	$(1 : 1)$	-1	96/2 $(96k4E1)$	$96B1$
275	$(A : B : C) = (1 : 1 : 4)$	-1	96/4 $(96k4B1)$	$96B1$

Note that for all listed arrangements but no. 244 we have $\text{Pic}(\tilde{X}) \cong \text{Pic}(Y)$, so Frob_p^* acts on $H^2_{\text{ét}}(\tilde{X})$ by multiplication with p. For arrangement no. 244 the rank of the skew–symmetric part of $\text{Pic}(\tilde{X})$ is one. It is generated by the divisor associated to the contact hyperplane $x+y-z+t$; so also in this case Frob_p^* acts on $H^2_{\text{ét}}(\tilde{X})$ by multiplication with p.

Now we want to prove the modularity at least in some cases. We consider an arrangement of eight planes and suppose that there is a plane (not belonging to the arrangement) which contains exactly two multiple lines of the arrangement. The intersection of the double covering X with this plane is then a double covering of $\mathbb{P}^2$ branched along the union of four lines. The preimage of this surface in $\tilde{X}$ is an elliptic surface as in 1.5.2 and the modularity of $\tilde{X}$ follows (with the help of corollary 1.6).

This construction works for two of the above families. In the other cases there does not seem to be a suitable configuration of double lines (but nonetheless there might be hidden elliptic surfaces).

Arrangement no. 244: A one-parameter family containing this arrangement is given by the equation

$$0 = xyzt(x + y + z + t)(Ax + Ay + Bz + Bt)(Ay + Bz + At)(Ax + Bz + At).$$

The plane containing the double lines $z = Ax + Ay + Bz + Bt = 0$ and $t = x + y + z + t = 0$ is given by

$$Ax + Ay + Az + Bt = 0.$$

The fourfold point which is the intersection of the remaining four planes,

$$x = y = Ay + Bz + At = Ax + Bz + At = 0,$$

has coordinates $(0 : 0 : A : -B)$. If we want the plane $Ax + Ay + Az + Bt = 0$ to contain this point then we get the condition $A^2 = B^2$. For $(A : B) = (1 : 1)$ the arrangement degenerates (there is a double plane), but for $(A : B) = (1 : -1)$ we are in the above situation.

Arrangement no. 4: A one-parameter family containing this arrangement is given by the equation

$$0 = xyzt(x + y)(y + z)(Ax + By + Bz - At)(Ax + Ay + Bz - At).$$

The plane containing the double lines $x = z = 0$ and $x + y = y + z = 0$ is given by

$$x - z = 0.$$

The fourfold point which is the intersection of the remaining four planes,

$$y = t = Ax + By + Bz - At = Ax + Ay + Bz - At = 0,$$

has coordinates $(B : 0 : -A : 0)$. If we want the plane $x - z = 0$ to contain this point then we get the condition $B = -A$.

The plane containing the double lines $x = Ax+Ay+Bz-At = 0$ and $x+y = Ax+By+Bz-At = 0$ is given by

$$(2A - B)x + Ay + Bz - At = 0.$$

The fourfold point which is the intersection of the remaining four planes,

$$y = z = t = y + z = 0,$$

has coordinates $(1 : 0 : 0 : 0)$. If we want the plane $(2A - B)x + Ay + Bz - At = 0$ to contain this point then we get the condition $B = 2A$.

The plane containing the double lines $z = Ax+By+Bz-At = 0$ and $y+z = Ax+Ay+Bz-At = 0$ is given by

$$Ax + By + (2B - A)z - At = 0.$$

The fourfold point which is the intersection of the remaining four planes,

$$x = y = t = x + y = 0,$$

has coordinates $(0 : 0 : 1 : 0)$. If we want the plane $Ax + By + (2B - A)z - At = 0$ to contain this point then we get the condition $2B = A$.

Modular examples with higher number of deformations

For some examples with higher number of deformations ($h^{2,1}(\tilde{X}) > 1$) I also found examples that seem to be modular. In all listed cases we have

$$a_p(\tilde{X}) = b_p + h^{2,1}(\tilde{X}) \cdot p \cdot c_p$$

for all primes $5 \leq p \leq 97$ where b_p are the coefficients of a weight four newform and c_p are the sums of coefficients of weight two newforms. Again I multiplied the equations of the branch divisors with λ to twist the modular form to reach a minimal level. Note that the search for modular examples was no longer systematic, so there might be many more.

no.	$h^{2,1}(\tilde{X})$	equation
6	2	$xyzt(x+y)(y+z)(y-t)(x-y-z-t)$
58	3	$xyzt(x+y)(z+t)(x-y-z+t)(x-y+z-t)$
269	2	$xyzt(x+y+z)(x+2y-z+t)(y+z-t)(x+y-2z+t)$
287	3	$xyzt(x+y+z-3t)(x+y-3z+t)(x-3y+z+t)(-3x+y+z+t)$
317	2	$xyzt(x+2y+z)(y+2z+t)(x+2t+z)(2x+y+t)$
385	3	$xyzt(x+y+z+t)(x-y+2z-2t)(x-3y+3z-3t)(x+y+2z)$

no.	λ	weight 4	weight 2
6	1	96/4 $(96k4B1)$	$2 \cdot 32A1$
58	1	32/1 $(32k4A1)$	$3 \cdot 32A1$
269	1	24/1 $(24k4A1)$	$2 \cdot 24A1$
287	1	6/1 $(6k4A1)$	$3 \cdot 24A1$
317	1	12/1 $(12k4A1)$	$2 \cdot 48A1$
385	-1	96/1 $(96k4D1)$	$96A1 + 2 \cdot 96B1$

At least for two examples the above construction works, showing certain elliptic surfaces in the resolution $\tilde{X}$. The modular form of the elliptic curves involved is the weight two newform in the table. To prove the modularity it remains to show that the elliptic surfaces span a subspace of $H^3_{\text{ét}}(\tilde{X}, \mathbb{Q}_\ell)$ of dimension $h^{2,1}(\tilde{X})$.

Arrangement no. 58: The plane given by $x + y + z - t = 0$ contains the two double lines $x = x - y - z + t = 0$ and $y = x - y + z - t = 0$ and the fourfold point $(1 : -1 : 0 : 0)$ which is the intersection of the remaining four planes.

The plane given by $x + y - z + t = 0$ contains the two double lines $x = x - y + z - t = 0$ and $y = x - y - z + t = 0$ and the fourfold point $(1 : -1 : 0 : 0)$ which is the intersection of the remaining four planes.

The plane given by $x - y + z + t = 0$ contains the two double lines $z = x - y - z + t = 0$ and $t = x - y + z - t = 0$ and the fourfold point $(0 : 0 : 1 : -1)$ which is the intersection of the remaining four planes.

The plane given by $-x+y+z+t=0$ contains the two double lines $z=x-y+z-t=0$ and $t=x-y-z+t=0$ and the fourfold point $(0:0:1:-1)$ which is the intersection of the remaining four planes.

Altogether there are at least four elliptic surfaces inside the double octic $\tilde{X}$.

Arrangement no. 287: The plane given by $x+y=3(z+t)$ contains the two double lines $z=x+y+z-3t=0$ and $t=x+y-3z+t=0$ and the fourfold point $(0:0:1:-1)$ which is the intersection of the remaining four planes. By permutation of coordinates we find six elliptic surfaces inside the double octic $\tilde{X}$.

Arrangements in finite characteristics

It is also interesting to search for arrangements of eight planes in finite characteristics. One purpose is to find arrangements that do not exist over $\mathbb{Q}$, the other is to check the list of examples defined over $\mathbb{Q}$ for completeness (since it is possible for small characteristic p to test *all* arrangements defined over $\mathbb{F}_p$).

I ran a computer search for the characteristics $p \in \{3,5,7,11,13,17\}$. Characteristic 2 makes no sense and larger characteristics are unlikely to produce new examples.

p	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	subarrangements of 6 planes
3		6	3	0	1	0	1	99909995959540393599999935095
3		11	0	0	0	0	0	99999909999099990990999000000
5		9	0	0	0	0	0	99899890999999909988999099998
7		6	3	0	1	0	1	99999090999999593359554099355
7		8	0	0	0	0	0	99999889899998989989998998908
7		9	0	0	0	0	0	99080998899999989999990989908
11		9	0	0	0	0	0	98899998998999999999998099900
13		6	3	0	1	0	1	99909995959540393599999935095
13		9	0	0	0	0	0	99980098999899989989998999900
13		8	0	0	0	0	0	99990999999898988999889899889

There are no new examples for characteristic 17. In this case I had to check about 1 billion of examples (which took about two weeks). Some of the examples in the table might lift to some finite extension of $\mathbb{Q}$. This will be discussed elsewhere.

4.3 Six planes and a quadric

The next interesting thing to consider are unions of six planes and a smooth quadric. Unfortunately the most interesting examples are no arrangements as defined in the last section since there is tangency of surfaces and tangency of intersection curves. Therefore we should have a closer look at the different possible types of multiple points. A priori we can exclude sixfold points and fourfold curves (i.e., fourfold lines in our case).

Note that we will only consider the local type of the multiple points, so the numerical data (numbers of multiple curves and points of certain types) will not determine the variety.

There are double points where one plane is tangent to the quadric and no other plane contains that point. In the following three lemmata we will prove that there are three types of triple points, eight types of fourfold points and fifteen types of fivefold points. Note that it is not a priori clear if all types of multiple points admit a Calabi–Yau resolution (but there is evidence since there will be modular examples with all types of multiple points).

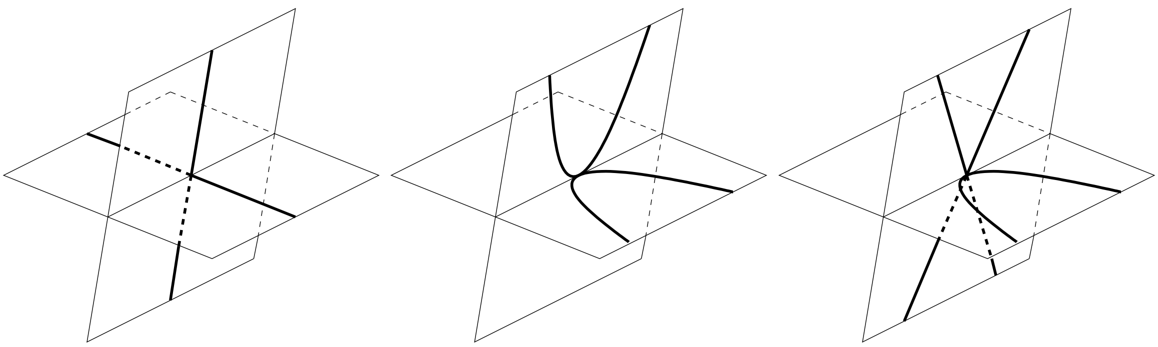
4.6 Lemma
There are exactly three different types of triple points (as intersection of three planes or two planes and a smooth quadric):

Type F_1**:** *Ordinary arrangement triple point: no two surfaces and no two curves are tangent at this point.*

Type F_2**:** *Intersection point of two planes and the quadric where the planes intersect the quadric in two conics and the conics are tangent to the intersection line of the planes.*

Type F_3**:** *Intersection point of two planes and the quadric where one plane is tangent to the quadric at this point and the other plane intersects the quadric in a conic which is tangent to the intersection line of the planes.*

The picture sketches the three different types. It shows two planes and the intersection curves with the third surface (plane or quadric).



Proof:
If a triple point P is no arrangement triple point (i.e., not of type F_1) then there must be tangency of surfaces or tangency of curves which means that one of the surfaces is the quadric.

Let there be no tangency of surfaces at P. If there is tangency of curves then one plane intersects the quadric in a conic which must be tangent to the intersection line of the two planes. In this case the point P is the only point of intersection of the second plane with the quadric. This means that the intersection curve is also a conic tangent at P to the intersection line of the two planes. The point P is then of type F_2.

Now let one plane be tangent to the quadric at P. Then again the point P is the only point of intersection of the second plane with the quadric and the intersection curve is a conic tangent at P to the intersection line of the two planes. The point P is then of type F_3. □

4.7 Lemma

There are exactly eight different types of 4–fold points (as intersection of four planes or three planes and a smooth quadric) not contained in a fourfold line:

Points not on triple lines:

Type G_1**:** *Ordinary arrangement* p_4^0 *point: no two surfaces and no two curves are tangent at this point.*

Type G_2**:** *Intersection point of three planes and the quadric where no plane is tangent to the quadric. Two planes intersect the quadric in conics which are tangent to the intersection line of the two planes at the point.*

Type G_3**:** *Intersection point of three planes and the quadric where one plane is tangent to the quadric at the point. The other planes intersect the quadric in conics which are tangent to the intersection lines with the first plane.*

Points on one triple line:

Type G_4**:** *Ordinary arrangement* p_4^1 *point: no two surfaces and no two curves are tangent at this point. Observe that the triple line can be the intersection of three planes or of two planes and the quadric.*

Type G_5**:** *Intersection point of three planes and the quadric where the planes intersect in a triple line. No plane is tangent to the quadric and the planes intersect the quadric in conics which are tangent to the triple line.*

Type G_6**:** *Intersection point of three planes and the quadric where the planes intersect in a triple line. One plane is tangent to the quadric at* P*, and the other two planes intersect the quadric in conics which are tangent to the triple line.*

Type G_7**:** *Intersection point of three planes and the quadric where two planes and the quadric intersect in a triple line. One of the two planes is tangent to the quadric at* P*. The third plane intersects the quadric in a conic which is tangent to the intersection line with the tangent plane.*

Points on two triple lines:

Type G_8**:** *Intersection point of three planes and the quadric where one plane is tangent to the quadric at the point and the other two planes pass through the two intersection lines. All intersection curves are lines.*

Proof:
We only have to consider intersection points of three planes and the quadric. Let P be the 4–fold point and let there be no triple lines through P. A priori there are the following sets of subconfigurations of two planes and the quadric: $\{F_1, F_1, F_1\}$, $\{F_1, F_1, F_2\}$, $\{F_1, F_2, F_2\}$, $\{F_2, F_2, F_2\}$, $\{F_1, F_1, F_3\}$, $\{F_1, F_2, F_3\}$, $\{F_2, F_2, F_3\}$, $\{F_1, F_3, F_3\}$, $\{F_2, F_3, F_3\}$, $\{F_3, F_3, F_3\}$.

Let the planes be called A, B, C and the quadric Q. If one of the subconfigurations is of type F_3 then without loss of generality plane A is tangent to the quadric at P and the two subconfigurations containing A are of type F_3. The conic $B \cap Q$ is tangent to the line $A \cap B$ and the conic $C \cap Q$ is tangent to the line $A \cap C$ so the subconfiguration not containing A is of type F_1 and the 4–fold point is of type G_3.

If none of the subconfigurations is of type F_3 than at most one can be of type F_2 (otherwise three intersection conics would be tangent to the same line). The set $\{F_1, F_1, F_1\}$ corresponds to type G_1, the set $\{F_1, F_1, F_2\}$ corresponds to type G_2.

Now let there be exactly one triple line through P. If the triple line is the intersection of the three planes then we can again consider the possible sets of subconfigurations of two planes and the quadric. If there is a subconfiguration of type F_3 then again there is a second one, and the third one must be of type F_2. This corresponds to type G_6. If there is no subconfiguration of type F_3 and if there is one of type F_2 then all three must be of type F_2. This corresponds to type G_5. The set $\{F_1, F_1, F_1\}$ corresponds to type G_4.

If the triple line is the intersection of two planes (say A, B) with the quadric then the subconfigurations of three surfaces corresponding to A, C, Q and to B, C, Q can only be of type F_1 or type F_3. If they are both of type F_1 then the 4–fold point is of type G_4. If one is of type F_3 and the other of type F_1 then the 4–fold point is of type G_7. If they were both of type F_3 then plane C would be tangent to the quadric and there would be a fourfold line.

Now let there be exactly two triple lines through P. Then one plane is tangent to the quadric at P and the other two planes pass through the two intersection lines. All intersection curves are lines so there is no tangency of curves. The 4–fold point is of type G_8. □

4.8 Lemma
There are exactly fifteen different types of 5–fold points (as intersection of five planes or four planes and a smooth quadric) not contained in a fourfold line:

Points not on triple lines:

Type H_1**:** *Ordinary arrangement* p_5^0 *point: no two surfaces and no two curves are tangent at this point.*

Type H_2**:** *Intersection point of four planes and the quadric where no plane is tangent to the quadric. Two planes intersect the quadric in conics which are tangent to the intersection line of the two planes at the point.*

Type H_3**:** *Intersection point of four planes and the quadric where no plane is tangent to the quadric. Two pairs of planes intersect the quadric in conics which are tangent to the intersection line of the two planes at the point.*

Type H_4**:** *Intersection point of four planes and the quadric where one plane is tangent to the quadric at the point. The other planes intersect the quadric in conics which are tangent to the intersection lines with the first plane.*

Points on one triple line:

Type H_5**:** *Ordinary arrangement* p_5^1 *point: no two surfaces and no two curves are tangent at this point. Observe that the triple line can be the intersection of three planes or of two planes and the quadric.*

Type H_6**:** *Intersection point of four planes and the quadric where no plane is tangent to the quadric. Two planes intersect the quadric in conics which are tangent to the intersection line of the two planes at the point. Observe that the triple line can be the intersection of three planes or of two planes and the quadric.*

Type H_7**:** *Intersection point of four planes and the quadric where three planes intersect in a triple line and the fourth plane is tangent to the quadric at the point. The first three planes intersect the quadric in conics which are tangent to the intersection lines with the fourth plane.*

Type H_8**:** *Intersection point of four planes and the quadric where three planes intersect in a triple line and the fourth plane is tangent to the quadric at the point. The first three planes intersect the quadric in conics which are tangent to the triple line.*

Type H_9**:** *Intersection point of four planes and the quadric where three planes intersect in a triple line and one of these planes is tangent to the quadric. The other two planes intersect the quadric in conics which are tangent to the triple line.*

Type H_{10}**:** *Intersection point of four planes and the quadric where two planes and the quadric intersect in a triple line. One of the two planes is tangent to the quadric at the point. The other two planes intersect the quadric in conics which are tangent to the intersection line with the tangent plane.*

Points on two triple lines:

Type H_{11}**:** *Ordinary arrangement* p_5^2 *point: no two surfaces and no two curves are tangent at this point. Observe that the triple lines can be the intersections of three planes or of two planes and the quadric.*

Type H_{12}**:** *Intersection point of four planes and the quadric where three planes intersect in a triple line and also two planes and the quadric intersect in a triple line. The plane* *containing only the second triple line is tangent to the quadric at the point. The two planes containing only the first triple line intersect the quadric in conics which are tangent to the intersection lines with the tangent plane.*

Type H_{13}**:** *Intersection point of four planes and the quadric where three planes intersect in a triple line and also two planes and the quadric intersect in a triple line. The plane containing both triple lines is tangent to the quadric at the point. The two planes containing only the first triple line intersect the quadric in conics which are tangent to the triple line.*

Type H_{14}**:** *Intersection point of four planes and the quadric where one plane is tangent to the quadric and two other planes pass through the intersection lines with the quadric. The fourth plane intersects the quadric in a conic which is tangent to the intersection line with the tangent plane.*

Points on three triple lines:

Type H_{15}**:** *Intersection point of four planes and the quadric where one plane is tangent to the quadric and two other planes pass through the intersection lines with the quadric. The fourth plane goes through the intersection line of these two planes and intersects the quadric in a conic which is tangent to the intersection line with the tangent plane.*

Proof:
The proof works like the one of lemma 4.7, by inspection of the possible sets of subconfigurations of four planes or three planes and the quadric. These are the sets of subconfigurations corresponding to the fifteen types of 5–fold points (where $ijklm$ stands for the set $\{G_i, G_j, G_k, G_l, G_m\}$):

H_1	11111
H_2	11122
H_3	12222
H_4	11333
H_5	11144

H_6	12244
H_7	33344
H_8	22245
H_9	23346
H_{10}	11377

H_{11}	14444
H_{12}	34477
H_{13}	24677
H_{14}	11778
H_{15}	44778

□

Experiments

Based on these results I performed some numerical experiments, counting points on double coverings of $\mathbb{P}^3$ branched along the union of six planes and a smooth quadric.

By lemma 4.4 there are only 10 possible arrangements of six planes containing no sixfold points and no fourfold lines. They all contain a triple point and can thus be given by an equation of the form

$$xyzt \cdot f(x, y, z, t) \cdot g(x, y, z, t) = 0$$

with certain linear polynomials f and g (cf. lemma 4.4 and the table below). For all 10 arrangements of six planes I investigated double coverings of $\mathbb{P}^3$ branched along the octic surface given by

$$xyzt \cdot f(x,y,z,t) \cdot g(x,y,z,t) \cdot Q(x,y,z,t) = 0$$

where

$$Q(x,y,z,t) = (a_0x^2 + a_1y^2 + a_2z^2 + a_3t^2 + a_4xy + a_5xz + a_6xt + a_7yz + a_8yt + a_9zt)$$

with $a_i \in \mathbb{Z}$, $|a_i| \leq 2$ such that $\{Q = 0\}$ is a smooth quadric surface. I determined the number of singular points of the different possible types, counted points over finite fields and compared with coefficients of weight four newforms. Many examples seem to be modular. They are listed in the tables in chapter B. If some examples are not separated by a horizontal line then they have the same numbers and types of singularities (I did not include the numbers and types in the table for layout reasons). Note that this does not mean that the geometry is the same. There are examples with the same numbers and types of singularities but different weight four newforms in their L-series.

The listed weight four newforms are always the twists of minimal level (they can be obtained by multiplying the equation of the octic by certain nonsquare numbers). I also predict if the (resolved) double octics will be rigid. This is based on numerical observations: If a Calabi–Yau threefold X defined over $\mathbb{Q}$ is rigid then most likely for good primes p the expression $X_p - a_p(X)$ will be a polynomial in p up to $p^2 + p$ times some Legendre symbol(s), and vice versa. I am pretty sure that the examples which are predicted to be rigid are really rigid. Some of the examples which are predicted to be non-rigid might also be rigid.

If we want to prove the modularity of these examples then we will have to resolve the different types of singularities. Note also that S. Cynk's programs for computing Hodge numbers currently do not work for octics that are no arrangements so the Hodge numbers would have to be determined by counting points. Some parts of this procedure could probably be automated but would still require an enormous amount of work.

In the following table I list the equations of the arrangements of six planes and the number of resulting double octics with different numbers and types of singularities (altogether 19258). An extension of the parameter space of the quadrics (there are about 20 millions of examples with $|a_i| \leq 2$) would produce even more results but I believe that there will not be many new examples with $|a_i| \geq 4$. Anyway the numbers are too large to raise expectations of a complete classification.

no.	equation	# octics
1	$xyzt(x+y)(x+z)$	1428
2	$xyzt(x+y)(z+t)$	486
3	$xyzt(x+y)(x-y+z)$	1379
4	$xyzt(x+y)(x+z+t)$	3894
5	$xyzt(x+y)(x+y+z+t)$	3774
6	$xyzt(x+y+z)(x-y+2z)$	567
7	$xyzt(x+y+z+t)(x-y+2z-2t)$	1039

no.	equation	# octics
8	$xyzt(x+y+z)(x+2y-z+t)$	2569
9	$xyzt(x+y+z+t)(x+y-z-t)$	2070
0	$xyzt(x+y+z)(x+y+t)$	2052

Note that if the quadric is not smooth but has a node then there are also many interesting examples but I did not include them here.

Note also that by a general construction (which will be explained in 4.6) there is a correspondence between the double octic given by the equation

$$u^2 = xyzt \cdot f(x,y,z,t) \cdot g(x,y,z,t) \cdot Q(x,y,z,t)$$

and the double octic given by the equation

$$u^2 = f(x^2,y^2,z^2,t^2) \cdot g(x^2,y^2,z^2,t^2) \cdot Q(x^2,y^2,z^2,t^2).$$

In general the Hodge numbers of the two double octics will be different but if a weight four newform occurs in the L-series of one of them then it should also occur in the L-series of the other.

If the six planes are the faces of the cube (sextic arrangement no. 0) then they can also be given by the equation

$$0 = (x-t)(x+t)(y-t)(y+t)(z-t)(z+t) = (x^2-t^2)(y^2-t^2)(z^2-t^2).$$

By the same construction as above there is a correspondence between the double octic given by the equation

$$u^2 = (x-t)(x+t)(y-t)(y+t)(z-t)(z+t)(ax^2+by^2+cz^2+dt^2)$$

and the double octic given by the equation

$$u^2 = xyzt(x-t)(y-t)(z-t)(ax+by+cz+dt).$$

If the six planes form a sextic arrangement of type no. 9 (two fourfold points) then they can also be given by the equation

$$0 = (x-t)(x+t)(y-t)(y+t)(x-z)(x+z) = (x^2-t^2)(y^2-t^2)(x^2-z^2).$$

By the same construction as above there is a correspondence between the double octic given by the equation

$$u^2 = (x-t)(x+t)(y-t)(y+t)(x-z)(x+z)(ax^2+by^2+cz^2+dt^2)$$

and the double octic given by the equation

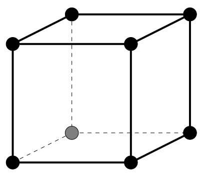
$$u^2 = xyzt(x-t)(y-t)(x-z)(ax+by+cz+dt).$$

We finish this section with some examples that have nice geometrical descriptions. All of them can be realized as the union of the faces of a cube and a quadric.

A ball through the vertices of a cube

Consider (as a real picture) the union of the faces of a symmetric cube (i.e., an arrangement of six planes of type 0) and a ball through its eight vertices. Such an octic surface can be given by the equation

$$(x-t)(x+t)(y-t)(y+t)(z-t)(z+t)(x^2+y^2+z^2-3t^2)=0.$$



The double covering X_1 of $\mathbb{P}^3$ branched along this surface is in correspondence with the double octic given by the equation

$$u^2 = xyzt(x-t)(y-t)(z-t)(x+y+z-3t)$$

which is arrangement no. 6 from 4.2. Consequently the weight four newform 96/4 ($96k4B1$) occurs in the L-series of a resolution $\tilde{X}_1$ of X_1. Moreover by numerical observation we have $h^{2,1}(\tilde{X}_1)=2$ and $a_p(\tilde{X}_1)=b_p+2p\cdot c_p$ where b_p are the coefficients of the newform 96/4 and c_p are the coefficients of the weight two newform $32A1$.

Indeed a general quadric surface through the vertices of the above cube is given by the equation

$$Ax^2+By^2+Cz^2-(A+B+C)t^2=0$$

with $(A:B:C)\in\mathbb{P}^2$; and an equation for a 2-dimensional family of double octics containing arrangement no. 6 is given by the equation

$$u^2 = xyzt(x-t)(y-t)(z-t)(Ax+By+Cz-(A+B+C)t).$$

Note that in these families there are examples with smaller number of deformations. If $A=0$ or $B=0$ or $C=0$ or $A+B+C=0$ then the quadric is nodal. For example, consider the double octic Y_1 given by the equation

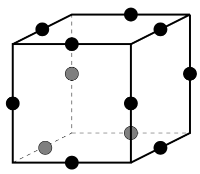
$$u^2=(x-t)(x+t)(y-t)(y+t)(z-t)(z+t)(x^2+y^2-2z^2).$$

By numerical observation we have $h^{2,1}(\tilde{Y}_1)=1$ for a resolution $\tilde{Y}_1$ of Y_1, and $a_p(\tilde{Y}_1)=b_p+p\cdot c_p$ where b_p are the coefficients of the weight four newform 32/1 ($32k4A1$) and c_p are the coefficients of the weight two newform $32A1$. The corresponding arrangement of eight planes is no. 4 (with the same L-series).

A ball through the midpoints of the edges of a cube

Consider (as a real picture) the union of the faces of a symmetric cube and a ball through the 12 midpoints of its edges. Such an octic surface can be given by the equation

$$(x-t)(x+t)(y-t)(y+t)(z-t)(z+t)(x^2+y^2+z^2-2t^2)=0.$$



The double covering X_2 of $\mathbb{P}^3$ branched along this surface is in correspondence with the double octic given by the equation

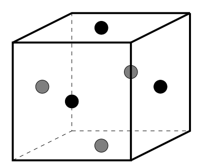
$$u^2 = xyzt(x-t)(y-t)(z-t)(x+y+z-2t)$$

which is the rigid arrangement no. 3 from 4.2. Consequently the weight four newform 32/2 ($32k4B1$) occurs in the L-series of a resolution $\tilde{X}_2$ of X_2. Moreover by numerical observation $\tilde{X}_2$ is rigid. Indeed the quadric surface is fixed by the 12 midpoints of the edges of the cube.

A ball through the midpoints of the faces of a cube

Consider (as a real picture) the union of the faces of a symmetric cube and a ball through the 6 midpoints of its faces. Such an octic surface can be given by the equation

$$(x-t)(x+t)(y-t)(y+t)(z-t)(z+t)(x^2+y^2+z^2-t^2)=0.$$



The double covering X_3 of $\mathbb{P}^3$ branched along this surface is in correspondence with the double octic given by the equation

$$u^2 = xyzt(x-t)(y-t)(z-t)(x+y+z-t)$$

which is also the rigid arrangement no. 3 from 4.2. Consequently the weight four newform 32/2 ($32k4B1$) occurs in the L-series of a resolution $\tilde{X}_3$ of X_3. Moreover by numerical observation we have $h^{2,1}(\tilde{X}_3)=3$ and $a_p(\tilde{X}_3)=b_p+3p\cdot c_p$ where b_p are the coefficients of the newform 32/2 and c_p are the coefficients of the weight two newform $32A1$. Indeed a general quadric surface through the midpoints of the faces of the above cube is given by the equation

$$A(x^2 + y^2 + z^2 - t^2) + Bxy + Cxz + Dyz = 0$$

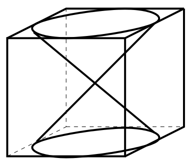
with $(A : B : C : D) \in \mathbb{P}^3$. It is rather interesting that in this case (unlike in the two previous cases) the Hodge numbers $h^{2,1}$ of the two relatives are not the same. Note that in the two previous cases the correspondences work for the whole families.

A cone and the faces of a cube

Consider the octic surface given by the equation

$$(x-t)(x+t)(y-t)(y+t)(z-t)(z+t)(x^2 + y^2 - z^2) = 0.$$

It can be desrcibed as the union of the faces of a symmetric cube and a cone through eight of the midpoints of its edges.



Let X_4 be a double covering of $\mathbb{P}^3$ branched along this surface. By numerical observation we have $h^{2,1}(\tilde{X}_4) = 1$ for a resolution $\tilde{X}_4$ of X_4, and $a_p(\tilde{X}_4) = b_p + p \cdot c_p$ where b_p are the coefficients of the weight four newform 32/2 ($32k4B1$) and c_p are the coefficients of the weight two newform $32A1$. The corresponding arrangement of eight planes (cf. 4.2) is the rigid arrangement no. 3.

Finally consider the octic surface given by the equation

$$(x-t)(x+t)(y-t)(y+t)(z-t)(z+t)(x^2 + y^2 + z^2) = 0.$$

The quadric is a complex cone and has no nice real geometric description. Let X_5 be a double covering of $\mathbb{P}^3$ branched along this surface. Then the weight four newform 96/4 ($96k4B1$) occurs in the L-series of a resolution $\tilde{X}_5$ of X_5. The corresponding arrangement of eight planes (cf. 4.2) is arrangement no. 6.

4.4 Four planes and two quadrics

If we investigate double coverings of $\mathbb{P}^3$ branched along the union of four planes and two quadrics then the situation becomes even more complicated. I confined myself to performing some numerical experiments with a special type of such double octics.

We consider unions of four planes and two quadrics of the form

$$xyzt \cdot (A(x^2+y^2+z^2+t^2)+B(xy+zt)+C(x+y)(z+t)) \\ \cdot (D(x^2+y^2+z^2+t^2)+E(xy+zt)+F(x+y)(z+t)) = 0.$$

with $(A:B:C) \in \mathbb{P}^2$, $(D:E:F) \in \mathbb{P}^2$. The group $(\mathbb{Z}/2\mathbb{Z})^3$ which is generated by the permutations (xy), (zt) and $(xz)(yt)$ acts on such octic surfaces. The surface with parameters $(A:B:-C)$, $(D:E:-F)$ corresponds to the coordinate change $z \mapsto -z$, $t \mapsto -t$. If we have $B=C$ and $E=D$ then the octic surface is even Σ_4-symmetric. Double coverings of $\mathbb{P}^3$ branched along surfaces of this type occur in 4.11 as relatives of double coverings of $\mathbb{P}^3$ branched along the union of two Heisenberg-invariant quartics.

The discriminant of the quadric surface given by the equation

$$A(x^2+y^2+z^2+t^2)+B(xy+zt)+C(x+y)(z+t)=0$$

is easily computed to be

$$(2A-B)^2(2A+B+2C)(2A+B-2C).$$

If exactly one factor vanishes then the quadric is nodal; if exactly two factors vanish then the quadric is the union of two planes; if more than two factors vanish then the quadric is a double plane.

For many values of $(A:B:C)$, $(D:E:F)$ the double covering of $\mathbb{P}^3$ branched along the corresponding octic surface seems to be modular (i.e., for each considered good prime p the number of points on the threefold agrees with the coefficient of a weight four newform modulo p). Here I multiplied the equations of the branch loci with a certain factor to get a twisted newform of minimal level. The table lists the parameters, the (twists of minimal level of the) occurring newforms and the types of the two quadrics (p means two planes, n means nodal and s means smooth). It contains all examples with $|A|,|B|,|C|,|D|,|E|,|F| \le 8$ and a few additional examples. If an example is also mentioned in 4.11 then there is a remark. If one of the quadrics is the union of two planes and the other quadric is smooth then the double octic should also occur in the tables in chapter B.

$(A:B:C)$	$(D:E:F)$	weight four newform	type	remark
$(0:0:1)$	$(0:1:0)$	32/1 $(32k4A1)$	ps	
$(0:0:1)$	$(0:2:1)$	8/1 $(8k4A1)$	pn	
$(0:0:1)$	$(1:-6:2)$	128/1 $(8k4A1)$	pn	
$(0:0:1)$	$(1:-2:0)$	32/1 $(32k4A1)$	pn	
$(0:0:1)$	$(1:-2:2)$	8/1 $(8k4A1)$	ps	
$(0:0:1)$	$(1:1:0)$	32/1 $(32k4A1)$	ps	
$(0:0:1)$	$(3:10:0)$	32/1 $(32k4A1)$	ps	
$(0:1:0)$	$(0:2:1)$	8/1 $(8k4A1)$	sn	
$(0:1:0)$	$(1:-6:2)$	32/1 $(32k4A1)$	sn	
$(0:1:0)$	$(1:-2:0)$	8/1 $(8k4A1)$	sn	

$(A:B:C)$	$(D:E:F)$	weight four newform		type	remark
$(0:1:0)$	$(1:-2:2)$	32/2	$(32k4B1)$	ss	
$(0:1:0)$	$(1:2:0)$	8/1	$(8k4A1)$	sp	
$(0:1:0)$	$(1:6:2)$	96/4	$(96k4B1)$	ss	
$(0:1:1)$	$(1:-2:-2)$	96/2	$(96k4E1)$	ss	4.11
$(0:1:1)$	$(1:-1:-1)$	24/1	$(24k4A1)$	ss	4.11
$(0:1:1)$	$(1:1:1)$	120/4	$(120k4F1)$	ss	4.11
$(0:1:1)$	$(3:-2:-2)$	96/1	$(96k4D1)$	sn	4.11
$(0:2:1)$	$(0:2:-1)$	8/1	$(8k4A1)$	nn	
$(0:2:1)$	$(1:-22:-2)$	360/2		ns	
$(0:2:1)$	$(1:-7:-2)$	120/2	$(120k4D1)$	ns	
$(0:2:1)$	$(1:-6:-2)$	8/1	$(8k4A1)$	nn	
$(0:2:1)$	$(1:-6:2)$	14/2	$(14k4A1)$	nn	
$(0:2:1)$	$(1:-2:-4)$	96/2	$(96k4E1)$	ns	
$(0:2:1)$	$(1:-2:-2)$	40/3	$(40k4A1)$	ns	
$(0:2:1)$	$(1:-2:0)$	32/2	$(32k4B1)$	nn	
$(0:2:1)$	$(1:-2:2)$	24/1	$(24k4A1)$	ns	
$(0:2:1)$	$(1:1:2)$	168/2	$(168k4E1)$	ns	
$(0:2:1)$	$(1:10:2)$	6/1	$(6k4A1)$	ns	
$(0:2:1)$	$(1:14:4)$	96/4	$(96k4B1)$	ns	
$(0:2:1)$	$(2:-4:1)$	30/2	$(30k4A1)$	ns	
$(0:2:1)$	$(2:4:-3)$	14/1	$(14k4B1)$	np	
$(0:2:1)$	$(2:4:5)$	6/1	$(6k4A1)$	np	
$(0:2:1)$	$(4:-10:1)$	168/1	$(168k4A1)$	nn	
$(0:2:1)$	$(4:-7:-2)$	120/3	$(120k4C1)$	ns	
$(0:3:1)$	$(1:1:3)$	120/4	$(120k4F1)$	ss	
$(0:3:2)$	$(2:2:3)$	168/2	$(168k4E1)$	sn	
$(0:4:1)$	$(1:-14:-6)$	32/2	$(32k4B1)$	sn	
$(0:4:1)$	$(1:-14:-2)$	96/2	$(96k4E1)$	ss	
$(0:4:1)$	$(1:-2:1)$	12/1	$(12k4A1)$	ss	
$(0:4:1)$	$(3:-10:2)$	96/4	$(96k4B1)$	sn	
$(0:6:1)$	$(1:10:2)$	8/1	$(8k4A1)$	ss	
$(0:8:1)$	$(1:-2:2)$	30/2	$(30k4A1)$	ss	
$(1:-6:2)$	$(1:-6:-2)$	32/2	$(32k4B1)$	nn	
$(1:-6:2)$	$(1:-2:-2)$	14/2	$(14k4A1)$	ns	
$(1:-6:2)$	$(1:-2:0)$	32/1	$(32k4A1)$	nn	
$(1:-6:2)$	$(1:-2:2)$	8/1	$(8k4A1)$	ns	
$(1:-2:0)$	$(1:-2:2)$	8/1	$(8k4A1)$	ns	
$(1:-2:0)$	$(1:0:1)$	96/4	$(96k4B1)$	nn	
$(1:-2:0)$	$(1:2:0)$	8/1	$(8k4A1)$	np	
$(1:-2:2)$	$(1:-2:-2)$	8/1	$(8k4A1)$	ss	
$(1:-2:2)$	$(1:-1:1)$	96/4	$(96k4B1)$	ss	4.11
$(1:-2:2)$	$(2:4:-3)$	14/1	$(14k4B1)$	sp	

$(A:B:C)$	$(D:E:F)$	weight four newform	type	remark
$(1:-2:2)$	$(2:4:5)$	$6/1$ $(6k4A1)$	sp	
$(1:-2:2)$	$(3:-2:2)$	$6/1$ $(6k4A1)$	sn	4.11
$(1:-1:1)$	$(3:-2:2)$	$96/1$ $(96k4D1)$	sn	4.11
$(1:-1:1)$	$(3:2:-2)$	$480/5$	ss	4.11
$(1:1:1)$	$(3:2:2)$	$480/2$	ss	4.11
$(3:-6:2)$	$(3:-2:2)$	$8/1$ $(8k4A1)$	sn	

Note also that by a general construction (which will be explained in 4.6) there is a correspondence between the double octic given by the equation

$$\begin{aligned}u^2 = xyzt \cdot (&A(x^2+y^2+z^2+t^2) + B(xy+zt) + C(x+y)(z+t)) \\ \cdot (&D(x^2+y^2+z^2+t^2) + E(xy+zt) + F(x+y)(z+t))\end{aligned}$$

and the double octic given by the equation

$$\begin{aligned}u^2 = (&A(x^4+y^4+z^4+t^4) + B(x^2y^2+z^2t^2) + C(x^2+y^2)(z^2+t^2)) \\ \cdot (&D(x^4+y^4+z^4+t^4) + E(x^2y^2+z^2t^2) + F(x^2+y^2)(z^2+t^2)).\end{aligned}$$

In general the Hodge numbers of the two double octics will be different but if a weight four newform occurs in the L-series of one of them then it should also occur in the L-series of the other.

If four planes meet in a point then they can also be given by the equation

$$0 = (x-t)(x+t)(y-t)(y+t) = (x^2-t^2)(y^2-t^2).$$

By the same construction as above there is a correspondence between the double octic given by the equation

$$u^2 = (x-t)(x+t)(y-t)(y+t)(ax^2+by^2+cz^2+dt^2)(a'x^2+b'y^2+c'z^2+d't^2)$$

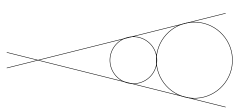
and the double octic given by the equation

$$u^2 = xyzt(x-t)(y-t)(ax+by+cz+dt)(a'x+b'y+c'z+d't).$$

We finish this section with certain explicit examples.

Two balls in a paperbag

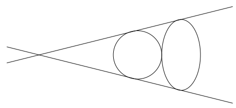
Consider (as a real picture) four planes which meet in a fourfold point and two balls which touch each plane and have one point in common. This is a picture of the two-dimensional analogon:



Such an octic surface can be given by the equation

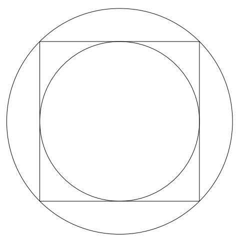
$$(x-t)(x+t)(y-t)(y+t)(x^2+y^2+(z-t)^2-t^2)(x^2+y^2+(z+t)^2-t^2)=0.$$

Let X_1 be a double covering of $\mathbb{P}^3$ branched along this surface and let $\tilde{X}_1$ be a resolution of X_1. By numerical observation we have $h^{2,1}(\tilde{X}_1)=1$ and $a_p(\tilde{X}_1)=b_p+p\cdot c_p$ where b_p are the coefficients of the weight four newform 32/1 (32k4A1) and c_p are the coefficients of the weight two newform 32A1. Indeed (in the real picture again) one of the quadrics can always be chosen to be a ball and the other will be an ellipsoid with radii depending only on its center.



More experiments

Consider (as a real picture) four planes which meet in a fourfold point and two balls with the same center, one touching all the planes and one through the intersection lines of the planes such that the lines are tangent in the intersection points. This is a picture of the situation "from above":



Such an octic surface can be given by the equation

$$(x-t)(x+t)(y-t)(y+t)(x^2+y^2+z^2-t^2)(x^2+y^2+z^2-2t^2)=0.$$

Let X_2 be a double covering of $\mathbb{P}^3$ branched along this surface and let $\tilde{X}_2$ be a resolution of X_2. By numerical observation $\tilde{X}_2$ is rigid and its L-series is given by the L-series of the weight four newform 32/2 (32k4B1). It is in correspondence with the double octic given by the equation

$$u^2=xyzt(x-t)(y-t)(x+y+z+t)(x+y+z-2t)$$

which is the rigid arrangement no. 3 from 4.2.

We can more generally investigate the double covering $X_{A,B,C,D}$ of $\mathbb{P}^3$ branched along the octic surface given by the equation

$$(x-t)(x+t)(y-t)(y+t)(A\cdot(x^2+y^2)+z^2+B\cdot t^2)(C\cdot(x^2+y^2)+z^2+D\cdot t^2)=0,$$

where $A \neq 0$ and $C \neq 0$. The quadrics are nodal for $B = 0$ resp. $D = 0$. There is always a relative of $X_{A,B,C,D}$ which is a double covering of $\mathbb{P}^3$ branched along the arrangement of planes given by the equation

$$xyzt(x-t)(y-t)(A\cdot(x+y)+z+B\cdot t)(C\cdot(x+y)+z+D\cdot t)=0.$$

For certain values of A, B, C, D the resolution $\tilde{X}_{A,B,C,D}$ of $X_{A,B,C,D}$ seems to be modular. In the table we list these values A, B, C, D, the occurring weight four newform, a prediction of the Hodge number $h^{2,1} = h^{2,1}(\tilde{X}_{A,B,C,D})$ (based on numerical observations) and the corresponding arrangement of planes.

A, B	C, D	weight four newform	$h^{2,1}$	arr. of planes
$1, -3$	$1, -2$	96/4 (96k4B1)	2	no. 6
$1, -2$	$1, -1$	32/2 (32k4B1)	0	no. 3
$1, -1$	$1, 0$	32/2 (32k4B1)	1	no. 3
$1, 0$	$1, 1$	96/4 (96k4B1)	2	no. 6
$1, -2$	$1, 0$	32/1 (32k4A1)	1	no. 4
$1, -2$	$2, -2$	8/1 (8k4A1)	2	no. 32
$1, -1$	$2, -2$	32/1 (32k4A1)	2	no. 13
$1, 0$	$2, -2$	8/1 (8k4A1)	0	no. 32

Playing with correspondences

We will display correspondences between various double octics. All correspondences are based on the general construction explained in 4.6.

Consider the double octic given by the equation

$$u^2 = xyzt(x^2+y^2+z^2+t^2)(x^2+y^2-z^2-t^2). \qquad (I)$$

By the coordinate change $x \mapsto x+y$, $y \mapsto x-y$, $z \mapsto z+t$, $t \mapsto z-t$ we get the equation

$$u^2 = (x+y)(x-y)(z+t)(z-t)(x^2+y^2+z^2+t^2)(x^2+y^2-z^2-t^2). \qquad (I^*)$$

There is a correspondence between $(I*)$ and the double octic given by the equation

$$u^2 = xyzt(x-y)(z-t)(x+y+z+t)(x+y-z-t). \qquad (II)$$

This is arrangement no. 58 from 4.2. By changing the sign of y and t we get the equation

$$u^2 = xyzt(x+y)(z+t)(x-y+z-t)(x-y-z+t). \qquad (II^*)$$

There is a correspondence between $(II*)$ and the double octic given by the equation

$$u^2 = (x^2+y^2)(z^2+t^2)(x^2-y^2+z^2-t^2)(x^2-y^2-z^2+t^2). \qquad (III)$$

By the coordinate change $x \mapsto x+y$, $y \mapsto x-y$, $z \mapsto z+t$, $t \mapsto z-t$ we get the equation

$$u^2 = (x^2+y^2)(z^2+t^2)(xy+zt)(xy-zt). \qquad (III^*)$$

There is a correspondence between $(III*)$ and the double octic given by the equation

$$u^2 = xyzt(x+y)(z+t)(xy-zt). \qquad (IV)$$

By changing the sign of y and t we get the equation

$$u^2 = -xyzt(x-y)(z-t)(xy-zt). \qquad (IV*)$$

There is a correspondence between $(IV*)$ and the double octic given by the equation

$$u^2 = -(x+y)(x-y)(z+t)(z-t)(xy-zt)(xy+zt). \qquad (V)$$

By the coordinate change $x \mapsto x+y$, $y \mapsto x-y$, $z \mapsto z+t$, $t \mapsto z-t$ and a sign change of x we get the equation

$$u^2 = xyzt(x^2-y^2+z^2-t^2)(x^2-y^2-z^2+t^2). \qquad (V*)$$

There is also a correspondence between (I) and the double octic given by the equation

$$u^2 = (x^4+y^4+z^4+t^4)(x^4+y^4-z^4-t^4), \qquad (VI)$$

and there is a correspondence between $(V*)$ and the double octic given by the equation

$$u^2 = (x^4-y^4+z^4-t^4)(x^4-y^4-z^4+t^4). \qquad (VII)$$

The weight four newform 32/1 ($32k4A1$) occurs in the L-series of (resolutions of) all listed double octics. The Hodge numbers seem to be different. The table predicts $h^{2,1}$ based on numerical observations (where possible):

$(I),(I^*)$	$(II),(II)^*$	$(III),(III^*)$	$(IV),(IV^*)$	$(V),(V^*)$	(VI)	(VII)
3	3	3	0	3	$>3?$	$>3?$

For all examples with $h^{2,1}=3$ the L-series seems to split into $b_p + 3p \cdot c_p$ where b_p are the coefficients of the newform 32/1 and c_p are the coefficients of the weight two newform $32A1$.

It is rather remarkable that we can see immediately that the examples (I) and (V) are birationally equivalent over $\mathbb{Q}[\sqrt{-1}]$ and that the examples (VI) and (VII) are birationally equivalent over $\mathbb{Q}[\sqrt[4]{-1}]$. The correspondences between them that we have found in this section are defined over $\mathbb{Q}$ but they do not seem to be induced by birational maps.

4.5 Four quadrics

In the general case of the union of four quadrics we will restrict ourselves to single examples. Most of them will be relatives of arrangements of planes, and a correspondence will be given by the general construction from 4.6.

Four nodal quadrics related to arrangement no. 239

Consider the octic surface given by the equation

$$(x^2+y^2+z^2)(y^2+z^2+t^2)(z^2+t^2+x^2)(t^2+x^2+y^2)=0.$$

It is the union of four nodal quadrics. The double covering X_1 of $\mathbb{P}^3$ branched along this surface is in correspondence with the double octic given by the equation

$$u^2=xyzt(x+y+z)(y+z+t)(z+t+x)(t+x+y)$$

which is the rigid arrangement no. 239 from 4.2. Consequently the weight four newform 12/1 ($12k4A1$) occurs in the L-series of a resolution $\tilde{X}_1$ of X_1. Moreover by numerical observation we have $h^{2,1}(\tilde{X}_1)=6$ and $a_p(\tilde{X}_1)=b_p+6p\cdot c_p$ where b_p are the coefficients of the newform 12/1 and c_p are the coefficients of the weight two newform $48A1$.

Four nodal quadrics related to arrangement no. 317

Consider the octic surface given by the equation

$$(x^2+2y^2+z^2)(y^2+2z^2+t^2)(z^2+2t^2+x^2)(t^2+2x^2+y^2)=0.$$

It is also the union of four nodal quadrics. The double covering X_2 of $\mathbb{P}^3$ branched along this surface is in correspondence with the double octic given by the equation

$$u^2=xyzt(x+2y+z)(y+2z+t)(z+2t+x)(t+2x+y)$$

which is arrangement no. 317 from 4.2. Consequently the weight four newform 12/1 ($12k4A1$) occurs in the L-series of a resolution $\tilde{X}_2$ of X_2. Moreover by numerical observation we have $h^{2,1}(\tilde{X}_2)=6$ and $a_p(\tilde{X}_2)=b_p+6p\cdot c_p$ where b_p are the coefficients of the newform 12/1 and c_p are the coefficients of the weight two newform $48A1$.

There should be a correspondence between X_1 and X_2 (and so a correspondence between arrangements no. 239 and no. 317) but I have not been able to find one.

Four smooth quadrics related to arrangement no. 239

Consider the octic surface given by the equation

$$(x^2+y^2+z^2-2t^2)(x^2+y^2-2z^2+t^2)(x^2-2y^2+z^2+t^2)(-2x^2+y^2+z^2+t^2)=0.$$

It is the union of four smooth quadrics. The double covering X_3 of $\mathbb{P}^3$ branched along this surface is in correspondence with the double octic given by the equation

$$u^2=xyzt(x+y+z-2t)(x+y-2z+t)(x-2y+z+t)(-2x+y+z+t)$$

which is again the rigid arrangement no. 239 from 4.2. Consequently the weight four newform 12/1 ($12k4A1$) occurs in the L-series of a resolution $\tilde{X}_3$ of X_3. Moreover by numerical observation $\tilde{X}_3$ seems to be rigid.

Four smooth quadrics related to arrangement no. 3

Consider the octic surface given by the equation

$$(x^2+y^2+z^2+t^2)(x^2+y^2+z^2-t^2)(x^2+y^2-z^2+t^2)(x^2-y^2+z^2+t^2).$$

It is the union of four smooth quadrics. The double covering X_4 of $\mathbb{P}^3$ branched along this surface is in correspondence with the double octic given by the equation

$$u^2 = xyzt(x+y+z+t)(x+y+z-t)(x+y-z+t)(x-y+z+t)$$

which is the rigid arrangement no. 3 from 4.2. Consequently the weight four newform 32/2 ($32k4B1$) occurs in the L-series of a resolution $\tilde{X}_4$ of X_4. Moreover by numerical observation $\tilde{X}_4$ seems to be rigid.

Two planes and three smooth quadrics related to arrangement no. 19

Consider the octic surface given by the equation

$$(x+y)(z+t)(xy+zt)(xz+yt)(xt+yz) = 0.$$

It is the union of two planes and three smooth quadrics. After the coordinate change $x \mapsto x+y+z+t$, $x \mapsto x-y+z-t$, $z \mapsto x+y-z-t$, $t \mapsto x-y-z+t$ this equation becomes

$$(x+z)(x-z)(x^2-y^2-z^2+t^2)(x^2-y^2+z^2-t^2)(x^2+y^2-z^2-t^2) = 0.$$

Thus the double covering X_5 of $\mathbb{P}^3$ branched along this surface is in correspondence with the double octic given by the equation

$$u^2 = xyzt(x-z)(x-y-z+t)(x-y+z-t)(x+y-z-t)$$

which is the rigid arrangement no. 19 from 4.2. Consequently the weight four newform 32/1 ($32k4A1$) occurs in the L-series of a resolution $\tilde{X}_5$ of X_5. Moreover by numerical observation $\tilde{X}_5$ seems to be rigid.

One smooth and three nodal quadrics related to arrangement no. 3

Consider the octic surface given by the equation

$$u^2 = (x^2+y^2+z^2+t^2)(x^2+y^2+z^2)(x^2+y^2+t^2)(x^2+z^2+t^2)$$

It is the union of one smooth and three nodal quadrics. The double covering X_6 of $\mathbb{P}^3$ branched along this surface is in correspondence with the double octic given by the equation

$$u^2 = xyzt(x+y+z+t)(x+y+z)(x+y+t)(x+z+t)$$

which is again the rigid arrangement no. 3 from 4.2. Consequently the weight four newform 32/2 ($32k4B1$) occurs in the L-series of a resolution $\tilde{X}_6$ of X_6. Moreover by numerical observation we have $h^{2,1}(\tilde{X}_6) = 3$ and $a_p(\tilde{X}_6) = b_p + 3p \cdot c_p$ where b_p are the coefficients of the newform 32/2 and c_p are the coefficients of the weight two newform $32A1$.

Four smooth quadrics related to arrangement no. 6

Consider the octic surface given by the equation

$$(x^2+y^2+z^2-2t^2)(x^2+y^2-2z^2+t^2)(x^2-2y^2+z^2+t^2)(x^2+y^2+z^2+t^2)=0.$$

It is the union of four smooth quadrics. The double covering X_7 of $\mathbb{P}^3$ branched along this surface is in correspondence with the double octic given by the equation

$$u^2=xyzt(x+y+z-t)(x+y-z+t)(x-y+z+t)(x+y+z+t)$$

which is arrangement no. 6 from 4.2. Consequently the weight four newform 96/4 ($96k4B1$) occurs in the L-series of a resolution $\tilde{X}_7$ of X_7. Moreover by numerical observation we have $h^{2,1}(\tilde{X}_7)=2$ and $a_p(\tilde{X}_7)=b_p+2p\cdot c_p$ where b_p are the coefficients of the newform 96/4 and c_p are the coefficients of the weight two newform $32A1$.

Four smooth quadrics related to an example by Nygaard and van Geemen

Consider the octic surface given by the equation

$$(xy-zt)(xy+zt)(xz+yt)(xt+yz)=0.$$

After the coordinate change $x\mapsto x+y$, $y\mapsto x-y$, $z\mapsto z+t$, $t\mapsto z-t$ this equation becomes

$$(x^2-y^2-z^2+t^2)(x^2-y^2+z^2-t^2)(xz+yt)(xz-yt)=0.$$

Thus the double covering X_8 of $\mathbb{P}^3$ branched along this surface is in correspondence with the double octic Y_8 given by the equation

$$u^2=xyzt(x-y-z+t)(x-y+z-t)(xz-yt).$$

The weight four newform 32/1 ($32k4A1$) occurs in the L-series of a resolution $\tilde{X}_8$ of X_8 and of a resolution $\tilde{Y}_8$ of Y_8. Moreover by numerical observation we have $h^{2,1}(\tilde{X}_8)=h^{2,1}(\tilde{Y}_8)=1$ and $a_p(\tilde{X}_8)=a_p(\tilde{Y}_8)=b_p+p\cdot c_p$ where b_p are the coefficients of the newform 32/1 and c_p are the coefficients of the weight two newform $32A1$.

There is an obvious correspondence between X_8 and the intersection Y of four quadrics in $\mathbb{P}^7$ with coordinates $(u_0:u_1:u_2:u_3:x:y:z:t)$ which is given by the following equations:

$$\begin{aligned}u_0^2&=2(xy+zt),\\u_1^2&=2(xz+yt),\\u_2^2&=2(xt+yt),\\u_3^2&=2(xy-zt).\end{aligned}$$

The variety Y has been examined by Nygaard and van Geemen in [75]. They show the following:

The singular locus of Y consists of the 16 ordinary nodes

$$(\pm\sqrt{2}:0:0:\pm\sqrt{2}:1:1:0:0),$$
$$(\pm\sqrt{-2}:0:0:\pm\sqrt{-2}:1:-1:0:0),$$
$$(\pm\sqrt{2}:0:0:\pm\sqrt{2}:0:0:1:1),$$
$$(\pm\sqrt{-2}:0:0:\pm\sqrt{-2}:0:0:1:-1),$$

and of the four plane conics (configured in a square) given by the equations

$$u_0 = u_2 = u_3 = x = z = 0,$$
$$u_0 = u_2 = u_3 = y = t = 0,$$
$$u_0 = u_1 = u_3 = x = t = 0,$$
$$u_0 = u_1 = u_3 = y = z = 0.$$

A (big) resolution Y' of Y can be obtained by first blowing up the 16 nodes and a pair of opposite sides in the square of conics and then blowing up the strict transforms of the other pair of conics. The Euler characteristic of such a resolution is $\chi(Y') = 80$, the Hodge numbers are $h^{1,1}(Y') = 41$ and $h^{2,1}(Y') = 1$. It is unknown if there exist projective small resolutions. There is an automorphism of Y defined by

$$(u_0 : u_1 : u_2 : u_3 : x : y : z : t) \mapsto (u_0 : u_2 : u_1 : \sqrt{-1}u_3 : z : t : x : y).$$

This lifts to an automorphism of Y' which induces a splitting of the L-series of Y' into two two-dimensional parts. Nygaard and van Geemen prove that

$$L(Y', s) = L(\psi^3, s)L(\psi, s-1)$$

where ψ is the Hecke character of $\mathbb{Q}[\sqrt{-1}]$. This means that

$$a_p(Y') = b_p + p \cdot c_p$$

where b_p are the coefficients of the newform 32/1 (32k4A1) and c_p are the coefficients of the weight two newform 32A1.

Nygaard and van Geemen also exhibited a correspondence between Y' and the triple product $E \times E \times E$ where $E = \{y^2 = 1 + x^4\}$ is the elliptic curve with complex multiplication by $\mathbb{Q}[\sqrt{-1}]$.

Four smooth quadrics related to arrangement no. 287

Let D_9 be the octic surface given by the equation

$$(x^2 + y^2 + z^2 - 3t^2)(x^2 + y^2 - 3z^2 + t^2)(x^2 - 3y^2 + z^2 + t^2)(-3x^2 + y^2 + z^2 + t^2) = 0,$$

and let X_9 be a double covering of $\mathbb{P}^3$ branched along D_9. The variety X_9 is in correspondence with the double octic given by the equation

$$u^2 = xyzt(x + y + z - 3t)(x + y - 3z + t)(x - 3y + z + t)(-3x + y + z + t)$$

which is arrangement no. 287 from 4.2.

The singular locus of D_9 consists of the 12 plane conics on the orbits under permutation of coordinates of the conics

$$x = \pm y, \qquad z^2 + t^2 = 2x^2.$$

Two conics (and two quadrics) meet at the 12 points on the orbits of the points $(1 : \pm\sqrt{-1} : 0 : 0)$, and six conics (and four quadrics) meet at the 8 points $(1 : \pm 1 : \pm 1 : \pm 1)$.

On the double covering X_9 the last 8 points look locally like arrangement p_4^0 points so they have to be blown up first. The first 12 points however leave us with 12 nodes after blowup of the double conics which also have to be resolved.

The Euler characteristic $\chi(X_9)$ of X_9 is

$$\chi(X_9) = 8 - \chi(D_9) = 8 - (4 \cdot 4 - 12 \cdot 2 + 12 + 3 \cdot 8) = -20.$$

Let $\tilde{X}_9$ be a small resolution of X_9. Then $\tilde{X}_9$ has Euler characteristic

$$\chi(\tilde{X}_9) = \chi(X_9) + 12 \cdot (4-2) + 8 \cdot (4-1) + 12 \cdot (2-1) = 40.$$

For $p \equiv 1 \mod 4$ all the double conics, the nodes and the rulings of their tangent cones are rational over $\mathbb{F}_p$ and the Lefschetz fixed point formula gives

$$\begin{aligned} &|\#\tilde{X}_{9,p} - 1 - p^3 - h^2(\tilde{X}_9) \cdot p(p+1)| \\ &\quad = |\#X_{9,p} + 12 \cdot p(p+1) + 12 \cdot p + 8 \cdot (\#N - 1) - 1 - p^3 - h^2(\tilde{X}_9) \cdot p(p+1)| \\ &\quad \leq p^{3/2} \cdot h^3(\tilde{X}_9) \\ &\quad = p^{3/2} \cdot (2 + 2h^2(\tilde{X}_9) - \chi(\tilde{X}_9)), \end{aligned}$$

where N is the normal cone at the point $(1 : 1 : 1 : 1)$ (which is a double covering of $\mathbb{P}^2$ branched along $(x+y+z)(x+y-3z)(x-3y+z)(-3x+y+z)$). Counting points over $\mathbb{F}_{37}$ and $\mathbb{F}_{41}$ gives $h^2(\tilde{X}_9) = 23$, $h^3(\tilde{X}_9) = 8$.

For $p \equiv 3 \mod 4$ the 12 nodes disappear. In this case we have the estimate

$$|\#\tilde{X}_{9,p} - 1 - p^3 - k \cdot p(p+1)| = |\#X_{9,p} + 12 \cdot p(p+1) + 8 \cdot (\#N - 1) - 1 - p^3 - k \cdot p(p+1)| \leq 8p^{3/2}$$

with a $k \in \mathbb{Z}$, $|k| \leq 23$. Counting points over $\mathbb{F}_{23}$ gives $k = 23$. We end up with the formula

$$a_p(\tilde{X}_9) = \begin{cases} p^3 + 11p^2 + 11p + 9 - 8 \cdot \#N - \#X_{9,p}, & p \equiv 3 \mod 4, \\ p^3 + 11p^2 - p + 9 - 8 \cdot \#N - \#X_{9,p}, & p \equiv 1 \mod 4. \end{cases}$$

Since N is birationally equivalent with the quadric surface given by the equation $t^2 + xy + xz + yz = 0$ (with discriminant 1) we can write

$$a_p(\tilde{X}_9) = \begin{cases} p^3 + 3p^2 - 5p + 1 - \#X_{9,p}, & p \equiv 3 \mod 4 \\ p^3 + 3p^2 - 17p + 1 - \#X_{9,p}, & p \equiv 1 \mod 4 \end{cases}$$

For all primes $5 \leq p \leq 97$ we find

$$a_p(\tilde{X}_9) = b_p + 3p \cdot c_p$$

where b_p are the coefficients of the weight four newform 6/1 ($6k4A1$) and c_p are the coefficients of the weight two newform $24A1$. The weight four newform 6/1 also occurs in the L-series of the corresponding double octic constructed from arrangement no. 287. Another relative of X_9 will be discussed in 5.9.

Four smooth quadrics related to arrangement no. 238

Let D be the octic surface given by the equation

$$(x^2 + y^2 + z^2 - t^2)(x^2 + y^2 - z^2 + t^2)(x^2 - y^2 + z^2 + t^2)(-x^2 + y^2 + z^2 + t^2) = 0,$$

and let Y be a double covering of $\mathbb{P}^3$ branched along D. The variety Y is in correspondence with the double octic given by the equation

$$u^2 = xyzt(x + y + z - t)(x + y - z + t)(x - y + z + t)(-x + y + z + t)$$

which is the rigid arrangement no. 238 from 4.2.

The singular locus of D consists of the lines on the orbit under permutation of coordinates of the lines

$$x = \pm y, \qquad z = \pm\sqrt{-1}t.$$

Two lines meet at the 24 points on the orbits of $(1 : \pm 1 : 0 : 0)$ and $(1 : \pm\sqrt{-1} : 0 : 0)$, and three lines meet at the 32 points on the orbits of $(1 : \pm\sqrt{-1} : \pm\sqrt{-1} : \pm\sqrt{-1})$.

On the double covering Y the last 32 points look locally like arrangement triple points so they disappear after blowup of the double lines. The first 24 points however leave us with 24 nodes after blowup of the double lines which also have to be resolved.

The Euler characteristic $\chi(Y)$ of Y is

$$\chi(Y) = 8 - \chi(D) = 8 - (4 \cdot 4 - 24 \cdot 2 + 24 + 32) = -16.$$

Let $\tilde{Y}$ be a small resolution of Y. Then $\tilde{Y}$ has Euler characteristic

$$\chi(\tilde{Y}) = \chi(Y) + 24 \cdot (4 - 2) + 24 = 56.$$

For $p \equiv 1 \mod 4$ all the double lines, the nodes and the rulings of their tangent cones are rational over $\mathbb{F}_p$ and the Lefschetz fixed point formula gives

$$\begin{aligned}
&|\#\tilde{Y}_p - 1 - p^3 - h^2(\tilde{Y}) \cdot p(p+1)| \\
&\quad = |\#Y_p + 24 \cdot p(p+1) + 24 \cdot p - 1 - p^3 - h^2(\tilde{Y}) \cdot p(p+1)| \\
&\quad \leq p^{3/2} \cdot h^3(\tilde{Y}) \\
&\quad = p^{3/2} \cdot (2 + 2h^2(\tilde{Y}) - \chi(\tilde{Y})).
\end{aligned}$$

Counting points over $\mathbb{F}_{13}$ gives $h^2(\tilde{Y}) = 28$, $h^3(\tilde{Y}) = 2$, so $\tilde{Y}$ is rigid.

For $p \equiv 3 \mod 4$ the singular locus of $\tilde{Y}$ consists of the 12 nodes on the orbits of the points $(1 : \pm 1 : 0 : 0)$ under permutation of coordinates. In this case we have the estimate

$$|\#\tilde{Y}_p - 1 - p^3 - k \cdot p(p+1)| = |\#Y_p + 12p - 1 - p^3 - k \cdot p(p+1)| \leq 2p^{3/2}$$

with a $k \in \mathbb{Z}$, $|k| \leq 28$. Counting points over $\mathbb{F}_{11}$ gives $k = 4$. We end up with the formula

$$a_p(\tilde{Y}) = \begin{cases} p^3 + 4p^2 - 8p + 1 - \#Y, & p \equiv 3 \mod 4, \\ p^3 + 4p^2 - 20p + 1 - \#Y, & p \equiv 1 \mod 4. \end{cases}$$

Counting points for $p \in \{3, 5, 7, 17\}$ we see that the $a_p(\tilde{Y})$ agree with the coefficients of the modular form 8/1 ($8k4A1$) and by corollary 1.6 they agree for all $p \geq 3$.

There is an obvious correspondence between Y and the complete intersection of four quadrics in $\mathbb{P}^7$ discussed in 5.4. For details about correspondences cf. also 6.1.4.

4.6 Segre's construction (squaring of coordinates)

We give an overview of a construction method invented by B. Segre ([90]):

Let u_i, $i = 0, \ldots, 3$ be four general linear forms on $\mathbb{P}^3$, and denote by T_u the tetrahedron determined by the four planes $u_i = 0$. Consider the map $\Omega : \mathbb{P}^3_v \longrightarrow \mathbb{P}^3_u$ given by $u_i = v_i^2$. It is ramified simply on $T_v = \Omega^{-1}(T_u)$ and has degree 8. The degree of Ω reduces to 4 on the faces, to 2 on the edges, and to 1 on the vertices of T_u. Let $F(u) \subset \mathbb{P}^3$ be a surface of degree n, then $G(v) = F(\Omega(v)) \subset \mathbb{P}^3$ is a surface of degree $2n$. Furthermore we have:

4.9 Theorem
$G(v)$ has only nodes as singularities if and only if

- *F has only nodes as singularities, and they lie outside T_u,*
- *if a face of T_u is tangent to F then it must be simply tangent, and the points of tangency must not lie on the edges,*
- *if an edge of T_u is tangent to F then it must be simply tangent in points which are not vertices.*

Moreover, if t is the number of nodes of F, r is the number of tangency points of the faces, s of the edges and m the number of vertices lying on F, then G has exactly $d = 8t + 4r + 2s + m$ nodes.

I have copied this version of the theorem from [21] where it was used to construct sextic surfaces with $1 \leq d \leq 64$ nodes. It was also used in [39] to construct two octic surfaces with 168 nodes (which is at present the world record for octic surfaces). The double coverings of $\mathbb{P}^3$ branched along these surfaces are rigid Calabi–Yau threefolds with defect 19 but the surfaces are not defined over $\mathbb{Q}$ but only over $\mathbb{Q}[\sqrt{2}]$.

Now let the polynomial $F(u)$ have degree 4. By extending the map Ω to the $8:1$ map

$$\begin{aligned}&\Omega' : \mathbb{P}^4(1,1,1,1,4) \longrightarrow \mathbb{P}^4(1,1,1,1,4),\\ &(v_0 : v_1 : v_2 : v_3 : w) \mapsto (v_0^2 : v_1^2 : v_2^2 : v_3^2 : v_0v_1v_2v_3w) =: (u_0 : u_1 : u_2 : u_3 : \tilde{w}),\end{aligned}$$

we get a correspondence between the two double octics given by

$$w^2 = G(v) = F(\Omega(v))$$

and by

$$\tilde{w}^2 = u_0u_1u_2u_3F(u).$$

This kind of correspondence has first been noticed by S. Cynk. We have already listed many examples in the preceding section and we will investigate some more.

4.7 Application to Kummer surfaces and other quartics

Consider the quartic surface $F_\lambda \subset \mathbb{P}^3$ given by the equation

$$x^4 + y^4 + z^4 + t^4 - \lambda \cdot xyzt = 0.$$

It is smooth except for $\lambda = 4\xi$ with ξ a fourth root of unity. In this case it is a Kummer surface (with 16 nodes as only singularities). Let $D_\lambda \subset \mathbb{P}^3$ be the octic surface constructed from F_λ with Segre's method which is given by the equation

$$x^8 + y^8 + z^8 + t^8 - \lambda \cdot x^2y^2z^2t^2 = 0.$$

By theorem 4.9 it is smooth except for $\lambda = 4\xi$ with ξ a fourth root of unity. In this case it has $128 = 8 \cdot 16$ nodes as only singularities. We will focus on $D := D_4$ which is defined over $\mathbb{Q}$. The 128 nodes are the points on the orbit of the point $(1:1:1:1)$ under the action of the group G generated by the coordinate transformations

$$(x : y : z : t) \mapsto (x : y \cdot \xi_8^a : z \cdot \xi_8^b : t \cdot \xi_8^c)$$

with $a, b, c \in \mathbb{Z}/8\mathbb{Z}$, $2(a + b + c) \equiv 0 \mod 8$ and ξ_8 a fixed primitive 8-th root of unity.

Let X be a double covering of $\mathbb{P}^3$ branched along D and let $\tilde{X}$ be a small resolution of X. Then $\tilde{X}$ has Euler characteristic

$$\chi(\tilde{X}) = -296 + 2 \cdot 128 = -40.$$

The defect of X is $d(X) = h^2(\tilde{X}) - 1 = 6 \neq 0$ (see the computation of $h^2(\tilde{X})$ below). Since G acts transitively on the set of nodes of D (and so of X) there exist projective small resolutions.

For $p \geq 3$ all the nodes and the rulings of their tangent cones are rational over $\mathbb{F}_p$ if primitive 8-th roots of unity exist. Thus for $p \equiv 1 \mod 8$ the Lefschetz fixed point formula gives

$$\begin{aligned}|\#\tilde{X}_p - 1 - p^3 - h^2(\tilde{X})p(p+1)| &= |\#X_p + 128p - 1 - p^3 - h^2(\tilde{X})p(p+1)| \\ &\leq p^{3/2}h^3(\tilde{X}) = p^{3/2}(2 + 2h^2(\tilde{X}) + 40).\end{aligned}$$

Counting points over $\mathbb{F}_{641}$ and $\mathbb{F}_{769}$ gives

$$h^2(\tilde{X}) = 7, \quad h^3(\tilde{X}) = 56.$$

For $p \equiv 3, 7 \mod 8$ only 8 and for $p \equiv 5 \mod 8$ only 24 of the nodes are defined over $\mathbb{F}_p$. The rulings of their tangent cones are rational over $\mathbb{F}_p$ if $\sqrt{-2}$ exists. We have the estimates

$$\begin{aligned}
|\#X_p + 8p - 1 - p^3 - k \cdot p(p+1)| &\leq 56p^{3/2}, \quad p \equiv 3 \mod 8,\\
|\#X_p - 24p - 1 - p^3 - l \cdot p(p+1)| &\leq 56p^{3/2}, \quad p \equiv 5 \mod 8,\\
|\#X_p - 8p - 1 - p^3 - m \cdot p(p+1)| &\leq 56p^{3/2}, \quad p \equiv 7 \mod 8,
\end{aligned}$$

with $k, l, m \in \mathbb{Z}$, $|k|, |l|, |m| \leq 7$. Counting points over $\mathbb{F}_{2683}$, $\mathbb{F}_{2707}$, $\mathbb{F}_{1669}$, $\mathbb{F}_{1949}$, $\mathbb{F}_{2711}$ and $\mathbb{F}_{2927}$ gives $k = 1$, $l = -5$ and $m = 1$. We end up with the formula

$$a_p(\tilde{X}) = \begin{cases} p^3 + 7p^2 - 33p + 1 - \#X_p, & p \equiv 1 \mod 8,\\ p^3 + p^2 - 7p + 1 - \#X_p, & p \equiv 3 \mod 8,\\ p^3 - 5p^2 + 19p + 1 - \#X_p, & p \equiv 5 \mod 8,\\ p^3 + p^2 + 9p + 1 - \#X_p, & p \equiv 7 \mod 8. \end{cases}$$

Now let b_p be the coefficients of the weight four newform 128/1 ($128k4A1$). For all primes $3 \leq p \leq 97$ we find by counting points

$$b_p - a_p \equiv 0 \mod 2p.$$

The following table lists the numbers $\frac{b_p - a_p}{p}$:

p	$\frac{b_p-a_p}{p}$	p	$\frac{b_p-a_p}{p}$	p	$\frac{b_p-a_p}{p}$	p	$\frac{b_p-a_p}{p}$
3	6	19	6	43	18	71	−36
5	−10	23	−12	47	24	73	−274
7	12	29	62	53	62	79	24
11	−6	31	0	59	42	83	−18
13	−10	37	−82	61	−10	89	158
17	14	41	−70	67	30	97	−178

The numbers in the table might be sums of coefficients of weight two newforms but this would be difficult to prove. I have not detected any weight four newforms in the L-series of the resolution of the double covering X_λ of $\mathbb{P}^3$ branched along D_λ for any rational values $\lambda \neq \pm 4$. The threefolds $X = X_4$ and X_{-4} are isomorphic over $\mathbb{Q}[i]$. Consequently the weight four newform 128/3 (128k4C1), which is a twist of the newform 128/1 by $\left(\frac{-1}{p}\right)$, occurs in the L-series of a small resolution of X_{-4}.

The threefold X_λ also occurs as the hypergeometric threefold $V_8(\varphi)$ in 5.11.

By the results of 4.6 there is a correspondence between X_λ and the double octic Y_λ given by the equation

$$u^2 = xyzt(x^4 + y^4 + z^4 + t^4 - \lambda \cdot xyzt).$$

Indeed the weight four newforms 128/1 resp. 128/3 seem to occur in the L-series of (resolutions of) Y_4 resp. Y_{-4}.

More experiments

There are also interesting numerical observations for the double octic Z_4 given by the equation

$$u^2 = 2 \cdot (x+y+z-t)(x+y-z+t)(x-y+z+t)(-x+y+z+t)(x^4+y^4+z^4+t^4-4\cdot xyzt).$$

Four of the sixteen nodes of the Kummer surface are not contained in any of the planes, and each intersection line of two planes contains two of the remaining twelve nodes. For all primes $3 \le p \le 97$ we find

$$c_p + 3p \cdot d_p = \begin{cases} p^3 + p^2 - 3p + 1 - \#Z_{4,p}, & p \equiv 1 \mod 4, \\ p^3 + p^2 + 5p + 1 - \#Z_{4,p}, & p \equiv 3 \mod 4, \end{cases}$$

where c_p are the coefficients of the weight four newform 32/2 (32k4B1) and d_p are the coefficients of the weight two newform 32A1.

For the double octic Z_{-4} given by the equation

$$u^2 = 2 \cdot (x+y+z-t)(x+y-z+t)(x-y+z+t)(-x+y+z+t)(x^4+y^4+z^4+t^4+4\cdot xyzt)$$

the geometry is quite different: Twelve of the nodes of the Kummer surface are not contained in any of the planes, and the other four nodes are the vertices of the tetrahedron. For all primes $3 \le p \le 97$ we find

$$\tilde{c}_p + 3p \cdot d_p = \begin{cases} p^3 + p^2 - 11p + 1 - \#Z_{-4,p}, & p \equiv 1 \mod 4, \\ p^3 + p^2 + p + 1 - \#Z_{-4,p}, & p \equiv 3 \mod 4, \end{cases}$$

where $\tilde{c}_p$ are the coefficients of the weight four newform 32/3 (32k4C1) which is a twist of 32/2 by $\left(\frac{-1}{p}\right)$. Conjecturally we have $h^3(\tilde{Z}_4) = h^3(\tilde{Z}_{-4}) = 8$ for Calabi–Yau resolutions $\tilde{Z}_4$ resp. $\tilde{Z}_{-4}$ of Z_4 resp. Z_{-4}, and the L-series of these varieties split as indicated above.

Applying the coordinate change $x \mapsto -x+y+z+t$, $y \mapsto x-y+z+t$, $z \mapsto x+y-z+t$, $t \mapsto x+y+z-t$, the equation for Z_4 becomes

$$u^2 = xyzt((x^2+y^2+z^2+t^2)^2 - 16 \cdot xyzt).$$

By the results of 4.6 there is a correspondence between Z_4 and the double octic W_4 given by the equation

$$u^2 = (x^4 + y^4 + z^4 + t^4)^2 - 16 \cdot x^2y^2z^2t^2.$$

The octic surface is the union of the two Kummer surfaces F_4 and F_{-4}. The surfaces have no common node. The weight four newform 32/2 ($32k4B1$) seems to occur in the L-series of W_4, as expected. By numerical observation we have $h^{2,1}(\tilde{W}_4) = 3$ for a resolution $\tilde{W}_4$ of W_4 and $a_p(\tilde{W}_4) = c_p + 3p \cdot d_p$.

A quartic with six A_3 singularities

Now consider the octic surface given by the equation

$$xyzt((x + y + z + t)^4 - 256 \cdot xyzt) = 0.$$

The quartic surface has 6 singularities of type A_3 (i.e., with local equation $x^2 + y^2 + z^4 = 0$) at the points on the orbit of the point $(0 : 0 : 1 : -1)$ under permutation of coordinates and one ordinary node at the point $(1 : 1 : 1 : 1)$. Each plane contains three of the A_3 singularities.

Let W be a double covering of $\mathbb{P}^3$ branched along this octic surface. By numerical observation a resolution $\tilde{W}$ of W is rigid and its L-series agrees with the L-series of the weight four newform 8/1 ($8k4A1$).

By the results of 4.6 there is a correspondence between W and the double octic W' given by the equation

$$\begin{aligned} u^2 &= (x^2 + y^2 + z^2 + t^2)^4 - 256 \cdot x^2y^2z^2t^2 \\ &= ((x^2 + y^2 + z^2 + t^2)^2 - 16 \cdot xyzt)((x^2 + y^2 + z^2 + t^2)^2 + 16 \cdot xyzt). \end{aligned}$$

The octic surface is again the union of two Kummer surfaces. The surfaces have 12 common nodes. By numerical observation a resolution $\tilde{W}'$ of W' is again rigid and its L-series agrees with the L-series of the weight four newform 8/1 ($8k4A1$), as expected.

4.8 Playing with cubic surfaces

There are rather nice examples of double octics constructed from a cubic surface and five planes. We will explain some constructions and report about numerical observations but not prove modularity in detail. Some of the material might be discussed elsewhere.

The Cayley cubic

The *Cayley cubic* C is one interesting cubic surface. It can be given in $\mathbb{P}^3$ by the equation

$$xyz + xyt + xzt + yzt = 0.$$

It corresponds to $\mathbb{P}^2$ blown up in the six intersection points of a configuration of four lines. It has 4 ordinary nodes as only singularities which is the maximal possible number for a cubic surface. There are only 9 lines on C, namely the lines on the orbits of the lines given by the equations

$$x = y = 0, \qquad x + y = z + t = 0,$$

under permutation of coordinates. Now consider the double covering X_1 of $\mathbb{P}^3$ branched along the Σ_4-symmetric octic surface given by the equation

$$xyzt(x + y + z + t)(xyz + xyt + xzt + yzt) = 0$$

or, in other coordinates, by the equation

$$\begin{aligned}(x + y + z - t)(x + y - z + t)(x - y + z + t)(-x + y + z + t)(x + y + z + t)\cdot\\ \cdot (4(x^3 + y^3 + z^3 + t^3) - (x + y + z + t)^3) = 0.\end{aligned}$$

Here each of the planes contains three of the lines of C and each of the first four planes contains three nodes of C. By numerical observation the double octic X_1 is rigid, and its L-series is equal to the L-series of the weight four newform 8/1 ($8k4A1$). In fact there is a birational correspondence between X_1 and other rigid Calabi–Yau threefolds connected with this newform (cf. 6.1.4).

Applying the Segre construction (cf. 4.6) we find a relative Y_1 of X_1 which is a double covering of $\mathbb{P}^3$ branched along the octic surface given by the equation

$$(x^2 + y^2 + z^2 + t^2)(x^2y^2z^2 + x^2y^2t^2 + x^2z^2t^2 + y^2z^2t^2) = 0.$$

The sextic surface has non-isolated singularities (theorem 4.9 does not apply here). By numerical observation the double octic Y_1 is also rigid, and its L-series is equal to the L-series of the weight four newform 8/1 ($8k4A1$), as expected.

Now consider the double covering X_2 of $\mathbb{P}^3$ branched along the Σ_4-symmetric octic surface given by the equation

$$xyzt(x + y + z + t)(4(x^3 + y^3 + z^3 + t^3) - (x + y + z + t)^3) = 0$$

or, in other coordinates, by the equation

$$\begin{aligned}(x + y + z - t)(x + y - z + t)(x - y + z + t)(-x + y + z + t)(x + y + z + t)\cdot\\ \cdot (xyz + xyt + xzt + yzt) = 0.\end{aligned}$$

Here the first four planes do not contain any lines or nodes of C, but there are six fourfold points of three planes and the cubic. By numerical observation the double octic X_2 is rigid, and its L-series is equal to the L-series of the weight four newform 40/3 ($40k4A1$).

Applying the Segre construction (cf. 4.6) we find a relative Y_2 of X_2 which is a double covering of $\mathbb{P}^3$ branched along the octic surface given by the equation

$$(x^2 + y^2 + z^2 + t^2)(4(x^6 + y^6 + z^6 + t^6) - (x^2 + y^2 + z^2 + t^2)^3) = 0.$$

Here theorem 4.9 applies: the sextic surface has 44 nodes as only singularities, namely the points on the orbits of the points $(1:1:0:0)$ and $(1:1:1:\sqrt{-1})$ under permutation of coordinates and sign change. By numerical observation the double octic Y_2 is non-rigid (I predict $h^{12}=6$), but its L-series still contains the L-series of the weight four newform 40/3 ($40k4A1$), as expected.

The Clebsch cubic

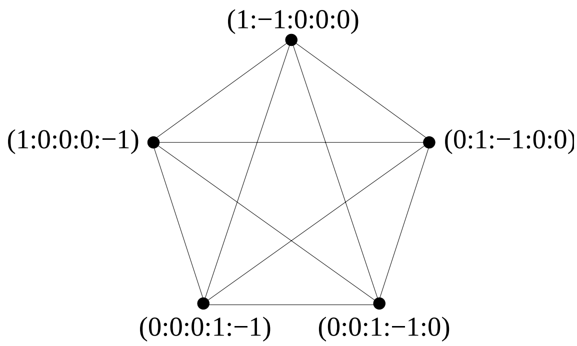
The *Clebsch cubic* is the only smooth cubic surface in $\mathbb{P}^3$ with 10 *Eckardt points*, i.e., points where three of the 27 lines on the surface meet. It corresponds to $\mathbb{P}^2$ blown up in the vertices and the center of a regular pentagon. It can be given in $\mathbb{P}^3$ by the equations

$$x^3+y^3+z^3+t^3+u^3=x+y+z+t+u=0.$$

Let X_3 be a double covering of $\mathbb{P}^3$ branched along the octic surface given by the equations

$$(x+y)(y+z)(z+t)(t+x)(x+u)(x^3+y^3+z^3+t^3+u^3)=x+y+z+t+u=0.$$

Consider the following (non-planar) pentagon with 5 Eckardt points as vertices where the 5 inner edges represent lines on the Clebsch cubic.



The five planes of the octic surface are chosen in such a way that each plane contains three consecutive vertices of the pentagon, with respect to the inner edges. Note that each plane also contains one Eckardt point which is not a vertex of the pentagon. By numerical observation the double octic X_3 is rigid, and its L-series is equal to the L-series of the weight four newform 5/1 ($5k4A1$).

Now consider the double covering X_4 of $\mathbb{P}^3$ branched along the octic surface given by the equations

$$xyztu(x^3+y^3+z^3+t^3+u^3)=x+y+z+t+u=0$$

Here the five planes of the octic surface are chosen in such a way that each plane contains three consecutive vertices of the pentagon, with respect to the *outer* edges. Note that each plane also contains three Eckardt points which are not vertices of the pentagon. The octic surface is Σ_5-symmetric; it occurs also as $X_{(5:6)}$ in 4.9. By numerical observation the double octic X_4 is rigid, and its L-series is equal to the L-series of the weight four newform 10/1 ($10k4A1$).

Applying the Segre construction (cf. 4.6) we find a relative Y_4 of X_4 which is a double covering of $\mathbb{P}^3$ branched along the octic surface given by the equation

$$(x^2+y^2+z^2+t^2)(x^6+y^6+z^6+t^6-(x^2+y^2+z^2+t^2)^3)=0.$$

Here theorem 4.9 applies: the sextic surface has 52 nodes as only singularities, namely the points on the orbits of the points $(1:0:0:0)$ and $(0:1:1:\sqrt{-1})$ under permutation of coordinates and sign change. By numerical observation the double octic Y_4 is non-rigid but its L-series still contains the L-series of the weight four newform 10/1 ($10k4A1$), as expected.

Σ_4-symmetric cubics

Modifying some of the above constructions I performed numerical experiments with double coverings of $\mathbb{P}^3$ branched along the union of the five planes $xyzt(x+y+z+t)=0$ and a cubic surface which is also invariant under permutation of coordinates. Such a surface is given by an equation of the form

$$F_{(f:g:h)}(x,y,z,t)=f\cdot S_3(x,y,z,t)+g\cdot S_1(x,y,z,t)\cdot S_2(x,y,z,t)+h\cdot S_1^3(x,y,z,t)=0$$

where $S_i(x,y,z,t)$ is the elementary symmetric polynomial of degree i in x, y, z, t, and $(f:g:h)\in\mathbb{P}^2$. To avoid a double plane we can assume that $f\neq 0$. For certain parameters there seem to occur weight four newforms in the L-series of the Calabi–Yau threefolds (i.e., for each considered good prime p the number of points on the threefold agrees with the coefficient of the newform modulo p). Here I multiplied the equations of the branch loci with a certain factor to get a twisted newform of minimal level.

Note that the coordinate transformation

$$\begin{aligned} x &\mapsto -x+y+z+t,\\ y &\mapsto x-y+z+t,\\ z &\mapsto x+y-z+t,\\ t &\mapsto x+y+z-t, \end{aligned}$$

maps the cubic with parameter $(f:g:h)$ to the one with parameter $(4f:-4(f+g):f-4h)$ and vice versa; the map $\phi:(f:g:h)\mapsto(4f:-4(f+g):f-4h)$ is an involution of $\mathbb{P}^2$. Outside $f=0$ it has only one fixed point, namely $(8:-4:1)$. The cubic degenerates to three planes; we get the rigid arrangement no. 238. For all other parameters there are two possibilities of choosing five planes which preserve the Σ_4-symmetry of the octic. In some examples (like for the Cayley cubic) we get different modular forms for the two choices.

In the table I also predict if the double octics are rigid. This is based on numerical observations.

$(f:g:h)$	weight four newform	$\phi(f:g:h)$	rigid?	comments
$(1:-1:0)$	10/1 ($10k4A1$)	$(4:0:1)$	y	Clebsch cubic, 4.9
$(1:0:0)$	8/1 ($8k4A1$)	$(4:-4:1)$	y	Cayley cubic
$(2:-3:0)$	8/1 ($8k4A1$)	$(4:2:1)$	n	

$(f:g:h)$	weight four newform		$\phi(f:g:h)$	rigid?	comments
$(2:-1:0)$	14/2	$(14k4A1)$	$(4:-2:1)$	n	
$(4:-4:1)$	40/3	$(40k4A1)$	$(1:0:0)$	y	Cayley cubic
$(6:-1:0)$	360/2		$(12:-10:3)$	n	Sarti cubic, 4.11
$(8:-4:1)$	8/1	$(8k4A1)$	$(8:-4:1)$	y	Arr. no. 238
$(9:-4:1)$	120/2	$(120k4D1)$	$(36:-20:5)$	y	
$(9:-1:0)$	42/2	$(42k4A1)$	$(36:-32:9)$	n	
$(16:-16:5)$	280/2	$(280k4D1)$	$(16:0:-1)$	n	
$(16:0:-1)$	88/2	$(88k4A1)$	$(16:-16:5)$	n	
$(18:-11:3)$	264/4	$(264k4D1)$	$(36:-14:3)$	n	
$(18:7:-3)$	210/6	$(210k4H1)$	$(36:-50:15)$	n	
$(24:-4:1)$	360/2		$(24:-20:5)$	n	
$(27:-27:8)$	210/6	$(210k4H1)$	$(108:0:-5)$	n	
$(27:0:-1)$	264/4	$(264k4D1)$	$(108:-108:31)$	y	
$(54:-27:7)$	42/2	$(42k4A1)$	$(108:-54:13)$	n	
$(54:-9:1)$	120/2	$(120k4D1)$	$(108:-90:25)$	n	
$(216:-36:5)$	120/2	$(120k4D1)$	$(216:-180:49)$	n	Sarti cubic, 4.11

The computer search ran over the parameter space $|f| \leq 100$, $|g| \leq 100$, $|h| \leq 100$. Some of the parameters $(f:g:h)$ might look rather strange but if we express the corresponding cubics in terms of sums of powers they become much nicer.

Note that applying the Segre construction (cf. 4.6) to the double octic constructed from the octic surface given by the equation

$$xyzt(x+y+z+t)F_{(f:g:h)}(x,y,z,t) = 0$$

we always find a relative (in general with different Hodge numbers) which is a double covering of $\mathbb{P}^3$ branched along the octic surface given by the equation

$$(x^2+y^2+z^2+t^2)F_{(f:g:h)}(x^2,y^2,z^2,t^2) = 0.$$

A cubic with three cusps

There is a cubic surface in $\mathbb{P}^3$ with 3 cusps (i.e., singularities given locally by $xy = z^3$; also called A_2 *singularities*) as only singularities. It can be given in $\mathbb{P}^3$ by the equation

$$xyz - t^3 = 0.$$

The coordinates of the cusps are then $(1:0:0:0)$, $(0:1:0:0)$, $(0:0:1:0)$. Let X_5 be a double covering of $\mathbb{P}^3$ branched along the octic surface given by the equation

$$xyzt(x+y+z-3t)(xyz-t^3) = 0.$$

The first three planes contain two of the cusps each, the fourth plane contains all three cusps and the third plane is tangent to the cubic at the point $(1:1:1:1)$. By numerical observation the double octic X_5 is rigid, and its L-series is equal to the L-series of the weight four newform 24/1 (24k4A1). Applying the Segre construction (cf. 4.6) we find a relative Y_5 of X_5 which is a double covering of $\mathbb{P}^3$ branched along the octic surface given by the equation

$$(x^2 + y^2 + z^2 - 3t^2)(xyz - t^3)(xyz + t^3) = 0.$$

By numerical observation the double octic Y_5 is also rigid, and its L-series is equal to the L-series of the weight four newform 24/1 (24k4A1), as expected.

Now let X_6 be a double covering of $\mathbb{P}^3$ branched along the octic surface given by the equation

$$(x - t)(y - t)(z - t)t(x + y + z - 3t)(xyz - t^3) = 0.$$

The first three planes contain one cusp each, the fourth plane contains all three cusps and the third plane is tangent to the cubic at the point $(1 : 1 : 1 : 1)$. The first three planes also contain that point. There are also three fourfold points of the cubic, the fourth and the fifth and one of the first three planes. By numerical observation the double octic X_6 is rigid, and its L-series is equal to the L-series of the weight four newform 12/1 (12k4A1).

After the change of coordinates $x \mapsto x + t$, $y \mapsto y + t$, $z \mapsto z + t$ the equation for X_6 becomes

$$xyzt(x + y + z)((x + t)(y + t)(z + t) - t^3) = 0.$$

Applying the Segre construction (cf. 4.6) we find a relative Y_6 of X_6 which is a double covering of $\mathbb{P}^3$ branched along the octic surface given by the equation

$$(x^2 + y^2 + z^2)((x^2 + t^2)(y^2 + t^2)(z^2 + t^2) - t^6) = 0.$$

By numerical observation the double octic Y_6 is non-rigid but its L-series still contains the L-series of the weight four newform 12/1 (12k4A1), as expected.

Now let X_7 be a double covering of $\mathbb{P}^3$ branched along the octic surface given by the equation

$$xyzt(x + y + z)(xyz - t^3) = 0.$$

The first three planes contain two of the cusps each, the fourth plane contains all three cusps and the fifth plane contains the intersection point of the first three planes. There are also three fourfold points of the cubic, the fourth and the fifth and one of the first three planes. By numerical observation the double octic X_7 is rigid, and its L-series is equal to the L-series of the weight four newform 108/3 (108k4A1). By applying an isomorphism defined over $\mathbb{Q}[\sqrt[3]{2}]$ we get the equation

$$xyzt(x + y + z)(xyz - 2t^3) = 0,$$

and the newform 9/1 (9k4A1) occurs in the L-series of the double octic.

Applying the Segre construction (cf. 4.6) we find a relative Y_7 of X_7 which is a double covering of $\mathbb{P}^3$ branched along the octic surface given by the equation

$$(x^2 + y^2 + z^2)(xyz - t^3)(xyz + t^3) = 0.$$

By numerical observation the double octic Y_7 is non-rigid but its L-series still contains the L-series of the weight four newform 108/3 (108k4A1), as expected. By applying an isomorphism defined over $\mathbb{Q}[\sqrt[3]{2}]$ we get the equation

$$(x^2 + y^2 + z^2)(xyz - 4t^3)(xyz + 4t^3) = 0,$$

and the newform 9/1 (9k4A1) occurs again in the L-series of the double octic.

Now let X_8 be a double covering of $\mathbb{P}^3$ branched along the octic surface given by the equation

$$xyz(x + y + z)(x + y + z - 3t)(xyz - t^3) = 0.$$

The first three planes contain two of the cusps each, the fourth plane contains the intersection point of the first three planes and the fifth plane is tangent to the cubic at the point $(1 : 1 : 1 : 1)$. There are also three fourfold points of the cubic, the fourth and the fifth and one of the first three planes. By numerical observation the double octic X_8 is non-rigid, and its L-series contains the L-series of the weight four newform 54/2 (54k4D1).

After the change of coordinates $t \mapsto (x + y + z - t)/3$ the equation for X_8 becomes

$$xyzt(x + y + z)(27xyz - (x + y + z - t)^3) = 0.$$

Applying the Segre construction (cf. 4.6) we find a relative Y_8 of X_8 which is a double covering of $\mathbb{P}^3$ branched along the octic surface given by the equation

$$(x^2 + y^2 + z^2)(27x^2y^2z^2 - (x^2 + y^2 + z^2 - t^2)^3) = 0.$$

By numerical observation the double octic Y_8 is non-rigid but its L-series still contains the L-series of the weight four newform 54/2 (54k4D1), as expected.

4.9 Σ_5-symmetric quintics and Barth's quintic with 15 cusps

Consider the power sums

$$C_i := C_i(x_0, x_1, \ldots, x_4) := \sum_{k=0}^{4} x_k^i,$$

and let the quintic surface $S_{(a:b)} \subset \mathbb{P}^3$ with $(a : b) \in \mathbb{P}^1$ be given by the equations

$$C_1 \ = \ aC_2C_3 - bC_5 \ = \ 0.$$

The varieties $S_{(a:b)} \subset \mathbb{P}^3$ define the pencil of quintic surfaces in $\mathbb{P}^3$ that are invariant under the operation of the symmetric group Σ_5 by permutation of coordinates.

The variety $S_{(a:b)}$ is the intersection of van Straten's quintic $\mathcal{M}_{(5b:6a-5b)}$ (cf. 3.6) with the hyperplane $x_5 = 0$. We will compute the singularities of $S_{(a:b)}$ in a similar way as in [101]. Let

$$D_i := D_i(x_0, x_1, x_2, x_3) := x_0^i + x_1^i + x_2^i + x_3^i.$$

Then $S_{(a:b)}$ is given in $\mathbb{P}^3$ by the equation

$$a(D_2 + D_1^2)(D_3 - D_1^3) - b(D_5 - D_1^5).$$

The singular locus of $S_{(1:0)}$ is clearly the smooth curve $D_2 + D_1^2 = D_3 - D_1^3 = 0$; the quintic $S_{(0:1)}$ is clearly non-singular. Thus we can assume that $a = 5$ and $b \neq 0$. For convenience, we will set $c := b - 6 \neq -6$. Let $(x_0 : x_1 : x_2 : x_3)$ be a singular point of $S_{(a:b)}$. Differentiating we get

$$F(x_0) = F(x_1) = F(x_2) = F(x_3) = 0$$

for the quartic polynomial

$$F(x) = (c+6)x^4 - 3(D_2 + D_1^2)x^2 - 2(D_3 - D_1^3)x - D_1(2D_3 - 3D_1D_2 + (c+1)D_1^3).$$

A priori and up to permutation there are five possibilities how the roots of F can be distributed over the four coordinates:

$$(x : x : x : x), \quad (x : x : x : y), \quad (x : x : y : y), \quad (x : x : y : z), \quad (x : y : z : t).$$

Case 1: We can assume that $(x_0 : x_1 : x_2 : x_3) = (1 : 1 : 1 : 1)$. We compute $D_1 = D_2 = D_3 = 4$ which leads to $(a : b) = (17 : 20)$. In fact the 5 points on the orbit of $(1 : 1 : 1 : 1 : -4)$ are nodes on $S_{(17:20)}$ (it is easy to check that they are really ordinary nodes).

Case 2: If we assume that $(x_0 : x_1 : x_2 : x_3) = (0 : 0 : 0 : 1)$ then we compute $D_1 = D_2 = D_3 = 1$ which leads to $(a : b) = (5 : 6)$. The variety $S_{(5:6)}$ is the intersection of five planes given by $xyzt(x + y + z + t) = 0$, and the point $(0 : 0 : 0 : 1)$ and its orbit are contained in the double lines.

Thus we can assume that $x = 1$ and so $(x_0 : x_1 : x_2 : x_3) = (1 : 1 : 1 : y)$. We compute $D_1 = 1 + t$, $D_2 = 1 + t^2$, $D_3 = 1 + t^3$ and (up to constants)

$$\begin{aligned} F(1) &= (y+4)(c(y^3 + 8y^2 + 22y + 20) + 6y) = 0, \\ F(y) &= (2y+3)(c(2y^2 + 6y + 9) + 2) = 0, \\ F(1) - F(y) &= (y-1)(c(y^3 + y^2 + y + 1) + 6(y+3)) = 0. \end{aligned}$$

The cases $y = 1$ and $y = -4$ lead us back to case 1. If we assume that $y = -3/2$ then we can compute $(a : b) = (13 : 30)$. In fact the 10 points on the orbit of $(1 : 1 : 1 : -3/2 : -3/2)$ are nodes on $S_{(13:30)}$ (it is easy to check that they are really ordinary nodes).

Inserting $c(2y^2 + 6y + 9) + 2 = 0$ into $c(y^3 + y^2 + y + 1) + 6(y + 3) = 0$ gives $(y + 4)(c + 2) = 0$. The first factor leads us back to case 1, the second factor gives $(a : b) = (5 : 4)$. In fact the 20 points on the orbit of $(1 : 1 : 1 : -3/2 + \sqrt{-7}/2 : -3/2 - \sqrt{-7}/2)$ are nodes on $S_{(5:4)}$ (it is easy to check that they are really ordinary nodes).

Case 3: If we assume that $(x_0 : x_1 : x_2 : x_3) = (0 : 0 : 1 : 1)$ then we compute $D_1 = D_2 = D_3 = 2$ which leads to $(a : b) = (5 : 6)$ again. Thus we can assume that $x = 1$ and so

$(x_0 : x_1 : x_2 : x_3) = (1 : 1 : y : y)$. We compute $D_1 = 2(1+y)$, $D_2 = 2(1+y^2)$, $D_3 = 2(1+y^3)$ and (up to constants)

$$\begin{aligned} F(1) &= (3+2y)(c(8y^3+20y^2+18y+5)+6y^2) = 0, \\ F(y) &= (2+3y)(c(5y^3+18y^2+20y+8)+6y) = 0, \\ F(1)-F(y) &= (y-1)(y+1)(cy^2+12y+c) = 0. \end{aligned}$$

The cases $y = -3/2$ and $y = -2/3$ lead us back to case 2, the case $y = 1$ leads us back to case 1. The case $y = -1$ leads to $(a : b) = (5 : 12)$. In fact the 15 points on the orbit of $(1 : 1 : -1 : -1 : 0)$ are cusps on $S_{(5:12)}$. A *cusp* is a singularity with local equation $x^2+y^2+z^3 = 0$ (also called an A_2 *singularity*).

Inserting $cy^2+12y+c = 0$ into $c(8y^3+20y^2+18y+5)+6y^2 = 0$ gives $(c-6)(2cy-3c-36y) = 0$. The first factor only gives again the 15 cusps on $S_{(5:12)}$. The second factor, together with $cy^2+12y+c = 0$, gives $-3cy^2 = 2cy$. All possibilities lead us back to previous cases.

Case 4: If we assume that $(x_0 : x_1 : x_2 : x_3) = (0 : 0 : 1 : y)$ then we compute $D_1 = 1+y$, $D_2 = 1+y^2$, $D_3 = 1+y^3$ and (up to constants)

$$\begin{aligned} F(0) &= c(y^4+4y^3+6y^2+4y+1) = 0, \\ F(1) &= c(y^4+4y^3+6y^2+4y) = 0, \\ F(y) &= c(4y^3+6y^2+4y+1) = 0. \end{aligned}$$

This is only possible for $c = 0$, i.e., $(a : b) = (5 : 6)$.

Thus we can assume that $(x_0 : x_1 : x_2 : x_3) = (1 : 1 : y : z)$. We compute (up to constant)

$$\begin{aligned} F(1)-F(y) &= (y-1)(c(y+1)(y^2+1)+6z(y+z+2)) = 0, \\ F(1)-F(z) &= (z-1)(c(z+1)(z^2+1)+6y(y+z+2)) = 0, \\ F(y)-F(z) &= (y-z)(c(y+z)(y^2+z^2)+6(y+z+2)) = 0. \end{aligned}$$

The cases $y = 1$, $z = 1$ and $y = z$ have been discussed before. Let

$$\begin{aligned} H_1 &= c(y+1)(y^2+1)+6z(y+z+2), \\ H_2 &= c(z+1)(z^2+1)+6z(y+z+2), \\ H_3 &= c(y+z)(y^2+z^2)+6(y+z+2). \end{aligned}$$

The case $c = 0$ leads back to the 15 cusps on $S_{(5:12)}$. If $c \neq 0$ then we compute (up to constant)

$$\begin{aligned} H_2 - yH_3 &= (y-1)((y+1)(y^2+1)+(z+1)(z^2+1)+yz(y+z+1)-1) = 0, \\ yH_1 - xH_2 &= (y-z)(yz(y+z+1)-1) = 0. \end{aligned}$$

Again the cases $y = 1$ and $y = z$ lead us to previous cases. Thus we find $(y+1)(y^2+1)+(z+1)(z^2+1) = 0$ and $H_1 + H_2 = 6(y+z)(y+z+2) = 0$. If $y+z = 0$ then $H_3 = 12 \neq 0$. If $y+z+2 = 0$ then we have either $y = z = -1$ (which leads us back to $c = 0$) or $y^2 = 1$ which includes $z^2+2z+3 = 0$, so $H_2 \neq 0$.

Case 5: If we assume that $(x_0 : x_1 : x_2 : x_3) = (0 : 1 : y : -1 - y)$ (note that the four roots of F must sum up to zero) then we compute

$$F(0) = 0, \quad F(1) = c, \quad F(y) = cy^4, \quad F(-1-y) = c(y^4 + 4y^3 + 6y^2 + 4y + 1),$$

so we conclude $c = 0$ and $(a : b) = (5 : 6)$ where we know the singular locus.

Thus we can assume that $(x_0 : x_1 : x_2 : x_3) = (1 : y : z : -1 - y - z)$. But then, considering the Σ_5-symmetry, the point $(0 : 1 : y : z)$ is also singular, and we conclude $z \in \{0, 1, y, -1 - y\}$ which leads to different previous cases.

The table collects the results of the above discussion. The general quintic $S_{(a:b)}$ is smooth, and the singular members are the following:

$(a : b)$	$(5b : 6a - 5b)$	singular locus
$(1 : 0)$	$(0 : 1)$	the curve $C_1 = C_2 = C_3 = 0$
$(5 : 6)$	$(1 : 0)$	10 lines given by $x_i = x_j = 0$
$(5 : 12)$	$(-2 : 1)$	15 cusps on the Σ_5-orbit of $(1 : 1 : -1 : -1 : 0)$
$(5 : 4)$	$(2 : 1)$	20 nodes on the Σ_5-orbit of $(2 : 2 : 2 : -3 + \sqrt{-7} : -3 - \sqrt{-7})$
$(17 : 20)$	$(50 : 1)$	5 nodes on the Σ_5-orbit of $(1 : 1 : 1 : 1 : -4)$
$(13 : 30)$	$(-25 : 12)$	10 nodes on the Σ_5-orbit of $(1 : 1 : 1 : -3/2 : -3/2)$

The surface $S_{(5:12)}$ was investigated by Barth in [5]. It is the quintic surface with the highest number of cusps that is currently known. The surface $S_{(5:6)}$ (which is the union of five planes) has already occurred in 4.8.

The surface $S_{(a:b)}$ contains the 15 lines given by

$$x_i + x_j = x_k + x_l = x_m = 0, \qquad \{i, j, k, l, m\} = \{0, 1, 2, 3, 4\}.$$

These lines are the complete intersection of $S_{(a:b)}$ with the *Clebsch cubic* C (cf. 4.8) which is given by the equations

$$C_1 = C_3 = 0.$$

Let $X_{(a:b)}$ be a double covering of $\mathbb{P}^3$ branched along the octic surface $S_{(a:b)} \cup C$. Counting points on $X_{(a:b),p}$ for small primes p we see that for some parameters $(a : b)$ we have

$$\# X_{(a:b),p} \equiv b_p \mod p$$

where b_p are the coefficients of certain weight four newforms for $\Gamma_0(N)$, suggesting that these newforms occur in the L-series of $X_{(a:b)}$. Here I multiplied the equations of the branch loci with a certain factor to get a twisted newform of minimal level.

$(a:b)$	$(5b:6a-5b)$	factor	newform
$(5:6)$	$(1:0)$	10	10/1 (10k4A1)
$(5:12)$	$(-2:1)$	-10	130/2 (130k4B1)
$(5:4)$	$(2:1)$	30	30/2 (30k4A1)
$(17:20)$	$(50:1)$	6	390/5
$(13:30)$	$(-25:12)$	2	10/1 (10k4A1)

The occurrence of the bad prime 13 in the levels of the newforms connected with $X_{(5:12)}$ and $X_{(17:20)}$ is remarkable. We will give some comments on the single examples.

- For $X_{(5:6)}$ and small good primes p we have the formula

$$b_p = p^3 + 7p^2 - 18p + 1 - \#X_{(5:6),p},$$

suggesting that $X_{(5:6)}$ is rigid. In fact S. Cynk computed

$$h^{1,2}(\tilde{X}_{(5:6)}) = 0, \quad h^{1,1}(\tilde{X}_{(5:6)}) = 42, \quad \chi(\tilde{X}_{(5:6)}) = 84$$

for a resolution $\tilde{X}_{(5:6)}$ of $X_{(5:6)}$. A modularity proof requires a closer look at the Picard group of $\tilde{X}_{(5:6)}$.

- For $X_{(5:12)}$ and small good primes p we have the formula

$$b_p + 4 \cdot c_p = p^3 + 6p^2 - 29p + 1$$

where c_p are the coefficients of the weight two newform $26B1$. S. Cynk computed

$$h^{1,2}(\tilde{X}_{(5:12)}) = 4, \quad h^{1,1}(\tilde{X}_{(5:12)}) = 21, \quad \chi(\tilde{X}_{(5:12)}) = 34$$

for a resolution $\tilde{X}_{(5:12)}$ of $X_{(5:12)}$. A modularity proof requires an explanation for the occurrence of the weight two newform and a closer look at the resolution of singularities and at the Picard group of $\tilde{X}_{(5:12)}$.

- For $X_{(5:4)}$ and small good primes p we have the formula

$$b_p = p^3 + 7p^2 + 1 - \#X_{(5:4),p} - \begin{cases} 28p, & p \equiv 3,5,6 \mod 7, \\ 48p, & p \equiv 1,2,4 \mod 7, \end{cases}$$

suggesting that $X_{(5:4)}$ is rigid. In fact S. Cynk computed

$$h^{1,2}(\tilde{X}_{(5:4)}) = 0, \quad h^{1,1}(\tilde{X}_{(5:4)}) = 22, \quad \chi(\tilde{X}_{(5:4)}) = 44$$

for a resolution $\tilde{X}_{(5:4)}$ of $X_{(5:4)}$. The nodes of $X_{(5:4)}$ (and the rulings of their tangent cones) are defined over $\mathbb{F}_p$ if $\sqrt{-7}$ exists which is the case exactly for $p \equiv 1,2,4 \mod 7$. A modularity proof requires a closer look at the resolution of singularities and at the Picard group of $\tilde{X}_{(5:4)}$.

• There can not be said much about $X_{(17:20)}$ at the moment. It seems to be non-rigid; S. Cynk conjectures $h^{1,2}(\tilde{X}_{(17:20)}) = 9$ for a resolution $\tilde{X}_{(17:20)}$ of $X_{(17:20)}$. The discriminant of the tangent cones at the nodes is -5.

• There can also not be said much about $X_{(13:30)}$ at the moment. It seems to be non-rigid; S. Cynk conjectures $h^{1,2}(\tilde{X}_{(13:30)}) = 4$ for a resolution $\tilde{X}_{(13:30)}$ of $X_{(13:30)}$. The discriminant of the tangent cones at the nodes is $105 = 3 \cdot 5 \cdot 7$ which is pretty unpleasant.

4.10 Σ_5-symmetric octics

We are going to have a glance at the general case of Σ_5-symmetric octic surfaces and double octics constructed from them. Consider again the power sums

$$C_i := C_i(x_0, x_1, \ldots, x_4) := \sum_{k=0}^{4} x_k^i,$$

and let the octic surface $T_{(a:b:c:d:e)} \subset \mathbb{P}^3$ with $(a:b:c:d:e) \in \mathbb{P}^4$ be given by the equations

$$C_1 \;=\; aC_5C_3 + bC_4^2 + cC_4C_2^2 + dC_3^2C_2 + eC_2^4 \;=\; 0.$$

The varieties $T_{(a:b:c:d:e)} \subset \mathbb{P}^3$ define the pencil of octic surfaces in $\mathbb{P}^3$ that are invariant under the action of the symmetric group Σ_5 by permutation of coordinates. There are three subfamilies where the octic splits into a sum of two symmetric surfaces of lower degree:

• For $b = c = e = 0$ the surface $T_{(a:b:c:d:e)}$ is the union of the Clebsch cubic and the quintic surface $S_{(d:-a)}$ from 4.9.

• For $a = d = 0$ the surface $T_{(a:b:c:d:e)}$ is the union of two Σ_5-symmetric quartic surfaces.

• For $a = b = 0$ the surface $T_{(a:b:c:d:e)}$ is the union of the Σ_5-symmetric quadric surface given by $C_1 = C_2 = 0$ and a Σ_5-symmetric sextic surface.

For many values of $(a : b : c : d : e)$ the double covering of $\mathbb{P}^3$ branched along the octic surface $T_{(a:b:c:d:e)}$ seems to be modular (i.e., for each considered good prime p the number of points on the threefold agrees with the coefficient of a weight four newform modulo p). Here I multiplied the equations of the branch loci with a certain factor to get a twisted newform of minimal level. The table lists the parameters and the (twists of minimal level of the) occurring newforms. The computer search ran over all parameters with $|a|, |b|, |c|, |d|, |e| \leq 25$. It took more than one month on a 3 Gigahertz machine to check these parameters (166.859.681 examples at a rate of $\sim$ 200.000 per hour). There might be many interesting examples missing.

$(a:b:c:d:e)$	weight four newform	remarks
$(0:0:4:2:-1)$	120/2 $(120k4D1)$	quadric and sextic, non-rigid
$(0:0:5:-5:-1)$	1920/3	quadric and sextic, non-rigid
$(0:0:6:-2:-3)$	360/2	quadric and sextic, non-rigid

$(a:b:c:d:e)$	weight four newform	remarks
$(0:8:-6:0:1)$	120/4 $(120k4F1)$	two quartics, non-rigid
$(4:0:0:-5:0)$	30/2 $(30k4A1)$	cubic and quintic, rigid, cf. 4.9
$(6:0:0:-5:0)$	10/1 $(10k4A1)$	cubic and quintic, rigid, cf. 4.9
$(12:-4:4:-10:-1)$	570/7	non-rigid, $570 = 2 \cdot 3 \cdot 5 \cdot 19$
$(12:0:0:-5:0)$	130/2 $(130k4B1)$	cubic and quintic, non-rigid, cf. 4.9
$(0:0:2:16:-1)$	120/1 $(120k4B1)$	quadric and sextic, non-rigid
$(0:0:0:20:-9)$	32/2 $(32k4B1)$	quadric and sextic, non-rigid
$(0:20:-20:-1:5)$	1110/2	non-rigid, $1110 = 2 \cdot 3 \cdot 5 \cdot 37$
$(20:0:-6:-15:3)$	330/4	non-rigid, $330 = 2 \cdot 3 \cdot 5 \cdot 11$
$(20:0:0:-17:0)$	390/5	cubic and quintic, non-rigid, cf. 4.9
$(0:0:0:30:-1)$	96/1 $(96k4D1)$	quadric and sextic, non-rigid
$(30:0:0:-13:0)$	10/1 $(10k4A1)$	cubic and quintic, non-rigid, cf. 4.9

The occurrence of the bad primes 11, 13, 19 and 37 is remarkable.

4.11 Sarti's Heisenberg-invariant surfaces

Construction of the surfaces

Consider two subgroups $G_1, G_2 \subset \mathrm{SO}(3)$. Let $\tilde{G}_1$, $\tilde{G}_2$ denote their inverse images in SU(2) under the universal covering $\mathrm{SU}(2) \longrightarrow SO(3)$ and let G_1G_2 denote the $2:1$ image of $\tilde{G}_1 \times \tilde{G}_2$ in SO(4) under the double covering $\mathrm{SU}(2) \times \mathrm{SU}(2) \longrightarrow \mathrm{SO}(4)$.

Consider the Klein four group $V \subset \mathrm{SO}(3)$. Then $H := VV \subset \mathrm{SO}(4)$ is called the *Heisenberg group* (with 32 elements).

Sarti ([83]) classified all subgroups $G \subset \mathrm{SO}(4)$ which contain H and studied their first nontrivial invariants in $\mathbb{C}[x, y, z, t]$. Since $H \subset G$ there is always the trivial invariant

$$Q(x, y, z, t) = x^2 + y^2 + z^2 + t^2,$$

and any nontrivial invariant f has even degree, say $\deg(f) = j$. A pencil of G-invariant surfaces of degree j in $\mathbb{P}^3$ is then given by

$$f(x, y, z, t) + \lambda \cdot Q(x, y, z, t)^{j/2} = 0, \qquad \lambda \in \mathbb{P}^1.$$

Sarti gave a list of those groups G where the above pencil with f the first nontrivial invariant for G contains surfaces with isolated singularities. In all cases the pencil contains *all* G-invariant surfaces of degree j in $\mathbb{P}^3$ (i.e., f is the only nontrivial invariant of degree j), there are exactly four singular surfaces (apart from the multiple quadric for $\lambda = \infty$), and in all but one case (in the case IO there are two additional double lines in the base locus) all the singularities are ordinary nodes that form one G-orbit.

G	order	j	λ				# of nodes			
$(OO)'$	192	4	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	4	12	16	8
$TT =: G_6$	288	6	-1	$-\frac{2}{3}$	$-\frac{7}{12}$	$-\frac{1}{4}$	12	48	48	12
$OO =: G_8$	1152	8	-1	$-\frac{3}{4}$	$-\frac{9}{16}$	$-\frac{5}{9}$	24	72	144	96
IO	2880	12	c_1	$-\frac{1}{8}$	c_2	0	240	360	240	120
$II =: G_{12}$	7200	12	$-\frac{3}{32}$	$-\frac{22}{243}$	$-\frac{2}{25}$	0	300	600	360	60

Here T, O, $I \subset \mathrm{SO}(3)$ denote the tetrahedral, octahedral and icosahedral groups (i.e., the rotation groups leaving invariant these platonic solids). The notation G_n is that of [82] and corresponds to the degree j (in [82] these groups were called *bi-polyhedral groups*). The group $(OO)'$ is a subgroup of OO (see [83] for generators). The numbers c_1 and c_2 are $-\frac{74}{972} + \frac{4}{243}\sqrt{10}$ and $-\frac{74}{972} - \frac{4}{243}\sqrt{10}$ (they did not look nice in the table).

We are interested in (double coverings of $\mathbb{P}^3$ branched along) octic surfaces, so we are going to have a look at the cases with $j \leq 8$:

G	first nontrivial invariant
$(OO)'$	$S_4(x,y,z,t) = x^4 + y^4 + z^4 + t^4$
TT	$S_6(x,y,z,t) = x^6 + y^6 + z^6 + t^6$ $+15(x^2y^2z^2 + x^2y^2t^2 + x^2z^2t^2 + y^2z^2t^2)$
OO	$S_8(x,y,z,t) = x^8 + y^8 + z^8 + t^8$ $+14(x^4y^4 + x^4z^4 + x^4t^4 + y^4z^4 + y^4t^4 + z^4t^4)$ $+168x^2y^2z^2t^2$

Let

$$\begin{aligned} D_\lambda &= \{S_8(x,y,z,t) + \lambda \cdot Q(x,y,z,t)^4 = 0\} \subset \mathbb{P}^3, \\ F_\lambda &= \{S_6(x,y,z,t) + \lambda \cdot Q(x,y,z,t)^3 = 0\} \subset \mathbb{P}^3, \\ H_\lambda &= \{S_4(x,y,z,t) + \lambda \cdot Q(x,y,z,t)^2 = 0\} \subset \mathbb{P}^3, \qquad \infty \neq \lambda \in \mathbb{P}^1, \end{aligned}$$

and let X_λ resp. Y_λ resp. $Z_{\lambda,\mu}$ be a double covering of $\mathbb{P}^3$ branched along D_λ resp. $F_\lambda \cup Q$ resp. $H_\lambda \cup H_\mu$ (which are all octic surfaces).

We will pay special attention to parameters λ, μ leading to surfaces with nodes. Let us start with H_λ. We list representatives of the nodes under permutation of coordinates:

Surface	Representative under permutation	# of nodes under permutation
H_{-1}	$(1:0:0:0)$	4
$H_{-\frac{1}{2}}$	$(1:\pm1:0:0)$	12
$H_{-\frac{1}{3}}$	$(1:\pm1:\pm1:0)$	16
$H_{-\frac{1}{4}}$	$(1:\pm1:\pm1:\pm1)$	8

To describe the singularities of D_λ and F_λ we consider the 24-cell in $\mathbb{C}^4$ (with Schläfli symbol $\{3,4,3\}$) and its reciprocal (denoted by $\{3,4,3\}'$). As in [24, p. 156] we choose the permutations of the points $(\pm1,\pm1,0,0)$ as vertices of $\{3,4,3\}$. The singularities of D_λ and F_λ (if there are any) are then given by the images in $\mathbb{P}^3$ of the following points:

Surface	Nodes
$F_{-\frac{1}{4}}$	Vertices of $\{3,4,3\}$
F_{-1}	Vertices of $\{3,4,3\}'$
$F_{-\frac{7}{12}}$	Middle points of the edges of $\{3,4,3\}$
$F_{-\frac{2}{3}}$	Middle points of the edges of $\{3,4,3\}'$
D_{-1}	Vertices of $\{3,4,3\}$ and vertices of $\{3,4,3\}'$
$D_{-\frac{5}{9}}$	Middle points of the edges of $\{3,4,3\}$ and middle points of the edges of $\{3,4,3\}'$
$D_{-\frac{3}{4}}$, $D_{-\frac{9}{16}}$	Certain middle points of the segments connecting the vertices of $\{3,4,3\}$ with those of $\{3,4,3\}'$

We also give a list of representatives of the nodes under permutation of coordinates.

Surface	Representative under permutation	# of nodes under permutation
$F_{-\frac{1}{4}}$	$(1:\pm1:0:0)$	12
F_{-1}	$(1:0:0:0)$ $(1:\pm1:\pm1:\pm1)$	4 8
$F_{-\frac{7}{12}}$	$(1:\pm1:\pm2:0)$	48

Surface	Representative under permutation	# of nodes under permutation
$F_{-\frac{2}{3}}$	$(1:\pm1:\pm1:\pm3)$	32
	$(1:\pm1:\pm1:0)$	16
D_{-1}	$(1:\pm1:0:0)$	12
	$(1:0:0:0)$	4
	$(1:\pm1:\pm1:\pm1)$	8
$D_{-\frac{5}{9}}$	$(1:\pm1:\pm2:0)$	48
	$(1:\pm1:\pm1:\pm3)$	32
	$(1:\pm1:\pm1:0)$	16
$D_{-\frac{3}{4}}$	$(1:\pm(1+\sqrt{2}):0:0)$	24
	$(1:\pm1:\pm(1+\sqrt{2}):\pm(1+\sqrt{2}))$	48
$D_{-\frac{9}{16}}$	$(1:\pm1:\pm\sqrt{2}:0)$	48
	$(1:\pm1:\pm(1+\sqrt{2}):\pm(1-\sqrt{2}))$	96

Nodal octics

Now we are ready to study the double coverings. Let $\hat{X}_\lambda$ be a big resolution of X_λ. Then $\hat{X}_\lambda$ has Euler characteristic

$$\chi(\hat{X}_\lambda) = -296 + 4 \cdot s_\lambda$$

where s_λ is the number of nodes of D_λ. The group $OO = G_8$ acts transitively on the sets of nodes in each of the four singular examples, so by corollary 1.9 there exist projective small resolutions exactly if the defect $d(X_\lambda)$ is not zero.

Let the four singular double octics $X_\lambda \subset \mathbb{P}^4(1,1,1,1,4)$ be given by

$$\begin{aligned} X_{-1} &= \{u^2 = S_8(x,y,z,t) - Q(x,y,z,t)^4\}, \\ X_{-\frac{5}{9}} &= \{u^2 = -(9S_8(x,y,z,t) - 5Q(x,y,z,t)^4)\}, \\ X_{-\frac{9}{16}} &= \{u^2 = 16S_8(x,y,z,t) - 9Q(x,y,z,t)^4\}, \\ X_{-\frac{3}{4}} &= \{u^2 = -(4S_8(x,y,z,t) - 3Q(x,y,z,t)^4)\}. \end{aligned}$$

For every good prime p such that all the nodes and the rulings of their tangent cones are rational over $\mathbb{F}_p$ (i.e., all the discriminants are squares in $\mathbb{F}_p$) the Lefschetz fixed point formula gives

$$\begin{aligned} &|\#\hat{X}_{\lambda,p} - 1 - p^3 - h^2(\hat{X}_\lambda)p(p+1)| \\ &\quad = |\#X_{\lambda,p} + s_\lambda \cdot p(p+2) - 1 - p^3 - h^2(\hat{X}_\lambda)p(p+1)| \\ &\quad \le p^{3/2} h^3(\hat{X}_\lambda) \\ &\quad = p^{3/2}(2 + 2h^2(\hat{X}_\lambda) - \chi(\hat{X}_\lambda)). \end{aligned}$$

With the above choice of equations this holds for all good primes in the cases $\lambda = -1$ and $\lambda = -5/9$ and for all good primes $p \equiv 1, 7 \mod 8$ (such that $\sqrt{2}$ exists) in the cases $\lambda = -9/16$ and $\lambda = -3/4$.

Counting points on X_λ for suitable primes (see the last column of the following table) we compute $h^2(\hat{X}_\lambda)$ and from that and the Euler characteristic all the other data. If there exist projective small resolutions then we also list the corresponding data.

λ	$\chi(\hat{X}_\lambda)$	$h^2(\hat{X}_\lambda)$	$h^3(\hat{X}_\lambda)$	$d(X_\lambda)$	$\chi(\tilde{X}_\lambda)$	$h^2(\tilde{X}_\lambda)$	primes
-1	-200	25	252	0			65089
$-\frac{5}{9}$	88	97	108	0			2687
$-\frac{9}{16}$	280	154	30	9	-8	10	$71, 353$
$-\frac{3}{4}$	-8	73	156	0			5711

Note that the counting of points on X_{-1} over $\mathbb{F}_{65089}$ took over three weeks on a 3 Gigahertz machine, using an $O(p^3)$ algorithm (and it took some months to find a suitable prime).

If $p \equiv 3, 5 \mod 8$ and $\lambda = -9/16$ or $\lambda = -3/4$ then none of the nodes are rational over $\mathbb{F}_p$. In this case we have the estimate

$$|\#\hat{X}_{\lambda,p} - 1 - p^3 - k_\lambda \cdot p(p+1)| = |\#X_{\lambda,p} - 1 - p^3 - k_\lambda \cdot p(p+1)| \leq p^{3/2} h^3(\hat{X}_\lambda)$$

with some $k_\lambda \in \mathbb{Z}$, $|k_\lambda| \leq h^2(\hat{X}_\lambda)$. Counting points over $\mathbb{F}_{349}$ and $\mathbb{F}_{421}$ gives $k_{-\frac{9}{16}} = 10$; counting points over $\mathbb{F}_{8093}$ and $\mathbb{F}_{10037}$ gives $k_{-\frac{3}{4}} = 1$.

We end up with the formulas

$$\begin{aligned}
a_p(\hat{X}_{-1}) &= p^3 + p^2 - 23p + 1 - \#X_{-1,p},\\
a_p(\hat{X}_{-\frac{5}{9}}) &= p^3 + p^2 - 95p + 1 - \#X_{-\frac{5}{9},p},\\
a_p(\hat{X}_{-\frac{9}{16}}) &= \begin{cases} p^3 + 10p^2 - 134p + 1 - \#X_{-\frac{9}{16},p}, & p \equiv 1, 7 \mod 8,\\ p^3 + 10p^2 + \ \ 10p + 1 - \#X_{-\frac{9}{16},p}, & p \equiv 3, 5 \mod 8,\end{cases}\\
a_p(\hat{X}_{-\frac{3}{4}}) &= \begin{cases} p^3 + p^2 - 71p + 1 - \#X_{-\frac{3}{4},p}, & p \equiv 1, 7 \mod 8,\\ p^3 + p^2 + \quad p + 1 - \#X_{-\frac{3}{4},p}, & p \equiv 3, 5 \mod 8.\end{cases}
\end{aligned}$$

We are also going to consider the smooth example

$$\hat{X}_0 = X_0 = \{u^2 = S_8(x, y, z, t)\}$$

with $\chi(\hat{X}_0) = -296$, $h^2(\hat{X}_0) = 1$, $h^3(\hat{X}_0) = 298$ and

$$a_p(\hat{X}_0) = p^3 + p^2 + p + 1 - \#X_{0,p}.$$

Now let $b_{p,\lambda}$ be the coefficients of the following weight four newforms:

λ	newform
-1	168/1 (168k4A1)
$-\frac{5}{9}$	336/10 (twist of 168/2)
$-\frac{9}{16}$	336/12 (twist of 42/2)
$-\frac{3}{4}$	336/3 (twist of 21/1)
0	120/5 (120k4A1)

For all good primes $p \leq 97$ we find by counting points

$$b_{p,\lambda} \equiv a_p(\hat{X}_\lambda) \mod 2p$$

and even

$$b_{p,-\frac{9}{16}} \equiv a_p(\hat{X}_{-\frac{9}{16}}) \mod 4p.$$

The following table lists the numbers $\dfrac{a_p(\hat{X}_\lambda) - b_{p,\lambda}}{p}$:

p	$\lambda = -1$	$\lambda = -\frac{5}{9}$	$\lambda = -\frac{9}{16}$	$\lambda = -\frac{3}{4}$	$\lambda = 0$
5	-50	-106	32	8	
7					0
11	60	-116	-4	34	-20
13	114	-74	36	26	102
17	-2	-126	-32	-120	-102
19	52	92	56	-92	-92
23	32	-48	-52	-54	-144
29	-70	-186	-100	-10	134
31	-128	128	-72	-144	32
37	162	-130	52	-78	-226
41	118	74	-24	100	-174
43	-172	-220	-16	308	276
47	272	-160	24	-36	184
53	-382	78	-108	-6	-258
59	-180	84	136	48	204
61	130	-234	12	62	502
67	60	-4	136	16	92
71	-144	-16	-172	-90	184
73	-146	-94	-124	-66	690
79	64	-176	-120	-388	-256

p	$\lambda=-1$	$\lambda=-\frac{5}{9}$	$\lambda=-\frac{9}{16}$	$\lambda=-\frac{3}{4}$	$\lambda=0$
83	228	76	128	-12	-132
89	790	-6	-24	-124	-94
97	-122	282	68	-306	298

The numbers in the table might be sums of coefficients of weight two modular forms but this would be difficult to prove. I have not detected any weight four newforms in the L-series of $\hat{X}_\lambda$ for any other values of λ.

Quadric and nodal sextics

Now we consider the surfaces $F_\lambda \cup Q$. By the results of [82] no singularities of F_λ are contained in Q. The intersection $F_\lambda \cap Q$ is reduced and consists of 12 lines, 6 of each ruling of Q.

The surfaces F_{-1} and $F_{-\frac{1}{4}}$ resp. $F_{-\frac{7}{12}}$ and $F_{-\frac{2}{3}}$ are isomorphic via the coordinate transformation given by the matrix (cf. [82])

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

Note that this transformation is not contained in G_6 but in G_8. It maps the 24-cell $\{3,4,3\}$ to its reciprocal $\{3,4,3\}'$.

The Euler characteristic $\chi(Y_\lambda)$ of Y_λ is

$$\begin{aligned} \chi(Y_\lambda) &= 2\cdot\chi(\mathbb{P}^3) - \chi(F_\lambda \cup Q) \\ &= 8 - (\chi(F_\lambda) + \chi(Q) - \chi(F_\lambda \cap Q)) \\ &= 8 - (108 - s_\lambda + 4 - (12\cdot 2 - 6\cdot 6)) \\ &= -116 + s_\lambda \end{aligned}$$

where s_λ is the number of nodes of F_λ.

The singularities of Y_λ can be resolved by blowing up the 12 singular lines and the double points. Let $\hat{Y}_\lambda$ denote such a big resolution. The Euler characteristic of $\hat{Y}_\lambda$ is then

$$\chi(\hat{Y}_\lambda) = \chi(Y_\lambda) + 3\cdot s_\lambda + 12\cdot(4-2) = -92 + 3\cdot s_\lambda.$$

The singular lines are defined over every $\mathbb{F}_p$ since they are rulings of the quadric Q (whose discriminant is a square). If we choose

$$\begin{aligned} Y_{-1} &= \{u^2 = -3(S_6(x,y,z,t) - Q(x,y,z,t)^3)\cdot Q(x,y,z,t)\}, \\ Y_{-\frac{7}{12}} &= \{u^2 = (12S_6(x,y,z,t) - 7Q(x,y,z,t)^3)\cdot Q(x,y,z,t)\} \end{aligned}$$

as equations for the double coverings then for every good prime p all the nodes and the rulings of their tangent cones are rational over $\mathbb{F}_p$ (i.e., all the discriminants are squares in $\mathbb{F}_p$) and the Lefschetz fixed point formula gives

$$\begin{aligned}&|\#\hat{Y}_{\lambda,p} - 1 - p^3 - h^2(\hat{Y}_\lambda)p(p+1)| \\ &\quad = |\#Y_{\lambda,p} + s_\lambda \cdot p(p+2) + 12 \cdot p(p+1) - 1 - p^3 - h^2(\hat{Y}_\lambda)p(p+1)| \\ &\quad \le p^{3/2} h^3(\hat{Y}_\lambda) \\ &\quad = p^{3/2}(2 + 2h^2(\hat{Y}_\lambda) - \chi(\hat{Y}_\lambda)).\end{aligned}$$

Counting points on Y_λ for suitable primes (see the last column of the following table) we compute $h^2(\hat{Y}_\lambda)$ and from that and the Euler characteristic all the other data. Note that it is an open question if there also exist projective small resolutions.

λ	$\chi(\hat{Y}_\lambda)$	$h^2(\hat{Y}_\lambda)$	$h^3(\hat{Y}_\lambda)$	primes
-1	-56	25	108	4271, 5039
$-\frac{7}{12}$	52	70	90	7333, 7703

We end up with the formulas

$$\begin{aligned}a_p(\hat{Y}_{-1}) &= p^3 + p^2 - 11p + 1 - \#Y_{-1,p}, \\ a_p(\hat{Y}_{-\frac{7}{12}}) &= p^3 + 10p^2 - 38p + 1 - \#Y_{-\frac{7}{12},p}.\end{aligned}$$

Now let $b_{p,-1}$ be the coefficients of the weight four newform 360/2 and $b_{p,-\frac{7}{12}}$ the coefficients of the weight four newform 120/2 ($120k4D1$). For all good primes $p \le 97$ we find by counting points

$$a_p(\hat{Y}_{-1}) \equiv b_{p,-1} \mod 2p$$

and even

$$a_p(\hat{Y}_{-\frac{7}{12}}) \equiv b_{p,-\frac{7}{12}} \mod 8p.$$

The following table lists the numbers $\dfrac{a_p(\hat{Y}_\lambda) - b_{p,\lambda}}{p}$:

p	$\lambda=-1$	$\lambda=-\frac{7}{12}$
3		
5		
7	-42	-8
11	-82	8
13	12	32
17	-30	40
19	4	8
23	0	8

p	$\lambda=-1$	$\lambda=-\frac{7}{12}$
29	-38	32
31	-148	-16
37	176	48
41	24	24
43	-72	-16
47	-16	8
53	-54	48
59	-114	-8

p	$\lambda=-1$	$\lambda=-\frac{7}{12}$
61	-26	32
67	20	16
71	164	0
73	-102	24
79	-148	0
83	132	-32
89	-188	24
97	-194	8

The numbers in the table might again be sums of coefficients of weight two modular forms but this would be difficult to prove. I have not detected any weight four newforms in the L-series of $\hat{Y}_\lambda$ for any other values of λ.

Two nodal quartics

Finally we consider the surfaces $H_\lambda \cup H_\mu$. If $\lambda \neq \mu$ then by the results of [83] the intersection $H_\lambda \cap H_\mu$ is the smooth curve of degree 8 given by

$$x^2 + y^2 + z^2 + t^2 = x^4 + y^4 + z^4 + t^4 = 0.$$

The Euler characteristic $\chi(Z_{\lambda,\mu})$ of $Z_{\lambda,\mu}$ is

$$\begin{aligned} \chi(Z_{\lambda,\mu}) &= 2 \cdot \chi(\mathbb{P}^3) - \chi(H_\lambda \cup H_\mu) \\ &= 8 - (\chi(H_\lambda) + \chi(H_\mu) - \chi(H_\lambda \cap H_\mu)) \\ &= 8 - (2 \cdot 24 - s_\lambda - s_\mu - (-64)) \\ &= -104 + s_\lambda + s_\mu \end{aligned}$$

where s_λ resp. s_μ is the number of nodes of F_λ resp. F_μ.

Since H_λ and H_μ do not intersect transversally the octic surface $H_\lambda \cup H_\mu$ is not an arrangement, and the resolution of the singular curve may be more complicated. We will not go through this in detail but concentrate on numerical observations.

Let us first have a closer look at the primes of bad reduction. Let $\lambda = (a : b) \in \mathbb{P}^1(\mathbb{Q})$, $a, b \in \mathbb{Z}$ with $\gcd(a, b) = 1$ and let $(\bar{x} : \bar{y} : \bar{z} : \bar{t})$ be a singular point of H_λ over $\mathbb{F}_p$. Differentiating we get

$$\begin{aligned} 4\bar{x}((a+b)\bar{x}^2 + b\bar{y}^2 + b\bar{z}^2 + b\bar{t}^2) &= 0, \\ 4\bar{y}(b\bar{x}^2 + (a+b)\bar{y}^2 + b\bar{z}^2 + b\bar{t}^2) &= 0, \\ 4\bar{z}(b\bar{x}^2 + b\bar{y}^2 + (a+b)\bar{z}^2 + b\bar{t}^2) &= 0, \\ 4\bar{t}(b\bar{x}^2 + b\bar{y}^2 + b\bar{z}^2 + (a+b)\bar{t}^2) &= 0, \end{aligned}$$

so the vector whose entries are the nonzero entries of $(\bar{x}^2, \bar{y}^2, \bar{z}^2, \bar{t}^2)$ must be in the kernel of one of the following matrices over $\mathbb{F}_p$ (the one with the correct size):

$$\begin{pmatrix} a+b & b & b & b \\ b & a+b & b & b \\ b & b & a+b & b \\ b & b & b & a+b \end{pmatrix}, \begin{pmatrix} a+b & b & b \\ b & a+b & b \\ b & b & a+b \end{pmatrix}, \begin{pmatrix} a+b & b \\ b & a+b \end{pmatrix}, \begin{pmatrix} a+b \end{pmatrix}.$$

The determinants of these matrices are

$$a^3(a+4b), \quad a^2(a+3b), \quad a(a+2b), \quad a+b,$$

so p has to divide one of the numbers $a + b^i$, $0 \leq i \leq 4$. The five examples which are singular over $\mathbb{C}$ (four examples with nodes and the double quadric Q) correspond to the vanishing of one of these numbers.

The kernel of the above matrices is $\langle(1,\dots,1)\rangle$ so whenever a prime p divides $a+b^i$ for some i, $0\le i\le 4$, it really *is* a bad prime.

For the following values of λ, μ we can detect congruences

$$\#Z_{\lambda,\mu,p} \equiv b_p \mod p$$

for all good primes $p\le 97$ where b_p are the coefficients of the listed weight four newform (here I twisted the equation for $H_\lambda \cup H_\mu$ by a nonsquare number to get a twisted newform of minimal level):

λ	μ	newform	
-1	1	120/4	$(120k4F1)$
-1	$-\frac{1}{2}$	96/2	$(96k4E1)$
-1	$-\frac{1}{3}$	24/1	$(24k4A1)$
-1	$-\frac{1}{4}$	96/1	$(96k4D1)$
1	$\frac{1}{2}$	480/2	

λ	μ	newform	
$-\frac{1}{2}$	$-\frac{1}{3}$	96/4	$(96k4B1)$
$-\frac{1}{2}$	$-\frac{1}{4}$	6/1	$(6k4A1)$
$-\frac{1}{3}$	$-\frac{1}{4}$	96/1	$(96k4D1)$
$\frac{1}{2}$	$-\frac{1}{3}$	480/5	

This is a list of the paramater values appearing above:

$\lambda=(a:b)$	$a+b$	$a+2b$	$a+3b$	$a+4b$	bad primes
$-1=(1:-1)$	0	-1	-2	-3	$2,3$
$-\frac{1}{2}=(2:-1)$	1	0	-1	-2	2
$-\frac{1}{3}=(3:-1)$	2	1	0	-1	$2,3$
$-\frac{1}{4}=(4:-1)$	3	2	1	0	$2,3$
$1=(1:1)$	2	3	4	5	$2,3,5$
$\frac{1}{2}=(2:1)$	3	4	5	6	$2,3,5$

These examples are special in the way that there are only very few and very small bad primes and they appear at small powers in the numbers $a+b^i$. I expect a weight four newform in the middle cohomology of *all* double octics $Z_{\lambda,\mu}$ but the level will be too high to be in my tables.

Remarks

In [7] Barth and Sarti studied the pencils of quotient surfaces F_λ/G_6, D_λ/G_8 and C_λ/G_{12} (where C_λ is given by

$$C_\lambda = \{S_{12}(x,y,z,t)+\lambda\cdot Q(x,y,z,t)^6=0\}\subset\mathbb{P}^3$$

with S_{12} being the nontrivial invariant of degree 12 for G_{12}). It turned out that the minimal resolution of a general member of each pencil is a K3 surface with Picard number 19, and in each case the four nodal examples lead to K3 surfaces with Picard number 20 (i.e., extremal K3 surfaces).

In each case with group G_n the number $n-1$ is a prime, and this prime occurs in a mysterious way in many places. This was first noted in [7], and we will add some more data.

We list again the special values for λ, the number s_λ of nodes on F_λ resp. D_λ resp. C_λ, the discriminant d_λ of the Picard lattice of F_λ/G_6 resp. D_λ/G_8 resp. C_λ/G_{12} and the level N_λ of the (twists of minimal level of the) weight four newforms associated to the double covering of $\mathbb{P}^3$ branched along the octic $F_\lambda \cup Q$ resp. D_λ.

$n=6$			
λ	s_λ	d_λ	N_λ
-1	12	-15	360
$-\frac{2}{3}$	48	-60	120
$-\frac{7}{12}$	48	-60	120
$-\frac{1}{4}$	12	-15	360

$n=8$			
λ	s_λ	d_λ	N_λ
-1	24	-28	168
$-\frac{3}{4}$	72	-84	21
$-\frac{9}{16}$	144	-168	42
$-\frac{5}{9}$	96	-112	168

$n=12$		
λ	s_λ	d_λ
$-\frac{3}{32}$	300	-660
$-\frac{22}{243}$	600	-440
$-\frac{2}{25}$	360	-792
0	60	-132

The primes 5, 7 and 11 also occur in the coefficients of the defining polynomials, in the cross ratio CR of the four special values for λ, in the absolute invariant j and in the sum of nodes $\sum s_\lambda$ over all special members:

n	6	8	12
CR	$\frac{5^2}{3^2}$	$\frac{7^2}{2^4 \cdot 3}$	$\frac{11^2}{2^5 \cdot 3}$
j	$\frac{13^3 \cdot 37^3}{2^8 \cdot 3^2 \cdot 7^4}$	$\frac{13^3 \cdot 181^3}{2^8 \cdot 3^2 \cdot 7^4}$	$\frac{12241^3}{2^{10} \cdot 3^2 \cdot 5^4 \cdot 11^4}$
$\sum s_\lambda$	$120 = 2^3 \cdot 3 \cdot 5$	$336 = 2^4 \cdot 3 \cdot 7$	$1320 = 2^3 \cdot 3 \cdot 5 \cdot 11$

It would be very interesting to find an explanation for this and to study the arithmetic of the K3 surfaces in detail.

Relatives

The polynomials Q, S_4, S_6 and S_8 are polynomials in x^2, y^2, z^2, t^2, so we can apply the results from 4.6 and find nice (double octic) relatives of the modular double octics constructed above. Let

$$T_2(x,y,z,t) = x^2 + y^2 + z^2 + t^2,$$
$$T_3(x,y,z,t) = x^3 + y^3 + z^3 + t^3 + 15(xyz + xyt + xzt + yzt),$$
$$T_4(x,y,z,t) = x^4 + y^4 + z^4 + t^4 + 14(x^2y^2 + x^2z^2 + x^2t^2 + y^2z^2 + y^2t^2 + z^2t^2) + 168xyzt,$$

such that $S_{2i}(x,y,z,t) = T_i(x^2,y^2,z^2,t^2)$.

In the S_8 case we get the double octic corresponding to the union of four planes and a (maybe nodal) quartic surface as a relative.

double octic	equation for relative	newform
X_{-1}	$u^2 = xyzt(T_4 - (x+y+z+t)^4)$	168/1 $(168k4A1)$
$X_{-\frac{5}{9}}$	$u^2 = xyzt(9T_4 - 5(x+y+z+t)^4)$	168/2 $(168k4E1)$
$X_{-\frac{3}{4}}$	$u^2 = xyzt(4T_4 - 3(x+y+z+t)^4)$	21/1 $(21k4B1)$
$X_{-\frac{9}{16}}$	$u^2 = xyzt(16T_4 - 9(x+y+z+t)^4)$	42/2 $(42k4A1)$
X_0	$u^2 = xyzt \cdot T_4$	120/5 $(120k4A1)$

In the S_6 case we get the double octic corresponding to the union of five planes and a (maybe nodal) cubic surface as a relative. Both examples also occur in 4.8.

double octic	equation for relative	newform
Y_{-1}	$u^2 = xyzt(x+y+z+t)(T_3 - (x+y+z+t)^3)$	360/2
$Y_{-\frac{7}{12}}$	$u^2 = xyzt(x+y+z+t)(12T_3 - 7(x+y+z+t)^3)$	120/2 $(120k4D1)$

In the S_4 case we get the double octic corresponding to the union of four planes and two quadric surfaces as a relative. The quadric surface given by $T_2 + \lambda \cdot (x+y+z+t)^2 = 0$ has a node exactly if $\lambda = -1/4$. All these examples also occur in 4.5.

double octic	equation for relative	newform
$Z_{-1,1}$	$xyzt(T_2 - (x+y+z+t)^2)(T_2 + (x+y+z+t)^2)$	120/4 $(120k4F1)$
$Z_{-\frac{1}{2},-\frac{1}{3}}$	$xyzt(2T_2 - (x+y+z+t)^2)(3T_2 - (x+y+z+t)^2)$	96/4 $(96k4B1)$
$Z_{-1,-\frac{1}{2}}$	$xyzt(T_2 - (x+y+z+t)^2)(2T_2 - (x+y+z+t)^2)$	96/2 $(96k4E1)$
$Z_{-\frac{1}{2},-\frac{1}{4}}$	$xyzt(2T_2 - (x+y+z+t)^2)(4T_2 - (x+y+z+t)^2)$	6/1 $(6k4A1)$
$Z_{-1,-\frac{1}{3}}$	$xyzt(T_2 - (x+y+z+t)^2)(3T_2 - (x+y+z+t)^2)$	24/1 $(24k4A1)$
$Z_{-\frac{1}{3},-\frac{1}{4}}$	$xyzt(3T_2 - (x+y+z+t)^2)(4T_2 - (x+y+z+t)^2)$	96/1 $(96k4D1)$
$Z_{-1,-\frac{1}{4}}$	$xyzt(T_2 - (x+y+z+t)^2)(4T_2 - (x+y+z+t)^2)$	96/1 $(96k4D1)$
$Z_{\frac{1}{2},-\frac{1}{3}}$	$xyzt(2T_2 + (x+y+z+t)^2)(3T_2 - (x+y+z+t)^2)$	480/5
$Z_{1,\frac{1}{2}}$	$xyzt(T_2 + (x+y+z+t)^2)(2T_2 + (x+y+z+t)^2)$	480/2

More experiments

I performed some more numerical experiments with the Sarti surfaces. Consider again the quartic surface H_λ given by the equation

$$S_4(x,y,z,t) + \lambda \cdot Q(x,y,z,t)^2 = x^4 + y^4 + z^4 + z^4 + \lambda \cdot (x^2 + y^2 + z^2 + t^2)^2 = 0.$$

For certain double octics X constructed from these surfaces I detected connections to weight four newforms in the sense that

$$\#X_p \equiv b_p \mod p$$

for all checked good primes where b_p are the coefficients of the respective newform. In the table we write $S = x + y + z + t$.

equation of double octic	newform
$u^2 = (S-x)(S-y)(S-z)(S-t)(3S_4 - Q^2)$	96/1 (96k4D1)
$u^2 = (S-x)(S-y)(S-z)(S-t)(4S_4 - Q^2)$	96/1 (96k4D1)
$u^2 = (S-2x)(S-2y)(S-2z)(S-2t)(4S_4 - Q^2)$	96/4 (96k4B1)
$u^2 = xyzt(4S_4 - Q^2)$	288/1
$u^2 = 4S_4(x^2, y^2, z^2, t^2) - Q(x^2, y^2, z^2, t^2)^2$	288/1

The last two examples in the table are again relatives by the construction from 4.6.

Chapter 5

Other examples

5.1 A rigid complete intersection with small Euler number

Let $X \subset \mathbb{P}^5$ be the complete intersection threefold defined by the equations

$$\begin{aligned} x_0^2 + x_1^2 + x_2^2 + x_3^2 &= 4x_4x_5, \\ x_4^4 + x_5^4 &= 2x_0x_1x_2x_3. \end{aligned}$$

It is invariant under the action of the group G which is generated by the permutations of the first four coordinates and by the transformations

$$\begin{aligned} (x_0 : x_1 : x_2 : x_3 : x_4 : x_5) &\mapsto (x_0 : -x_1 : x_2 : x_3 : \xi_8 x_4 : \xi_8^{-1} x_5), \\ (x_0 : x_1 : x_2 : x_3 : x_4 : x_5) &\mapsto (x_0 : x_1 : x_2 : x_3 : \xi_4 x_4 : \xi_4^{-1} x_5), \\ (x_0 : x_1 : x_2 : x_3 : x_4 : x_5) &\mapsto (x_0 : -x_1 : -x_2 : x_3 : x_4 : x_5), \end{aligned}$$

where ξ_4 is a 4-th root of unity and ξ_8 is a primitive 8-th root of unity.

The singular locus of X consists of 12 singularities of type $(2, 2, 4, 4)$ on the orbit of the point $(1 : \sqrt{-1} : 0 : 0 : 0 : 0)$ and 32 nodes on the orbit of the point $(1 : 1 : 1 : 1 : 1 : 1)$ under the action of G.

The Euler characteristic of X is

$$\chi(X) = -176 + 32 + 12 \cdot 9 = -36.$$

Let $\tilde{X}$ be a small resolution of X. Then

$$\chi(\tilde{X}) = -36 + 32 + 12 \cdot (4 - 1) = 32.$$

To my knowledge, this is the smallest known Euler number for a rigid Calabi–Yau threefold (we will check rigidity below).

There exist projective small resolutions. The singularities of type $(2,2,4,4)$ are contained in the smooth divisors

$$x_4 = \xi_8 x_5, \quad x_i^2 + x_j^2 + x_k^2 = 4\xi_8 x_5^2, \quad x_l = 0$$

where ξ_8 is a primitive 8th root of unity and $\{i,j,k,l\} = \{0,1,2,3\}$.

To see that the nodes are also contained in smooth divisors we rewrite the first equation for X as

$$(x_0 - x_1)^2 + (x_2 - x_3)^2 + 2(x_0x_1 + x_2x_3 - 2x_4x_5) = 0$$

and the second equation as

$$2(x_4x_5 - x_0x_1)(x_4x_5 - x_2x_3) + 2x_4x_5(x_0x_1 + x_2x_3 - 2x_4x_5) = (x_4^2 - x_5^2)^2.$$

Thus the smooth surface given by the equations

$$\begin{aligned} x_0 - x_1 &= \sqrt{-1}(x_2 - x_3), \\ x_0x_1 + x_2x_3 &= 2x_4x_5, \\ \sqrt{-2}(x_4x_5 - x_0x_1) &= x_4^2 - x_5^2 \end{aligned}$$

is contained in X. Moreover it contains the node $(1:1:1:1:1:1)$ of X.

Over the finite field $\mathbb{F}_p$ not all of the singularities may appear, depending on the existence of 4-th and 8-th roots of unity:

$p \mod 8$	# of nodes	# of $(2,2,4,4)$-points
1	32	12
5	16	12
3, 7	8	0

The tangent cones at the nodes are given by the quadric surface defined by

$$5(x^2 + y^2 + z^2) + 2(xy + xz + yz) - 16w(x + y + z) + 32w^2 = 0$$

with discriminant $2 \cdot 64^2$, so all rulings are defined over fields where 2 is a square.

At the singular points of type $(2,2,4,4)$ the variety X looks locally like

$$xy(2 + x^2 - y^2 - 4zt) + z^4 + t^4 = 0.$$

It seems that one of the resolving curves is defined over $\mathbb{Q}[\sqrt{-1}]$ and the remaining two only over $\mathbb{Q}[\sqrt{-2}]$. This has still to be checked.

Then for $p \equiv 1 \mod 8$ all singularities and all resolving curves are rational over $\mathbb{F}_p$. We apply the Lefschetz fixed point formula:

$$\begin{aligned} |\#\tilde{X}_p - 1 - p^3 - h^2(\tilde{X}) \cdot p(p+1)| &= |\#X_p + 12 \cdot 3p + 32p - 1 - p^3 - h^2(\tilde{X}) \cdot p(p+1)| \\ &\leq p^{3/2} h^3(\tilde{X}) = p^{3/2}(2 + 2h^2(\tilde{X}) - 32). \end{aligned}$$

Counting points over $\mathbb{F}_{17}$ we find

$$h^2(\tilde{X}) = 16, \quad h^3(\tilde{X}) = 2,$$

so $\tilde{X}$ is rigid.

For $p \not\equiv 1 \mod 8$ we have the estimates

$$\begin{aligned}
|\#X_p - 12p - 16p - 1 - p^3 - \ k \cdot p(p+1)| &\le 2p^{3/2}, \quad p \equiv 5 \mod 8, \\
|\#X_p + 8p - 1 - p^3 - \ l \cdot p(p+1)| &\le 2p^{3/2}, \quad p \equiv 7 \mod 8, \\
|\#X_p - 8p - 1 - p^3 - m \cdot p(p+1)| &\le 2p^{3/2}, \quad p \equiv 3 \mod 8,
\end{aligned}$$

with $k, l, m \in \mathbb{Z}$, $|k|, |l|, |m| \le 16$. Counting points over $\mathbb{F}_{13}$, $\mathbb{F}_{23}$ and $\mathbb{F}_{11}$ gives $k = -8$, $l = -2$ and $m = -2$. We end up with the formula

$$a_p(\tilde{X}) = \begin{cases} p^3 + 16p^2 - 52p + 1 - \#X_p, & p \equiv 1 \mod 8, \\ p^3 - \ 8p^2 + 20p + 1 - \#X_p, & p \equiv 5 \mod 8, \\ p^3 - \ 2p^2 - 10p + 1 - \#X_p, & p \equiv 7 \mod 8, \\ p^3 - \ 2p^2 + \ 6p + 1 - \#X_p, & p \equiv 3 \mod 8. \end{cases}$$

Counting points for all primes $3 \le 97$ we find that the $a_p(\tilde{X})$ agree with the coefficients of the weight four newform 16/1 (16k4A1, twist of 8/1 by $\left(\frac{-1}{p}\right)$), and by corollary 1.6 they agree for all primes $p \ge 5$.

A related double octic

Eliminating x_5 from the second equation for X we obtain the equation

$$256 \cdot x_4^8 + (x_0^2 + x_1^2 + x_2^2 + x_3^2)^4 = 2 \cdot 256 \cdot x_0 x_1 x_2 x_3 x_4^4$$

which we rewrite as

$$-256 \cdot (x_4^4 - x_0 x_1 x_2 x_3)^2 = (x_0^2 + x_1^2 + x_2^2 + x_3^2)^4 - 256 \cdot x_0^2 x_1^2 x_2^2 x_3^2.$$

Thus there is a correspondence defined over $\mathbb{Q}$ between X and the double octic given by the equation

$$u^2 = 256 \cdot x^2 y^2 z^2 t^2 - (x^2 + y^2 + z^2 + t^2)^4.$$

This double octic is also discussed in 4.7. Note that by multiplying the branch locus with -1 we twist the occurring newform 16/1 by $\left(\frac{-1}{p}\right)$, thus obtaining the newform 8/1.

5.2 A family of nodal complete intersections

Let $a = (a_0 : a_1 : a_2 : a_3 : a_4 : a_5) \in \mathbb{P}^5(\mathbb{Q})$ and let $X_a \subset \mathbb{P}^5$ be the complete intersection threefold defined by

$$\sum_{i=0}^{5} a_i x_i^2 = \sum_{i=0}^{5} a_i x_i^4 = 0.$$

If $a_0 \cdot \ldots \cdot a_5 \neq 0$ then X_a has only isolated singularities, namely the points with coordinates

$$x_i = \begin{cases} \pm 1, & i \in I \\ 0, & i \notin I \end{cases} \quad \text{where} \quad I \subset \{0, 1, \ldots, 5\} \quad \text{with} \quad \sum_{i \in I} a_i = 0.$$

The group $(\mathbb{Z}/2\mathbb{Z})^6$ acts on X_a by sign change of the coordinates.

Now let $\zeta = (\zeta_0 : \zeta_1 : \zeta_2 : \zeta_3 : \zeta_4 : \zeta_5)$ be a singular point of X_a. Since $a_0 \cdot \ldots \cdot a_5 \neq 0$ we can assume that $a_0 = 1$, $\zeta_0^2 = 1$ and $\zeta_5 = 1$. The tangent cone at ζ is then given by the quadric surface

$$2 \sum_{i=1}^{4} a_i (3\zeta_i^2 - 1) x_i^2 + 4 \left(\sum_{i=1}^{4} a_i \zeta_i x_i \right)^2 = 0$$

with discriminant

$$a_1 a_2 a_3 a_4 (3\zeta_1^2 - 1)(3\zeta_2^2 - 1)(3\zeta_3^2 - 1)(3\zeta_4^2 - 1) \cdot \\ \cdot \left(1 + \frac{2a_1\zeta_1^2}{3\zeta_1^2 - 1} + \frac{2a_2\zeta_2^2}{3\zeta_2^2 - 1} + \frac{2a_3\zeta_3^2}{3\zeta_3^2 - 1} + \frac{2a_4\zeta_4^2}{3\zeta_4^2 - 1} \right).$$

Since $\zeta_i^2 \in \{0, 1\}$, the last factor can be written as

$$1 + a_1\zeta_1^2 + a_2\zeta_2^2 + a_3\zeta_3^2 + a_4\zeta_4^2.$$

But $a_1\zeta_1^2 + a_2\zeta_2^2 + a_3\zeta_3^2 + a_4\zeta_4^2 + a_5 = -1$ with $a_5 \neq 0$, so ζ is an ordinary node with discriminant

$$-a_1 a_2 a_3 a_4 a_5 (3\zeta_1^2 - 1)(3\zeta_2^2 - 1)(3\zeta_3^2 - 1)(3\zeta_4^2 - 1).$$

Using a computer we find examples with 0, 2, 4, 6, ..., 76, 80, 82, 90, 96 and 122 nodes. Some numbers seem to occur only finitely many times. In the table we list the corresponding examples.

# of nodes	$(a_0 : a_1 : a_2 : a_3 : a_4 : a_5)$
54	$(1 : 1 : 2 : -2 : 3 : -4)$
54	$(1 : -1 : 2 : 2 : -2 : -3)$
68	$(1 : 1 : -1 : -2 : -2 : 3)$
72	$(1 : 1 : 1 : 1 : -1 : -3)$
74	$(1 : 1 : 1 : -1 : 2 : -3)$
76	$(1 : 1 : 1 : 2 : -2 : -2)$
80	$(1 : 1 : 1 : 1 : 1 : -3)$
80	$(1 : 1 : 1 : 1 : 1 : -4)$
80	$(1 : 1 : 1 : 1 : -2 : -2)$
80	$(1 : 1 : 1 : -1 : -1 : -2)$
90	$(1 : 1 : -1 : -1 : 2 : -2)$
96	$(1 : 1 : 1 : 1 : -1 : -2)$
122	$(1 : 1 : 1 : -1 : -1 : -1)$

Over the finite field $\mathbb{F}_p$ with $p \geq 5$ the singularities of X_a are given by the points with coordinates

$$x_i = \begin{cases} \pm 1, & i \in I, \quad a_i \not\equiv 0 \mod p \\ 0, & i \notin I, \quad a_i \not\equiv 0 \mod p \end{cases} \quad \text{where} \quad I \subset \{0, 1, \ldots, 5\} \quad \text{with} \quad \sum_{i \in I} a_i \equiv 0 \mod p,$$

so it is possible to detect the bad primes by looking at the coefficients a_i. Note that 2 and 3 are always bad primes.

Lemma: Up to permutation of coordinates there are only seven sets of coefficients $a = (a_0 : a_1 : a_2 : a_3 : a_4 : a_5) \in \mathbb{P}^5(\mathbb{Q})$ with $a_0 \cdot \ldots \cdot a_5 \neq 0$ such that 2 and 3 are the only bad primes for X_a. These are

$$\begin{aligned} (1 : 1 : 1 : 1 : -1 : -3), \\ (1 : 1 : 1 : 1 : -2 : -2), \\ (1 : 1 : 1 : -1 : -1 : -2), \\ (2 : 1 : 1 : -1 : -1 : -2), \\ (1 : 1 : 1 : 1 : -1 : -2), \\ (1 : 1 : -1 : -1 : -1 : -1), \\ (1 : 1 : 1 : -1 : -1 : -1). \end{aligned}$$

Proof: We may assume that $a_i = \pm 2^{\alpha_i} 3^{\beta_i} \in \mathbb{Z}$ and $\gcd(a_0, \ldots, a_5) = 1$. There are two cases:

Case 1: $a_0 = 1$.

Let $i \in \{1, \ldots, 5\}$. Then $a_0 + a_i = 1 \pm 2^{\alpha_i} 3^{\beta_i} = \pm 2^{\gamma} 3^{\delta}$ from which we conclude that $\alpha_i \cdot \gamma = \beta_i \cdot \delta = 0$. The cases $\alpha_i = \beta_i = 0$ and $\gamma = \delta = 0$ give $a_i \in \{\pm 1, \pm 2\}$. The cases $\alpha_i = \delta = 0$ and $\gamma = \beta_i = 0$ lead to special cases of Catalán's conjecture (which has been proven by P. Mihăilescu in [70]) and give $a_i \in \{-9, -4, -3, 2, 3, 8\}$.

Case 2: $a_i \neq \pm 1$ for all $i \in \{0, \dots, 5\}$.

Because $\gcd(a_0, \dots, a_5) = 1$ there are $i, j \in \{0, \dots, 5\}$, $i \neq j$ with $a_i = \pm 2^\alpha$, $a_j = \pm 3^\beta$. We have $a_i + a_j = \pm 2^\gamma 3^\delta$ which is only possible if $\gamma = \delta = 0$. If $k \in \{0, \dots, 5\}$, $i \neq k \neq j$, then $a_i + a_j + a_k = \pm 1 + a_k = \pm 2^\lambda 3^\mu$ from which we conclude that $|a_k| \in \{1, 2, 3, 4, 6, 8, 9\}$ like in case 1.

We are left with a finite problem that can easily be solved with the help of a computer. □

I also performed a computer search for sets of coefficients $a = (a_0 : a_1 : a_2 : a_3 : a_4 : a_5) \in \mathbb{P}^5(\mathbb{Q})$ such that 2, 3 and 5 are the only bad primes for X_a; and it seems that there are only finitely many of them. A proof could require generalizations of Catalán's conjecture. This is a list of the examples that I found (I checked all coefficients with absolute value ≤ 20):

$$
\begin{array}{lll}
(3:3:3:3:-4:-8), & (4:4:1:1:-4:-6), & (4:4:4:-3:-3:-6), \\
(1:1:1:1:1:-6), & (2:1:1:1:1:-6), & (4:4:1:1:-5:-5), \\
(2:2:2:2:-5:-5), & (4:2:2:2:-5:-5), & (5:4:1:-1:-4:-5), \\
(3:3:3:-1:-3:-5), & (2:2:2:2:-3:-5), & (1:1:1:1:-1:-5), \\
(2:1:1:1:-1:-5), & (3:1:1:1:-1:-5), & (2:2:1:1:-1:-5), \\
(1:1:1:1:1:-5), & (2:1:1:1:1:-5), & (2:2:2:2:2:-5), \\
(3:3:3:-4:-4:-4), & (3:3:3:-1:-4:-4), & (3:3:3:3:-4:-4), \\
(3:3:2:-2:-2:-4), & (1:1:1:1:-2:-4), & (2:1:1:1:-2:-4), \\
(3:1:1:1:-2:-4), & (2:2:1:1:-2:-4), & (1:1:1:-1:-1:-4), \\
(2:1:1:-1:-1:-4), & (3:1:1:-1:-1:-4), & (4:1:1:-1:-1:-4), \\
(2:2:1:-1:-1:-4), & (3:2:1:-1:-1:-4), & (2:2:2:-1:-1:-4), \\
(1:1:1:1:-1:-4), & (2:1:1:1:-1:-4), & (3:1:1:1:-1:-4), \\
(2:2:1:1:-1:-4), & (1:1:1:1:1:-4), & (2:1:1:1:1:-4), \\
(2:2:2:-2:-3:-3), & (3:3:2:-2:-3:-3), & (1:1:1:1:-3:-3), \\
(2:1:1:1:-3:-3), & (3:1:1:1:-3:-3), & (2:2:1:1:-3:-3), \\
(2:2:2:2:-3:-3), & (1:1:1:-1:-2:-3), & (2:1:1:-1:-2:-3), \\
(3:1:1:-1:-2:-3), & (2:2:1:-1:-2:-3), & (3:2:1:-1:-2:-3), \\
(2:2:2:-1:-2:-3), & (1:1:1:1:-2:-3), & (2:1:1:1:-2:-3), \\
(3:1:1:1:-2:-3), & (2:2:1:1:-2:-3), & (2:2:2:2:-2:-3), \\
(-1:-1:1:1:1:3), & (-2:-1:1:1:1:3), & (-3:-1:1:1:1:3), \\
(-2:-2:1:1:1:3), & (1:1:1:-1:-1:-3), & (2:1:1:-1:-1:-3), \\
(3:1:1:-1:-1:-3), & (2:2:1:-1:-1:-3), & (2:2:2:-1:-1:-3), \\
(2:1:1:1:-1:-3), & (2:2:1:1:-1:-3), & (1:1:1:1:1:-3), \\
(2:1:1:1:1:-3), & (1:1:1:-2:-2:-2), & (2:1:1:-2:-2:-2), \\
(2:2:1:-2:-2:-2), & (-1:-1:1:1:2:2), & (-2:-1:1:1:2:2), \\
(-2:-2:1:1:2:2), & (1:1:1:-1:-2:-2), & (2:1:1:-1:-2:-2), \\
(2:2:1:-1:-2:-2), & (2:1:1:1:-2:-2), & (-1:1:1:1:1:2), \\
(2:1:1:1:1:-2), & (-1:-1:1:1:1:2), & (-2:-1:1:1:1:2), \\
(1:1:1:1:1:-2), & (1:1:1:1:1:1), & (1:1:1:1:1:-1).
\end{array}
$$

Now we are going to investigate the members of the family for modularity. We will consider a small resolution $\tilde{X}_a$ of X_a and compute $a_p(\tilde{X}_a)$. For some values $a \in \mathbb{P}^5$ we find

$$a_p(\tilde{X}_a) \equiv b_p \mod 2p$$

for the coefficients of certain weight four newforms and all considered good primes p, suggesting that the newform occurs in the L-series of $\tilde{X}_a$. In particular, all the examples with bad primes only 2 and 3 seem to be modular. I conjecture that a weight four newform for some $\Gamma_0(N)$ can be found in the L-series of *all* examples $\tilde{X}_a$ but the level will be too high to be in my tables. I have not been able to detect if the remaining part of the L-series of the modular examples is a sum of weight two newforms.

The computation of $a_p(\tilde{X})$ is done in the usual way, using the Lefschetz fixed point formula. I am going to omit the details. Note that the computation of $h^2(\tilde{X}_a)$ requires counting of points over rather large fields (like $\mathbb{F}_{2579}$ and $\mathbb{F}_{3853}$), so the counting program had to be highly optimized.

In most examples it is not clear if there exist projective small resolutions.

The following table summarizes the results, listing the coefficients a, the number of nodes, the Hodge numbers, information about projective small resolutions and the weight four newform. The single examples are discussed in more detail afterwards.

a	#nodes	$h^{1,1}$	$h^{2,1}$	proj.	weight four newform	
$(1:1:1:1:1:1)$	0	1	89	yes	480/2	
$(1:1:1:1:1:-1)$	10	1	79	no	240/11	$(240k4H1,$ twist of $120/4)$
$(1:1:1:1:1:-5)$	32	2	58	yes	600/10	(twist of 600/2)
$(1:1:1:1:1:-2)$	40	1	49	no	1920/10	(twist of 1920/2)
$(1:1:1:-1:-1:2)$	40	1	49	no	1920/6	(twist of 1920/2)
$(1:1:1:-1:-1:-3)$	44	1	45	no	1440/7	
$(1:1:1:3:-3:-3)$	52	4	40	?	360/10	(twist of 40/2)
$(1:1:1:1:-1:-1)$	64	3	27	?	96/2	$(96k4E1)$
$(1:1:1:2:-2:-3)$	66	3	25	?	720/25	(twist of 360/2)
$(1:1:1:1:-1:-3)$	72	6	22	?	72/1	$(72k4C1)$
$(1:1:1:1:1:-3)$	80	11	19	yes	1440/7	
$(1:1:1:1:-2:-2)$	80	4	12	?	192/7	$(192k4C1,$ twist of $6/1)$
$(1:1:1:-1:-1:-2)$	80	2	10	?	384/4	(twist of 384/3)
$(1:1:-1:-1:2:-2)$	90	15	13	?	48/3	$(48k4A1,$ twist of $6/1)$
$(1:1:1:1:-1:-2)$	96	15	7	?	384/5	(twist of 384/3)
$(1:1:1:-1:-1:-1)$	122	34	0	yes	12/1	$(12k4A1)$

No. 1: $a = (1:1:1:1:1:1)$**:**

The variety X_a is smooth with

$$\chi(X_a) = -176, \quad h^2(X_a) = 1, \quad h^3(X_a) = 180$$

and

$$a_p(X_a) = p^3 + p^2 + p + 1 - \#X_{a,p}.$$

The table lists the numbers $(b_p - a_p(X_a))/p$ where b_p are the coefficients of the newform 480/2.

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(X_a))/p$	104	76	198	74	184	−300	382	176	278	−298	−124

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(X_a))/p$	212	−370	84	−334	−444	−144	574	776	748	−450	286

No. 2: $a = (1 : 1 : 1 : 1 : 1 : -1)$**:**

The variety X_a has 10 nodes as only singularities, namely the points on the orbit of

$$(1 : 0 : 0 : 0 : 0 : 1)$$

under sign change and permutation of the first five coordinates. We have

$$\chi(\tilde{X}_a) = -156, \quad h^2(\tilde{X}_a) = 1, \quad h^3(\tilde{X}_a) = 160$$

and

$$a_p(\tilde{X}_a) = p^3 + p^2 - 9p + 1 - \#X_{a,p}.$$

There do not exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 240/11 ($240k4H1$).

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$	100	−24	98	182	84	−152	−294	−8	66	−6	−84

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	−280	−422	376	−122	−116	832	−30	−472	−596	−206	642

No. 3: $a = (1 : 1 : 1 : 1 : 1 : -5)$**:**

The variety X_a has 32 nodes as only singularities, namely the points on the orbit of the point

$$(1 : 1 : 1 : 1 : 1 : 1).$$

under sign change. We have

$$\chi(\tilde{X}_a) = -112, \quad h^2(\tilde{X}_a) = 2, \quad h^3(\tilde{X}_a) = 118$$

and

$$a_p(\tilde{X}_a) = \begin{cases} p^3 + 2p^2 - 30p + 1 - \#X_{a,p}, & p \equiv 1,4 \mod 5, \\ p^3 \qquad\quad + 32p + 1 - \#X_{a,p}, & p \equiv 2,3 \mod 5. \end{cases}$$

By a generalization of corollary 1.9 there exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 600/10.

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$	4	184	−46	−54	−92	20	−32	372	2	184	48

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	−108	−74	144	184	160	328	382	212	232	−24	374

No. 4: $a = (1 : 1 : 1 : 1 : 1 : -2)$**:**

The variety X_a has 40 nodes as only singularities, namely the points on the orbit of the point

$$(1 : 1 : 0 : 0 : 0 : 1)$$

under sign change and permutation of the first 5 coordinates. We have

$$\chi(\tilde{X}_a) = -96, \quad h^2(\tilde{X}_a) = 1, \quad h^3(\tilde{X}_a) = 100$$

and

$$a_p(\tilde{X}_a) = \begin{cases} p^3 + p^2 - 39p + 1 - \#X_{a,p}, & p \equiv 1 \mod 4, \\ p^3 + p^2 + 41p + 1 - \#X_{a,p}, & p \equiv 3 \mod 4. \end{cases}$$

There do not exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 1920/10.

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$	12	−26	152	−152	48	64	−10	134	48	66	−100

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	−256	−90	−54	174	−76	−88	250	−50	−332	−94	−50

No. 5: $a = (1 : 1 : 1 : -1 : -1 : 2)$**:**

The variety X_a has 40 nodes as only singularities, namely the points on the orbits of the points

$$(0 : 0 : 0 : 1 : 1 : 1), \quad (1 : 1 : 0 : 1 : 1 : 0), \quad (1 : 0 : 0 : 1 : 0 : 0)$$

under sign change and permutation of the first 3 resp. the next 2 coordinates. We have

$$\chi(\tilde{X}_a) = -96, \quad h^2(\tilde{X}_a) = 1, \quad h^3(\tilde{X}_a) = 100$$

and

$$a_p(\tilde{X}_a) = \begin{cases} p^3 + p^2 - 39p + 1 - \#X_{a,p}, & p \equiv 1,3 \mod 8, \\ p^3 + p^2 + 33p + 1 - \#X_{a,p}, & p \equiv 5,7 \mod 8. \end{cases}$$

There do not exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 1920/6.

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$	28	2	8	40	112	−96	−134	134	−48	−30	20

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	−240	−70	102	34	28	184	194	−290	−204	2	126

No. 6: $a = (1 : 1 : 1 : -1 : -1 : -3)$**:**

The variety X_a has 44 nodes as only singularities, namely the points on the orbits of the points

$$(1:1:1:0:0:1), \quad (1:1:0:1:1:0), \quad (1:0:0:1:0:0)$$

under sign change and permutation of the first three resp. the next two coordinates. We have

$$\chi(\tilde{X}_a) = -88, \quad h^2(\tilde{X}_a) = 1, \quad h^3(\tilde{X}_a) = 92$$

and

$$a_p(\tilde{X}_a) = \begin{cases} p^3 + p^2 - 43p + 1 - \#X_{a,p}, & p \equiv 1,11 \mod 12, \\ p^3 + p^2 + 45p + 1 - \#X_{a,p}, & p \equiv 5,\ 7 \mod 12. \end{cases}$$

There do not exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 1440/7.

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$	−62	34	76	−102	76	−112	158	−112	48	12	56

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	−48	110	106	−2	−52	276	−102	144	−156	−184	206

No. 7: $a = (1:1:1:3:-3:-3)$**:**

The variety X_a has 52 nodes as only singularities, namely the points on the orbits of the points

$$(0:0:0:1:1:0), \quad (0:0:0:1:0:1), \quad (1:1:1:0:1:0),$$
$$(1:1:1:0:0:1), \quad (1:1:1:1:1:1)$$

under sign change and permutation of the last two coordinates. We have

$$\chi(\tilde{X}_a) = -72, \quad h^2(\tilde{X}_a) = 4, \quad h^3(\tilde{X}_a) = 82$$

and

$$a_p(\tilde{X}_a) = \begin{cases} p^3 + 4p^2 - 48p + 1 - \#X_{a,p}, & p \equiv 1 \mod 4, \\ p^3 - 2p^2 + 50p + 1 - \#X_{a,p}, & p \equiv 3 \mod 4. \end{cases}$$

It is not clear if there exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 360/10.

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$	34	−44	−12	−16	−24	34	16	36	44	−84	−30

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	46	−108	−136	−132	118	−12	104	80	−106	−96	−144

No. 8: $a = (1:1:1:1:-1:-1)$**:**

The variety X_a has 64 nodes as only singularities, namely the points on the orbits of the points

$$(1:1:0:0:1:1), \quad (1:0:0:0:1:0)$$

under sign change and permutation of the first four resp. the last two coordinates. We have

$$\chi(\tilde{X}_a) = -48, \quad h^2(\tilde{X}_a) = 3, \quad h^3(\tilde{X}_a) = 56$$

and

$$a_p(\tilde{X}_a) = \begin{cases} p^3 + 3p^2 - 61p + 1 - \#X_{a,p}, & p \equiv 1 \mod 4, \\ p^3 - p^2 + 63p + 1 - \#X_{a,p}, & p \equiv 3 \mod 4. \end{cases}$$

It is not clear if there exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 96/2 ($96k4E1$).

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$	−4	−100	54	−46	100	208	−158	−204	−144	144	−100

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	8	42	−108	46	108	−224	−254	212	116	146	−38

No. 9: $a = (1:1:1:2:-2:-3)$**:**

The variety X_a has 66 nodes as only singularities, namely the points on the orbits of the points

$$(0:0:0:1:1:0), \quad (1:1:1:0:0:1), \quad (1:1:1:1:1:1),$$

$$(1:1:0:0:1:0), \quad (1:0:0:1:0:1)$$

under sign change and permutation of the first three coordinates. We have

$$\chi(\tilde{X}_a) = -44, \quad h^2(\tilde{X}_a) = 3, \quad h^3(\tilde{X}_a) = 52$$

and

$$a_p(\tilde{X}_p) = \begin{cases} p^3 + 3p^2 - 63p + 1 - \#X_{a,p}, & p \equiv 1, 19 \mod 24, \\ p^3 + 3p^2 - 15p + 1 - \#X_{a,p}, & p \equiv 7, 13 \mod 24, \\ p^3 - \ p^2 + 17p + 1 - \#X_{a,p}, & p \equiv 5, 23 \mod 24, \\ p^3 - \ p^2 + 65p + 1 - \#X_{a,p}, & p \equiv 11, 17 \mod 24. \end{cases}$$

It is not clear if there exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 720/25.

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$	22	−18	4	−18	16	48	−18	4	112	−36	28

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	48	78	−42	−26	88	−48	130	4	−60	24	−38

No. 10: $a = (1:1:1:1:-1:-3)$**:**

The variety X has 72 nodes as only singularities, namely the points on the orbits of the points

$$(1:0:0:0:1:0), \quad (1:1:1:0:0:1), \quad (1:1:1:1:1:1)$$

under sign change and permutation of the first four coordinates. We have

$$\chi(\tilde{X}_a) = -32, \quad h^2(\tilde{X}_a) = 6, \quad h^3(\tilde{X}_a) = 46$$

and

$$a_p(\tilde{X}_a) = \begin{cases} p^3 + 6p^2 - 66p + 1 - \#X_{a,p}, & p \equiv 1 \mod 6, \\ p^3 - 4p^2 + 68p + 1 - \#X_{a,p}, & p \equiv 5 \mod 6. \end{cases}$$

It is not clear if there exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 72/1 (72k4C1).

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$	42	−12	16	−42	−116	24	42	76	124	30	−92

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	0	18	−12	−116	196	−24	64	28	12	−114	−80

No. 11: $a = (1 : 1 : 1 : 1 : 1 : -3)$**:**

The variety X_a has 80 nodes as only singularities, namely the points on the orbit of the point

$$(1 : 1 : 1 : 0 : 0 : 1)$$

under sign change and permutation of the first five coordinates. We have

$$\chi(\tilde{X}_a) = -16, \quad h^2(\tilde{X}_a) = 11, \quad h^3(\tilde{X}_a) = 40$$

and

$$a_p(\tilde{X}_a) = \begin{cases} p^3 + 11p^2 - 69p + 1 - \#X_{a,p}, & p \equiv 1, 11 \mod 12, \\ p^3 - 9p^2 + 71p + 1 - \#X_{a,p}, & p \equiv 5, 7 \mod 12. \end{cases}$$

By a generalization of corollary 1.9 there exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 1440/7.

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$		10	−16	−42	−60	80	−94	−80	28	72	40

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	120	26	−110	170	−20	60	−74	80	60	−124	162

No. 12: $a = (1 : 1 : 1 : 1 : -2 : -2)$**:**

The variety X_a has 80 nodes as only singularities, namely the points on the orbit of the points

$$(1 : 1 : 1 : 1 : 1 : 1), \quad (1 : 1 : 0 : 0 : 1 : 0)$$

under sign change and permutation of the first four resp. the last two coordinates. We have

$$\chi(\tilde{X}_a) = -16, \quad h^2(\tilde{X}_a) = 4, \quad h^3(\tilde{X}_a) = 26$$

and

$$a_p(\tilde{X}_a) = \begin{cases} p^3 + 4p^2 - 76p + 1 - \#X_{a,p}, & p \equiv 1 \mod 8, \\ p^3 \qquad\quad + 16p + 1 - \#X_{a,p}, & p \equiv 5 \mod 8, \\ p^3 + 2p^2 - 14p + 1 - \#X_{a,p}, & p \equiv 7 \mod 8, \\ p^3 - 2p^2 + 78p + 1 - \#X_{a,p}, & p \equiv 3 \mod 8. \end{cases}$$

It is not clear if there exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 192/7 (192k4C1).

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$	24	0	−12	24	0	0	12	−24	−12	24	0

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	−48	60	−48	36	48	96	−24	−24	96	−24	72

Note that in this case we even have

$$a_p(\tilde{X}_a) \equiv b_p \mod 12p.$$

No. 13: $a = (1:1:1:-1:-1:-2)$**:**

The variety X_a has 80 nodes as only singularities, namely the points on the orbits of the points

$$(1:1:0:1:1:0), \quad (1:0:0:1:0:0), \quad (1:1:0:0:0:1), \quad (1:1:1:1:0:1)$$

under sign change and permutation of the first three resp. the next two coordinates. We have

$$\chi(\tilde{X}_a) = -16, \quad h^2(\tilde{X}_a) = 2, \quad h^3(\tilde{X}_a) = 22$$

and

$$a_p = \begin{cases} p^3 + 2p^2 - 78p + 1 - \#X, & p \equiv 1 \mod 8, \\ p^3 + 2p^2 - \;\;6p + 1 - \#X, & p \equiv 5 \mod 8, \\ p^3 \qquad\quad + \;\;8p + 1 - \#X, & p \equiv 7 \mod 8, \\ p^3 \qquad\quad + 80p + 1 - \#X, & p \equiv 3 \mod 8. \end{cases}$$

It is not clear if there exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 384/4.

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$	2	−40	20	−16	28	76	−54	−78	−56	48	−36

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	4	18	−32	32	32	−60	−80	82	48	68	−28

No. 14: $a = (1 : 1 : -1 : -1 : 2 : -2)$**:**

The variety X_a has 90 nodes as only singularities, namely the points on the orbits of the points

$$(1:1:0:0:0:1), \quad (0:0:1:1:1:0), \quad (1:1:1:1:1:1),$$

$$(1:1:1:1:0:0), \quad (1:0:1:0:0:0), \quad (1:0:1:0:1:1), \quad (0:0:0:0:1:1)$$

under sign change and permutation of the first two resp. the second two coordinates. We have

$$\chi(\tilde{X}_a) = 4, \quad h^2(\tilde{X}_a) = 15, \quad h^3(\tilde{X}_a) = 28$$

and

$$a_p(\tilde{X}_a) = \begin{cases} p^3 + 15p^2 - 75p + 1 - \#X_{a,p}, & p \equiv 1,3 \mod 8, \\ p^3 + 15p^2 - 59p + 1 - \#X_{a,p}, & p \equiv 5,7 \mod 8. \end{cases}$$

The planes given by

$$x_0 = \pm x_i, \quad x_1 = \pm x_j, \quad x_4 = \pm x_5$$

where $\{i, j\} = \{2, 3\}$ are contained in X and contain 82 of the 90 nodes, but it is still not clear if there exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 48/3 (48k4A1).

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$	16	36	10	6	12	−40	−46	24	−30	14	20

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	0	−6	20	−54	−20	136	−18	−24	−100	78	−26

No. 15: $a = (1 : 1 : 1 : 1 : -1 : -2)$**:**

The variety X_a has 96 nodes as only singularities, namely the points on the orbits of the points

$$(1:0:0:0:1:0), \quad (1:1:1:0:1:1), \quad (1:1:0:0:0:1)$$

under sign change and permutation of the first four coordinates. We have

$$\chi(\tilde{X}_a) = 16, \quad h^2(\tilde{X}_a) = 15, \quad h^3(\tilde{X}_a) = 16$$

and

$$a_p(\tilde{X}_a) = \begin{cases} p^3 + 15p^2 - 81p + 1 - \#X_{a,p}, & p \equiv 1,3 \mod 8, \\ p^3 + 15p^2 - 65p + 1 - \#X_{a,p}, & p \equiv 5,7 \mod 8. \end{cases}$$

It is not clear if there exist projective small resolutions. The table lists the numbers $(b_p - a_p(\tilde{X}_a))/p$ where b_p are the coefficients of the newform 384/5.

p	7	11	13	17	19	23	29	31	37	41	43
$(b_p - a_p(\tilde{X}_a))/p$	10	20	-6	6	48	20	4	-46	14	-34	8

p	47	53	59	61	67	71	73	79	83	89	97
$(b_p - a_p(\tilde{X}_a))/p$	36	-12	-28	-82	28	76	70	66	-60	82	-54

No. 16: $a = (1 : 1 : 1 : -1 : -1 : -1)$:

This variety was investigated by van Geemen and Werner in [99] where also its modularity was proven. It was denoted there by V_{24}.

The variety $V_{24} = X_a$ has 122 nodes as only singularities, namely the points on the orbits of the points

$$(1 : 0 : 0 : 1 : 0 : 0), \quad (1 : 1 : 0 : 1 : 1 : 0), \quad (1 : 1 : 1 : 1 : 1 : 1)$$

under sign change and permutation of the first three resp. the last three coordinates. We have

$$\chi(\tilde{X}_a) = 68, \quad h^2(\tilde{X}_a) = 34, \quad h^3(\tilde{X}_a) = 2$$

and

$$a_p(\tilde{X}_a) = p^3 + 34p^2 - 88p + 1 - \#X_{a,p}.$$

The planes given by

$$x_0 = \pm x_i, \quad x_1 = \pm x_j, \quad x_2 = \pm x_k$$

with $i, j, k \in \{3, 4, 5\}$ pairwise disjoint are contained in X_a and contain all the nodes so there exist projective small resolutions. For all primes $5 \leq p \leq 97$ the $a_p(\tilde{X}_a)$ agree with the coefficients of the weight four newform 12/1 ($12k4A1$), and by corollary 1.6 they agree for all $p \geq 5$.

5.3 Van Geemen's and Werner's complete intersections

Van Geemen and Werner ([99]) discuss two rigid complete intersection Calabi–Yau threefolds and prove their modularity. We will study their examples in some detail.

Complete intersection of a quadric and a quartic in $\mathbb{P}^5$

Let $V_{24} \subset \mathbb{P}^5$ be the threefold given by the equations

$$\begin{aligned} x_0^2 + x_1^2 + x_2^2 &= x_3^2 + x_4^2 + x_5^2, \\ x_0^4 + x_1^4 + x_2^4 &= x_3^4 + x_4^4 + x_5^4. \end{aligned}$$

Then V_{24} has 122 nodes as only singularities. There exist projective small resolutions. They are rigid, and their L-series is given by the weight four newform 12/1 ($12k4A1$). This example is a special member of the family discussed in 5.2. In fact my study of that family was inspired by van Geemen's and Werner's example.

Complete intersection of two cubics in $\mathbb{P}^5$

Let $V_{33} \subset \mathbb{P}^5$ be the threefold given by the equations

$$x_0^3 + x_1^3 + x_2^3 + x_3^3 = 0,$$
$$x_2^3 + x_3^3 + x_4^3 + x_5^3 = 0.$$

This variety can be constructed as a covering of $\mathbb{P}^3$ of degree 3^5, branched along a configuration of the six planes of a cube. The construction is due to Hirzebruch ([48]), but he does not give it explicitly.

Consider the triple cover T of $\mathbb{P}^3$ branched along a cube given by the equation

$$u^3 = (x-t)(x+t)(y-t)(y+t)(z-t)(z+t).$$

There is a $3^4 : 1$ map $V_{33} \dashrightarrow T$ induced by the map $\mathbb{P}^5 \dashrightarrow \mathbb{P}^4(1,1,1,1,3)$,

$$(x_0 : x_1 : x_2 : x_3 : x_4 : x_5) \mapsto (x_0^3 - x_1^3 : x_2^3 - x_3^3 : x_4^3 - x_5^3 : x_2^3 + x_3^3 : 4x_0x_1x_2x_3x_4x_5).$$

The variety V_{33} has 9 singularities of type $(3,3,3,3)$, namely the points

$$(-1 : \xi_3 : 0 : 0 : 0 : 0), \quad (0 : 0 : -1 : \xi_3 : 0 : 0), \quad (0 : 0 : 0 : 0 : -1 : \xi_3)$$

where ξ_3 is a third root of unity. The Euler characteristic of V_{33} is $\chi(V_{33}) = -144 + 9 \cdot 16 = 0$. Let $\tilde{V}_{33}$ be a big resolution of V_{33} (which is Calabi–Yau, cf. 1.6.3). Then $\tilde{V}_{33}$ has Euler characteristic $\chi(\tilde{V}_{33}) = \chi(V_{33}) + 9 \cdot 8 = 72$.

The tangent cone at the singularities is locally isomorphic to the cone over the Del Pezzo surface

$$x^3 + y^3 + z^3 + t^3 = 0.$$

This surface is isomorphic to $\mathbb{P}^2$ blown up in the 6 points

$$\begin{aligned} &(-\xi_3 : 1 : 1), \quad (-\xi_3^2 : 1 : 1), \\ &(0 : 1 : -\xi_3), \quad (0 : 1 : -\xi_3^2), \\ &(1 : -\xi_3^2 : -\xi_3), \quad (1 : -\xi_3 : -\xi_3^2), \end{aligned}$$

where ξ_3 is a primitive third root of unity (cf. [38]), so over $\mathbb{F}_p$ it contains

$$\begin{cases} p^2 + 7p + 1, & p \equiv 1 \mod 3, \\ p^2 + p + 1, & p \equiv 2 \mod 3 \end{cases}$$

points. Now for $p \equiv 1 \mod 3$ the Lefschetz fixed point formula gives

$$\begin{aligned}|\#\tilde{V}_{33,p} - 1 - p^3 - h^2(\tilde{V}_{33}) \cdot p(p+1)| &= |\#V_{33,p} + 9(p^2+7p) - 1 - p^3 - h^2(\tilde{V}_{33}) \cdot p(p+1)| \\ &\leq p^{3/2} h^3(\tilde{V}_{33}) = p^{3/2}(2 + 2h^2(\tilde{V}_{33}) - 72).\end{aligned}$$

Counting points over $\mathbb{F}_7$ gives

$$h^2(\tilde{V}_{33}) = 36, \quad h^3(\tilde{V}_{33}) = 2,$$

so $\tilde{V}_{33}$ is rigid. For $p \equiv 2 \mod 3$ we have the estimate

$$|\#\tilde{V}_{33,p} - 1 - p^3 - k \cdot p(p+1)| = |\#V_{33,p} + 3(p^2+p) - 1 - p^3 - k \cdot p(p+1)| \leq 2p^{3/2}$$

for a $k \in \mathbb{Z}$, $|k| \leq 36$. Counting points over $\mathbb{F}_{11}$ gives $k = 4$. We end up with the formula

$$a_p(\tilde{V}_{33}) = \begin{cases} p^3 + 27p^2 - 27p + 1 - \#V_{33,p}, & p \equiv 1 \mod 3, \\ p^3 + p^2 + p + 1 - \#V_{33,p}, & p \equiv 2 \mod 3. \end{cases}$$

Counting points we find that for all primes $5 \leq p \leq 97$ the $a_p(\tilde{V}_{33})$ agree with the coefficients of the weight four newform 9/1 ($9k4A1$), and by corollary 1.6 they agree for all $p \geq 5$.

Van Geemen and Werner note ([99]) that because the automorphism of $\tilde{V}_{33}$ induced by

$$(x_0 : x_1 : x_2 : x_3 : x_4 : x_5) \mapsto (\xi_3 x_0 : x_1 : x_2 : x_3 : x_4 : x_5)$$

acts nontrivially on $H^3(\tilde{V}_{33})$, the Galois representation comes from a Hecke character of $\mathbb{Q}(\sqrt{-3})$. As both Galois representations are unramified outside 3 it is then easy to check the isomorphism.

5.4 Nygaard's and van Geemen's complete intersection

The threefold in this section was studied by Nygaard and van Geemen in [75] who already proved its modularity.

Let the complete intersection threefold $X \subset \mathbb{P}^7$ be given by the equations

$$\begin{aligned} 2y_0^2 &= +x_0^2 - x_1^2 - x_2^2 - x_3^2, \\ 2y_1^2 &= -x_0^2 + x_1^2 - x_2^2 - x_3^2, \\ 2y_2^2 &= -x_0^2 - x_1^2 + x_2^2 - x_3^2, \\ 2y_3^2 &= -x_0^2 - x_1^2 - x_2^2 + x_3^2. \end{aligned}$$

Then X is invariant under the action of the group G generated by the following transformations:

- permute the x_i and the y_i simultaneously.
- change the sign of some x_i or y_i.

- $(x_0 : x_1 : x_2 : x_3 : y_0 : y_1 : y_2 : y_3) \mapsto (y_0 : y_1 : y_2 : y_3 : x_0 : x_1 : x_2 : x_3)$
- $(x_0 : x_1 : x_2 : x_3 : y_0 : y_1 : y_2 : y_3) \mapsto$
  $(x_0 \cdot \sqrt{-1} : x_1 \cdot \sqrt{-1} : x_2 : x_3 : y_1 : y_0 : y_3 \cdot \sqrt{-1} : y_2 \cdot \sqrt{-1})$

The variety X has 96 ordinary nodes as only singularities, namely the points on the orbit of

$$(1 : 1 : 0 : 0 : 0 : 0 : \sqrt{-1} : \sqrt{-1})$$

under the action of G. Let $\tilde{X}$ be a small resolution of X. Then $\tilde{X}$ has Euler characteristic

$$\chi(\tilde{X}) = -128 + 2 \cdot 96 = 64.$$

The divisor on X given by

$$y_1 = y_0 \cdot \sqrt{-1}, \quad x_3 = x_2 \cdot \sqrt{-1}$$

is smooth in the above singular point so there exist projective small resolutions (cf. [99]). Note also that the defect of X is $d(X) = h^2(\tilde{X}) - 1 = 31 \neq 0$ (see the computation of $h^2(\tilde{X})$ below). The existence of projective small resolutions could also be deduced from a generalization of corollary 1.9.

For $p \geq 3$ all the nodes and the rulings of their tangent cones are rational over $\mathbb{F}_p$ if $p \equiv 1 \mod 4$. We apply the Lefschetz fixed point formula:

$$\begin{aligned} |\#\tilde{X} - 1 - p^3 - h^2(\tilde{X}) \cdot p(p+1)| &= |\#X + 96p - 1 - p^3 - h^2(\tilde{X}) \cdot p(p+1)| \\ &\leq p^{3/2} h^3(\tilde{X}) = p^{3/2}(2 + 2h^2(\tilde{X}) - 64). \end{aligned}$$

Counting points over $\mathbb{F}_{13}$ we find

$$h^2(\tilde{X}) = 32, \quad h^3(\tilde{X}) = 2,$$

so $\tilde{X}$ is rigid. For $p \equiv 3 \mod 4$ none of the nodes are rational over $\mathbb{F}_p$ and we have the estimate

$$|\#\tilde{X} - 1 - p^3 - k \cdot p(p+1)| = |\#X - 1 - p^3 - k \cdot p(p+1)| \leq 2p^{3/2}$$

for some $k \in \mathbb{Z}$, $|k| \leq 32$. Counting points over $\mathbb{F}_{11}$ gives $k = 8$. We end up with the formula

$$a_p(\tilde{X}) = \begin{cases} p^3 + 32p^2 - 64p + 1 - \#X, & p \equiv 1 \mod 4, \\ p^3 + 8p^2 - 8p + 1 - \#X, & p \equiv 3 \mod 4. \end{cases}$$

Counting points we find that for all primes $3 \leq p \leq 97$ the $a_p(\tilde{X})$ agree with the coefficients of the weight four newform 8/1 ($8k4A1$), and by corollary 1.6 they agree for all $p \geq 3$. For correspondences between X and other threefolds connected with the same newform, cf. 6.1.4.

5.5 Libgober's and Teitelbaum's complete intersection

Let the threefold $X_\lambda \subset \mathbb{P}^5$ be given by the equations

$$\begin{aligned} x_1^3+x_2^3+x_3^3 &= 3\lambda x_4x_5x_6, \\ x_4^3+x_5^3+x_6^3 &= 3\lambda x_1x_2x_3. \end{aligned}$$

This is a complete intersection which is invariant under the group $G_{81} \subset PGL(5)$ (of order 81) of transformations $g_{\alpha,\beta,\delta,\epsilon,\mu}$ where $\alpha,\beta,\delta,\epsilon \in \mathbb{Z}/3\mathbb{Z}$, $\mu \in \mathbb{Z}/9\mathbb{Z}$, and $\mu \equiv \alpha+\beta \equiv \delta+\epsilon \mod 3$ (Note the misprint in [61]). These transformations act as

$$\begin{aligned} g_{\alpha,\beta,\delta,\epsilon,\mu} &: (x_1:x_2:x_3:x_4:x_5:x_6) \\ &\mapsto (\xi_3^\alpha\xi_9^\mu x_1 : \xi_3^\beta\xi_9^\mu x_2 : \xi_9^\mu x_3 : \xi_3^{-\delta}\xi_9^{-\mu}x_4 : \xi_3^{-\epsilon}\xi_9^{-\mu}x_5 : \xi_9^{-\mu}x_6) \end{aligned}$$

where ξ_i is a fixed primitive i-th root of unity. For generic λ the variety X_λ is a smooth Calabi–Yau threefold with Euler characteristic $\chi(X_\lambda) = -144$. Libgober and Teitelbaum ([61]) prove that the mirror partner of X_λ can be described as a resolution of the quotient X_λ/G_{81}. Bernardara ([13]) notes that on X_λ there are more than the expected 1053 lines.

The special member X_1, however, has 81 nodes as only singularities, namely the points on the orbit of the point $(1:1:1:1:1:1)$ under the action of G_{81}. To see that the nodes are contained in smooth divisors we rewrite the equations for X_1 as

$$\begin{aligned} 2(x_1^3+x_2^3+x_3^3-x_4^3-x_5^3-x_6^3) &= 3(x_1+x_2+x_3)(x_1^2+x_2^2+x_3^2)-(x_1+x_2+x_3)^3 \\ &= 3(x_4+x_5+x_6)(x_4^2+x_5^2+x_6^2)-(x_4+x_5+x_6)^3. \end{aligned}$$

The smooth cubic surface given by the equations

$$x_1+x_2+x_3 = x_4+x_5+x_6 = x_1^3+x_2^3+x_3^3-x_4^3-x_5^3-x_6^3 = 0$$

is contained in X_1 and contains 27 of the nodes (and the surfaces on its G_{81}-orbit contain all 81 nodes). Thus there exist projective small resolutions. Since the defect of X_1 is $d(X_1) = h^2(\tilde{X}_1) - 1 = 12 \neq 0$ (see the computation of $h^2(\tilde{X}_1)$ below) the existence of projective small resolutions could also be deduced from a generalization of corollary 1.9.

Now let $\tilde{X}_1$ be a small resolution of X_1. Then $\tilde{X}_1$ has Euler characteristic

$$\chi(\tilde{X}_1) = -144 + 2\cdot 81 = 18.$$

Over $\mathbb{F}_p$ not all of the 81 nodes may appear, depending on the existence of 9-th and 3-rd roots of unity:

$p \mod 9$	# of nodes
1	81
4, 7	27
2, 5, 8	1

If $p \equiv 1 \mod 9$ then all the nodes and the rulings of their tangent cones are rational over $\mathbb{F}_p$ and thus the Lefschetz fixed point formula gives

$$\begin{aligned}|\#\tilde{X}_{1,p} - 1 - p^3 - h^2(\tilde{X}_1)\cdot p(p+1)| &= |\#X_1, p + 81p - 1 - p^3 - h^2(\tilde{X}_1)\cdot p(p+1)| \\ &\leq p^{3/2} h^3(\tilde{X}_1) = p^{3/2}(2 + 2h^2(\tilde{X}_1) - 18).\end{aligned}$$

Counting points on X_1 over $\mathbb{F}_{19}$ and $\mathbb{F}_{127}$ gives

$$h^2(\tilde{X}_1) = 13, \quad h^3(\tilde{X}_1) = 10.$$

Otherwise only 27 resp. 1 of the nodes (and the rulings of their tangent cones) are rational over $\mathbb{F}_p$ and we have the estimates

$$\begin{aligned}|\#X_{1,p} + 27p - 1 - p^3 - \ k\cdot p(p+1)| &\leq p^{3/2}h^3, \quad p \equiv 4,7 \mod 9,\\ |\#X_{1,p} + \ \ p - 1 - p^3 - \ l\cdot p(p+1)| &\leq p^{3/2}h^3, \quad p \equiv 2,5,8 \mod 9,\end{aligned}$$

for some $k, l \in \mathbb{Z}, |k|, |l| \leq h^2(\tilde{X}_1) = 13$. Counting points on X_1 over $\mathbb{F}_{97}$, $\mathbb{F}_{89}$ and $\mathbb{F}_{101}$ gives $k = 13$, $l = 1$. We end up with the formula

$$a_p(\tilde{X}_1) = \begin{cases} p^3 + 13p^2 - 68p + 1 - \#X_1, & p \equiv 1 \mod 9,\\ p^3 + 13p^2 - 14p + 1 - \#X_1, & p \equiv 4,7 \mod 9,\\ p^3 + \ \ p^2 \qquad\quad + 1 - \#X_1, & p \equiv 2,5,8 \mod 9.\end{cases}$$

For all primes $5 \leq p \leq 97$ we find

$$a_p(\tilde{X}_1) = b_p + 4\cdot p \cdot c_p$$

where b_p are the coefficients of the weight four newform 27/2 ($27k4B1$) and c_p are the coefficients of the weight two newform $27A1$.

To prove that this formula holds true for all good primes p we use the fact that X_1 is birationally equivalent with the twisted fibre product $(Y_{\Gamma(3)}, \mathrm{pr}) \times_{\mathbb{P}^1} (Y_{\Gamma(3)}, \pi\circ \mathrm{pr})$ where $(Y_{\Gamma(3)}, \mathrm{pr})$ is the Hesse pencil and $\pi(t) = 9/t$ is an automorphism of $\mathbb{P}^1$ (cf. chapter 2). The elliptic surface $(Y_{\Gamma(3)}, \mathrm{pr})$ has four singular fibres of type I_3 over the cusps ∞, -3, $-3w$, $-3w^2$, where w is a primitive third root of unity. The automorphism π maps the cusps to 0, -3, $-3w$, $-3w^2$.

In [89] (cf. also chapter 2) Schütt considered the automorphism $\pi'(t) = 3 - t$ instead. It maps the cusps to ∞, 6, $-3w^2$, $-3w$, so we are in a completely analogous situation and the modularity proof can be copied from [89].

In analogy to the case of the Schoen quintic (3.1 and 3.2) we can also study the complete intersection $Y_\lambda \subset \mathbb{P}^5$ given by the equations

$$\begin{aligned}(x_1 + x_2 + x_3)^3 &= 3^3\lambda x_4x_5x_6,\\ (x_4 + x_5 + x_6)^3 &= 3^3\lambda x_1x_2x_3.\end{aligned}$$

Note that in this case the map

$$\phi : \mathbb{P}^5 \longrightarrow \mathbb{P}^5, \quad (z_0 : z_1 : z_2 : z_3 : z_4 : z_5) \mapsto (z_0^3 : z_1^3 : z_2^3 : z_3^3 : z_4^3 : z_5^3)$$

does not divide out the whole group G_{81} but only a subgroup with 27 elements so that Y_λ is not the mirror of X_λ. According to numerical experiments (i.e., counting of points) the L-series of Y_1 seems to agree with that of X_1 (but the resolution of singularities will be much more complicated).

5.6 An intersection of two cubics in $\mathbb{P}^5$ with 108 nodes

Let the complete intersection threefold $X \subset \mathbb{P}^5$ be defined by the equations

$$x_0^3 + x_1^3 + x_2^3 = x_3^3 + x_4^3 + x_5^3$$
$$x_0x_1x_2 = x_3x_4x_5.$$

Then X has 108 ordinary nodes as only singularities, namely the 27 points on the orbits of

$$(1:0:0:1:0:0), \quad (1:0:0:\xi:0:0), \quad (1:0:0:\xi^2:0:0)$$

under permutation of the first three resp. the last three coordinates and the 81 points

$$(1:\xi^a:\xi^b:\xi^c:\xi^d:\xi^e)$$

with $a, b, c, d, e \in \mathbb{Z}/3\mathbb{Z}$, $a + b \equiv c + d + e \mod 3$ where ξ is a primitive third root of unity.

The planes given by

$$x_i = \xi^a x_l, \quad x_j = \xi^b x_m, \quad x_k = \xi^c x_n$$

with $a, b, c \in \mathbb{Z}/3\mathbb{Z}$, $a + b + c \equiv 0 \mod 3$, where ξ is a primitive third root of unity and $\{i, j, k\} = \{0, 1, 2\}$, $\{l, m, n\} = \{3, 4, 5\}$, are contained in X and contain all the nodes so there exist projective small resolutions.

Let $\tilde{X}$ be a small resolution of X. Then $\tilde{X}$ has Euler characteristic

$$\chi(\tilde{X}) = -144 + 2 \cdot 108 = 72.$$

If $p \equiv 1 \mod 3$ then all the nodes and the rulings of their tangent cones are rational over $\mathbb{F}_p$ and thus the Lefschetz fixed point formula gives

$$\begin{aligned} |\#\tilde{X}_p - 1 - p^3 - h^2(\tilde{X}) \cdot p(p+1)| &= |\#X_p + 108p - 1 - p^3 - h^2(\tilde{X}) \cdot p(p+1)| \\ &\leq p^{3/2} h^3(\tilde{X}) = p^{3/2}(2 + 2h^2(\tilde{X}) - 72). \end{aligned}$$

Counting points over $\mathbb{F}_{19}$ gives

$$h^2(\tilde{X}) = 36, \quad h^3(\tilde{X}) = 2,$$

so $\tilde{X}$ is rigid.

If $p \equiv 2 \mod 3$ then only 10 nodes (and the rulings of their tangent cones) are rational over $\mathbb{F}_p$. In this case we have the estimate

$$|\#X_p + 10p - 1 - p^3 - k \cdot p(p+1)| \leq 2p^{3/2}$$

with $k \in \mathbb{Z}$, $|k| \leq h^2(\tilde{X}) = 36$. Counting points over $\mathbb{F}_{11}$ gives $k = 6$. We end up with the formula

$$a_p(\tilde{X}) = \begin{cases} p^3 + 36p^2 - 72p + 1 - \#X_p, & p \equiv 1 \mod 3, \\ p^3 + \ \ 6p^2 - \ \ 4p + 1 - \#X_p, & p \equiv 2 \mod 3. \end{cases}$$

Counting points on X_p for all good primes $p \leq 97$ we detect that the $a_p(\tilde{X})$ agree with the coefficients of the weight 4 newform 9/1 ($9k4A1$) and by corollary 1.6 they agree for all $p \geq 5$.

The threefold X is birationally equivalent with the self-fibre product $(Y_{\Gamma(3)}, \mathrm{pr}) \times_{\mathbb{P}^1} (Y_{\Gamma(3)}, \mathrm{pr})$ where $(Y_{\Gamma(3)}, \mathrm{pr})$ is again the Hesse pencil (cf. chapter 2). It has been studied here because it seems to be the intersection of two cubics in $\mathbb{P}^5$ with the highest known number of nodes.

5.7 Verrill's threefolds

We consider a root system $\mathcal{R}$ of rank n. Let $\mathcal{L}_{\mathcal{R}}$ be the root lattice generated by $\mathcal{R}$, and let $\mathcal{L}^*_{\mathcal{R}}$ be its dual lattice. Define the *Weyl chambers* of $\mathcal{R}$ as follows: For $r \in \mathcal{R}$, let $H_r := \{s \in \mathcal{L}^*_{\mathcal{R}} \otimes \mathbb{Q} \,|\, \langle s, r\rangle = 0\}$. A Weyl chamber is the closure of any connected component of $\mathcal{L}^*_{\mathcal{R}} \otimes \mathbb{Q} \setminus \bigcup_{r\in\mathcal{R}} H_r$.

Let $\Sigma_{\mathcal{R}}$ be the fan in $\mathcal{L}^*_{\mathcal{R}} \otimes \mathbb{Q}$ consisting of the Weyl chambers, together with all their subfaces, and let $X(\Sigma_{\mathcal{R}})$ be the toric variety associated to the fan $\Sigma_{\mathcal{R}}$. Let $\Delta_{\mathcal{R}}$ be the polyhedron in $\mathcal{L}_{\mathcal{R}}\otimes\mathbb{Q}$ with vertices in $\mathcal{R}$, and let $L(\Delta_{\mathcal{R}})$ denote the space of Laurent polynomials with support in $\Delta_{\mathcal{R}}$. Let the notation e^x denote the passing from $\mathcal{L}_{\mathcal{R}}$ to $\mathbb{C}[\mathcal{L}_{\mathcal{R}}]$, $x \mapsto e^x$, so that each root $r \in \mathcal{R}$ gives a monomial e^r. We define a Laurent polynomial

$$\chi_{\mathcal{R}} := \sum_{r\in\mathcal{R}} e^r \in L(\Delta_{\mathcal{R}})$$

and so obtain a rational function $\chi_{\mathcal{R}} : X(\Sigma_{\mathcal{R}}) \longrightarrow \mathbb{P}^1$. Blowing up the base locus of this map and resolving singularities we obtain a variety $\mathcal{X}_{\mathcal{R}}$ with a fibration $\mathcal{X}_{\mathcal{R}} \longrightarrow \mathbb{P}^1$. In [103] it is shown that if $\mathcal{R}$ is of type A_n or a product of A_n type lattices then the general fibre is a Calabi–Yau variety. The variety $\mathcal{X}_{\mathcal{R}}$ itself does not have canonical class, and so can not be a Calabi–Yau variety, but for $\mathcal{R} = A_3$, $A_1^3 := A_1 \times A_1 \times A_1$ or $A_1 \times A_2$, we can obtain a Calabi–Yau variety $\mathcal{Z}_{\mathcal{R}}$, which is the desingularization of a double cover of $\mathcal{X}_{\mathcal{R}}$ and is given by the pullback in the following diagram, where $F(t)$ is a certain rational function of degree 2 (cf. [104], Theorem 2.1.):

$$\begin{array}{ccc} \mathcal{Z}_{\mathcal{R}} & \longrightarrow & \mathcal{X}_{\mathcal{R}} \\ \downarrow & & \downarrow \chi_{\mathcal{R}} \\ \mathbb{P}^1 & \xrightarrow{t \mapsto \lambda = F(t)} & \mathbb{P}^1 \end{array}$$

The A_3 case

Let $\{E_1, E_2, E_3, E_4\}$ be the standard basis for $\mathbb{R}^4$. The root lattice A_3 is a sublattice of $\mathbb{R}^4$ of rank 3 generated by $v_1 := E_1 - E_2$, $v_2 := E_2 - E_3$, $v_3 := E_3 - E_4$, and the collection of all roots is given by the set

$$\{E_i - E_j \mid 1 \le i, j \le 4,\, i \ne j\}.$$

By putting $x_i = e^{E_i}$ we associate the monomial $x_i x_j^{-1}$ to the root $E_i - E_j$. The Laurent polynomial χ_{A_3} is then given by

$$\chi_{A_3} = \sum_{i \ne j} x_i x_j^{-1} = (x_1 + x_2 + x_3 + x_4)(x_1^{-1} + x_2^{-1} + x_3^{-1} + x_4^{-1}) - 4.$$

The variety $\mathcal{X}_{A_3}$ is given by the desingularization of $\{\chi_{A_3} = \lambda\} \subset \mathbb{P}^3 \times \mathbb{P}^1$. We obtain the Calabi–Yau variety $\mathcal{Z}_{A_3}$ by taking the double cover $\lambda = (t-1)^2/t$ and resolving singularities. Verrill computes

$$\chi(\mathcal{Z}_{A_3}) = 100, \quad h^2(\mathcal{Z}_{A_3}) = 50, \quad h^3(\mathcal{Z}_{A_3}) = 2,$$

so $\mathcal{Z}_{A_3}$ is rigid. Verrill uses theorem 1.5 to prove that $a_p(\mathcal{Z}_{A_3}) = b_p$ for all $p \ge 5$ where b_p are the coefficients of the weight four newform 6/1 (6k4A1). There are two more proofs of this fact in the literature; the one in [81] uses a correspondence (see also 6.1.2), and the one in [107] uses Wiles' results on comparison of Galois representations.

The A_1^3 case

Let again $\{E_1, E_2, E_3, E_4\}$ be the standard basis for $\mathbb{R}^4$. The root lattice A_1^3 is a sublattice of $\mathbb{R}^4$ of rank 3 generated by E_1, E_2, E_3, and the collection of all roots is given by the set

$$\{\pm E_1, \pm E_2, \pm E_3\}.$$

By putting $x_i = e^{E_i}$ we associate the monomial $x_i^{\pm 1}$ to the root $\pm E_i$. The Laurent polynomial $\chi_{A_1^3}$ is then given by

$$\chi_{A_1^3} = x + x^{-1} + y + y^{-1} + z + z^{-1}.$$

The variety $\mathcal{X}_{A_1^3}$ is given by the desingularization of $\{\chi_{A_1^3} = \lambda\} \subset (\mathbb{P}^1)^4$. We obtain the Calabi–Yau variety $\mathcal{Z}_{A_1^3}$ by taking the double cover $\lambda = -t - t^{-1}$ and resolving singularities. Verrill computes

$$\chi(\mathcal{Z}_{A_1^3}) = 140, \quad h^2(\mathcal{Z}_{A_1^3}) = 70, \quad h^3(\mathcal{Z}_{A_1^3}) = 2,$$

so $\mathcal{Z}_{A_1^3}$ is rigid. Verrill uses theorem 1.5 to prove that $a_p(\mathcal{Z}_{A_1^3}) = b_p$ for all $p \ge 3$ where b_p are the coefficients of the weight four newform 8/1 (8k4A1). A different proof of this fact using character sum calculations can be found in [3]. There are various correspondences between $\mathcal{Z}_{A_1^3}$ and other varieties with the same L-series, cf. 6.1.4.

5.8 Hulek's and Verrill's threefolds

In [51] Hulek and Verrill investigated the geometry and arithmetic of a family of Calabi–Yau threefolds $X_{\boldsymbol{a}}$, $\boldsymbol{a} = (a_1 : \cdots : a_6) \in \mathbb{P}^5$, given by

$$X_{\boldsymbol{a}} \cap T: \quad (x_1 + \cdots + x_5)\left(\frac{a_1}{x_1} + \cdots + \frac{a_5}{x_5}\right) = a_6.$$

where $T := \mathbb{P}^4 \setminus \{x_1 \cdots x_5 = 0\}$. The variety $X_{\boldsymbol{a}}$ is the closure of $X_{\boldsymbol{a}} \cap T$ in the toric variety $\tilde{P}$ associated to the root lattice A_4 (cf. 5.7). It is birational to a variety in $\mathbb{P}^5$ defined by the two equations

$$\sum_{i=1}^{6} \frac{a_i}{x_i} = \sum_{i=1}^{6} x_i = 0.$$

This follows immediately from setting $x_6 = -\sum_{i=1}^{5} x_i$.

The advantage of the toric realisation is that the resulting varieties have only ordinary nodes as singularities (which are easier to resolve). Hulek and Verrill compute the number of nodes and the Hodge numbers of the desingularizations in each case. The Hodge numbers can be computed since by rewriting the equations for $X_{\boldsymbol{a}}$ as

$$\begin{aligned} x_1 + x_2 + x_3 &= -(x_4 + x_5 + x_6), \\ \frac{a_1}{x_1} + \frac{a_2}{x_2} + \frac{a_3}{x_3} &= -\left(\frac{a_4}{x_4} + \frac{a_5}{x_5} + \frac{a_6}{x_6}\right), \end{aligned}$$

we see that $X_{\boldsymbol{a}}$ is birational to the fibre product of the elliptic surfaces given by

$$\begin{aligned} (x_1 + x_2 + x_3)\left(\frac{a_1}{x_1} + \frac{a_2}{x_2} + \frac{a_3}{x_3}\right)\lambda_0 &= \lambda_1, \\ (x_4 + x_5 + x_6)\left(\frac{a_4}{x_4} + \frac{a_5}{x_5} + \frac{a_6}{x_6}\right)\mu_0 &= \mu_1, \end{aligned}$$

and these fibre products can be investigated with the help of Schoen's results ([84], cf. also chapter 2). It is also possible to determine if $X_{\boldsymbol{a}}$ has a projective small resolution or not.

In the following table we list a number of cases where Hulek and Verrill determined the L-series. We give the number of nodes on $X_{\boldsymbol{a}}$, the Euler number $\chi(\tilde{X}_{\boldsymbol{a}})$ and the Hodge number $h^{2,1}(\tilde{X}_{\boldsymbol{a}})$ of a big resolution $\tilde{X}_{\boldsymbol{a}}$.

$\boldsymbol{a}$	# of nodes	$\chi(\tilde{X}_{\boldsymbol{a}})$	$h^{2,1}(\tilde{X}_{\boldsymbol{a}})$
$(1:1:1:1:1:1)$	40	180	0
$(1:1:1:1:1:9)$	35	160	0
$(1:1:1:1:4:4)$	37	168	0
$(1:1:1:4:4:9)$	35	160	0
$(1:1:1:1:1:25)$	31	144	4
$(1:1:1:9:9:9)$	33	152	2
$(1:1:4:4:4:16)$	34	156	1

Note that there do not exist projective small resolutions except for the case $\boldsymbol{a} = (1:1:1:1:1:1)$. This variety is birationally equivalent with the Barth-Nieto quintic (cf. 3.7). Thus the Euler number of a small resolution $\hat{X}_{\boldsymbol{a}}$ in this case is $\chi(\hat{X}_{\boldsymbol{a}}) = 100$.

Hulek and Verrill prove in each case that the L-series splits into two-dimensional pieces. Let $\boldsymbol{a} = (a_1 : \cdots : a_6) \in \mathbb{P}^5$ and $k < l < m$, with $\{i, j, k, l, m\} = \{1, 2, 3, 4, 5\}$. Let H_{ij} denote the hyperplane in T given by $x_i + x_j = 0$. We define

$$E^{ij}_{\boldsymbol{a}} := \overline{(X_{\boldsymbol{a}} \cap H_{ij} \cap T)} \subset X_{\boldsymbol{a}}.$$

Substituting $x_i = -x_j$ in the equation for $X_{\boldsymbol{a}} \cap T$ gives the curve

$$E_{ij} := \{(x_k + x_l + x_m)\left(\frac{a_k}{x_k} + \frac{a_l}{x_l} + \frac{a_m}{x_m}\right) = a_6\},$$

so $E^{ij}_{\boldsymbol{a}}$ is birational to $\overline{E_{ij} \times \mathbb{P}^1} \subset X_{\boldsymbol{a}}$. Now suppose that

$$\prod_{i=1}^{6} a_n \neq 0,\ a_i = a_j \quad \text{and} \quad \sqrt{a_k} \pm \sqrt{a_l} \pm \sqrt{a_m} \pm \sqrt{a_6} \neq 0.$$

Then $E^{ij}_{\boldsymbol{a}}$ is smooth and contains no singularities of $X_{\boldsymbol{a}} \cap T$. Hulek and Verrill consider the induced homomorphism (cf. 1.5.2)

$$H^3_{\text{ét}}(\bar{X}_{\boldsymbol{a}}, \mathbb{Q}_\ell) \longrightarrow \bigoplus_{i,j \text{ as above}} H^3_{\text{ét}}(\bar{E}^{ij}_{\boldsymbol{a}}, \mathbb{Q}_\ell) \quad \simeq \quad \bigoplus_{i,j \text{ as above}} H^1_{\text{ét}}(\bar{E}_{ij}, \mathbb{Q}_\ell) \otimes H^2_{\text{ét}}(\bar{\mathbb{P}}^1, \mathbb{Q}_\ell)$$

Let $W_{\boldsymbol{a}}$ be the kernel of the above map. It turns out that for the non-rigid examples in the table its dimension is equal to $h^{2,1}(\tilde{X}_{\boldsymbol{a}})$. Thus in these cases the L-series of $X_{\boldsymbol{a}}$ splits into two-dimensional parts and (with the help of theorem 1.5)

$$a_p(\tilde{X}_{\boldsymbol{a}}) = b_p + h^{2,1}(\tilde{X}_{\boldsymbol{a}}) \cdot p \cdot c_p$$

where b_p are the coefficients of a weight four newform for some $\Gamma_0(N)$ and c_p the coefficients of a weight two newform for some $\Gamma_0(N)$ associated to the elliptic curves E_{ij}. We list the occurring newforms for the rigid and non-rigid examples.

$\boldsymbol{a}$	b_p	c_p
$(1:1:1:1:1:1)$	6/1 $(6k4A1)$	–
$(1:1:1:1:1:9)$	6/1 $(6k4A1)$	–
$(1:1:1:1:4:4)$	12/1 $(12k4A1)$	–
$(1:1:1:4:4:9)$	60/1 $(60k4A1)$	–
$(1:1:1:1:1:25)$	30/1 $(30k4B1)$	$30A1$
$(1:1:1:9:9:9)$	90/2 $(90k4A1$, twist of 10/1)	$30A1$
$(1:1:4:4:4:16)$	30/2 $(30k4A1)$	$30A1$

By numerical experimentation I found some more parameters $\boldsymbol{a}$ such that the L-series of $X_{\boldsymbol{a}}$ seems to split into a weight four part and certain weight two parts. Verrill computed the dimension of $W_{\boldsymbol{a}}$ and the levels of the corresponding weight two newforms. In all examples we have $\dim(W_{\boldsymbol{a}}) = h^{2,1}(\tilde{X}_{\boldsymbol{a}}) - 1$. The numerical experiments suggest formulas of the type

$$a_p(\tilde{X}_{\boldsymbol{a}}) = b_p + (h^{2,1}(\tilde{X}_{\boldsymbol{a}}) - 1) \cdot p \cdot c_p + p \cdot d_p$$

where b_p are the coefficients of a weight four newform for some $\Gamma_0(N)$, c_p are the coefficients of a weight two newform for some $\Gamma_0(N)$ associated to the elliptic curves E_{ij} and d_p are the coefficients of another weight two newform for $\Gamma_0(N)$.

$\boldsymbol{a}$	$\chi(\tilde{X}_{\boldsymbol{a}})$	$h^{2,1}(\tilde{X}_{\boldsymbol{a}})$	b_p		c_p	d_p
$(1:1:1:1:1:-7)$	140	5	14/1	(14k4B1)	14A1	14A1
$(1:1:-1:-1:4:-4)$	148	3	40/2	(40k4B1)	160A1	20A1
$(1:1:1:-1:-1:-1)$	140	5	60/1	(60k4A1)	20A1	30A1
$(1:1:9:9:9:81)$	148	3	210/6	(210k4H1)	210A1	30A1
$(1:9:9:9:9:25)$	156	1	30/1	(30k4B1)	–	30A1

There is no explanation yet for the occurrence of the later weight two newforms.

Note that the second and the third example in the table are birationally equivalent with twisted self-fibre products of elliptic surfaces. The second example corresponds to $(Y, \mathrm{pr}) \times_{\mathbb{P}^1} (Y, \pi \circ \mathrm{pr})$ where Y is given by

$$(x+y+z)(xy+xz+4yz) = txyz$$

and $\pi(t) = -t$. The third example corresponds to $(Y_{\Gamma_1(6)}, \mathrm{pr}) \times_{\mathbb{P}^1} (Y_{\Gamma_1(6)}, \pi \circ \mathrm{pr})$ (cf. the tables in chapter 2).

5.9 Bernadara's complete intersections

Let the complete intersection threefold $V_\lambda \subset \mathbb{P}^7$ with $\lambda = (\lambda_{01}, \lambda_{23}, \lambda_{45}, \lambda_{67}) \in (\mathbb{P}^1)^4$ be given by the equations

$$\begin{aligned}
x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 &= \lambda_{67} \cdot x_6 x_7, \\
x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_6^2 + x_7^2 &= \lambda_{45} \cdot x_4 x_5, \\
x_0^2 + x_1^2 + x_4^2 + x_5^2 + x_6^2 + x_7^2 &= \lambda_{23} \cdot x_2 x_3, \\
x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 + x_7^2 &= \lambda_{01} \cdot x_0 x_1.
\end{aligned}$$

Bernardara ([13]) considered the subfamily with $\lambda_{23} = \lambda_{45} = \lambda_{67}$. He showed that on generic X_λ there are more than the expected 512 lines.

I performed some numerical experiments with V_λ. For certain values of the parameter λ we have

$$\#V_{\lambda,p} \equiv b_p \mod p$$

for all considered primes p and the coefficients b_p of certain weight four newforms, suggesting that these newforms appear in the L-series of V_λ:

λ	newform	
$(6, 6, 6, 6)$	6/1	(6k4A1)
$(1, 1, 1, 4)$	48/3	(48k4A1, twist of 6/1)
$(2, 10, 10, 10)$	2400/5	(twist of 96/4)
$(6, 2/3, 6, 2)$	288/7	(288k4F1, twist of 32/2)

Let $V = V_{(6,6,6,6)}$. Then V is invariant under the action of the group $G \subset PGL(7)$ of order $3072 = 24 \cdot 16 \cdot 8$ which is generated by the permutations $(x_{2i}x_{2i+1})$, $(x_{2i}x_{2j})(x_{2i+1}x_{2j+1})$ and the sign changing transformations $x_{2i} \mapsto -x_{2i}$, $x_{2i+1} \mapsto -x_{2i+1}$.

V has 56 ordinary nodes as only singularities, namely the 48 points on the orbit of $(1 : -3+\sqrt{8} : \sqrt{-1} : \sqrt{-1} \cdot (-3+\sqrt{8}) : 0 : 0 : 0 : 0)$ and the 8 points on the orbit of $(1 : 1 : 1 : 1 : 1 : 1 : 1 : 1)$ under the action of G. I expect V to have a large number of deformations (since Nygaard's and van Geemen's complete intersection of four quadrics in $\mathbb{P}^7$ (cf. 5.4) is rigid with 96 nodes). The other examples in the table are less symmetric and should have even larger number of deformations but I did not check this.

Let us consider the coordinate change

$$\begin{aligned} y_0 &= x_0 + x_1, & y_1 &= x_0 - y_1, \\ y_2 &= x_2 + x_3, & y_3 &= x_2 - y_3, \\ y_4 &= x_4 + x_5, & y_5 &= x_4 - y_5, \\ y_6 &= x_6 + x_7, & y_7 &= x_6 - y_7. \end{aligned}$$

In these new coordinates V_λ is given by the equations

$$\begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & -\lambda_{67} & \lambda_{67} \\ 2 & 2 & 2 & 2 & -\lambda_{45} & \lambda_{45} & 2 & 2 \\ 2 & 2 & -\lambda_{23} & \lambda_{23} & 2 & 2 & 2 & 2 \\ -\lambda_{01} & \lambda_{01} & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} y_0^2 \\ \vdots \\ y_7^2 \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}.$$

For $\lambda = (6, 6, 6, 6)$ the above matrix can be transformed such that V_λ is given by the equations

$$\begin{pmatrix} -3 & 1 & 1 & 1 & 2 & 0 & 0 & 0 \\ 1 & -3 & 1 & 1 & 0 & 2 & 0 & 0 \\ 1 & 1 & -3 & 1 & 0 & 0 & 2 & 0 \\ 1 & 1 & 1 & -3 & 0 & 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} y_0^2 \\ \vdots \\ y_7^2 \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix},$$

so we get a correspondence between V_λ and the double octic X_9 given by the equation

$$u^2 = (x^2 + y^2 + z^2 - 3t^2)(x^2 + y^2 - 3z^2 + t^2)(x^2 - 3y^2 + z^2 + t^2)(-3x^2 + y^2 + z^2 + t^2)$$

from 4.5, induced by the $8 : 1$ rational map

$$\mathbb{P}^7 \longrightarrow \mathbb{P}^4(1, 1, 1, 1, 4), \qquad (y_0 : \cdots : y_7) \mapsto (y_0 : y_1 : y_2 : y_3 : y_4y_5y_6y_7).$$

Another correspondence between V_λ and the double octic X given by the equation

$$u^2 = xyzt(x + y + z - 3t)(x + y - 3z + t)(x - 3y + z + t)(-3x + y + z + t)$$

(arrangement no. 287 from 4.2) is induced by the 64:1 rational map

$$\mathbb{P}^7 \longrightarrow \mathbb{P}^4(1, 1, 1, 1, 4), \qquad (y_0 : \cdots : y_7) \mapsto (y_0^2 : y_1^2 : y_2^2 : y_3^2 : y_0y_1y_2y_3y_4y_5y_6y_7).$$

Similar correspondences between V_λ for other values of λ and double octics can easily be constructed.

5.10 Σ_6-symmetric complete intersections

Consider the power sums

$$C_i := C_i(x_0, x_1, \ldots, x_5) := \sum_{k=0}^{5} x_k^i$$

and let the complete intersection threefold $X_{\lambda,\mu} \subset \mathbb{P}^5$ be given by the equations

$$aC_1^2 + bC_2 = 0,$$
$$cC_4 + dC_3C_1 + eC_1^4 = 0,$$

with $\lambda := (a : b) \in \mathbb{P}^1$, $\mu := (c : d : e) \in \mathbb{P}^2$. The pencil $\{X_{\lambda,\mu}\}$ is the pencil of Σ_6-symmetric complete intersections of a quadric and a quartic in $\mathbb{P}^5$. This construction is inspired by van Straten's Σ_6-symmetric quintics in $\mathbb{P}^4$ (cf. 3.6). I have not classified the members of this pencil but performed some numerical experiments. For certain values of the parameters λ, μ we have

$$\#X_{\lambda,\mu,p} \equiv b_p \mod p$$

for all considered primes p and the coefficients b_p of certain weight four newforms, suggesting that these newforms appear in the L-series of $X_{\lambda,\mu}$. I ran the computer search for integer values of a, b, c, d, e with $|a|$, $|b|$, $|c|$, $|d|$, $|e| \leq 20$. Note that for $\lambda = (a : b) = (0 : 1)$, $\mu = (c : d : e) = (1 : 0 : 0)$ we find $X_{\lambda,\mu} = X_a$ with $a = (1:1:1:1:1:1)$ from 5.2.

$\lambda = (a : b)$	$\mu = (c : d : e)$	newform		level
$(-5:3)$	$(3:4:-5)$	465/2		$465 = 3 \cdot 5 \cdot 31$
$(-1:1)$	$(3:-4:1)$	15/1	$(15k4B1)$	$15 = 3 \cdot 5$
$(-1:1)$	$(3:-2:-1)$	300/2		$300 = 2^2 \cdot 3 \cdot 5^2$
$(0:1)$	$(1:0:0)$	480/2		$480 = 2^5 \cdot 3 \cdot 5$
$(1:1)$	$(1:0:1)$	1365/1		$1365 = 3 \cdot 5 \cdot 7 \cdot 13$
$(4:3)$	$(3:4:-2)$	480/2		$480 = 2^5 \cdot 3 \cdot 5$
$(-1:1)$	$(3:4:-7)$	15/1	$(15k4B1)$	$15 = 3 \cdot 5$
$(1:3)$	$(9:-6:-1)$	180/2	$(180k4A1,$ twist of 180/1)	$180 = 2^2 \cdot 3^2 \cdot 5$
$(-5:3)$	$(9:-6:-1)$	180/1	$(180k4B1)$	$180 = 2^2 \cdot 3^2 \cdot 5$
$(-1:2)$	$(12:-10:1)$	930/3		$930 = 2 \cdot 3 \cdot 5 \cdot 31$
$(-1:3)$	$(9:-12:1)$	465/2		$465 = 3 \cdot 5 \cdot 31$
$(-9:4)$	$(12:-13:1)$	930/3		$930 = 2 \cdot 3 \cdot 5 \cdot 31$
$(7:3)$	$(9:-6:-13)$	900/1	(twist of 180/1)	$900 = 2^2 \cdot 3^2 \cdot 5^2$
$(1:2)$	$(8:-16:-7)$	480/2		$480 = 2^5 \cdot 3 \cdot 5$

To my knowledge, the bad primes 13 and 31 have not appeared in examples of this kind before (the bad prime 13 occurs also in 4.9). It would be interesting to study the varieties $X_{\lambda,\mu}$ in detail, i.e., determine the singularities and describe a resolution. This could be done along the lines of [101].

As an example we will investigate $X := X_{\lambda,\mu}$ for $\lambda = (1:1)$, $\mu = (1:0:1)$. This variety is even Σ_7-symmetric since it can be given in $\mathbb{P}^6$ by the equations

$$\sum_{i=0}^{6} x_i = \sum_{i=0}^{6} x_i^2 = \sum_{i=0}^{6} x_i^4 = 0.$$

We will prove that X is smooth over $\mathbb{C}$ and find the bad primes on the way. Differentiating we see that if $(x_0 : \ldots : x_6)$ is a singular point of X then there is $(A : B : C) \in \mathbb{P}^2$ with

$$Cx_i^3 + Bx_i + A = 0$$

for all $i \in \{0, \ldots, 6\}$. If we assume that $A = 0$ then we can conclude $x_i \in \{-1, 0, 1\}$ for all $i \in \{0, \ldots, 6\}$. This leads to singular points in characteristic 2,3,5,7 (so these primes are primes of bad reduction) but no other characteristics.

Now let $A \neq 0$. Then we can assume (for all characteristics except 7) that also $C \neq 0$ and thus $C = 1$. We can further assume that the roots of $x^3 + Bx + A$ are $\{1, \beta, -1-\beta\}$. In particular, considering symmetry we have $x_i = 1$ for $i \in \{0, \ldots, m\}$, and we can restrict ourselves to the cases with $m \geq 2$ (by the pigeonhole principle).

If $m = 6$ then $(x_0 : \ldots : x_6) = (1:1:1:1:1:1:1)$ and this point is non-singular except in characteristic 2.

If $m = 5$ then $(x_0 : \ldots : x_6) = (1:1:1:1:1:1:\beta)$ and so $\beta = -6$. The other two equations for X give $6 + 6^2 = 42 = 0$ and $6 + 6^4 = 1302 = 0$. The common prime divisors of 42 and 1302 are 2, 3 and 7, so only in these characteristics we can get singular points.

If $m = 4$ then $(x_0 : \ldots : x_6) = (1:1:1:1:1:\beta:\beta)$ or $(x_0 : \ldots : x_6) = (1:1:1:1:1:\beta:-1-\beta)$. In the first case we conclude that $5 + 2\beta = 5 + 2\beta^2 = 0$ which can only happen in characteristics 5 and 7. In the second case we find $4 = 0$.

If $m = 3$ then $(x_0 : \ldots : x_6) = (1:1:1:1:\beta:\beta:\beta)$ or $(x_0 : \ldots : x_6) = (1:1:1:1:\beta:\beta:-1-\beta)$. In the first case we conclude that $4 + 3\beta = 4 + 3\beta^2 = 0$ which can only happen in characteristics 2 and 7. In the second case we find $\beta = -3$. The other two equations for X give $26 = 0$ and $182 = 0$. Thus in characteristic 13 we have 105 isolated singular points on the Σ_7-orbit of the point

$$(1:1:1:1:2:-3:-3).$$

If $m = 2$ then we have $(x_0 : \ldots : x_6) = (1:1:1:\beta:\beta:\beta:\beta)$ or $(x_0 : \ldots : x_6) = (1:1:1:\beta:\beta:-1-\beta:-1-\beta)$ or $(x_0 : \ldots : x_6) = (1:1:1:\beta:\beta:\beta:-1-\beta)$. In the first case we conclude $3 + 4\beta = 3 + 4\beta^2 = 0$ which can only happen in characteristics 3 and 7. In the second case we find $1 = 0$. In the third case we find $2(1+\beta) = 0$. This leads to singular points only for the bad primes 2 and 3.

It is nice to see that all the bad primes except 2 appear in the level of the modular form 1365/1 which is conjectured to occur in the L-series of X.

Note that there are no Σ_6-symmetric complete intersections of two cubics in $\mathbb{P}^5$ since all Σ_6-symmetric cubic polymials are linear combinations of C_1^3, C_1C_2 and C_3.

5.11 Rodriguez-Villegas' hypergeometric threefolds

A *hypergeometric weight system* is a formal linear combination

$$\gamma = \sum_{\mu \geq 1} \gamma_\mu [\mu],$$

where $\gamma_\mu \in \mathbb{Z}$ are zero for all but finitely many μ, satisfying the following two conditions:

$$(i) \qquad \sum_{\mu \geq 1} \gamma_\mu \mu = 0,$$

$$(ii) \qquad d = d(\gamma) := -\sum_{\mu \geq 1} \gamma_\mu > 0.$$

The number d is called the *dimension* of the weight system γ. To γ we associate the hypergeometric function

$$u(\lambda) := \sum_{n \geq 0} u_n \lambda^n$$

where

$$u_n = \prod_{\mu \geq 1} (\mu n)!^{\gamma_\mu}.$$

The weight system γ is called *integral* if $u_n \in \mathbb{Z}$ for all $n > 0$.

We consider the weight systems generated by those of the form

$$\phi(n)[1] - \sum_{m|n} \mu\left(\frac{n}{m}\right)[m], \qquad n \geq 2,$$

where ϕ is Euler's phi-function and μ is the Möbius function. Exactly for these weight systems we have $d = r$ where r is another invariant called the *rank*. They are all integral. By toric geometry we can associate to each such weight system a one-parameter family X_φ of Calabi–Yau $d-1$-folds as subvarieties of a product of weighted projective spaces of total dimension d (with $u(\lambda)$ as one of its periods). For $d = 4$ we find exactly 14 families of Calabi–Yau threefolds. The generic member X_φ of each family is smooth and has $h^{1,1}(X_\varphi) = 1$. For finitely many values of φ the threefold X_φ becomes singular and the resolution of singularities is again a Calabi–Yau threefold defined over $\mathbb{Q}$. Rodriguez-Villegas ([80]) claims that these threefolds are rigid but this is not always the case.

Thirteen of the families were mentioned by Batyrev and van Straten in [11]. The 14-th example (last one in the table) was found later by Rodriguez-Villegas in [80] who also dealt with the modularity of the singular members. In the table we list the weight systems, the threefolds (where $V_{d_1,\ldots,d_s}$ denotes a complete intersection of hypersurfaces of degrees $d_1, \ldots, d_s$), the Euler characteristic of the general members, the ambient (weighted) projective spaces and the weight four newform associated to the singular members. To save space, "$\sim 128/1$" is used as an abbreviation for “twist of 128/1”.

weight system	threefold	χ	ambient space	weight four newform
$[5]-5[1]$	V_5	-200	$\mathbb{P}^4$	25/1 $(25k4A1)$
$[6]-[2]-4[1]$	V_6	-204	$\mathbb{P}^4(1,1,1,1,2)$	108/2 $(108k4D1)$
$[8]-[4]-4[1]$	V_8	-296	$\mathbb{P}^4(1,1,1,1,4)$	128/3 $(128k4C1, \sim 128/1)$
$[10]-[5]-[2]-3[1]$	V_{10}	-288	$\mathbb{P}^4(1,1,1,2,5)$	200/10 $(200k4A1, \sim 200/1)$
$2[3]-6[1]$	$V_{3,3}$	-144	$\mathbb{P}^5$	27/2 $(27k4B1, \sim 27/1)$
$[4]+[2]-6[1]$	$V_{2,4}$	-176	$\mathbb{P}^5$	16/1 $(16k4A1, \sim 8/1)$
$[3]+2[2]-7[1]$	$V_{2,2,3}$	-144	$\mathbb{P}^6$	36/1 $(36k4A1, \sim 12/1)$
$4[2]-8[1]$	$V_{2,2,2,2}$	-128	$\mathbb{P}^7$	8/1 $(8k4A1)$
$[4]+[3]-[2]-5[1]$	$V_{3,4}$	-156	$\mathbb{P}^5(1,1,1,1,1,2)$	9/1 $(9k4A1)$
$2[4]-2[2]-4[1]$	$V_{4,4}$	-144	$\mathbb{P}^5(1,1,1,1,2,2)$	32/3 $(32k4C1, \sim 32/2)$
$[6]+[2]-[3]-5[1]$	$V_{2,6}$	-256	$\mathbb{P}^5(1,1,1,1,1,3)$	72/2 $(72k4B1, \sim 24/1)$
$[6]+[4]-[3]-2[2]-3[1]$	$V_{4,6}$	-156	$\mathbb{P}^5(1,1,1,2,2,3)$	144/1 $(144k4E1, \sim 72/1)$
$2[6]-2[3]-2[2]-2[1]$	$V_{6,6}$	-120	$\mathbb{P}^5(1,1,2,2,3,3)$	216/3 $(216k4D1, \sim 216/1)$
$[12]+[2]-[6]-[4]-4[1]$	$V_{2,12}$		$\mathbb{P}^5(1,1,1,1,4,6)$	864/4 $(\sim 864/1)$

The Euler characteristics of the first thirteen examples can be found, for example, in [56] and [57]. The Euler characteristic of $V_{2,12}$ has not been computed yet but this could be done with standard methods (cf. [57]).

For each family $\{X_\varphi\}$ there is a group G operating on X_φ such that the mirror of X_φ can be described as a resolution of the quotient X_φ/G. The singular members have one orbit of ordinary nodes under the action of G and the resolution of the quotient X_φ/G is a rigid Calabi–Yau threefold.

It is possible to write down equations for all families. We give some examples. For the special value of φ the points on the orbit of the points $(1 : \ldots : 1)$ under the action of the respective group become singular.

$$V_5(\varphi) = \{x_0^5 + x_1^5 + x_2^5 + x_3^5 + x_4^5 - 5\varphi \cdot x_0x_1x_2x_3x_4 = 0\}$$
$$V_6(\varphi) = \{x_0^6 + x_1^6 + x_2^6 + x_3^6 + x_4^3 - 6\varphi \cdot x_0x_1x_2x_3x_4 = 0\}$$
$$V_8(\varphi) = \{x_0^8 + x_1^8 + x_2^8 + x_3^8 + x_4^2 - 8\varphi \cdot \begin{cases} x_0x_1x_2x_3x_4 \\ x_0^2x_1^2x_2^2x_3^2 \end{cases} = 0\}$$
$$V_{10}(\varphi) = \{x_0^{10} + x_1^{10} + x_2^{10} + x_3^5 + x_4^2 - 10\varphi \cdot \begin{cases} x_0x_1x_2x_3x_4 \\ x_0^2x_1^2x_2^2x_3^2 \end{cases} = 0\}$$
$$V_{3,3}(\varphi) = \{x_0^3 + x_1^3 + x_2^3 = 3\varphi \cdot x_3x_4x_5, \quad x_3^3 + x_4^3 + x_5^3 = 3\varphi \cdot x_0x_1x_2\}$$
$$V_{4,4}(\varphi) = \{x_0^4 + x_1^4 + 2x_2^2 = 4\varphi \cdot x_3x_4x_5, \quad x_3^4 + x_4^4 + 2x_5^2 = 4\varphi \cdot x_0x_1x_2\}$$

The family $V_5(\varphi)$ is investigated in 3.1. The special members have 125 nodes and are rigid.

The special members of the family $V_6(\varphi)$ have 108 nodes (cf. [56]).

The different equations for $V_8(\varphi)$ are equivalent. The family $V_8(\varphi)$ is discussed in 4.7. The special members have 128 nodes and $h^{2,1} = 27$.

The different equations for $V_{10}(\varphi)$ are equivalent. The special members have 100 nodes (cf. [56]).

The family $V_{3,3}(\varphi)$ is investigated in 5.5. The special members have 81 nodes and $h^{2,1} = 4$.

The family $V_{4,4}(\varphi)$ is investigated in [57]. The special members have 64 nodes.

Note that the resolutions of X_φ/G for the special members X_φ will be rigid Calabi–Yau threefolds with large Euler characteristics. For the family $V_5(\varphi)$ this is explicitly computed in 3.1. The largest possible Euler number seems to be 298 (constructed from the family $V_8(\varphi)$).

Chapter 6

Tables, correspondences, conclusions

6.1 Modular threefolds with small levels

This section collects information about modular threefolds whose L-series contains the L-series of a weight four newform of small level (≤ 12) or twists of these newforms. The data includes Hodge numbers (as far as they have been computed or conjectured) and internal and external references. If any correspondences are known then they are given explicitly.

In the tables, "CI" is an abbreviation for "complete intersection".

Whenever examples from appendix B occur in the tables the number in brackets is the total number. The number in front gives the number of examples with different numerical data (which ensures that the geometry is different).

Denote by

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n \in \mathbb{N}} (1 - q^n), \qquad q = e^{2\pi i \tau}$$

the *Dedekind η function*. Some weight four newforms can be written as products or quotients of η functions. Martin ([67]) gives a complete list of these cases, and we will mention them here.

6.1.1 Level 5

There is only one newform f of weight four with rational coefficients for $\Gamma_0(5)$. In this thesis it is denoted by 5/1; Stein ([97]) denotes it by $5k4A1$. It can be written as an eta product

$$f(q) = \eta(\tau)^4 \eta(5\tau)^4.$$

By twisting with certain Legendre symbols we obtain the following newforms:

25/2	45/5	80/4	225/4	245/1	320/10	320/14	400/13
605/4	720/5	845/1	1225/11	1445/8	1600/13	1600/16	1805/6

The following table lists all currently known examples of Calabi–Yau threefolds whose L-series contains the L-series of f. Hodge numbers are included as far as they have been computed.

no.	symbol	comments	$h^{1,1}$	$h^{1,2}$	ref.	external ref.
1	$W_1(5)$	projective small resolution of self-fibre product of $Y_{\Gamma_1(5)}$	52	0	ch. 2	[81], [84], [111]
2	$W_1(5)^\pi$	projective small resolution of twisted self-fibre product of $Y_{\Gamma_1(5)}$	52	0	ch. 2	[87]
3	X_3	double octic from Clebsch cubic and five planes		0	4.8	
4		8 (29) double octics from six planes and quadric			4.3, app.B	
5		relatives of double octics and self-fibre products by various constructions			1.7.2, 4.6, ch. 2,	

Correspondences

Examples no. 1 and no. 2 are birational. The twist $t \mapsto -1/t$ (cf. chapter 2) corresponds to the coordinate change $x \mapsto x + y$, $y \mapsto x + y - z$, $z \mapsto y - z$, which transforms the equation

$$(x + y)(x + y - z)(y - z) = txyz$$

for the elliptic surface $Y_{\Gamma_1(5)}$ into the equation

$$xyz = -t(x + y)(x + y - z)(y - z).$$

(cf. [87]). No correspondences involving any of the other examples are known. In particular, it would be interesting to investigate if the symmetry of the pentagon behind example no. 3 can also be found in other examples.

6.1.2 Level 6

There is only one newform f of weight four with rational coefficients for $\Gamma_0(6)$. In this thesis it is denoted by 6/1; Stein ([97]) denotes it by $6k4A1$. It can be written as an eta product

$$f(q) = [\eta(\tau)\eta(2\tau)\eta(3\tau)\eta(6\tau)]^2.$$

By twisting with certain Legendre symbols we obtain the following newforms:

18/1	48/3	144/6	150/9	192/4	192/7	294/9	450/8
576/9	576/10	726/5	882/1	1014/3	1200/29	1734/4	

The following table lists all currently known examples of Calabi–Yau threefolds whose L-series contains the L-series of f. Hodge numbers are included as far as they have been computed.

no.	symbol	comments	$h^{1,1}$	$h^{1,2}$	ref.	external ref.
1	$W_1(6)$	projective small resolution of self-fibre product of $Y_{\Gamma_1(6)}$	50	0	ch. 2	[50], [81], [84], [111]
2	$\mathcal{Z}_{A_3}$	desingularization of toric variety connected with A_3	50	0	5.7	[50], [81], [103], [104], [107]
3	Y	desingularization of Barth-Nieto quintic N	50	0	3.7	[6], [50], [68], [107]
4	Z	desingularization of double cover of Barth-Nieto quintic N	40	0	3.7	[50]
5	$\tilde{\mathcal{M}}_{(1:1)}$	proj. small resolution of 130-nodal van Straten quintic $\mathcal{M}_{(1:1)}$	30	0	3.6	[101], [68], [69]
6	$\tilde{\mathcal{M}}_{(-2:1)}$	desingularization of van Straten quintic $\mathcal{M}_{(-2:1)}$		0?	3.6	[101], [68], [69]
7	$\tilde{X}$	double octic from arrangement no. 240 with 10 fourfold points	40	0	4.2	[28]
8	$\tilde{X}$	double octic from arrangement no. 245 with 9 fourfold points	38	0	4.2	[28]
9	$\tilde{Z}_{\lambda,\mu}$	double octic from two Sarti quartics, $\lambda = -\frac{1}{2}$, $\mu = -\frac{1}{4}$		> 0	4.11	[82], [83]
10		double octic from four planes and two quadrics related to no. 9		> 0	4.11	
11	$\hat{X}_1$, $\hat{X}_{\boldsymbol{a}}$	proj. small resol. of toric variety connected with A_4, $\boldsymbol{a}$ =(1:1:1:1:1:1)	50	0	5.8	[51]
12	$\hat{X}_9$, $\hat{X}_{\boldsymbol{a}}$	non-proj. small resol. of toric var. connected with A_4, $\boldsymbol{a}$ =(1:1:1:1:1:9)	45	0	5.8	[51]
13	$\tilde{X}$	double octic from arrangement no. 287	37	3	4.2	[28]
14	$\tilde{X}_9$	double octic from four smooth quadrics related to arr. no. 287	23	3	4.5	
15	$\tilde{V}_\lambda$	resolution of CI of quadric and quartic, $\lambda = (6,6,6,6)$		> 0	5.9	[13]
16	$\tilde{V}_\lambda$	resolution of CI of quadric and quartic, $\lambda = (1,1,1,4)$		> 0	5.9	[13]
17	$\tilde{X}_a$	small resolution of CI of quadric and quartic, a =(1:1:1:1:-2:-2)	4	12	5.2	
18	$\tilde{X}_a$	small resolution of CI of quadric and quartic, a =(1:1:-1:-1:2:-2)	15	13	5.2	
19		resol. of various twisted self-fibre prod. of $Y_{\Gamma_1(4)\cap\Gamma(2)}$ and $Y_{\Gamma_0(8)\cap\Gamma_1(4)}$			ch. 2	
20		resolutions of various twisted self-fibre products of $Y_{\Gamma_1(6)}$			ch. 2	[87]

no.	symbol	comments	$h^{1,1}$	$h^{1,2}$	ref.	external ref.
21		3 double octics from four planes and two quadrics			4.4	
22		39 (68) double octics from six planes and quadric			4.3, app.B	

Correspondences

Consider the singular Barth-Nieto quintic $N \subset \mathbb{P}^4$ (no. 3) given by the equations

$$x + y + z + r + s + t = \frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{r} + \frac{1}{s} + \frac{1}{t} = 0.$$

Writing these equations as

$$x + y + z = -(r + s + t), \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = -\left(\frac{1}{r} + \frac{1}{s} + \frac{1}{t}\right)$$

and multiplying them we obtain the equation

$$(x + y + z)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = (r + s + t)\left(\frac{1}{r} + \frac{1}{s} + \frac{1}{t}\right)$$

which we rewrite as

$$(x + y + z)(xy + xz + yz)rst = (r + s + t)(rs + rt + st)xyz$$

which is nothing but an equation for Schoen's fibre product $W_1(6)$ from chapter 2 (no. 1), so the rational map given by

$$\mathbb{P}^5 \dashrightarrow \mathbb{P}^2 \times \mathbb{P}^2, \quad (x : y : z : r : s : t) \mapsto (x : y : z), (r : s : t)$$

induces a birational equivalence between N and $W_1(6)$.

By using the equations

$$x + y + z + r = -(s + t), \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{r} = -\left(\frac{1}{s} + \frac{1}{t}\right)$$

we obtain

$$(x + y + z + r)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{r}\right) = (s + t)\left(\frac{1}{s} + \frac{1}{t}\right).$$

Since

$$(s + t)\left(\frac{1}{s} + \frac{1}{t}\right) = \frac{(t + s)^2}{st} = \frac{(t - s)^2}{st} + 4$$

the above equation is nothing but the one for the singular model of $\mathcal{Z}_{A_3}$ from 5.7 (no. 2), so the rational map given by

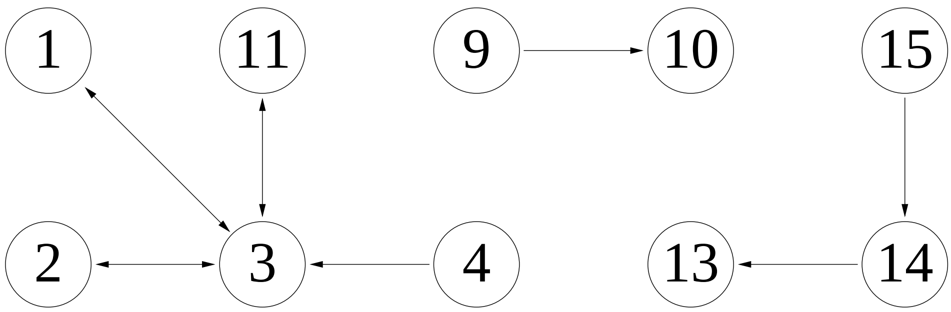
$$\mathbb{P}^5 \dashrightarrow \mathbb{P}^3 \times \mathbb{P}^1, \quad (x:y:z:r:s:t) \mapsto (x:y:z:r), (s:t)$$

induces a birational equivalence between N and $\mathcal{Z}_{A_3}$.

Thus the Barth-Nieto quintic N, the fibre product $W_1(6)$, the variety $\mathcal{Z}_{A_3}$ and the variety X_1 (no. 11, cf. 5.8) are birationally equivalent over $\mathbb{Q}$.

The above birational equivalence between N and $W_1(6)$ was constructed in [50], section 4. The authors also give a birational equivalence between N and $\mathcal{Z}_{A_3}$ but it seems to be different from ours. Another explicit birational map between these varieties was constructed in [81, section 5]. Note also that in [10, Problem 7] K. Hulek poses the problem to exhibit a correspondence between no. 11 and no. 12.

Examples no. 13, no. 14 and no. 15 are related by correspondences (cf. 4.5 and 5.9). Examples no. 9 and no. 10 are related by the Segre construction (cf. 4.6). Apart from such standard constructions no other correspondences seem to be known. This is a picture of the situation so far:



In particular, it would be interesting to determine the Hodge numbers of the double octics constructed from six planes and a quadric.

6.1.3 Level 7

There is only one newform f of weight four for $\Gamma_0(7)$ with rational coefficients. In this thesis it is denoted by 7/1; Stein ([97]) denotes it by $7k4A1$. It can not be written as an eta product. By twisting with certain Legendre symbols we obtain the following newforms:

49/2	63/1	112/5	175/2	441/9	448/5	448/9
784/6	847/1	1008/19	1183/1	1225/1	1575/3	

At present there are no known examples of Calabi–Yau threefolds whose L-series contains the L-series of f.

6.1.4 Level 8

There is only one newform f of weight four with rational coefficients for $\Gamma_0(8)$. In this thesis it is denoted by 8/1; Stein ([97]) denotes it by $8k4A1$. It can be written as an eta product

$$f(q) = \eta(2\tau)^4\eta(4\tau)^4.$$

By twisting with certain Legendre symbols we obtain the following newforms:

16/1	64/1	64/5	72/4	144/3	200/4	392/5	400/9	576/5
576/6	784/10	968/4	1352/2	1600/22	1600/25	1800/30	1936/5	

The following table lists all currently known examples of Calabi–Yau threefolds whose L-series contains the L-series of f. Hodge numbers are included as far as they have been computed.

no.	symbol	comments	$h^{1,1}$	$h^{1,2}$	ref.	external ref.
1	$W_1(4)$	proj. small resolution of self-fibre product of Y_Γ, $\Gamma = \Gamma_1(4) \cap \Gamma(2)$	40	0	ch. 2	[81], [84], [111]
2	$W_0(8)$	proj. small resolution of self-fibre product of Y_Γ, $\Gamma = \Gamma_0(8) \cap \Gamma_1(4)$	70	0	ch. 2	[81], [84], [111]
3	$\mathcal{Z}_{A_1^3}$	desingularization of toric variety connected with A_1^3	70	0	5.7	[3], [14], [75], [76], [103], [104]
4	$\tilde{X}$	projective small resolution of CI of quadric and quartic	16	0	5.1	
5	$\tilde{X}$	projective small resolution of CI of four quadrics	32	0	5.4	[99]
6	$\tilde{X}$	double octic from arrangement no. 1 with 4 triple lines	70	0	4.2	[28]
7	$\tilde{X}$	double octic from arrangement no. 32	50	0	4.2	[28]
8	$\tilde{X}$	double octic from arrangement no. 69	50	0	4.2	[28]
9	$\tilde{X}$	double octic from arrangement no. 93	46	0	4.2	[28]
10	$\tilde{X}$	double octic from arrangement no. 238 with 12 fourfold points	44	0	4.2, 4.8	[28]
11	$\tilde{X}$	double octic from arrangement no. 241 with 10 fourfold points	40	0	4.2	[28]
12	$\tilde{X}$	double octic from arrangement no. 154 with parameter $(2 : -3)$	41	1	4.2	[28]
13	$\tilde{X}_1$	double octic from Cayley cubic and five planes	70	0	4.8	

no.	symbol	comments	$h^{1,1}$	$h^{1,2}$	ref.	external ref.
14	$\tilde{Y}_1$	Relative of no. 13 by the Segre construction		0	4.8	
15	$\tilde{Y}$	double octic from four smooth quadrics related to arr. no. 238	28	0	4.5	
16	W	double octic from four planes and quartic with six A_3 singularities		0	4.7	
17	W'	double octic from two Kummer quartics with 12 common nodes		0	4.7	
18		double octic from Σ_4-symm. cubic with param. (2:−3:0) and 5 planes		> 0	4.8	
19	$V_{2,4}$	special member of family of hypergeometric threefolds			5.11	[80], [111]
20	$V_{2,2,2,2}$	special member of family of hypergeometric threefolds			5.11	[80], [111]
21		resol. of various twisted self-fibre prod. of $Y_{\Gamma_1(4)\cap\Gamma(2)}$ and $Y_{\Gamma_0(8)\cap\Gamma_1(4)}$			ch. 2	[87]
22		13 double octics from four planes and two quadrics			4.4	
23		88 (254) double octics from six planes and quadric			4.3, app.B	
24		relatives of double octics and self-fibre products by various constructions			1.7.2, 4.6, ch. 2,	

Correspondences

An open part of $W_0(8)$ (no. 2) is given by the equation

$$\frac{(x+y)(xy+1)}{xy} = \frac{(z+t)(zt+1)}{zt}.$$

Because of

$$x + \frac{1}{x} + y + \frac{1}{y} = \frac{(x+y)(xy+1)}{xy}$$

and after a sign change of z and t this equation becomes

$$x + \frac{1}{x} + y + \frac{1}{y} + z + \frac{1}{z} + t + \frac{1}{t} = 0.$$

This is also an equation for an open part of Verrill's threefold $\mathcal{Z}_{A_1^3}$ (no. 3, cf. 5.7).

By homogenizing this equation we find a birational model as a quintic in $\mathbb{P}^4$ given by

$$w^2(xyz + xyt + xzt + yzt) = xyzt(x + y + z + t).$$

This quintic is birationally equivalent with the double covering X_1 of $\mathbb{P}^3$ branched along the union of five planes and a Cayley cubic (no. 13, cf. 4.8) which can be given by the equation

$$u^2 = xyzt(x+y+z+t)(xyz+xyt+xzt+yzt).$$

By the Segre construction (4.6) there is a correspondence between no. 13 and no. 14.

Now consider Nygaard's and van Geemen's complete intersection X of four quadrics in $\mathbb{P}^7$ (no. 5) given by the equations

$$\begin{aligned} 2y_0^2 &= +x_0^2 - x_1^2 - x_2^2 - x_3^2, \\ 2y_1^2 &= -x_0^2 + x_1^2 - x_2^2 - x_3^2, \\ 2y_2^2 &= -x_0^2 - x_1^2 + x_2^2 - x_3^2, \\ 2y_3^2 &= -x_0^2 - x_1^2 - x_2^2 + x_3^2. \end{aligned}$$

A dominant rational map $X \longrightarrow W_0(8)$ is then given by

$$x = \frac{y_0 + x_0}{y_0 - x_0}, \qquad y = \frac{y_1 + x_1}{y_1 - x_1}, \qquad z = \frac{y_2 + x_2}{y_2 - x_2}, \qquad t = \frac{y_3 + x_3}{y_3 - x_3}.$$

This map was constructed by J. Stienstra. It can also be found in [75, page 60] but there are some misprints. Implicitly Nygaard and van Geemen give the correspondence induced by the Segre construction between X and the double octic constructed from arrangement no. 238 (no. 10 in the above table) which can be given by the equation

$$u^2 = xyzt(x+y+z-t)(x+y-z+t)(x-y+z+t)(-x+y+z+t).$$

The explicit map is given by

$$(y_0 : y_1 : y_2 : y_3 : x_0 : x_1 : x_2 : x_3) \mapsto (x_0^2 : x_1^2 : x_2^2 : x_3^2 : 4x_0x_1x_2x_3y_0y_1y_2y_3).$$

The double octic Y (no. 15, cf. 4.5) can be given by the equation

$$u^2 = (x^2+y^2+z^2-t^2)(x^2+y^2-z^2+t^2)(x^2-y^2+z^2+t^2)(-x^2+y^2+z^2+t^2).$$

There are immediate correspondences between Y and Nygaard's and van Geemen's complete intersection X (no. 5) and the double octic constructed from arrangement no. 238 (no. 10). Note that the equations for X in [75] differ from ours by the factors 2 at the y_i, so a priori the two threefolds are isomorphic over $\mathbb{Q}[\sqrt{2}]$. Nevertheless their L-series are exactly the same because both varieties correspond with the double octic Y.

The double octic constructed from arrangement no. 238 (no. 10 in the above table) can also be given (after a change of coordinates) by the equation

$$\begin{aligned} u^2 &= (x-y)(x+y)(y-z)(y+z)(z-t)(z+t)(t-x)(t+x) \\ &= (x^2-y^2)(y^2-z^2)(z^2-t^2)(t^2-x^2). \end{aligned}$$

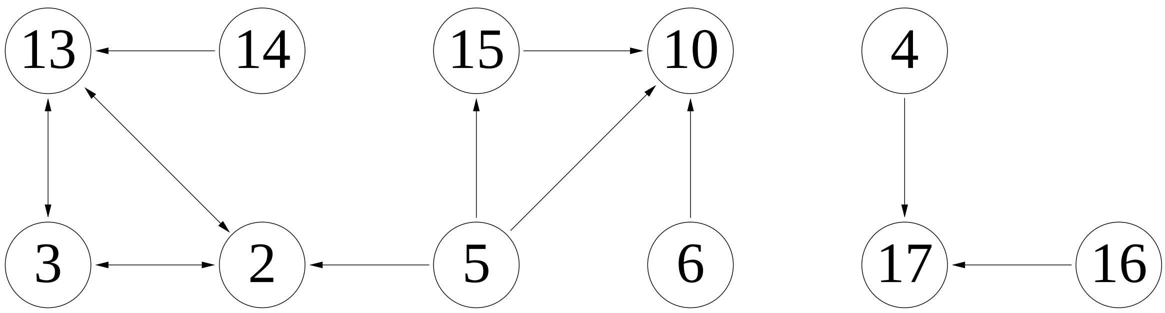
By the Segre construction there is a correspondence between this double octic and the double octic given by the equation

$$u^2 = xyzt(x-y)(y-z)(z-t)(t-x)$$

which is the double octic constructed from arrangement no. 1 (no. 6 in the above table). This correspondence was first noticed by S. Cynk. Note that although the Hodge numbers of the later double octic are the same as those of examples no. 2, no. 3 and no. 13, the correspondence between them is not given by a birational map (but there might exist such a map).

There is a correspondence between no. 4 and no. 17, cf. 5.1. Examples no. 16 and no. 17 are related by the Segre construction.

This is a picture of the situation so far:



No correspondences between any of the other examples seem to be known, except standard constructions. In particular, it would be interesting to study the (many!) examples of double octics constructed from six planes and a quadric and to determine their Hodge numbers.

Note also that among the listed rigid examples there are many with different Hodge numbers $h^{1,1}$. The occurring values are at least 16, 28, 32, 40, 44, 46, 50, 70.

6.1.5 Level 9

There is only one newform f of weight four with rational coefficients for $\Gamma_0(9)$. In this thesis it is denoted by 9/1; Stein ([97]) denotes it by $9k4A1$. It can be written as an eta product

$$f(q) = \eta(3\tau)^8.$$

By twisting with certain Legendre symbols we obtain the following newforms:

144/5	225/6	441/6	576/1	576/2	1089/1	1521/1

The following table lists all currently known examples of Calabi–Yau threefolds whose L-series contains the L-series of f. Hodge numbers are included as far as they have been computed.

no.	symbol	comments	$h^{1,1}$	$h^{1,2}$	ref.	external ref.
1	$W(3)$	proj. small resolution of self-fibre product of Y_Γ, $\Gamma = \Gamma(3)$	36	0	ch. 2	[81], [84], [111]
2	$W_0(9)$	proj. small resolution of self-fibre product of Y_Γ, $\Gamma = \Gamma_0(9) \cap \Gamma_1(3)$	84	0	ch. 2	[81], [84], [111]
3	$\tilde{V}_{33}$	big resolution of CI of two cubics with 9 sing. of type $(3,3,3,3)$	36	0	5.3	[99]
4	$\tilde{X}$	projective small resolution of CI of two cubics with 108 nodes	36	0	5.6, ch. 2	
5	$\tilde{T}$	resolution of triple cover of $\mathbb{P}^3$ branched along the faces of a cube			5.3	[48], [99]
6	$\tilde{E}^3$	resol. of quotient of triple product of the elliptic curve $x^3+y^3+z^3=0$	36	0	below	[111]
7	X_7	double octic from five planes and cubic with three cusps		0	4.8	
8	$V_{3,4}$	special member of family of hypergeometric threefolds			5.11	[80], [111]
9		resol. of various twisted self-fibre prod. of $Y_{\Gamma(3)}$ and $Y_{\Gamma_0(9)\cap\Gamma_1(3)}$			ch. 2	[87]
10		4 (6) double octics from six planes and quadric			4.3, app.B	
11		relatives of double octics and self-fibre products by various constructions			1.7.2, 4.6, ch. 2,	

Correspondences

Singular birational models for $W(3)$ (no. 1) and $W_0(9)$ (no. 2) are given by the equations

$$(x^3+y^3+z^3)rst = (r^3+s^3+t^3)xyz,$$
$$(x^2z+y^2x+z^2y)rst = (r^2t+s^2r+t^2s)xyz,$$

with $(x:y:z, r:s:t)$ coordinates of $\mathbb{P}^2 \times \mathbb{P}^2$.

A correspondence between these two varieties is given by the $3:1$ map $W_0(9) \longrightarrow W(3)$ induced by (cf. [85], Theorem 13.2.)

$$\mathbb{P}^2 \times \mathbb{P}^2 \longrightarrow \mathbb{P}^2 \times \mathbb{P}^2, \quad (x:y:z,\ r:s:t) \mapsto (x^2y : y^2z : z^2x,\ r^2s : s^2t : t^2r).$$

Now consider the triple product E^3 of the elliptic curve

$$E := \{x^3+y^3+z^3=0\} \subset \mathbb{P}^2$$

with complex multiplication by $\mathbb{Z}[\sqrt{-3}]$. Let the equations of the three factors be given by

$$u_i^3 + v_i^3 + w_i^3 = 0, \quad i = 1, 2, 3.$$

The variety E^3 is not Calabi–Yau but we obtain a Calabi–Yau threefold by dividing out the group of automorphisms generated by $w_1 \mapsto \xi \cdot w_1$, $w_2 \mapsto \xi \cdot w_2$, $v_3 \mapsto \xi \cdot v_3$ and resolving singularities. According to [111], ex. 5.22, a smooth Calabi–Yau model $\tilde{E}^3$ (no. 6) of E^3 has

$$h^{2,1}(\tilde{E}^3) = 0, \quad h^{1,1}(\tilde{E}^3) = 36, \quad \chi(\tilde{E}^3) = 72.$$

Recently Kimura ([55]) constructed a correspondence $E^3 \longrightarrow \tilde{V}_{33}$ as follows:

Consider the affine piece $\{x_3 \neq 0\}$ of the singular model V_{33} (no. 3). It is given by the equations

$$x_0^3 + x_1^3 + x_2^3 + 1 = x_2^3 + 1 + x_4^3 + x_5^3 = 0.$$

On the other hand, the equations of $(E - \{z = 0\})^3$ are given by

$$u_i^3 + v_i^3 + 1 = 0, \quad i = 1, 2, 3.$$

Then there is a $3:1$ rational map

$$(E - \{z = 0\})^3 \longrightarrow V_{33}, \quad x_0 = -u_1 v_3, x_1 = -v_1 v_3, x_2 = u_3, x_4 = -u_2 v_3, x_5 = -v_2 v_3.$$

Actually this map induces a $1:1$ rational map $\tilde{E}^3 \dashrightarrow V_{33}$. Note that in [111], ex. 8.6, it was conjectured that the L-series of $\tilde{E}^3$ is connected with a weight four newform for $\Gamma_0(27)$ instead of $\Gamma_0(9)$.

A correspondence (given by an $81:1$ map) between V_{33} and a triple cover of $\mathbb{P}^3$ branched along the faces of a cube (no. 5) is given in 5.3.

The curve E is isogenous to the curve given by

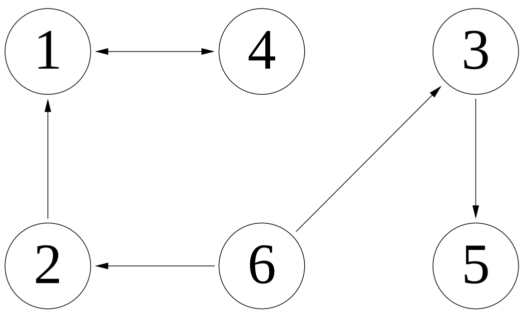
$$\{t_0^3 - t_2 ab = 0\} \subset \mathbb{P}^2$$

where $a = t_1 + t_2/2$, $b = t_1 - t_2/2$. Schoen ([85], Theorem 13.2.) establishes the correspondence $E^3 \longrightarrow W_0(9)$ induced by the rational map $E^3 \longrightarrow \mathbb{P}^2 \times \mathbb{P}^2$ given by

$$(x:y:z) = (-t_2 a (t_0')^2 t_0'' b'' : -t_0 a t_2' b' (t_0'')^2 : t_0^2 t_0' t_2' t_2'' b''),$$
$$(r:s:t) = (-t_0 a t_0' t_2' t_0'' b'' : -t_0^2 (t_0')^2 (t_0'')^2 : t_2 a t_2' b' t_2'' b''),$$

where the number of dashes distinguishes between the variables of the different factors of E^3.

This is a picture of the situation so far:



Note that it is still unknown if the examples with the same Hodge numbers in the above discussion are all birational. It would also be interesting to study the double octics (and their relatives) and find correspondences between them and the other examples.

6.1.6 Level 10

There is only one newform f of weight four for $\Gamma_0(10)$ with rational coefficients. In this thesis it is denoted by 10/1; Stein ([97]) denotes it by $10k4A1$. It can not be written as an eta product. By twisting with certain Legendre symbols we obtain the following newforms:

50/2	80/6	90/2	320/1	320/2	400/3	450/17
490/15	720/19	1210/12	1600/44	1600/46	1690/13	

The following table lists all currently known examples of Calabi–Yau threefolds whose L-series contains the L-series of f. Hodge numbers are included as far as they have been computed.

no.	symbol	comments	$h^{1,1}$	$h^{1,2}$	ref.	external ref.
1	W_3	projective small resolution of twisted self-fibre product of $Y_{\Gamma_1(6)}$	33	0	ch. 2	[88]
2	$\tilde{X}_4$, $\tilde{X}_{(5:6)}$	double octic from Clebsch cubic and five planes	42	0	4.8, 4.9	
3	$\tilde{Y}_4$	Relative of no. 2 by Segre construction		> 0	4.8	
4	$\tilde{X}_{(13:30)}$	double octic from Clebsch cubic and Barth quintic with 10 nodes		4?	4.9	
5	$\hat{X}_{\boldsymbol{a}}$	non-proj. small resol. of toric variety connected with A_4, $\boldsymbol{a}$ =(1:1:1:9:9:9)	45	2	5.8, ch. 2	[51]
6		resolutions of various twisted self-fibre products of $Y_{\Gamma_1(6)}$			ch. 2	
7		3 double octics from six planes and quadric			4.3, app.B	
8		relatives of double octics and self-fibre products by various constructions			1.7.2, 4.6, ch. 2,	

Correspondences

There are correspondences between no. 1 resp. the twisted self-fibre products in no. 6 and complete intersections of two cubics (contained in no. 8), cf. chapter 2. A correspondence between no. 2 and no. 3 is given by the Segre construction, cf. 4.6. There are various correspondences between no. 2 resp. no. 4 resp. the double octics in no. 7 and other varieties (like quintics or complete intersections, contained in no. 8), cf. 1.7.2. There is a birational correspondence between no. 5 and certain (twisted) self-fibre products (contained in no. 6), cf. 5.8.

6.1.7 Level 12

There is only one newform of weight four for $\Gamma_0(12)$ with rational coefficients. In this thesis it is denoted by 12/1; Stein ([97]) denotes it by $12k4A1$. It can not be written as an eta product. By twisting with certain Legendre symbols we obtain the following newforms:

36/1	48/2	144/7	192/3	192/12	300/3	576/23
576/24	588/5	900/5	1200/8	1452/6	1764/11	

The following table lists all currently known examples of Calabi–Yau threefolds whose L-series contains the L-series of f. Hodge numbers are included as far as they have been computed.

no.	symbol	comments	$h^{1,1}$	$h^{1,2}$	ref.	external ref.
1	$\tilde{V}_{24}$, $\tilde{X}_a$	proj. small resolution of CI of quadric and quartic, a =(1:1:1:−1:−1:−1)	34	0	5.2	[99]
2	$\hat{X}_{\boldsymbol{a}}$	non-proj. small resol. of toric variety connected with A_4, $\boldsymbol{a}$ =(1:1:1:1:4:4)	47	0	5.8	[51]
3		non-projective small resolution of twisted self-fibre product of ell. surface	47	0	ch. 2, 5.8	[51]
4	$\tilde{X}$	double octic from arrangement no. 239 with 10 fourfold points	40	0	4.2	[28]
5	$\tilde{X}$	double octic from arrangement no. 244 with parameter $(1:-1)$	39	1	4.2	[28]
6	$\tilde{X}$	double octic from arrangement no. 317	36	2	4.2	[28]
7	X_6	double octic from five planes and cubic with three cusps		0	4.8	
8	$V_{2,2,3}$	special member of family of hypergeometric threefolds			5.11	[80], [111]
9	$\tilde{X}_1$	double octic from four smooth quadrics related to arrangement no. 239		6	4.5	
10	$\tilde{X}_2$	double octic from four smooth quadrics related to arrangement no. 317		6	4.5	
11	$\tilde{X}_3$	double octic from four smooth quadrics related to arrangement no. 239		0	4.5	

no.	symbol	comments	$h^{1,1}$	$h^{1,2}$	ref.	external ref.
12		double octic from four planes and two smooth quadrics			4.4	
13		resolutions of various twisted self-fibre products of $Y_{\Gamma_1(4)\cap\Gamma(2)}$ and $Y_{\Gamma_0(8)\cap\Gamma_1(4)}$			ch. 2	
14		19 (58) double octics from six planes and quadric			4.3, app.B	

Correspondences

Examples no. 2 and no. 3 are birationally equivalent, cf. 5.8. By the Segre construction there is a correspondence between no. 6 and no. 10. Examples no. 4, no. 9 and no. 11 are also related by the same construction. Apart from such standard correspondences not much is known.

6.2 Modular threefolds with large levels

For the next few weight four newforms with rational coefficients (up to level 32) we list the threefolds containing the L-series of these newforms in their L-series.

newform	known examples
13/1	no examples known
14/1	double octics constructed from four planes and two quadrics, cf. 4.4, toric varieties (resp. twisted self-fibre products of elliptic surfaces), cf. 5.8
14/2	double octics constructed from four planes and two quadrics, cf. 4.4, double octics constructed from six planes and a quadric, cf. 4.3, app.B, double octic constructed from five planes and Σ_4-symmetric cubic, cf. 4.8, twisted self-fibre products of elliptic surfaces, cf. ch. 2
15/1	Σ_6-symmetric complete intersections, cf. 5.10
15/2	no examples known
16/1	just a twist of 8/1, cf. 6.1.4
17/1	twisted self-fibre products of elliptic surfaces, cf. ch. 2
18/1	just a twist of 6/1, cf. 6.1.2
19/1	no examples known
20/1	double octics constructed from six planes and a quadric, cf. 4.3, app.B,
21/1	double octic constructed from Sarti octic, cf. 4.11, van Straten's quintic $\mathcal{M}_{(-3:1)}$, cf. 3.6
21/2	twisted self-fibre products of elliptic surfaces, cf. ch. 2
22/1	no examples known
22/2	twisted self-fibre products of elliptic surfaces, cf. ch. 2
22/3	no examples known

newform	known examples
23/1	no examples known
24/1	double octics constructed from eight planes, cf. 4.2, double octics constructed from four planes and two quadrics, cf. 4.4, double octic constructed from two Sarti quartics, cf. 4.11, double octic constructed from five planes and cubic with 3 cusps, cf. 4.8, hypergeometric threefold, cf. 5.11
25/1	Schoen's quintic and its relative, cf. 3.1, 5.11
25/2	just a twist of 5/1, cf. 6.1.1
25/3	just a twist of 25/1
26/1	no examples known
26/2	no examples known
26/3	no examples known
27/1	twisted self-fibre products of elliptic surfaces, cf. ch. 2, Libgober's and Teitelbaum's complete intersection of two cubics, cf. 5.5, hypergeometric threefold, cf. 5.11
27/2	just a twist of 27/1
28/1	twisted fibre product of two elliptic surfaces, cf. ch. 2
28/2	double octic constructed from six planes and a quadric, cf. 4.3, app.B
30/1	toric varieties (resp. twisted self-fibre products of elliptic surfaces), cf. 5.8
30/2	twisted self-fibre products of elliptic surfaces, cf. ch. 2, toric varieties (resp. twisted self-fibre products of elliptic surfaces), cf. 5.8, double octic constructed from Σ_5-symmetric quintic and cubic, cf. 4.9, double octics constructed from four planes and two quadrics, cf. 4.4
32/1	double octics constructed from eight planes, cf. 4.2, double octic constructed from six planes and a quadric, cf. 4.3, app.B, double octics constructed from four planes and two quadrics, cf. 4.4, double octics constructed from four quadrics, cf. 4.5
32/2	double octics constructed from eight planes, cf. 4.2, double octic constructed from six planes and a quadric, cf. 4.3, app.B, double octics constructed from four planes and two quadrics, cf. 4.4, double octics constructed from four quadrics, cf. 4.5, double octics constructed from four planes and a Kummer surface, cf. 4.7, double octics constructed two Kummer surfaces, cf. 4.7, complete intersection of four quadrics, cf. 5.9, hypergeometric threefold, cf. 5.11
32/3	just a twist of 32/2

The other weight four newforms (not regarding twists) known or conjectured to occur in the L-series of Calabi–Yau threefolds are

35/1,	40/2,	40/3,	42/2,	50/3,	54/1,	54/2,
55/1,	60/1,	68/1,	72/1,	73/1,	78/2,	88/2,
96/1,	96/2,	96/4,	102/3,	108/2,	110/5,	120/1,
120/2,	120/3,	120/4,	120/5,	128/1,	130/2,	168/1,
168/2,	180/1,	200/1,	210/6,	210/9,	216/1,	256/1,
256/3,	256/7,	264/4,	280/2,	288/1,	300/2,	330/4,
360/2,	384/1,	384/3,	390/5,	465/2,	480/2,	480/5,
544/1,	570/7,	600/2,	864/1,	930/3,	1110/2,	1365/1,
1440/7,	1568/1,	1920/2,	1920/3.			

In 6.4.3 it is further discussed which newforms might occur.

6.3 Hodge and Euler numbers

It is still an open question if the value of the Euler number $\chi(X)$ of a Calabi–Yau threefold X is bounded by two constants (if there are such constants then their absolute values will be the same because of mirror symmetry). It is therefore also unknown which pairs of Hodge numbers $h^{1,1}(X)$, $h^{2,1}(X)$ may occur. The currently known bounds for the Euler number (based on constructions of Calabi–Yau threefolds in weighted projective spaces) are -960 and 960. For a general survey of the classification of threefolds, cf. [78].

This question can also be restricted to *rigid* Calabi–Yau threefolds. In this case we have $h^{2,1}(X) = 0$ and $2h^{1,1}(X) = \chi(X)$ so the Euler number $\chi(X)$ must be positive. The smallest Euler number for a rigid Calabi–Yau threefold that I am aware of is 32 (cf. 5.1), the largest is 202 (cf. 3.2). Both examples have been constructed in this thesis. Examples with even larger Euler characteristics may be constructed as described in 5.11.

There are (projective) rigid Calabi–Yau threefolds with Euler numbers 32, 50, 52, 56, 60, 64, 66, 68, 72, 76, 80, 84, 88, 92, 96, 100, 104, 108, 124, 140, 168, 202. In particular, every number between 52 and 108 which is divisible by four is realized. It is not surprising if the Hodge number $h^{2,1}$ of a modular Calabi–Yau threefold and its level have many common divisors because in most cases there is some symmetry which affects both the bad primes and the Picard group. Examples without symmetries are very difficult to construct.

I believe that some gaps in the above list could be filled by computing Euler numbers of resolutions of the double octics listed in appendix B. Most if not all of these numbers will also be divisible by four but we can avoid this by allowing nodal quadrics (a node and its small resolution will increase the Euler number by 2).

It is also interesting to investigate which Hodge numbers can occur for modular Calabi–Yau threefolds with the same weight four newform. Between two such threefolds there should be a correspondence. If they are birationally equivalent then by Batyrev ([8]) the Hodge numbers are equal but if the correspondence is not given by a birational map then there are more possibilities.

For example, in most cases the Segre construction (4.6) produces relatives with different Hodge numbers. In 4.4 there are examples of modular Calabi–Yau threefolds connected with the newform 32/1 (32k4A1) which are related by correspondences. The occurring Hodge numbers $h^{2,1}$ are at least 0, 1, 2, 3.

Note also that the Hodge numbers do not determine the newform. For example, there are three rigid double octics constructed from eight planes with 10 fourfold points (cf. 4.2). They all have $h^{1,1} = 40$ but the newforms are different (6/1, 8/1 resp. 12/1). By the Tate conjecture these threefolds can not be in correspondence (and, in particular, not birationally equivalent).

6.4 Bad primes

6.4.1 Problems

There are two (closely related) main problems concerned with primes of bad reduction for Calabi–Yau threefolds. We will present them and afterwards collect some material which might help shedding some light.

6.1 Problem
What determines the level of (the weight four newform connected with) a modular Calabi–Yau threefold? More specifically, if p is a prime of bad reduction for the modular Calabi–Yau threefold X, at what power does it occur in the level of the newform whose L-series is contained in the L-series of X? Generalize the notion of a conductor (of an elliptic curve) to Calabi–Yau threefolds.

This problem has also been posed in [10, Problem6] by R. Schimmrigk.

6.2 Problem
For which weight four newforms f does there exist a rigid Calabi–Yau variety X such that the L-series of X equals the L-series of f? More generally, does there exist a Calabi–Yau variety Y such that some part of the L-series of Y is equal to the L-series of f?

This problem has also been posed, for the rigid case, in [10, Problem 8] by K. Hulek (he became aware of this problem when B. Mazur asked him this question in 2003) and in [108, Problem 7.1] by N. Yui. At an earlier occasion, D. van Straten had also asked the question to B. van Geemen.

6.4.2 Powers of bad primes

Let X be a modular Calabi–Yau threefold, and let the L-series of the weight four newform f for $\Gamma_0(N)$ occur in the L-series of X. Then by theorem 1.4 the exponent e_p of a prime p dividing N is bounded by $e_p \leq 2$ if $p > 3$, $e_3 \leq 5$ and $e_2 \leq 8$. Thus the primes 2 and 3 play a special role and it is extremely difficult to predict which powers will occur in a level.

A bad prime $p \geq 5$ can occur to the power 0, 1 or 2. There are only very few examples known where a bad prime $p \geq 5$ occurs to the power 2 in the level of a modular Calabi–Yau threefold (apart from twists). Unfortunately it is almost always the prime $p = 5$:

• Consider the Schoen quintic X in $\mathbb{P}^4$ which is given by the equation

$$x_0^5 + x_1^5 + x_2^5 + x_3^5 + x_4^5 - 5x_0x_1x_2x_3x_4 = 0.$$

In 3.1 it was shown that the L-series of a resolution $\tilde{X}$ of X is equal to the L-series of the weight four newform 25/1 ($25k4A1$). Over $\mathbb{F}_5$ the threefold X degenerates into

$$(x_0 + x_1 + x_2 + x_3 + x_4)^5 = 0.$$

The same degeneration happens to the relative Y of X which is given by the equation (cf. 3.1)

$$(x_0 + x_1 + x_2 + x_3 + x_4)^5 - 5^5 x_0x_1x_2x_3x_4 = 0.$$

• Consider the Hirzebruch quintic V in $\mathbb{P}^4$ which is given by the homogenisation of the equation

$$f(x, y) - f(z, w) = 0$$

where

$$f(x, y) = (2x + 1)(5y^4 - 5y^2(2x^2 - 2x + 1) + (x^2 + x - 1)^2).$$

In 3.3 it was shown that the L-series of a resolution $\tilde{V}$ of V is equal to the L-series of the weight four newform 50/3 ($50k4B1$). Over $\mathbb{F}_5$ the threefold V degenerates into

$$(x - z)^5 = 0.$$

• Consider the complete intersection X of a quartic and a quadric in $\mathbb{P}^5$ which is given by the equations

$$x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 = 5x_5^2,$$
$$x_0^4 + x_1^4 + x_2^4 + x_3^4 + x_4^4 = 5x_5^4.$$

In 5.2 it is conjectured that the weight four newform 600/10 occurs in the L-series of X. Over $\mathbb{F}_5$ the threefold X has additional non-isolated singularities, namely the 16 lines given by

$$x_0 = \pm x_1 = \pm x_2 = \pm x_3 = \pm x_4.$$

• Consider the Σ_6-symmetric complete intersection X of a quartic and a quadric in $\mathbb{P}^5$ which is given by the equations

$$C_1^2 - C_2 = 0,$$
$$3C_4 - 2C_3C_1 - C_1^4 = 0,$$

where $C_i := \sum_{k=0}^5 x_k^i$ are the power sums. In 5.10 it is conjectured that the weight four newform 300/2 occurs in the L-series of X. In fact over fields with characteristic zero or ≥ 7 the threefold

X is smooth. Over $\mathbb{F}_5$ it has non-isolated singularities, namely the 6 lines on the Σ_6-orbit of the line given by the equation

$$x_0 = x_1 = x_2 = x_3 = x_4.$$

• Consider the hypergeometric threefold V_{10} from 5.11. In [80] Villegas showed that its L-series is determined by the weight four newform 200/10 ($200k4A1$). This example can be interpreted as a double covering of $\mathbb{P}^3(1, 1, 1, 2)$ branched along a surface of degree 10. Over $\mathbb{F}_5$ the surface develops multiple components.

• Very recently S. Cynk constructed a double covering of $\mathbb{P}^3$ branched along an arrangement of eight planes with the L-series of the weight four newform 49/1 ($49k4B1$) in its L-series (unpublished). Over $\mathbb{F}_7$ the threefold degenerates; two of the eight planes coincide.

Thus in all examples of modular Calabi–Yau threefolds with a prime $p \geq 5$ to the second power in the level of the modular form the threefold degenerates modulo p or develops non-isolated singularities.

In most examples of modular Calabi–Yau threefolds with a prime $p \geq 5$ to the first power in the level of the modular form the threefold develops only isolated singularities modulo p. The main problem here is to find a suitable birational model which shows only the "really" bad primes. We will discuss some examples:

• Consider the double coverings of $\mathbb{P}^3$ branched along the union of six planes and a smooth quadric investigated in 4.3 and tabulated in appendix B. I computed the discriminants of all the quadric surfaces. There are only six examples where the discriminant is divisible by a prime $p \geq 5$ (it is always $p = 5$ and the discriminants are 5 or 25). In fact the quadric surfaces develop a node in characteristic 5. The prime 5 occurs to the power 1 in the levels of the corresponding weight four newforms.

• Consider the double coverings of $\mathbb{P}^3$ branched along the union of five planes and the Clebsch cubic investigated in 4.8. The bad prime 5 occurs in the levels of the two weight four newforms associated with these examples. In fact the Clebsch cubic develops an extra node modulo 5.

• The (twisted) fibre products of elliptic surfaces investigated in chapter 2 develop only isolated singularities modulo the bad primes. Consequently the bad primes occur to the power ≤ 1 in the levels of the (twists of minimal level of the) corresponding weight four newforms. Note that M. Schütt ([89, section 6.2]) made some very interesting observations. There are examples where there does not exist a projective small resolution. A big resolution develops additional nodes modulo some primes. If there was a projective small resolution then it would have good reduction at these primes. In this case the primes do not seem to occur in the level.

• Consider the family of nodal complete intersections investigated in 5.2. For many examples the prime 5 occurs to the power 1 in the levels of the associated weight four newforms. In all these cases the threefold develops additional nodes modulo 5. Note that there are also many examples with set of bad primes $\{2, 3, 5\}$ where I could not detect a weight four newform in the L-series. Maybe the powers of 2 and 3 in the levels are too large so that the newforms are not contained in the tables in appendix C.

• The Σ_7-symmetric complete intersection investigated in 5.10 develops additional isolated singularities modulo the primes 5, 7 and 13. All three primes occur to the power 1 in the level of the corresponding weight four newform.

Based on these obeservations we may formulate the following conjecture. Note that it is still very vague (but it can already be used as a "rule of thumb"). It would be helpful to construct more examples of modular Calabi–Yau threefolds with large bad primes but this seems to be an extremely difficult task.

6.3 Conjecture
("rule of thumb") Let X be a modular Calabi–Yau threefold and let $p \geq 5$ be a prime. Let f be the twist of minimal level of the weight four newform associated with X. If p occurs to the power 2 in the level of f then X develops non-isolated singularities over $\mathbb{F}_p$. If p occurs to the power 1 in the level of f then X is singular modulo p and there is a birational model of X with only isolated singularities modulo p (note that it is very difficult to find examples where these singularities are not ordinary nodes). If p does not occur in the level of f then there is a birational model of X with good reduction modulo p.

There is a certain analogy between the above conjecture and the conductor of an elliptic curve E (i.e., a one-dimensional Calabi–Yau manifold) defined over $\mathbb{Q}$. The conductor of an elliptic curve is equal to the level of the associated weight two newform and is given by

$$N = \prod_{p \text{ prime}} p^{f_p}$$

where

$$f_p = \begin{cases} 0, & \text{if } E \text{ has good reduction modulo } p, \\ 1, & \text{if } E \text{ has a node modulo } p, \\ 2, & \text{if } E \text{ has a cusp modulo } p, \text{ and } p \neq 2, 3, \\ 2 + \delta_p, & \text{if } E \text{ has a cusp at } p = 2 \text{ or } p = 3. \end{cases}$$

Here δ_p depends on wild ramification in the action of the inertia group at p of $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on the Tate module $T_p(E)$ of E.

6.4.3 Which newforms do occur?

At present, the following primes have occurred in levels of weight four newforms connected with *rigid* Calabi–Yau threefolds (cf. 6.1, 6.2):

$$2, 3, 5, 7, 11, 17, 73$$

The following primes have occurred in levels of weight four newforms connected with (not necessarily rigid) Calabi–Yau threefolds (cf. 6.1, 6.2):

$$2, 3, 5, 7, 11, 13, 17, 19, 31, 37, 73$$

Up to twist there are only 7 weight four newforms with rational coefficients for $\Gamma_0(N)$ where N is a power of 2. These are 8/1, 32/1, 32/2, 128/1, 256/1, 256/3, 256/7. All of them are known to occur in the L-series of some Calabi–Yau threefolds; all of them but 256/3 are known to occur in the L-series of some *rigid* Calabi–Yau threefolds.

The table below summarizes the current situation for weight four newforms with level divisible only by 2 and 3. The symbols in the table stand for the following:

- $-$: There are no newforms of this level.
- T : There are newforms of this level but they are all twists of newforms of lower level.
- ? : There might be newforms of this level but this has not been investigated yet because of lack of computer power.
- M : There are newforms of this level. Until now none of them has occurred in the L-series of a Calabi–Yau threefold.
- $(\surd)$: There are newforms of this level. Some of them but not all have occurred in the L-series of a Calabi–Yau threefold.
- $\surd$: There are newforms of this level. They all have occurred in the L-series of a Calabi–Yau threefold.

	3 $-$	9 $\surd$	27 $\surd$	81 $-$	243 $(\surd)$
2 $-$	6 $\surd$	18 T	54 $\surd$	162 M	486 M
4 $-$	12 $\surd$	36 T	108 $\surd$	324 M	972 $\surd$
8 $\surd$	24 $\surd$	72 $\surd$	216 $(\surd)$	648 M	1944 M
16 T	48 T	144 T	432 T	1296 T	3888 ?
32 $\surd$	96 $\surd$	288 $\surd$	864 $\surd$	2592 ?	7776 ?
64 T	192 T	576 T	1728 T	5184 ?	15552 ?
128 $\surd$	384 $\surd$	1152 T	3456 ?	10368 ?	31104 ?
256 $\surd$	768 M	2304 T	6912 ?	20736 ?	62208 ?

It is possible to produce similar tables for different primes but this does not make sense in the current situation. We should perform large computer searches for modular Calabi–Yau threefolds and weight four newforms first. In only a few years time computers will be powerful enough for this. The problem with examples constructed by human beings is that they usually exhibit too much symmetry and obstruct the view towards the general case.

In the meantime we are restricted to making conjectures. I am not sure if every weight four newform will occur in the L-series of some Calabi–Yau threefold (this is only possible if there are infinitely many families of Calabi–Yau threefolds, cf. [78]) but I am pretty sure that this will be the case for every newform the computation of which comes into the range of computers.

6.5 Other aspects and questions

There are some aspects concerning modularity of Calabi–Yau threefolds that have not been discussed in this thesis but that promise to be interesting:

• What can be said about the L-series of a (rigid) Calabi–Yau threefold which is not defined over $\mathbb{Q}$ but over a finite extension of $\mathbb{Q}$, e.g. a double covering of $\mathbb{P}^3$ branched along Endraß' octic surface with 168 nodes (cf. 4.6)?

• What is the connection between modularity of a rigid Calabi–Yau threefold and modularity of its intermediate Jacobian (which is an elliptic curve)? A precise conjecture can be found in [111, Conjecture 8.4]. Note that there are some numerical computations by H. Verrill, suggesting that the conjecture might be wrong.

• What kinds of modular or automorphic forms can occur in the L-series of a non-rigid Calabi–Yau threefold? E.g., there seem to be examples involving weight three and weight two modular forms (cf. [64]).

• Can (rigid) Calabi–Yau threefolds be classified somehow, and will the classification shed light on the modularity question? Some attempts in this direction can be found in [111].

Appendix A

Arrangements of eight planes

The following table lists 450 examples of arrangements of eight planes defined over $\mathbb{Q}$ that have been found with a computer search. Many aspects of these examples are discussed in 4.2.

We give the numerical data of the arrangements, the Hodge numbers $h^{1,1} = h^{1,1}(\tilde{X})$ and $h^{1,2} = h^{1,2}(\tilde{X})$ and the Euler number $\chi = \chi(\tilde{X})$ of the Calabi–Yau resolution $\tilde{X}$ of the double coverings X of $\mathbb{P}^3$ branched along the arrangements, and the list of types of subarrangements of six planes (in lexicographical order with respect to some numbering of the planes, i.e., from $D_0 \cup \cdots \cup D_5$ to $D_2 \cup \cdots \cup D_7$).

no.	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	$h^{1,2}$	$h^{1,1}$	χ	subarrangements of 6 planes
1	4	1	4	0	0	4	4	0	70	140	505155155110515155211115152 1
2	8	0	4	0	0	4	4	1	69	136	4941551491155451155511121121
3	8	3	3	0	0	3	3	0	62	124	9551441000990551515511551411
4	12	2	3	0	0	3	3	1	61	120	9411554909999151451111445541
5	12	2	3	0	0	3	3	1	61	120	9411554909908151541111454541
6	16	1	3	0	0	3	3	2	60	116	8441441989890451514411451411
7	20	0	3	0	0	3	3	3	59	112	8441441888880441414411441411
8	9	1	5	0	1	2	3	1	61	120	4951541995595303555311551421
9	13	0	5	0	1	2	3	2	60	116	4941551884595493354411551321
10	8	2	7	0	0	2	3	1	57	112	5951441995495594554411451421
11	12	1	7	0	0	2	3	2	56	108	4441841495885444425511941451
12	16	0	7	0	0	2	3	3	55	104	4841441784485484454411441421
13	6	0	7	0	2	1	3	1	61	120	3352455303300542555511355551
14	9	0	9	0	1	1	3	2	56	108	4442444393499532555411455441
15	12	0	11	0	0	1	3	3	51	96	4442444484488442444411444441
16	14	2	2	0	2	1	2	1	57	112	9053953909999593355511053351
17	18	1	2	0	2	1	2	2	56	108	8953943999899303555311953451
18	22	0	2	0	2	1	2	3	55	104	8943953888899493354411953351
19	9	4	4	0	1	1	2	0	54	108	0933955990900594553511955551
20	13	3	4	0	1	1	2	1	53	104	0953953899909484554511953551

no.	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	$h^{1,2}$	$h^{1,1}$	χ	subarrangements of 6 planes
21	13	3	4	0	1	1	2	1	53	104	9455599999909151545133499543
22	17	2	4	0	1	1	2	2	52	100	8843843890899494555511953451
23	17	2	4	0	1	1	2	2	52	100	9943943889999485454411953551
24	17	2	4	0	1	1	2	2	52	100	7854844999999393455311955541
25	17	2	4	0	1	1	2	2	52	100	9933854899808584543511954541
26	17	2	4	0	1	1	2	2	52	100	4858549958458394539113999154
27	21	1	4	0	1	1	2	3	51	96	8843843889889495454411943451
28	21	1	4	0	1	1	2	3	51	96	4441184895584334883598195947
29	25	0	4	0	1	1	2	4	50	92	4441184884574343884388184957
30	14	1	6	1	0	1	2	2	52	100	5849548656558565559114099154
31	18	0	6	1	0	1	2	3	51	96	4451195784485544885566195956
32	8	5	6	0	0	1	2	0	50	100	9955954909999595455511945541
33	12	4	6	0	0	1	2	1	49	96	8945854989809495554411955541
34	12	4	6	0	0	1	2	1	49	96	4549588549588101558144099554
35	12	4	6	0	0	1	2	1	49	96	4489558589548491159554099154
36	12	4	6	0	0	1	2	1	49	96	8445958999998154155514949554
37	16	3	6	0	0	1	2	2	48	92	4478548589458491158454999154
38	16	3	6	0	0	1	2	2	48	92	4849558848448494558114999154
39	16	3	6	0	0	1	2	2	48	92	5445988454988114481549549597
40	16	3	6	0	0	1	2	2	48	92	9551144998088845844548144557
41	16	3	6	0	0	1	2	2	48	92	9151445988888415544859444958
42	16	3	6	0	0	1	2	2	48	92	9454588484585411844589184058
43	20	2	6	0	0	1	2	3	47	88	7845844788898485444411954541
44	20	2	6	0	0	1	2	3	47	88	1844144844144899888448585547
45	20	2	6	0	0	1	2	3	47	88	4481144784584845488088144557
46	20	2	6	0	0	1	2	3	47	88	4441184484587854875594194947
47	20	2	6	0	0	1	2	3	47	88	9145154888988441447548484457
48	24	1	6	0	0	1	2	4	46	84	4841144774474898785448145447
49	24	1	6	0	0	1	2	4	46	84	8745844788888474444411844441
50	24	1	6	0	0	1	2	4	46	84	7845844787788485444411844441
51	28	0	6	0	0	1	2	5	45	80	8441144778877744474474144447
52	15	0	4	0	3	0	2	2	56	108	4993943993934303550355033552
53	10	2	6	0	2	0	2	1	53	104	3095935905935395539553945542
54	14	1	6	0	2	0	2	2	52	100	4993853895854394350535953452
55	14	1	6	0	2	0	2	2	52	100	4595959594858353249333959453
56	18	0	6	0	2	0	2	3	51	96	4893953784854484459335953352
57	18	0	6	0	2	0	2	3	51	96	4993943884844393459345943452
58	18	0	6	0	2	0	2	3	51	96	3993944993944393449344944442
59	18	0	6	0	2	0	2	3	51	96	4895584895584334483339245358
60	13	2	8	0	1	0	2	2	48	92	4485859495858444249533959453
61	17	1	8	0	1	0	2	3	47	88	4883384444244595489548385547

no.	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	$h^{1,2}$	$h^{1,1}$	χ	subarrangements of 6 planes
62	21	0	8	0	1	0	2	4	46	84	884338444485474854742443845 47
63	21	0	8	0	1	0	2	4	46	84	388384488384448444844484444 2
64	12	3	10	0	0	0	2	2	44	84	945485848458542444485944495 8
65	16	2	10	0	0	0	2	3	43	80	444244487458444857485844495 8
66	20	1	10	0	0	0	2	4	42	76	874548444845747447424448444 7
67	24	0	10	0	0	0	2	5	41	72	774447447444774447447424444 7
68	24	0	1	1	2	0	1	3	51	96	699690999690949334943390934 3
69	14	5	1	0	2	0	1	0	50	100	090909009990059353953399053 3
70	18	4	1	0	2	0	1	1	49	96	899090998980950335943390035 3
71	18	4	1	0	2	0	1	1	49	96	899900999899930355933399055 3
72	18	4	1	0	2	0	1	1	49	96	355309955309933099309999999 7
73	18	4	1	0	2	0	1	1	49	96	999909985339490099530339503 8
74	22	3	1	0	2	0	1	2	48	92	889999999998993934593339904 53
75	22	3	1	0	2	0	1	2	48	92	889809999890839354933399955 3
76	22	3	1	0	2	0	1	2	48	92	999009843539833598339959999 7
77	22	3	1	0	2	0	1	2	48	92	549339839439895389909938900 8
78	22	3	1	0	2	0	1	2	48	92	898909834834850359933909930 8
79	22	3	1	0	2	0	1	2	48	92	899090988880859334943390935 3
80	26	2	1	0	2	0	1	3	47	88	888990988880848334843390935 3
81	26	2	1	0	2	0	1	3	47	88	844338899908833598358939999 7
82	26	2	1	0	2	0	1	3	47	88	899899999789858334843399935 3
83	26	2	1	0	2	0	1	3	47	88	799999988789859334943399935 3
84	26	2	1	0	2	0	1	3	47	88	889999989889848334943399935 3
85	30	1	1	0	2	0	1	4	46	84	883348479887899988339359493 8
86	30	1	1	0	2	0	1	4	46	84	888808888808843384438338403 8
87	30	1	1	0	2	0	1	4	46	84	889898889898838344933398844 3
88	34	0	1	0	2	0	1	5	45	80	789788889788838344933388844 3
89	19	2	3	1	1	0	1	2	48	92	988999969690956555943399055 3
90	23	1	3	1	1	0	1	3	47	88	489339589548599888096639595 6
91	27	0	3	1	1	0	1	4	46	84	478444858933958888899663959 56
92	27	0	3	1	1	0	1	4	46	84	449339948444888448899663999 6
93	13	6	3	0	1	0	1	0	46	92	999099999099994955594339005 53
94	17	5	3	0	1	0	1	1	45	88	888990989999949455953399055 3
95	17	5	3	0	1	0	1	1	45	88	988999989090949555495339995 43
96	17	5	3	0	1	0	1	1	45	88	889999999999983945305539995 54
97	17	5	3	0	1	0	1	1	45	88	980999089899938443955399955 5
98	17	5	3	0	1	0	1	1	45	88	845594899908833949305905990 8
99	17	5	3	0	1	0	1	1	45	88	899899945859459595930399903 8
100	17	5	3	0	1	0	1	1	45	88	855489499999833095390599409 8
101	21	4	3	0	1	0	1	2	44	84	878989989089948445953399045 3
102	21	4	3	0	1	0	1	2	44	84	879898989999948445853399055 3

no.	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	$h^{1,2}$	$h^{1,1}$	χ	subarrangements of 6 planes
103	21	4	3	0	1	0	1	2	44	84	989998989999847544943399953
104	21	4	3	0	1	0	1	2	44	84	888990988899849554843399953
105	21	4	3	0	1	0	1	2	44	84	888989899089839435953499454
106	21	4	3	0	1	0	1	2	44	84	889898899999839435853499554
107	21	4	3	0	1	0	1	2	44	84	789998898998839345935409954
108	21	4	3	0	1	0	1	2	44	84	988909834939459394859499895 7
109	21	4	3	0	1	0	1	2	44	84	589339488459489988999938505 8
110	21	4	3	0	1	0	1	2	44	84	588549489339480888999939595 8
111	21	4	3	0	1	0	1	2	44	84	845854989898859548933008939 8
112	21	4	3	0	1	0	1	2	44	84	448955838834899349999998950 8
113	21	4	3	0	1	0	1	2	44	84	935389498999843985489599499 8
114	21	4	3	0	1	0	1	2	44	84	989998989999848444853399955 3
115	21	4	3	0	1	0	1	2	44	84	998990088789948455843389955 3
116	25	3	3	0	1	0	1	3	43	80	878989988989848544843399945 3
117	25	3	3	0	1	0	1	3	43	80	388348498359489888088859594 7
118	25	3	3	0	1	0	1	3	43	80	488338487458499098998839495 7
119	25	3	3	0	1	0	1	3	43	80	388348498359488988998859495 7
120	25	3	3	0	1	0	1	3	43	80	448338848458795588909839899 7
121	25	3	3	0	1	0	1	3	43	80	983539488899798997539449494 7
122	25	3	3	0	1	0	1	3	43	80	978998843935858448934998949 7
123	25	3	3	0	1	0	1	3	43	80	398539488438489888898948595 8
124	25	3	3	0	1	0	1	3	43	80	388438488438489988898958505 8
125	25	3	3	0	1	0	1	3	43	80	589339478448489887990839495 8
126	25	3	3	0	1	0	1	3	43	80	849548588887883384999939495 8
127	25	3	3	0	1	0	1	3	43	80	493359874447789988359958999 8
128	25	3	3	0	1	0	1	3	43	80	943395878888745848395994999 8
129	25	3	3	0	1	0	1	3	43	80	888988988989848444843399955 3
130	25	3	3	0	1	0	1	3	43	80	879898989889848454853389945 3
131	29	2	3	0	1	0	1	4	42	76	488338477447499988998838594 7
132	29	2	3	0	1	0	1	4	42	76	388438488438498988898847594 7
133	29	2	3	0	1	0	1	4	42	76	388438488438488878998858594 7
134	29	2	3	0	1	0	1	4	42	76	388438488438487978808858495 7
135	29	2	3	0	1	0	1	4	42	76	983539478888788987438459494 7
136	29	2	3	0	1	0	1	4	42	76	838438487787795394899848495 8
137	29	2	3	0	1	0	1	4	42	76	778888988879848454843389945 3
138	29	2	3	0	1	0	1	4	42	76	888988988878848444843388945 3
139	33	1	3	0	1	0	1	5	41	72	778898878788744447433854839 8
140	33	1	3	0	1	0	1	5	41	72	878888878888847444843388844 3
141	33	1	3	0	1	0	1	5	41	72	878879878879847444843379844 3
142	33	1	3	0	1	0	1	5	41	72	778778888888848544843388844 3
143	37	0	3	0	1	0	1	6	40	68	778778878778847444843378844 3

no.	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	$h^{1,2}$	$h^{1,1}$	χ	subarrangements of 6 planes
144	14	4	5	1	0	0	1	1	45	88	5945599655586999065599499990
145	18	3	5	1	0	0	1	2	44	84	8889909455498545985566599996
146	22	2	5	1	0	0	1	3	43	80	8798989494558844588966559596
147	22	2	5	1	0	0	1	3	43	80	8485495888808845958866495956
148	22	2	5	1	0	0	1	3	43	80	9986698889888485549545689554
149	26	1	5	1	0	0	1	4	42	76	8484595787898844858866485956
150	30	0	5	1	0	0	1	5	41	72	8886688778877485448544688554
151	30	0	5	1	0	0	1	5	41	72	6886888886888484448444888444
152	16	6	5	0	0	0	1	1	41	80	9595594998998955948998494957
153	16	6	5	0	0	0	1	1	41	80	8809989484448955489990559598
154	16	6	5	0	0	0	1	1	41	80	5599549488549894589999989598
155	16	6	5	0	0	0	1	1	41	80	5599459488549895489999989598
156	20	5	5	0	0	0	1	2	40	76	8845594889997899975594594947
157	20	5	5	0	0	0	1	2	40	76	8449558889988595488549989497
158	20	5	5	0	0	0	1	2	40	76	8549548889988594488549989597
159	20	5	5	0	0	0	1	2	40	76	5489548498548884480989989597
160	20	5	5	0	0	0	1	2	40	76	8455849888989448484859958908
161	20	5	5	0	0	0	1	2	40	76	8444884888088459855985984098
162	20	5	5	0	0	0	1	2	40	76	8889988989888485549455899454
163	20	5	5	0	0	0	1	2	40	76	9888988999997584548544988554
164	20	5	5	0	0	0	1	2	40	76	8879998889997495548454998554
165	24	4	5	0	0	0	1	3	39	72	8979988845447889884548548597
166	24	4	5	0	0	0	1	3	39	72	8945584879887898975484594947
167	24	4	5	0	0	0	1	3	39	72	4748548847548899898448989597
168	24	4	5	0	0	0	1	3	39	72	4875484874584898988889485958
169	24	4	5	0	0	0	1	3	39	72	4885594774474888889899485958
170	24	4	5	0	0	0	1	3	39	72	7445848888987448575958948998
171	24	4	5	0	0	0	1	3	39	72	8455948788987447474958958998
172	24	4	5	0	0	0	1	3	39	72	8898889484457745488899448598
173	24	4	5	0	0	0	1	3	39	72	8444884778988458844885985098
174	24	4	5	0	0	0	1	3	39	72	9789988799888474448545889454
175	24	4	5	0	0	0	1	3	39	72	8879998788888484548445889554
176	24	4	5	0	0	0	1	3	39	72	8889988798878484548545889454
177	28	3	5	0	0	0	1	4	38	68	7788988474447845487988548597
178	28	3	5	0	0	0	1	4	38	68	8745484788897788874584594947
179	28	3	5	0	0	0	1	4	38	68	9845584878887798874474584947
180	28	3	5	0	0	0	1	4	38	68	4874584774474888978898484958
181	28	3	5	0	0	0	1	4	38	68	7879888788778484548445889454
182	28	3	5	0	0	0	1	4	38	68	8778988788888474448445879554
183	28	3	5	0	0	0	1	4	38	68	8880888778787584448444888454
184	28	3	5	0	0	0	1	4	38	68	8789878889787484548444888454

no.	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	$h^{1,2}$	$h^{1,1}$	χ	subarrangements of 6 planes
185	28	3	5	0	0	0	1	4	38	68	8788978888887484547544888454
186	32	2	5	0	0	0	1	5	37	64	7778878744447878784548548497
187	32	2	5	0	0	0	1	5	37	64	7778878788787474448454888454
188	32	2	5	0	0	0	1	5	37	64	7789888777777474448454888454
189	32	2	5	0	0	0	1	5	37	64	8779888778787474447444888454
190	36	1	5	0	0	0	1	6	36	60	7778878777777474447444878454
191	36	1	5	0	0	0	1	6	36	60	8778877778877474447444877444
192	40	0	5	0	0	0	1	7	35	56	7777777777777474447444777444
193	24	3	0	2	0	0	0	2	44	84	6996909996990990990966999996
194	28	2	0	2	0	0	0	3	43	80	9889999699609986998966999906
195	32	1	0	2	0	0	0	4	42	76	6898699898699889988966998096
196	36	0	0	2	0	0	0	5	41	72	8896699788888888889966699996
197	22	6	0	1	0	0	0	1	41	80	9999099899998999989066099096
198	22	6	0	1	0	0	0	1	41	80	8899999899999980999066009906
199	22	6	0	1	0	0	0	1	41	80	8990909899999989990966999906
200	22	6	0	1	0	0	0	1	41	80	9799999969690999999609099999
201	26	5	0	1	0	0	0	2	40	76	8990909889898999989866999996
202	26	5	0	1	0	0	0	2	40	76	8889909888899980999966999906
203	26	5	0	1	0	0	0	2	40	76	8798989898099989899066099906
204	26	5	0	1	0	0	0	2	40	76	8899999889990879999966999096
205	26	5	0	1	0	0	0	2	40	76	9788899899090889999966999096
206	26	5	0	1	0	0	0	2	40	76	8899999909889989999966880996
207	26	5	0	1	0	0	0	2	40	76	8899999889799999998966900996
208	26	5	0	1	0	0	0	2	40	76	8989999999998989988966999096
209	26	5	0	1	0	0	0	2	40	76	9988999889999899998866999006
210	30	4	0	1	0	0	0	3	39	72	8789899888808989989966999996
211	30	4	0	1	0	0	0	3	39	72	8789899899908878989966999996
212	30	4	0	1	0	0	0	3	39	72	8789899799898888989966009996
213	30	4	0	1	0	0	0	3	39	72	9889999888898898988866999096
214	30	4	0	1	0	0	0	3	39	72	8789899888899889998866909906
215	30	4	0	1	0	0	0	3	39	72	8788999887999989899966989906
216	30	4	0	1	0	0	0	3	39	72	8888998899999998889866989996
217	30	4	0	1	0	0	0	3	39	72	7899988899988668886099099997
218	30	4	0	1	0	0	0	3	39	72	8799889999899888899866880996
219	30	4	0	1	0	0	0	3	39	72	8898099898889888998966880996
220	30	4	0	1	0	0	0	3	39	72	8999989888788999988966999996
221	30	4	0	1	0	0	0	3	39	72	8888989888998989889966099996
222	34	3	0	1	0	0	0	4	38	68	8788999888998888888866989996
223	34	3	0	1	0	0	0	4	38	68	8787898788999888988866999096
224	34	3	0	1	0	0	0	4	38	68	7788889988899888998866880996
225	34	3	0	1	0	0	0	4	38	68	8888989988889888898866880896

no.	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	$h^{1,2}$	$h^{1,1}$	χ	subarrangements of 6 planes
226	34	3	0	1	0	0	0	4	38	68	8799889898898878989866889996
227	34	3	0	1	0	0	0	4	38	68	7898989787788989988966999996
228	34	3	0	1	0	0	0	4	38	68	7788889898998978888966999996
229	34	3	0	1	0	0	0	4	38	68	7788889798888888989966099996
230	34	3	0	1	0	0	0	4	38	68	8889889988898888988866889996
231	38	2	0	1	0	0	0	5	37	64	7888978888987686888688088997
232	38	2	0	1	0	0	0	5	37	64	7788889887898878988866889996
233	38	2	0	1	0	0	0	5	37	64	7788889787788988889966989896
234	38	2	0	1	0	0	0	5	37	64	7788889888888888888866989896
235	42	1	0	1	0	0	0	6	36	60	7788889787788877888866889896
236	42	1	0	1	0	0	0	6	36	60	7888888787788888888866888886
237	46	0	0	1	0	0	0	7	35	56	7787788787788877888866788886
238	8	12	0	0	0	0	0	0	44	88	9990000990000900000000999999
239	16	10	0	0	0	0	0	0	40	80	0900099899988909880090099098
240	16	10	0	0	0	0	0	0	40	80	0999999999089900899990999008
241	16	10	0	0	0	0	0	0	40	80	9999099999900999999999099999
242	16	10	0	0	0	0	0	1	41	80	0098999890999909999800999008
243	20	9	0	0	0	0	0	1	39	76	0999089889989809899980089008
244	20	9	0	0	0	0	0	1	39	76	8889909889990908998099099999
245	20	9	0	0	0	0	0	0	38	76	8989999889889999009890990990
246	20	9	0	0	0	0	0	1	39	76	9990999899989999989099999998
247	24	8	0	0	0	0	0	1	37	72	9889089889089990899089979997
248	24	8	0	0	0	0	0	1	37	72	9990098889987899879098098098
249	24	8	0	0	0	0	0	1	37	72	9990999889888898889890999098
250	24	8	0	0	0	0	0	1	37	72	8899098888088889899999999008
251	24	8	0	0	0	0	0	1	37	72	8999998888998899999908998098
252	24	8	0	0	0	0	0	1	37	72	9899899899989808889899999098
253	24	8	0	0	0	0	0	1	37	72	9999099788989898899980989908
254	24	8	0	0	0	0	0	1	37	72	8879899988990898098999999999
255	24	8	0	0	0	0	0	1	37	72	9899998998998098989999899997
256	24	8	0	0	0	0	0	1	37	72	9990098798988998880089999997
257	24	8	0	0	0	0	0	1	37	72	9989998808998098989999899997
258	24	8	0	0	0	0	0	1	37	72	9989998900998999979898999987
259	24	8	0	0	0	0	0	1	37	72	9909999998998989989999798997
260	24	8	0	0	0	0	0	2	38	72	9799999979999999999999799997
261	24	8	0	0	0	0	0	1	37	72	9889999889999999999999799997
262	24	8	0	0	0	0	0	1	37	72	9099999999898998888998998998
263	24	8	0	0	0	0	0	2	38	72	9989899998998898999999989998
264	24	8	0	0	0	0	0	1	37	72	9889999088998998899809989998
265	24	8	0	0	0	0	0	1	37	72	9089099888988998899089989998
266	24	8	0	0	0	0	0	1	37	72	8999989888878999890099099998

no.	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	$h^{1,2}$	$h^{1,1}$	χ	subarrangements of 6 planes
267	24	8	0	0	0	0	0	1	37	72	988999908899899889999899998
268	24	8	0	0	0	0	0	1	37	72	998899988889899899980999098
269	24	8	0	0	0	0	0	2	38	72	988099808899899889980988990 8
270	24	8	0	0	0	0	0	1	37	72	889999998888989098999998908
271	24	8	0	0	0	0	0	2	38	72	809099988088888889890999098
272	24	8	0	0	0	0	0	2	38	72	988099809999988888980988990 8
273	24	8	0	0	0	0	0	1	37	72	888998889908989988908909990 8
274	24	8	0	0	0	0	0	1	37	72	998899989808999989898088990 8
275	24	8	0	0	0	0	0	1	37	72	898999989999999998989997998 9
276	24	8	0	0	0	0	0	1	37	72	898999989999999998899998988 9
277	28	7	0	0	0	0	0	2	36	68	878999888999798998909899899 7
278	28	7	0	0	0	0	0	2	36	68	887999888998889988998909999 7
279	28	7	0	0	0	0	0	2	36	68	988898899808809988898997999 7
280	28	7	0	0	0	0	0	2	36	68	978998889908889878098908999 7
281	28	7	0	0	0	0	0	2	36	68	998999888989789897880899809 8
282	28	7	0	0	0	0	0	2	36	68	998908888998789987808808809 8
283	28	7	0	0	0	0	0	2	36	68	888988989898889898998098999 8
284	28	7	0	0	0	0	0	2	36	68	880998978777899988999099999 8
285	28	7	0	0	0	0	0	2	36	68	889999988887889988989098999 8
286	28	7	0	0	0	0	0	2	36	68	988999988898880888888098999 8
287	28	7	0	0	0	0	0	3	37	68	088998988888880888888098909 8
288	28	7	0	0	0	0	0	2	36	68	888989888899889889889999990 8
289	28	7	0	0	0	0	0	2	36	68	887999888808889889999899899 08
290	28	7	0	0	0	0	0	2	36	68	888898989897988998999998890 8
291	28	7	0	0	0	0	0	2	36	68	798989887898889089999899809 8
292	28	7	0	0	0	0	0	2	36	68	889979889899789988899999999 8
293	28	7	0	0	0	0	0	2	36	68	899999878889789889999999899 8
294	28	7	0	0	0	0	0	2	36	68	899989978889789988999999899 8
295	28	7	0	0	0	0	0	2	36	68	898999778988899889989989990 8
296	28	7	0	0	0	0	0	2	36	68	878999897989899999899889999 7
297	28	7	0	0	0	0	0	2	36	68	989908899997989888999888989 7
298	28	7	0	0	0	0	0	2	36	68	889099899998979898898888999 7
299	28	7	0	0	0	0	0	2	36	68	897998888808899998898999999 7
300	28	7	0	0	0	0	0	2	36	68	889099889898889898998988999 7
301	28	7	0	0	0	0	0	2	36	68	998999889999889888888998999 7
302	28	7	0	0	0	0	0	2	36	68	888998899999889888999988999 7
303	28	7	0	0	0	0	0	2	36	68	989999899989898897989889998 7
304	28	7	0	0	0	0	0	2	36	68	888998888808899999898899999 7
305	28	7	0	0	0	0	0	2	36	68	989999899999798897999799898 7
306	28	7	0	0	0	0	0	2	36	68	997999988889898999899889899 7
307	28	7	0	0	0	0	0	2	36	68	908909987788899889998899998 7

no.	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	$h^{1,2}$	$h^{1,1}$	χ	subarrangements of 6 planes
308	28	7	0	0	0	0	0	2	36	68	98899999889998889889988898997
309	28	7	0	0	0	0	0	2	36	68	98990888990888897790880889997
310	28	7	0	0	0	0	0	2	36	68	99989989989989899889988797997
311	28	7	0	0	0	0	0	2	36	68	98999989899897989899888889997
312	28	7	0	0	0	0	0	2	36	68	99889998888988889888089980 98
313	28	7	0	0	0	0	0	2	36	68	89899998788888898899998098098
314	28	7	0	0	0	0	0	2	36	68	90890998889888888880889889 98
315	28	7	0	0	0	0	0	2	36	68	99990998878788898890989889 98
316	28	7	0	0	0	0	0	2	36	68	89899990888888899889988898998
317	28	7	0	0	0	0	0	2	36	68	98908899888888898889890889998
318	28	7	0	0	0	0	0	2	36	68	88899888880888998880880989 08
319	28	7	0	0	0	0	0	2	36	68	79899888889789998990899898 08
320	28	7	0	0	0	0	0	2	36	68	88799988889988988998998999 08
321	28	7	0	0	0	0	0	2	36	68	78999887888779998800990999 98
322	28	7	0	0	0	0	0	2	36	68	89998997998988878888909999 98
323	28	7	0	0	0	0	0	2	36	68	88898898808889988898909998 98
324	28	7	0	0	0	0	0	2	36	68	89908898899888888988899989 08
325	28	7	0	0	0	0	0	2	36	68	88898898899888898989890989 08
326	28	7	0	0	0	0	0	2	36	68	88998978999997888899999989 98
327	28	7	0	0	0	0	0	2	36	68	99889998999987998888998979 98
328	28	7	0	0	0	0	0	2	36	68	98808999898998980987888898 89
329	32	6	0	0	0	0	0	3	35	64	87899987898878898880880989 97
330	32	6	0	0	0	0	0	3	35	64	89799888798879999888889989 97
331	32	6	0	0	0	0	0	3	35	64	98799888899878998889780889 97
332	32	6	0	0	0	0	0	3	35	64	87798888889988888899899999 97
333	32	6	0	0	0	0	0	3	35	64	98799888889888088888799899 97
334	32	6	0	0	0	0	0	3	35	64	87789888880889898999889799 97
335	32	6	0	0	0	0	0	3	35	64	88898987888978988789980989 98
336	32	6	0	0	0	0	0	3	35	64	98809988788877988788989980 98
337	32	6	0	0	0	0	0	3	35	64	87998898878797898888999989 08
338	32	6	0	0	0	0	0	3	35	64	88899888779887998989889880 98
339	32	6	0	0	0	0	0	3	35	64	88899887788778999898999899 98
340	32	6	0	0	0	0	0	3	35	64	88898987889877988899899899 98
341	32	6	0	0	0	0	0	3	35	64	88880887778878998999899880 98
342	32	6	0	0	0	0	0	3	35	64	87898789898888898999889899 97
343	32	6	0	0	0	0	0	3	35	64	78899988888888898989988899 97
344	32	6	0	0	0	0	0	3	35	64	88899888897788898999889899 97
345	32	6	0	0	0	0	0	3	35	64	87798889798988898989889999 97
346	32	6	0	0	0	0	0	3	35	64	87798889809988898989789898 97
347	32	6	0	0	0	0	0	3	35	64	87798888898988998979889999 97
348	32	6	0	0	0	0	0	3	35	64	87899988797888998979889999 97

no.	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	$h^{1,2}$	$h^{1,1}$	χ	subarrangements of 6 planes
349	32	6	0	0	0	0	0	3	35	64	787898887988899999888999997
350	32	6	0	0	0	0	0	3	35	64	878999888998888879998898 9897
351	32	6	0	0	0	0	0	3	35	64	888808888887899999898888 9897
352	32	6	0	0	0	0	0	3	35	64	778898898988889898998889997
353	32	6	0	0	0	0	0	3	35	64	878987889098988888898898 9997
354	32	6	0	0	0	0	0	3	35	64	788999888998978888888889999 97
355	32	6	0	0	0	0	0	3	35	64	978998889998978778998898 9997
356	32	6	0	0	0	0	0	3	35	64	879998889987989878908897 9897
357	32	6	0	0	0	0	0	3	35	64	879998889987979888898898 9997
358	32	6	0	0	0	0	0	3	35	64	887999888988989988897898 9897
359	32	6	0	0	0	0	0	3	35	64	888998889987989988997897 9897
360	32	6	0	0	0	0	0	3	35	64	988099888998779887988709 8987
361	32	6	0	0	0	0	0	3	35	64	987998888989789988888899 8997
362	32	6	0	0	0	0	0	3	35	64	888998888897788088998809 8897
363	32	6	0	0	0	0	0	3	35	64	888998898989788988899888 8997
364	32	6	0	0	0	0	0	3	35	64	888887899897899878998908 9897
365	32	6	0	0	0	0	0	3	35	64	899999899888888897888888 9987
366	32	6	0	0	0	0	0	3	35	64	898898890989888897888888 9987
367	32	6	0	0	0	0	0	3	35	64	888808888808888989887898 9997
368	32	6	0	0	0	0	0	3	35	64	988988898888808989888888 9897
369	32	6	0	0	0	0	0	3	35	64	888899888899888989878898 9997
370	32	6	0	0	0	0	0	3	35	64	997999988889888898889789 8987
371	32	6	0	0	0	0	0	3	35	64	888898988887898889909898 8897
372	32	6	0	0	0	0	0	3	35	64	987988997889887989898899 8997
373	32	6	0	0	0	0	0	3	35	64	987988988988888989897809 8897
374	32	6	0	0	0	0	0	3	35	64	988888988889898889889989 8897
375	32	6	0	0	0	0	0	3	35	64	987898998808989888888898 8897
376	32	6	0	0	0	0	0	3	35	64	888808888808888977808808 8997
377	32	6	0	0	0	0	0	3	35	64	888999788999789997999799 7977
378	32	6	0	0	0	0	0	3	35	64	889988898989789987099888 8977
379	32	6	0	0	0	0	0	3	35	64	897988998898887889889889 8998
380	32	6	0	0	0	0	0	3	35	64	988888998888889888880898 8898
381	32	6	0	0	0	0	0	3	35	64	088998988888888878808898 8898
382	32	6	0	0	0	0	0	3	35	64	888998899888889798799888 9898
383	32	6	0	0	0	0	0	3	35	64	808888889988888888888898 89088
384	32	6	0	0	0	0	0	4	36	64	088888888888888808888880 888088
385	32	6	0	0	0	0	0	3	35	64	997889888798989789998989 8987
386	36	5	0	0	0	0	0	4	34	60	877988887889787988898899 8997
387	36	5	0	0	0	0	0	4	34	60	778898877888798888899899 8997
388	36	5	0	0	0	0	0	4	34	60	877898887898788978898808 8997
389	36	5	0	0	0	0	0	4	34	60	778988878887778887999809 8998

no.	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	$h^{1,2}$	$h^{1,1}$	χ	subarrangements of 6 planes
390	36	5	0	0	0	0	0	4	34	60	7778878978888999998888889897
391	36	5	0	0	0	0	0	4	34	60	7789888888888888798998889897
392	36	5	0	0	0	0	0	4	34	60	7788988878778998898998889897
393	36	5	0	0	0	0	0	4	34	60	7788988878778899897988989997
394	36	5	0	0	0	0	0	4	34	60	8779888889898888797988889897
395	36	5	0	0	0	0	0	4	34	60	8789998879788888797988889897
396	36	5	0	0	0	0	0	4	34	60	7789888879888888798888999897
397	36	5	0	0	0	0	0	4	34	60	8778988878888899897878989997
398	36	5	0	0	0	0	0	4	34	60	8879998878888898897878889897
399	36	5	0	0	0	0	0	4	34	60	8889988978878889798888889797
400	36	5	0	0	0	0	0	4	34	60	8878978978878889798988989897
401	36	5	0	0	0	0	0	4	34	60	7788988889878888798088979897
402	36	5	0	0	0	0	0	4	34	60	8788888889898888797888989897
403	36	5	0	0	0	0	0	4	34	60	7778878989988889898978979897
404	36	5	0	0	0	0	0	4	34	60	8889988889778898989787889887
405	36	5	0	0	0	0	0	4	34	60	8888088889988788887877989987
406	36	5	0	0	0	0	0	4	34	60	8888088878888888788987989987
407	36	5	0	0	0	0	0	4	34	60	8778988889988798888877089987
408	36	5	0	0	0	0	0	4	34	60	8888088978878898888897889887
409	36	5	0	0	0	0	0	4	34	60	8788888889898888887897989987
410	36	5	0	0	0	0	0	4	34	60	8880888889877798879887098887
411	36	5	0	0	0	0	0	4	34	60	8889988788777989989888888897
412	36	5	0	0	0	0	0	4	34	60	8898988998888878878888879987
413	36	5	0	0	0	0	0	4	34	60	9789988798988777779988988997
414	36	5	0	0	0	0	0	4	34	60	9988089887878778888987988987
415	36	5	0	0	0	0	0	4	34	60	9878989887989887888888888897
416	36	5	0	0	0	0	0	4	34	60	9898888988888988888888797797
417	36	5	0	0	0	0	0	4	34	60	8788888888088898879889888887
418	36	5	0	0	0	0	0	4	34	60	8880888878888888788888888808
419	36	5	0	0	0	0	0	4	34	60	7989988777777998888989889898
420	36	5	0	0	0	0	0	4	34	60	8799889787778788878899998998
421	40	4	0	0	0	0	0	5	33	56	7888978777777889877988988998
422	40	4	0	0	0	0	0	5	33	56	7778878978888889897878879897
423	40	4	0	0	0	0	0	5	33	56	7778878878778899897878989897
424	40	4	0	0	0	0	0	5	33	56	8788978888878888797978879797
425	40	4	0	0	0	0	0	5	33	56	7788988878778888797988879897
426	40	4	0	0	0	0	0	5	33	56	7778878888888888797988879897
427	40	4	0	0	0	0	0	5	33	56	7778878889878888798978979797
428	40	4	0	0	0	0	0	5	33	56	7778878978888788888887989987
429	40	4	0	0	0	0	0	5	33	56	7788988978888788887887879987
430	40	4	0	0	0	0	0	5	33	56	77898888888888787788897889887

no.	p_3	p_4^0	p_4^1	p_5^0	p_5^1	p_5^2	l_3	$h^{1,2}$	$h^{1,1}$	χ	subarrangements of 6 planes
431	40	4	0	0	0	0	0	5	33	56	8789878889778888888887879887
432	40	4	0	0	0	0	0	5	33	56	8789878889778798888777989887
433	40	4	0	0	0	0	0	5	33	56	7778878889878798888877089887
434	40	4	0	0	0	0	0	5	33	56	8878978978878889887887879887
435	40	4	0	0	0	0	0	5	33	56	8888878088888888887777879887
436	40	4	0	0	0	0	0	5	33	56	8878879887888887888897888887
437	40	4	0	0	0	0	0	5	33	56	8789887888088778788897887887
438	40	4	0	0	0	0	0	5	33	56	8788897877988888799788788887
439	44	3	0	0	0	0	0	6	32	52	7778878878778888797878879797
440	44	3	0	0	0	0	0	6	32	52	7788988878778787787887879887
441	44	3	0	0	0	0	0	6	32	52	7778878889788787788787889787
442	44	3	0	0	0	0	0	6	32	52	8778988878888787787777879887
443	44	3	0	0	0	0	0	6	32	52	7778878877887888978787888887
444	44	3	0	0	0	0	0	6	32	52	7778878878877788778887088887
445	44	3	0	0	0	0	0	6	32	52	7777777888978888799878878787
446	48	2	0	0	0	0	0	7	31	48	7778878878778787787777879787
447	48	2	0	0	0	0	0	7	31	48	7778878877887778877777878887
448	48	2	0	0	0	0	0	7	31	48	7788787877878778788787787787
449	52	1	0	0	0	0	0	8	30	44	7778878777777777777777878787
450	56	0	0	0	0	0	0	9	29	40	7777777777777777777777777777

Appendix B

Modular double octics

This apppendix contains tables of modular double octics constructed from six planes and a smooth quadric surface. For a rather detailed discussion cf. 4.3. There are ten different types of arrangements of six planes. The equations that I used are given before each table. The tables contain the parameter $(a_0 : \ldots : a_9)$ of the quadric surface given by

$$a_0x^2 + a_1y^2 + a_2z^2 + a_3t^2 + a_4xy + a_5xz + a_6xt + a_7yz + a_8yt + a_9zt = 0,$$

the (twists of minimal level of the) weight four newforms occuring in the L-series of the double octics, and a prediction if the double octics are rigid. Double octics that are separated by a horizontal line have different numbers or types of singularities. Examples with the same numbers and types of singularities do not have to be isomorphic (there are examples with different newforms occuring in the L-series).

The weight four newforms occuring in the tables are 5/1, 6/1, 8/1, 9/1, 10/1, 12/1, 14/2, 20/1, 24/1, 28/2, 32/1, 32/2, 40/2, 40/3, 72/1, 96/2, 96/4, 128/1, 168/1, 256/1, 256/3, 256/7, 288/1, 544/1, 1568/1. The occurence of the bad prime 17 in the level 544 is remarkable.

Sextic arrangement no. 1:

Equation for the arrangement of six planes:

$$xyzt(x+z)(y+z) = 0$$

Note: Some examples are isomorphic over $\mathbb{Q}[\sqrt{-1}]$.

parameter	weight four newform	rigid?
$(0:0:0:1:-2:-1:0:-1:0:2)$	256/1 $(256k4G1)$	y
$(0:0:1:1:2:1:0:1:0:2)$	256/1 $(256k4G1)$	y
$(0:0:0:1:-1:-1:2:0:-2:0)$	6/1 $(6k4A1)$	y
$(0:0:0:1:1:0:2:0:2:2)$	6/1 $(6k4A1)$	y
$(0:0:1:1:1:1:2:1:2:2)$	6/1 $(6k4A1)$	y

parameter	weight four newform	rigid?
$(0:0:0:1:-1:-1:1:0:-1:0)$	12/1 $(12k4A1)$	y
$(0:0:0:1:1:0:1:0:1:1)$	12/1 $(12k4A1)$	y
$(0:0:1:1:1:1:1:1:1:1)$	12/1 $(12k4A1)$	y
$(0:0:0:1:-1:-1:0:0:0:2)$	8/1 $(8k4A1)$	y
$(0:0:0:1:1:0:0:0:0:2)$	8/1 $(8k4A1)$	y
$(0:0:1:1:1:1:0:1:0:2)$	8/1 $(8k4A1)$	y
$(0:0:0:1:-1:0:0:0:1:1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:1:0:0:1:1:1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:1:0:1:1:0:0)$	8/1 $(8k4A1)$	y
$(0:0:0:1:0:-1:0:0:2:0)$	32/2 $(32k4B1)$	y
$(0:0:0:1:0:-1:0:0:2:2)$	32/2 $(32k4B1)$	y
$(0:0:1:1:0:0:-2:1:0:-2)$	32/2 $(32k4B1)$	y
$(0:0:1:1:0:0:-2:1:0:0)$	32/2 $(32k4B1)$	y
$(0:0:0:1:0:-1:0:0:-2:2)$	96/4 $(96k4B1)$	n
$(0:0:0:1:1:1:1:1:1:1)$	40/2 $(40k4B1)$	y
$(0:0:1:-1:-1:0:1:0:1:1)$	40/2 $(40k4B1)$	y
$(0:0:1:-1:1:0:1:1:-1:0)$	40/2 $(40k4B1)$	y
$(0:0:0:1:2:1:1:1:1:1)$	32/1 $(32k4A1)$	y
$(0:0:1:-1:2:1:-1:1:1:0)$	32/1 $(32k4A1)$	y
$(0:0:1:1:1:1:1:1:1:-2)$	6/1 $(6k4A1)$	n
$(0:1:-2:1:0:-2:2:-1:2:1)$	8/1 $(8k4A1)$	n
$(0:1:0:1:0:2:2:-1:-2:1)$	8/1 $(8k4A1)$	y
$(0:1:0:1:-2:-1:0:0:0:0)$	32/1 $(32k4A1)$	y
$(0:1:0:2:-2:-1:0:0:0:0)$	256/7 $(256k4B1)$	y
$(0:1:1:-2:2:1:0:2:0:0)$	256/7 $(256k4B1)$	y
$(0:1:1:-1:2:1:0:2:0:0)$	32/1 $(32k4A1)$	y
$(0:1:0:-2:0:0:-1:1:-1:-1)$	32/1 $(32k4A1)$	n
$(0:1:0:-1:0:0:-1:1:0:1)$	96/4 $(96k4B1)$	n
$(0:1:0:-1:0:0:2:1:0:1)$	32/1 $(32k4A1)$	n
$(0:1:0:-1:0:0:1:1:0:0)$	32/2 $(32k4B1)$	y
$(0:1:0:-1:0:0:1:1:0:1)$	32/2 $(32k4B1)$	y
$(0:1:0:-1:0:2:2:1:0:-1)$	8/1 $(8k4A1)$	n
$(0:1:0:1:-2:-2:1:1:-2:-2)$	32/1 $(32k4A1)$	y
$(0:1:0:1:-2:0:1:-1:-2:1)$	32/1 $(32k4A1)$	y
$(0:1:0:1:2:0:1:1:2:0)$	32/1 $(32k4A1)$	y
$(0:1:0:1:-1:-1:1:1:-2:-2)$	40/2 $(40k4B1)$	y
$(0:1:0:1:-1:0:1:0:-2:1)$	40/2 $(40k4B1)$	y
$(0:1:0:1:1:0:1:1:2:0)$	40/2 $(40k4B1)$	y
$(0:1:0:1:-1:-1:0:1:2:1)$	12/1 $(12k4A1)$	y
$(0:1:0:1:-1:0:0:0:2:1)$	12/1 $(12k4A1)$	y
$(0:1:0:1:1:0:0:1:2:1)$	12/1 $(12k4A1)$	y
$(0:1:0:1:-1:-1:0:1:2:-2)$	6/1 $(6k4A1)$	y

parameter	weight four newform	rigid?
$(0:1:1:1:1:1:0:2:2:-2)$	6/1 $(6k4A1)$	y
$(0:1:0:1:0:0:2:1:-2:0)$	32/1 $(32k4A1)$	y
$(0:1:0:1:0:0:2:1:2:2)$	32/1 $(32k4A1)$	y
$(0:1:0:1:0:0:-1:1:-2:0)$	96/4 $(96k4B1)$	n
$(0:1:0:1:0:0:-1:1:2:-1)$	96/4 $(96k4B1)$	n
$(0:1:0:1:0:0:-1:1:2:2)$	96/4 $(96k4B1)$	n
$(0:1:0:1:0:0:-1:1:-2:-2)$	32/2 $(32k4B1)$	y
$(0:1:0:1:0:0:-1:1:-2:-1)$	32/2 $(32k4B1)$	y
$(0:1:0:1:0:0:1:1:-2:-1)$	32/2 $(32k4B1)$	y
$(0:1:0:1:0:0:1:1:-2:0)$	32/2 $(32k4B1)$	y
$(0:1:1:1:1:1:1:2:2:-2)$	40/3 $(40k4A1)$	n
$(0:2:0:-2:-2:-1:0:1:0:1)$	32/2 $(32k4B1)$	n
$(0:2:0:1:-2:-1:0:1:0:2)$	256/3 $(256k4F1)$	n
$(1:-1:0:-1:0:1:0:-1:0:1)$	128/1 $(128k4A1)$	n
$(1:-1:0:-1:0:1:0:-1:2:-1)$	8/1 $(8k4A1)$	n
$(1:1:-2:1:-2:-1:2:-1:2:-1)$	96/2 $(96k4E1)$	y
$(1:1:0:1:-2:1:2:1:2:1)$	544/1	n
$(1:1:0:-1:0:1:0:1:0:1)$	128/1 $(128k4A1)$	y
$(1:1:0:1:0:1:-2:1:2:-1)$	128/1 $(128k4A1)$	n
$(1:1:0:1:0:1:-2:1:2:1)$	128/1 $(128k4A1)$	n
$(1:1:0:1:0:1:2:1:2:1)$	128/1 $(128k4A1)$	n
$(1:1:1:1:-2:-2:2:-2:2:2)$	40/2 $(40k4B1)$	y
$(2:2:1:1:0:2:0:2:0:2)$	256/1 $(256k4G1)$	n
$(2:2:0:-1:-2:1:1:1:1:1)$	288/1 $(288k4C1)$	n
$(2:2:1:2:0:2:0:2:0:2)$	128/1 $(128k4A1)$	n

Sextic arrangement no. 2:

Equation for the arrangement of six planes:

$$xyzt(x+y)(z+t) = 0$$

parameter	weight four newform	rigid?
$(0:0:0:0:1:0:2:2:2:1)$	6/1 $(6k4A1)$	y
$(0:0:0:1:-1:0:-2:2:0:1)$	6/1 $(6k4A1)$	y
$(0:0:0:1:-1:2:0:2:2:1)$	6/1 $(6k4A1)$	y
$(0:0:0:0:1:0:0:0:2:1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:-1:0:0:0:2:1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:-1:0:0:2:2:1)$	8/1 $(8k4A1)$	y
$(0:0:0:0:1:-1:-1:0:0:-1)$	8/1 $(8k4A1)$	y
$(0:0:0:0:1:-1:0:-1:0:-1)$	8/1 $(8k4A1)$	y

parameter	weight four newform	rigid?
$(0:0:0:1:1:-1:-1:-1:-1:1)$	8/1 (8k4A1)	y
$(0:0:0:1:1:0:-1:0:-1:1)$	8/1 (8k4A1)	y
$(0:0:0:1:1:0:0:1:0:1)$	8/1 (8k4A1)	y
$(0:0:0:0:1:0:0:0:0:-2)$	256/7 (256k4B1)	y
$(0:0:0:0:1:0:0:0:0:-1)$	32/1 (32k4A1)	y
$(0:0:0:0:1:0:0:0:0:1)$	32/1 (32k4A1)	y
$(0:0:0:0:1:0:0:0:0:2)$	256/7 (256k4B1)	y
$(0:0:0:1:-2:0:0:0:0:1)$	256/7 (256k4B1)	y
$(0:0:0:1:-1:0:0:0:0:1)$	32/1 (32k4A1)	y
$(0:0:0:1:-2:-2:-1:2:2:2)$	32/2 (32k4B1)	n
$(0:0:0:1:1:-1:-2:-1:-2:0)$	96/4 (96k4B1)	n
$(0:0:0:1:1:1:-1:1:-1:0)$	96/4 (96k4B1)	n
$(0:1:-1:0:0:0:-1:0:1:-1)$	96/4 (96k4B1)	n
$(0:0:0:1:-2:1:-1:1:2:0)$	32/1 (32k4A1)	n
$(0:1:-1:0:0:0:-2:0:-1:-1)$	32/1 (32k4A1)	n
$(0:0:0:1:1:2:1:2:1:0)$	32/1 (32k4A1)	n
$(0:0:0:2:-1:1:-1:1:2:0)$	32/1 (32k4A1)	n
$(0:0:0:1:-1:-2:0:2:2:0)$	128/1 (128k4A1)	y
$(0:0:1:1:-1:0:2:2:0:2)$	128/1 (128k4A1)	y
$(0:1:1:1:1:0:2:-2:2:2)$	128/1 (128k4A1)	y
$(0:0:0:1:1:1:0:1:0:0)$	32/2 (32k4B1)	y
$(0:0:0:1:1:1:1:1:1:0)$	32/2 (32k4B1)	y
$(0:0:1:1:1:0:1:0:1:2)$	32/2 (32k4B1)	y
$(0:0:1:1:1:1:1:1:1:-2)$	8/1 (8k4A1)	y
$(0:1:-1:-1:1:1:1:0:0:2)$	8/1 (8k4A1)	y
$(0:1:-1:0:2:0:0:-2:0:-2)$	128/1 (128k4A1)	n
$(0:1:-1:1:2:0:0:2:2:0)$	128/1 (128k4A1)	n
$(0:1:0:1:2:0:0:0:2:2)$	128/1 (128k4A1)	n
$(1:-1:-1:1:0:-2:-2:-2:-2:0)$	128/1 (128k4A1)	n
$(0:1:0:1:0:0:-2:2:0:0)$	32/1 (32k4A1)	n
$(0:1:1:1:0:2:2:0:0:-2)$	32/2 (32k4B1)	n
$(0:1:1:1:0:2:2:2:2:-2)$	32/2 (32k4B1)	n
$(1:1:1:1:-2:0:2:0:2:2)$	32/2 (32k4B1)	n
$(0:1:1:1:0:-2:-2:2:2:-2)$	96/4 (96k4B1)	n
$(0:1:1:1:1:2:2:-2:2:2)$	8/1 (8k4A1)	y
$(1:1:1:1:-2:2:2:2:2:-2)$	8/1 (8k4A1)	y
$(1:1:1:1:-2:-2:2:-2:2:2)$	32/1 (32k4A1)	y
$(1:1:1:1:1:-2:-2:-2:0:1)$	32/1 (32k4A1)	n
$(1:1:1:1:1:-2:0:0:2:1)$	32/1 (32k4A1)	n
$(1:1:1:1:2:-2:2:2:-2:2)$	32/1 (32k4A1)	y

Sextic arrangement no. 3:

Equation for the arrangement of six planes:

$$xyzt(x+y)(x-y+z) = 0$$

Note: Some examples are isomorphic over $\mathbb{Q}[\sqrt{-1}]$.

parameter	weight four newform	rigid?
$(0:0:0:1:-1:0:0:0:2:-2)$	128/1 (128k4A1)	y
$(0:0:0:1:-1:0:2:0:0:2)$	128/1 (128k4A1)	y
$(0:0:0:1:0:-2:2:-2:0:-1)$	32/2 (32k4B1)	n
$(0:0:0:1:0:-1:2:1:2:0)$	8/1 (8k4A1)	y
$(1:1:0:1:-2:1:2:-1:2:0)$	8/1 (8k4A1)	y
$(0:0:0:1:1:0:0:0:2:-2)$	128/1 (128k4A1)	n
$(0:0:0:1:1:0:2:0:0:2)$	128/1 (128k4A1)	n
$(0:0:1:-1:0:1:1:-1:1:0)$	8/1 (8k4A1)	y
$(0:0:1:1:0:-1:1:-1:0:-2)$	8/1 (8k4A1)	n
$(0:0:1:1:0:1:0:1:1:2)$	8/1 (8k4A1)	n
$(0:0:1:1:0:1:0:-1:2:-2)$	8/1 (8k4A1)	y
$(0:0:1:1:0:1:2:-1:0:2)$	8/1 (8k4A1)	y
$(0:1:0:-2:-1:-1:1:-1:1:0)$	32/2 (32k4B1)	n
$(0:2:0:-1:-2:-2:1:-2:1:0)$	32/2 (32k4B1)	n
$(1:0:0:-2:-1:1:1:1:1:0)$	32/2 (32k4B1)	n
$(0:1:0:-2:0:-1:0:-1:0:0)$	256/7 (256k4B1)	y
$(0:1:0:-1:0:-1:0:-1:0:0)$	32/1 (32k4A1)	y
$(0:1:0:1:0:-1:0:-1:0:0)$	32/1 (32k4A1)	y
$(0:1:0:2:0:-1:0:-1:0:0)$	256/7 (256k4B1)	y
$(0:2:0:-1:0:-2:0:-2:0:0)$	256/7 (256k4B1)	y
$(0:2:0:1:0:-2:0:-2:0:0)$	256/7 (256k4B1)	y
$(0:1:0:-1:-1:-1:1:-1:0:1)$	8/1 (8k4A1)	n
$(1:0:0:-1:-1:1:0:1:1:-1)$	8/1 (8k4A1)	n
$(0:1:0:1:-1:-1:2:-1:2:0)$	256/3 (256k4F1)	n
$(1:0:0:1:-1:1:2:1:2:0)$	256/3 (256k4F1)	n
$(1:1:1:-1:2:2:0:-2:0:0)$	32/1 (32k4A1)	y
$(1:1:1:-2:2:2:0:-2:0:0)$	256/7 (256k4B1)	y
$(1:1:1:1:2:2:0:-2:0:0)$	32/1 (32k4A1)	y
$(1:1:1:2:2:2:0:-2:0:0)$	256/7 (256k4B1)	y

Sextic arrangement no. 4:

Equation for the arrangement of six planes:

$$xyzt(x+y)(x+z+t) = 0$$

parameter	weight four newform	rigid?
$(0:0:0:0:0:0:1:-1:1:-1)$	96/4 $(96k4B1)$	n
$(0:0:0:0:0:0:1:-1:1:1)$	32/2 $(32k4B1)$	y
$(0:0:0:1:-1:0:0:-1:-2:1)$	32/2 $(32k4B1)$	y
$(1:0:0:1:1:1:2:1:2:1)$	32/2 $(32k4B1)$	y
$(0:0:0:0:0:0:1:-1:1:2)$	32/1 $(32k4A1)$	n
$(0:0:0:0:0:0:1:2:1:-1)$	32/1 $(32k4A1)$	n
$(0:0:0:0:0:0:2:1:2:1)$	32/1 $(32k4A1)$	n
$(0:0:0:0:0:0:1:1:1:-1)$	32/2 $(32k4B1)$	y
$(0:0:0:0:0:0:1:1:1:1)$	32/2 $(32k4B1)$	y
$(0:0:0:1:-1:0:2:-1:0:1)$	32/2 $(32k4B1)$	y
$(0:0:0:1:1:0:0:1:0:1)$	32/2 $(32k4B1)$	y
$(1:0:-1:0:1:0:1:0:1:-1)$	32/2 $(32k4B1)$	y
$(1:0:0:1:1:1:2:1:0:1)$	32/2 $(32k4B1)$	y
$(0:0:0:0:2:-1:2:1:0:2)$	32/2 $(32k4B1)$	n
$(0:0:0:0:1:-1:-1:0:0:-1)$	12/1 $(12k4A1)$	y
$(0:0:0:1:-1:1:1:0:0:1)$	12/1 $(12k4A1)$	y
$(1:0:0:0:1:1:1:0:0:1)$	12/1 $(12k4A1)$	y
$(1:0:0:1:1:1:1:0:0:1)$	12/1 $(12k4A1)$	y
$(0:0:0:0:2:-2:1:0:1:-2)$	32/2 $(32k4B1)$	n
$(0:0:0:0:2:0:2:1:-2:2)$	96/4 $(96k4B1)$	n
$(0:0:0:0:1:-1:0:0:0:-1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:-1:1:2:0:0:1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:1:0:0:0:0:1)$	8/1 $(8k4A1)$	y
$(1:0:-1:0:1:0:0:0:0:-1)$	8/1 $(8k4A1)$	y
$(1:0:0:0:1:0:1:0:0:1)$	8/1 $(8k4A1)$	y
$(1:0:0:1:1:1:2:0:0:1)$	8/1 $(8k4A1)$	y
$(0:0:0:0:1:0:0:0:1:1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:-1:0:1:0:-1:1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:0:-1:0:-1:-1:1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:0:0:1:1:1:1)$	8/1 $(8k4A1)$	y
$(1:0:0:0:1:0:1:0:1:-1)$	8/1 $(8k4A1)$	y
$(1:0:0:1:1:0:2:0:1:1)$	8/1 $(8k4A1)$	y
$(0:0:0:0:1:0:0:1:1:-1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:0:0:1:-1:0:1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:0:1:1:1:0:1)$	8/1 $(8k4A1)$	y
$(1:0:0:0:1:1:1:1:1:1)$	8/1 $(8k4A1)$	y
$(0:0:0:0:1:0:1:0:1:1)$	12/1 $(12k4A1)$	y
$(0:0:0:1:-1:0:0:0:-1:1)$	12/1 $(12k4A1)$	y
$(1:0:0:0:1:0:0:0:1:-1)$	12/1 $(12k4A1)$	y
$(1:0:0:1:0:0:1:-1:-1:1)$	12/1 $(12k4A1)$	y
$(1:0:0:1:0:1:2:1:1:1)$	12/1 $(12k4A1)$	y
$(1:0:0:1:1:0:1:0:1:1)$	12/1 $(12k4A1)$	y

parameter	weight four newform	rigid?
$(0:0:0:0:2:-2:0:2:1:-2)$	32/2 $(32k4B1)$	n
$(0:0:0:2:1:0:0:-1:1:2)$	32/2 $(32k4B1)$	n
$(1:0:-2:0:1:1:-1:1:-1:-2)$	32/2 $(32k4B1)$	n
$(0:0:0:0:2:-1:0:1:1:-1)$	32/1 $(32k4A1)$	y
$(0:0:0:1:-1:2:2:1:0:1)$	32/1 $(32k4A1)$	y
$(0:0:0:1:1:0:0:-1:0:1)$	32/1 $(32k4A1)$	y
$(1:0:-1:0:1:0:-1:0:-1:-1)$	32/1 $(32k4A1)$	y
$(1:0:0:1:1:1:2:-1:0:1)$	32/1 $(32k4A1)$	y
$(2:0:0:0:2:1:2:1:1:1)$	32/1 $(32k4A1)$	y
$(0:0:2:-2:0:0:-2:-1:-1:0)$	32/2 $(32k4B1)$	n
$(0:0:2:-2:0:1:-1:-1:-1:0)$	32/2 $(32k4B1)$	n
$(0:0:0:1:1:1:1:0:1:0)$	96/4 $(96k4B1)$	n
$(0:0:0:1:-2:-1:1:-2:-2:1)$	32/1 $(32k4A1)$	n
$(0:0:0:1:0:-1:-1:0:-2:1)$	32/1 $(32k4A1)$	n
$(0:0:0:1:0:-1:1:0:2:1)$	32/1 $(32k4A1)$	n
$(0:0:0:1:-2:1:1:0:-2:0)$	32/1 $(32k4A1)$	n
$(0:0:0:1:1:-2:1:0:1:0)$	32/1 $(32k4A1)$	n
$(0:0:0:2:-1:-1:1:-1:-2:2)$	8/1 $(8k4A1)$	n
$(1:0:0:0:0:0:1:-1:1:-2)$	8/1 $(8k4A1)$	n
$(0:0:0:1:-2:1:2:0:-1:1)$	32/1 $(32k4A1)$	y
$(0:0:0:1:1:-1:-1:-1:-1:1)$	32/1 $(32k4A1)$	y
$(1:0:-1:0:1:0:0:-1:-1:-1)$	32/1 $(32k4A1)$	y
$(1:0:0:0:2:-1:1:0:1:-1)$	32/1 $(32k4A1)$	y
$(1:0:0:0:2:0:1:1:0:1)$	32/1 $(32k4A1)$	y
$(0:0:0:1:0:-1:1:0:-1:1)$	96/4 $(96k4B1)$	n
$(0:0:0:2:-1:-2:2:-1:-1:-2)$	32/2 $(32k4B1)$	n
$(0:0:0:1:2:-2:-1:0:0:2)$	32/2 $(32k4B1)$	n
$(0:0:0:1:2:-2:1:0:0:2)$	32/2 $(32k4B1)$	n
$(0:0:0:1:-1:-2:2:-1:0:-1)$	8/1 $(8k4A1)$	n
$(1:0:0:1:1:-1:2:1:0:-1)$	8/1 $(8k4A1)$	n
$(0:0:0:1:-1:-1:1:-2:-1:0)$	32/1 $(32k4A1)$	y
$(0:0:1:1:0:0:1:-1:1:2)$	32/1 $(32k4A1)$	y
$(1:0:0:1:1:1:2:2:1:0)$	32/1 $(32k4A1)$	y
$(0:0:0:1:-1:-1:1:-1:-1:-1)$	32/1 $(32k4A1)$	y
$(1:0:0:1:1:0:2:1:1:-1)$	32/1 $(32k4A1)$	y
$(0:0:0:1:-1:-1:1:-1:-1:1)$	32/2 $(32k4B1)$	y
$(0:0:0:1:0:-1:0:0:-1:1)$	32/2 $(32k4B1)$	y
$(0:0:0:1:0:-1:1:0:1:1)$	32/2 $(32k4B1)$	y
$(1:0:0:0:0:0:1:-1:0:-1)$	32/2 $(32k4B1)$	y
$(1:0:0:0:0:1:1:0:1:-1)$	32/2 $(32k4B1)$	y
$(1:0:0:1:1:0:2:1:1:1)$	32/2 $(32k4B1)$	y
$(0:0:0:1:-1:-1:1:0:-1:0)$	32/2 $(32k4B1)$	y

parameter	weight four newform	rigid?
$(0:0:1:1:0:-1:0:-1:-1:2)$	32/2 $(32k4B1)$	y
$(0:0:1:1:0:0:1:1:1:2)$	32/2 $(32k4B1)$	y
$(1:0:0:1:-1:1:2:0:-1:0)$	32/2 $(32k4B1)$	y
$(1:0:0:1:1:-1:2:0:1:0)$	32/2 $(32k4B1)$	y
$(0:0:0:1:-1:0:0:-2:-2:2)$	8/1 $(8k4A1)$	y
$(0:0:0:1:-1:0:2:-2:0:2)$	8/1 $(8k4A1)$	y
$(0:0:1:-1:-1:0:0:-2:0:0)$	8/1 $(8k4A1)$	y
$(1:0:-1:1:1:0:2:0:2:0)$	8/1 $(8k4A1)$	y
$(1:0:0:1:1:2:2:2:0:2)$	8/1 $(8k4A1)$	y
$(1:0:0:1:1:2:2:2:2:2)$	8/1 $(8k4A1)$	y
$(0:0:0:1:-1:0:1:-1:-1:0)$	8/1 $(8k4A1)$	y
$(0:0:0:1:0:-1:1:-1:0:0)$	8/1 $(8k4A1)$	y
$(0:0:0:1:0:0:1:1:0:0)$	8/1 $(8k4A1)$	y
$(0:0:1:1:0:0:1:-1:0:2)$	8/1 $(8k4A1)$	y
$(0:0:1:1:0:1:1:0:1:2)$	8/1 $(8k4A1)$	y
$(1:0:0:1:1:1:2:1:1:0)$	8/1 $(8k4A1)$	y
$(0:0:0:1:-1:0:1:-1:0:0)$	12/1 $(12k4A1)$	y
$(0:0:0:1:0:-1:0:-1:-1:0)$	12/1 $(12k4A1)$	y
$(0:0:0:1:0:0:1:1:1:0)$	12/1 $(12k4A1)$	y
$(0:0:1:1:-1:1:1:-1:0:2)$	12/1 $(12k4A1)$	y
$(1:0:0:1:1:1:1:1:0:0)$	12/1 $(12k4A1)$	y
$(1:0:1:1:1:1:2:0:1:2)$	12/1 $(12k4A1)$	y
$(0:0:0:1:-1:1:-2:0:0:1)$	6/1 $(6k4A1)$	n
$(1:0:0:1:1:1:-2:0:0:1)$	6/1 $(6k4A1)$	n
$(0:0:0:1:-1:1:1:-1:-1:1)$	32/2 $(32k4B1)$	y
$(0:0:0:1:0:1:0:0:-1:1)$	32/2 $(32k4B1)$	y
$(0:0:0:1:0:1:1:0:1:1)$	32/2 $(32k4B1)$	y
$(1:0:0:0:0:1:1:-1:0:1)$	32/2 $(32k4B1)$	y
$(1:0:0:0:0:1:2:0:1:1)$	32/2 $(32k4B1)$	y
$(1:0:0:1:1:2:2:1:1:1)$	32/2 $(32k4B1)$	y
$(0:0:0:1:-1:1:1:0:-1:0)$	32/2 $(32k4B1)$	y
$(0:0:0:1:1:-1:1:0:1:0)$	32/2 $(32k4B1)$	y
$(0:0:1:1:0:0:1:-1:-1:2)$	32/2 $(32k4B1)$	y
$(0:0:1:1:0:1:2:1:1:2)$	32/2 $(32k4B1)$	y
$(1:0:-1:0:1:0:1:1:0:0)$	32/2 $(32k4B1)$	y
$(1:0:0:1:1:1:2:0:1:0)$	32/2 $(32k4B1)$	y
$(0:0:0:1:-1:1:1:0:-1:2)$	32/1 $(32k4A1)$	y
$(0:0:1:-1:0:0:-1:-1:-1:0)$	32/1 $(32k4A1)$	y
$(1:0:0:1:1:1:2:0:1:2)$	32/1 $(32k4A1)$	y
$(0:0:0:1:-1:1:1:1:-1:1)$	32/1 $(32k4A1)$	y
$(0:0:0:1:0:-1:0:-2:-1:1)$	32/1 $(32k4A1)$	y
$(0:0:0:1:0:1:1:2:1:1)$	32/1 $(32k4A1)$	y

parameter	weight four newform	rigid?
$(1:0:0:0:2:0:1:1:2:-1)$	32/1 (32k4A1)	y
$(1:0:0:0:2:1:1:1:2:1)$	32/1 (32k4A1)	y
$(1:0:0:1:1:0:2:-1:1:1)$	32/1 (32k4A1)	y
$(0:0:0:1:0:-1:-2:0:-1:1)$	96/4 (96k4B1)	n
$(0:0:0:1:0:-1:-1:0:1:1)$	96/4 (96k4B1)	n
$(0:0:0:1:0:-2:-1:0:-1:1)$	32/1 (32k4A1)	n
$(0:0:0:1:0:-2:0:0:1:1)$	32/1 (32k4A1)	n
$(1:0:0:0:0:-1:1:-1:0:1)$	96/4 (96k4B1)	n
$(1:0:0:0:0:0:1:1:0:1)$	96/4 (96k4B1)	n
$(0:0:0:1:0:1:2:0:-1:1)$	96/4 (96k4B1)	n
$(0:0:0:1:0:1:-1:0:-1:1)$	96/4 (96k4B1)	n
$(0:0:0:1:0:1:0:0:1:1)$	96/4 (96k4B1)	n
$(0:0:0:2:0:0:-2:2:2:1)$	96/4 (96k4B1)	n
$(0:0:0:1:0:-1:-1:0:-1:1)$	32/2 (32k4B1)	y
$(0:0:0:1:0:-1:0:0:1:1)$	32/2 (32k4B1)	y
$(1:0:0:0:0:-1:1:-1:0:-1)$	32/2 (32k4B1)	y
$(1:0:0:0:0:0:1:1:0:-1)$	32/2 (32k4B1)	y
$(1:0:0:1:-1:0:2:-1:-1:1)$	32/2 (32k4B1)	y
$(0:0:0:1:0:1:1:0:-1:1)$	32/2 (32k4B1)	y
$(0:0:0:1:0:1:2:0:1:1)$	32/2 (32k4B1)	y
$(0:0:0:1:1:1:1:1:1:1)$	32/2 (32k4B1)	y
$(1:0:-1:0:1:0:0:1:1:-1)$	32/2 (32k4B1)	y
$(1:0:0:0:0:0:1:-1:0:1)$	32/2 (32k4B1)	y
$(1:0:0:0:0:1:1:0:1:1)$	32/2 (32k4B1)	y
$(0:0:0:1:1:-2:1:-1:1:-1)$	8/1 (8k4A1)	y
$(0:0:0:2:-2:1:2:-1:-2:1)$	8/1 (8k4A1)	y
$(1:0:-1:0:1:0:1:1:-1:1)$	8/1 (8k4A1)	y
$(0:0:0:1:1:0:-1:-1:-1:-1)$	8/1 (8k4A1)	y
$(0:0:0:2:2:0:0:1:0:1)$	8/1 (8k4A1)	y
$(1:0:-1:0:1:0:-1:-1:-1:1)$	8/1 (8k4A1)	y
$(2:0:-2:0:2:0:1:0:1:-1)$	8/1 (8k4A1)	y
$(0:0:0:1:1:0:0:1:0:-1)$	32/1 (32k4A1)	y
$(0:0:0:2:0:0:1:-1:-1:1)$	32/1 (32k4A1)	y
$(0:0:0:2:0:1:2:1:1:1)$	32/1 (32k4A1)	y
$(1:0:-1:0:1:0:1:0:1:1)$	32/1 (32k4A1)	y
$(0:0:0:1:1:1:-1:1:-1:-1)$	8/1 (8k4A1)	y
$(1:0:-2:0:0:-1:0:-2:-1:-1)$	8/1 (8k4A1)	n
$(1:0:-2:0:0:1:1:2:1:-1)$	8/1 (8k4A1)	n
$(1:0:-1:0:1:0:0:-1:1:1)$	8/1 (8k4A1)	n
$(0:0:0:1:1:1:1:1:0:1)$	5/1 (5k4A1)	y
$(1:0:-1:0:1:-1:0:0:1:-1)$	5/1 (5k4A1)	y
$(1:0:0:0:0:0:0:-1:-1:1)$	5/1 (5k4A1)	y

parameter	weight four newform	rigid?
$(1:0:0:0:0:1:1:1:1:1)$	5/1 $(5k4A1)$	y
$(0:0:0:1:1:2:1:2:1:2)$	8/1 $(8k4A1)$	y
$(0:0:1:-1:0:-1:-1:-1:1:0)$	8/1 $(8k4A1)$	y
$(0:0:1:-1:0:0:-2:1:-1:0)$	8/1 $(8k4A1)$	y
$(1:0:-1:0:1:0:0:1:2:-2)$	8/1 $(8k4A1)$	y
$(1:0:0:1:-1:0:2:-2:-1:2)$	8/1 $(8k4A1)$	y
$(0:0:0:1:2:1:1:2:0:1)$	8/1 $(8k4A1)$	n
$(1:0:0:0:0:-1:-1:-2:-2:1)$	8/1 $(8k4A1)$	n
$(1:0:0:0:0:1:1:2:2:1)$	8/1 $(8k4A1)$	n
$(2:0:-1:0:2:-1:1:0:2:-1)$	8/1 $(8k4A1)$	n
$(0:0:0:1:2:2:2:2:1:1)$	8/1 $(8k4A1)$	n
$(1:0:-1:0:-1:0:-1:1:-1:-1)$	8/1 $(8k4A1)$	n
$(2:0:-1:0:1:-1:0:-1:1:-1)$	8/1 $(8k4A1)$	n
$(2:0:-1:0:2:-1:0:1:2:-1)$	8/1 $(8k4A1)$	n
$(2:0:0:0:0:0:0:-2:-1:1)$	8/1 $(8k4A1)$	n
$(2:0:0:0:0:1:2:1:2:1)$	8/1 $(8k4A1)$	n
$(0:0:0:2:-2:-1:2:-1:-2:1)$	8/1 $(8k4A1)$	y
$(1:0:0:1:-1:1:2:1:-1:-1)$	8/1 $(8k4A1)$	y
$(0:0:0:2:-1:1:1:-1:-2:2)$	8/1 $(8k4A1)$	y
$(1:0:0:0:0:1:2:-1:1:2)$	8/1 $(8k4A1)$	y
$(1:0:-1:0:2:0:-1:2:-2:-1)$	32/2 $(32k4B1)$	n
$(0:0:0:2:0:-1:2:-2:2:0)$	32/2 $(32k4B1)$	n
$(0:0:0:2:0:1:0:2:-2:0)$	32/2 $(32k4B1)$	n
$(0:0:0:2:1:1:1:1:0:2)$	8/1 $(8k4A1)$	n
$(1:0:-2:0:1:-1:0:0:1:-2)$	8/1 $(8k4A1)$	y
$(1:0:0:0:0:0:1:-1:-1:2)$	8/1 $(8k4A1)$	y
$(1:0:0:0:0:1:2:1:1:2)$	8/1 $(8k4A1)$	y
$(0:0:1:1:0:0:1:-1:1:0)$	128/1 $(128k4A1)$	n
$(0:0:1:2:-1:1:1:-1:-2:2)$	128/1 $(128k4A1)$	n
$(0:0:1:1:-1:0:0:-1:-1:-1)$	24/1 $(24k4A1)$	n
$(1:0:1:1:1:1:1:1:1:-1)$	24/1 $(24k4A1)$	n
$(0:0:1:1:-1:-2:1:0:-1:2)$	24/1 $(24k4A1)$	y
$(1:0:1:1:1:-2:2:0:1:2)$	24/1 $(24k4A1)$	y
$(0:0:1:1:-1:1:1:-1:-1:-2)$	32/2 $(32k4B1)$	y
$(1:0:1:1:1:2:2:1:1:-2)$	32/2 $(32k4B1)$	y
$(0:0:1:1:0:-2:-1:-1:-1:2)$	96/4 $(96k4B1)$	n
$(0:0:1:1:0:-1:0:1:1:2)$	96/4 $(96k4B1)$	n
$(0:0:1:1:0:-2:0:-2:2:2)$	96/4 $(96k4B1)$	n
$(1:0:-1:-1:-1:0:0:-1:-1:2)$	96/4 $(96k4B1)$	n
$(2:0:-1:-1:1:1:1:1:1:2)$	96/4 $(96k4B1)$	n
$(0:0:1:1:1:1:1:1:1:-2)$	32/2 $(32k4B1)$	n
$(1:0:-1:-1:1:0:0:1:1:2)$	32/2 $(32k4B1)$	n

parameter	weight four newform	rigid?
$(0:0:1:1:0:0:1:-1:1:-2)$	8/1 $(8k4A1)$	y
$(0:0:1:1:0:0:2:-2:2:2)$	32/2 $(32k4B1)$	n
$(0:0:1:1:1:-1:-1:-1:-1:-2)$	32/1 $(32k4A1)$	y
$(1:0:-1:-1:1:0:0:-1:-1:2)$	32/1 $(32k4A1)$	y
$(0:1:-2:0:0:-2:0:-1:-2:-2)$	32/2 $(32k4B1)$	n
$(0:1:-2:0:1:-2:0:1:-1:-2)$	32/2 $(32k4B1)$	n
$(0:1:-2:0:-1:-1:0:-1:-2:0)$	32/2 $(32k4B1)$	n
$(0:1:-2:0:1:-2:-2:-1:-2:0)$	32/2 $(32k4B1)$	n
$(0:1:-2:0:1:-1:0:1:2:0)$	32/2 $(32k4B1)$	n
$(0:1:-2:0:-1:-1:1:-1:-1:-2)$	32/2 $(32k4B1)$	n
$(0:1:0:0:2:-2:0:-1:0:2)$	32/2 $(32k4B1)$	n
$(1:-1:0:0:0:0:1:0:-1:-2)$	32/2 $(32k4B1)$	n
$(0:1:-2:0:0:-2:0:-1:-1:-1)$	8/1 $(8k4A1)$	n
$(0:1:-1:0:1:-1:-1:0:-1:1)$	8/1 $(8k4A1)$	n
$(0:1:-1:0:1:-1:0:0:1:1)$	8/1 $(8k4A1)$	n
$(1:1:-2:0:2:-1:1:1:1:-1)$	8/1 $(8k4A1)$	n
$(0:1:-2:0:0:0:-1:1:2:-2)$	6/1 $(6k4A1)$	y
$(0:1:0:1:0:1:0:0:0:1)$	128/1 $(128k4A1)$	n
$(0:1:-1:0:0:0:-1:0:0:-1)$	128/1 $(128k4A1)$	y
$(0:1:-1:0:2:-2:-2:0:0:-1)$	128/1 $(128k4A1)$	y
$(0:1:-2:0:1:-2:-1:-1:-1:-2)$	8/1 $(8k4A1)$	n
$(0:1:-2:0:1:-1:0:1:1:-2)$	8/1 $(8k4A1)$	n
$(0:1:-2:0:2:-2:-1:1:0:-2)$	8/1 $(8k4A1)$	n
$(0:1:0:0:0:0:1:-1:0:2)$	8/1 $(8k4A1)$	n
$(0:1:-1:-1:1:-2:-1:-1:0:-2)$	40/2 $(40k4B1)$	y
$(0:1:-1:-1:1:-1:-1:0:1:-2)$	40/2 $(40k4B1)$	y
$(0:1:-1:0:0:-1:0:-1:-1:0)$	40/2 $(40k4B1)$	y
$(0:1:-1:0:1:-1:-1:0:-1:0)$	40/2 $(40k4B1)$	y
$(0:1:-1:0:1:-1:0:0:1:0)$	40/2 $(40k4B1)$	y
$(1:1:-1:0:2:0:1:1:1:0)$	40/2 $(40k4B1)$	y
$(0:1:-1:-1:1:-1:-1:0:0:-1)$	24/1 $(24k4A1)$	y
$(0:1:-1:-1:1:-1:-1:0:0:2)$	6/1 $(6k4A1)$	y
$(0:1:-1:-1:1:-1:1:0:0:-2)$	8/1 $(8k4A1)$	n
$(0:1:-1:0:1:-1:2:0:0:0)$	8/1 $(8k4A1)$	n
$(0:1:-1:-1:2:-2:-2:0:0:2)$	128/1 $(128k4A1)$	y
$(1:-1:1:1:0:2:2:0:0:-2)$	128/1 $(128k4A1)$	y
$(0:1:-1:0:-2:2:-1:0:0:-1)$	96/2 $(96k4E1)$	y
$(1:1:0:0:-2:1:1:1:1:1)$	544/1	y
$(0:1:-1:0:-1:-1:-1:0:-1:-1)$	128/1 $(128k4A1)$	n
$(1:1:0:0:0:1:1:1:1:1)$	128/1 $(128k4A1)$	n
$(2:1:0:0:2:0:0:-1:-1:1)$	128/1 $(128k4A1)$	n
$(0:1:-1:0:-1:-1:1:0:-1:-1)$	8/1 $(8k4A1)$	n

parameter	weight four newform	rigid?
$(0:1:0:0:2:-2:-2:-1:-1:1)$	8/1 (8k4A1)	n
$(1:-1:0:0:0:1:1:-1:-1:-1)$	8/1 (8k4A1)	y
$(0:1:-1:0:0:-1:0:0:-1:-1)$	40/2 (40k4B1)	y
$(0:1:0:0:1:-1:-1:-1:-1:1)$	40/2 (40k4B1)	y
$(0:1:0:0:1:0:0:1:1:1)$	40/2 (40k4B1)	y
$(1:1:-1:0:2:-1:1:0:1:-1)$	40/2 (40k4B1)	y
$(0:1:-1:0:0:0:1:0:0:-1)$	32/1 (32k4A1)	y
$(0:1:-1:0:2:-2:0:0:0:-1)$	32/1 (32k4A1)	y
$(0:1:0:0:2:-1:0:0:0:1)$	32/1 (32k4A1)	y
$(1:-1:0:0:0:0:1:0:0:-1)$	32/1 (32k4A1)	y
$(1:-1:0:1:0:0:2:0:0:1)$	32/1 (32k4A1)	y
$(1:1:-1:0:2:0:1:0:0:-1)$	32/1 (32k4A1)	y
$(0:1:-1:0:1:-1:-2:0:-1:-1)$	128/1 (128k4A1)	y
$(0:1:-1:0:1:-1:-1:0:1:-1)$	128/1 (128k4A1)	y
$(1:1:0:0:0:1:1:-1:-1:1)$	128/1 (128k4A1)	y
$(2:1:0:0:2:2:2:1:1:1)$	128/1 (128k4A1)	y
$(0:1:-1:0:1:-1:-1:-1:-1:-1)$	5/1 (5k4A1)	y
$(0:1:-1:0:1:0:0:1:1:-1)$	5/1 (5k4A1)	y
$(0:1:-1:0:2:-1:-1:1:0:-1)$	5/1 (5k4A1)	y
$(0:1:0:0:0:0:1:-1:0:1)$	5/1 (5k4A1)	y
$(1:-1:0:1:0:1:2:0:1:1)$	5/1 (5k4A1)	y
$(1:1:0:0:2:1:1:0:1:1)$	5/1 (5k4A1)	y
$(0:1:-1:0:1:-1:-1:0:0:-1)$	6/1 (6k4A1)	y
$(1:1:0:0:1:1:1:0:0:1)$	6/1 (6k4A1)	y
$(0:1:-1:0:1:-1:0:0:-1:-1)$	32/1 (32k4A1)	y
$(0:1:-1:0:1:-1:1:0:1:-1)$	32/1 (32k4A1)	y
$(0:1:0:0:2:0:0:1:1:1)$	32/1 (32k4A1)	y
$(1:-1:0:0:0:1:1:1:1:-1)$	32/1 (32k4A1)	y
$(0:1:-1:0:1:-1:0:0:0:-1)$	8/1 (8k4A1)	y
$(0:1:0:0:1:0:0:0:0:1)$	8/1 (8k4A1)	y
$(0:1:0:0:0:-1:0:-2:1:-2)$	8/1 (8k4A1)	y
$(1:1:0:0:2:-1:1:-1:2:-2)$	8/1 (8k4A1)	y
$(0:1:0:0:1:-1:0:-1:0:-1)$	12/1 (12k4A1)	y
$(0:1:0:0:1:0:0:0:1:-1)$	12/1 (12k4A1)	y
$(0:1:0:1:0:0:1:-1:-1:1)$	12/1 (12k4A1)	y
$(0:1:0:1:1:0:0:0:-1:1)$	12/1 (12k4A1)	y
$(0:1:0:1:1:0:1:0:1:1)$	12/1 (12k4A1)	y
$(1:1:0:1:2:1:2:1:1:1)$	12/1 (12k4A1)	y
$(0:1:1:1:1:0:0:-1:-1:-1)$	72/1 (72k4C1)	n
$(0:1:1:1:1:1:1:1:1:-1)$	72/1 (72k4C1)	n
$(0:2:-1:-1:1:-1:-1:-1:-1:2)$	128/1 (128k4A1)	n
$(0:2:-1:-1:2:-2:-1:-1:1:-2)$	8/1 (8k4A1)	n

parameter	weight four newform	rigid?
$(0:2:-1:0:1:-1:-1:-1:-2:0)$	8/1 (8k4A1)	n
$(0:2:-1:0:1:-1:-1:-1:-1:-1)$	128/1 (128k4A1)	n
$(0:2:-1:0:2:-2:-1:-1:0:-1)$	128/1 (128k4A1)	n
$(0:2:-1:0:2:-1:-1:1:0:-1)$	128/1 (128k4A1)	n
$(1:2:0:0:2:0:1:-1:0:1)$	128/1 (128k4A1)	n
$(1:2:0:0:2:1:1:0:1:1)$	128/1 (128k4A1)	n
$(0:2:-1:0:1:1:0:1:1:-1)$	8/1 (8k4A1)	n
$(0:2:0:0:0:0:1:-2:1:-1)$	8/1 (8k4A1)	n
$(0:2:0:0:0:0:2:-1:0:1)$	8/1 (8k4A1)	n
$(0:2:-1:0:2:-1:-2:-1:-2:-1)$	8/1 (8k4A1)	n
$(0:2:-1:0:2:0:0:1:2:-1)$	8/1 (8k4A1)	n
$(0:2:0:0:0:0:1:-2:-1:1)$	8/1 (8k4A1)	n
$(1:-2:0:1:-1:1:2:-1:1:1)$	8/1 (8k4A1)	y
$(1:0:-1:-1:0:0:0:-1:-1:2)$	40/2 (40k4B1)	n
$(1:0:-1:-1:0:1:1:1:1:2)$	40/2 (40k4B1)	n
$(1:0:-1:-1:2:0:0:2:2:2)$	32/1 (32k4A1)	y
$(1:0:1:1:2:2:2:2:2:-2)$	32/1 (32k4A1)	y
$(1:0:0:1:1:0:-2:0:1:1)$	6/1 (6k4A1)	n
$(1:0:1:1:-1:2:2:-1:-1:-2)$	96/4 (96k4B1)	n
$(1:0:1:1:1:-2:2:1:1:2)$	8/1 (8k4A1)	y
$(1:1:0:1:2:1:2:1:-2:1)$	24/1 (24k4A1)	y
$(1:1:1:1:0:2:2:0:0:-2)$	128/1 (128k4A1)	y
$(2:1:1:1:2:2:2:0:0:-2)$	128/1 (128k4A1)	y
$(1:2:0:0:2:0:1:-1:2:-1)$	128/1 (128k4A1)	y
$(1:2:0:0:2:-1:1:-2:1:-1)$	128/1 (128k4A1)	y

Sextic arrangement no. 5:

Equation for the arrangement of six planes:

$$xyzt(x+y)(x+y+z+t)=0$$

parameter	weight four newform	rigid?
$(0:0:0:0:0:0:1:-2:0:-1)$	32/1 (32k4A1)	n
$(0:0:0:0:0:0:1:1:0:2)$	32/1 (32k4A1)	n
$(0:0:0:0:0:0:1:1:0:-1)$	96/4 (96k4B1)	n
$(0:0:0:0:0:0:1:-1:0:-1)$	32/2 (32k4B1)	y
$(0:1:-1:0:1:0:0:0:1:-1)$	32/2 (32k4B1)	y
$(0:1:0:1:1:0:2:1:2:1)$	32/2 (32k4B1)	y
$(0:0:0:0:0:0:1:1:0:1)$	32/2 (32k4B1)	y
$(0:1:0:1:1:0:0:1:2:1)$	32/2 (32k4B1)	y

parameter	weight four newform	rigid?
(0 : 0 : 0 : 0 : 1 : 0 : 0 : 0 : 0 : −1)	8/1 (8k4A1)	y
(0 : 0 : 0 : 1 : 1 : 0 : 1 : 0 : 1 : 1)	8/1 (8k4A1)	y
(0 : 0 : 0 : 0 : 1 : 0 : 0 : 0 : 1 : 1)	12/1 (12k4A1)	y
(0 : 0 : 0 : 1 : −1 : 0 : 0 : 0 : 1 : 1)	12/1 (12k4A1)	y
(0 : 1 : 0 : 1 : 0 : 0 : 1 : 1 : 2 : 1)	12/1 (12k4A1)	y
(0 : 0 : 0 : 0 : 1 : 0 : 0 : 1 : 1 : −1)	40/2 (40k4B1)	y
(0 : 1 : −1 : 0 : 0 : −1 : 0 : −1 : 1 : −1)	40/2 (40k4B1)	y
(0 : 0 : 0 : 1 : −2 : −2 : −1 : 0 : 2 : 1)	8/1 (8k4A1)	n
(0 : 1 : −1 : 0 : −1 : 1 : 0 : 0 : −1 : −1)	8/1 (8k4A1)	n
(0 : 2 : 0 : 0 : 0 : 0 : 1 : 2 : 0 : −1)	8/1 (8k4A1)	n
(0 : 2 : 0 : 0 : 0 : 0 : 2 : 1 : 2 : 1)	8/1 (8k4A1)	n
(0 : 0 : 0 : 2 : 1 : 0 : 1 : 1 : 2 : 2)	8/1 (8k4A1)	n
(0 : 0 : 0 : 1 : 0 : −1 : −1 : −1 : 1 : 1)	32/1 (32k4A1)	n
(0 : 0 : 0 : 1 : 0 : 2 : 1 : 2 : 2 : 1)	32/1 (32k4A1)	n
(0 : 0 : 0 : 1 : 0 : −1 : −2 : −1 : −1 : 1)	96/4 (96k4B1)	n
(0 : 0 : 0 : 1 : 0 : −2 : −2 : 2 : 0 : 1)	96/4 (96k4B1)	n
(0 : 0 : 0 : 1 : 0 : −2 : −1 : −2 : 0 : 1)	32/1 (32k4A1)	n
(0 : 0 : 0 : 1 : 0 : 1 : 0 : 1 : 2 : 1)	32/1 (32k4A1)	n
(0 : 2 : −1 : 0 : 2 : −1 : 0 : −1 : 2 : 1)	32/2 (32k4B1)	n
(0 : 0 : 0 : 1 : 0 : 1 : −1 : 1 : 0 : 1)	96/4 (96k4B1)	n
(0 : 0 : 0 : 1 : 0 : −2 : 0 : 2 : 2 : 1)	32/2 (32k4B1)	n
(1 : −1 : −1 : 0 : 0 : 0 : −1 : −2 : 1 : −1)	32/2 (32k4B1)	n
(2 : −2 : 0 : 0 : 0 : 1 : 2 : −1 : −2 : −1)	32/2 (32k4B1)	n
(0 : 0 : 0 : 1 : 0 : −1 : 1 : −1 : 2 : 1)	96/4 (96k4B1)	n
(0 : 0 : 0 : 1 : 0 : −1 : −1 : −1 : 0 : 1)	32/2 (32k4B1)	y
(1 : 1 : 0 : 0 : 2 : −1 : 1 : 0 : 1 : −1)	32/2 (32k4B1)	y
(0 : 0 : 0 : 1 : 0 : −1 : 0 : −1 : 1 : 1)	32/2 (32k4B1)	y
(0 : 1 : 0 : 1 : 1 : −1 : 1 : 0 : 2 : 1)	32/2 (32k4B1)	y
(1 : 1 : 0 : 0 : 2 : 0 : 1 : 1 : 1 : −1)	32/2 (32k4B1)	y
(0 : 0 : 0 : 1 : 0 : −1 : 0 : 0 : 1 : 0)	12/1 (12k4A1)	y
(0 : 1 : 0 : 1 : 1 : 0 : 1 : 1 : 1 : 0)	12/1 (12k4A1)	y
(0 : 1 : 1 : 1 : 1 : 1 : 1 : 1 : 2 : 2)	12/1 (12k4A1)	y
(0 : 0 : 0 : 1 : 0 : −1 : 0 : 0 : 1 : 1)	8/1 (8k4A1)	y
(0 : 1 : 0 : 0 : 1 : 0 : 0 : 0 : 1 : −1)	8/1 (8k4A1)	y
(0 : 1 : 0 : 1 : 1 : 0 : 1 : 0 : 2 : 1)	8/1 (8k4A1)	y
(0 : 0 : 0 : 1 : 0 : −1 : 0 : 1 : 1 : 1)	32/1 (32k4A1)	y
(0 : 1 : 0 : 1 : 1 : 1 : 1 : 0 : 2 : 1)	32/1 (32k4A1)	y
(1 : −1 : 0 : 0 : 0 : 0 : 1 : −1 : −1 : −1)	32/1 (32k4A1)	y
(0 : 0 : 0 : 1 : 0 : −1 : 1 : 0 : 1 : 0)	8/1 (8k4A1)	y
(0 : 0 : 1 : 1 : 0 : 0 : 1 : 1 : 1 : 2)	8/1 (8k4A1)	y
(0 : 1 : 0 : 1 : 1 : 0 : 1 : 1 : 2 : 0)	8/1 (8k4A1)	y
(0 : 0 : 0 : 1 : 0 : 0 : 1 : 1 : 1 : 1)	8/1 (8k4A1)	y

parameter	weight four newform	rigid?
$(0:1:0:0:1:0:0:1:1:1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:0:1:0:1:1:1)$	32/2 $(32k4B1)$	y
$(0:1:0:1:1:1:1:2:2:1)$	32/2 $(32k4B1)$	y
$(1:1:0:0:2:1:1:1:2:1)$	32/2 $(32k4B1)$	y
$(0:0:0:1:0:1:1:1:2:1)$	32/2 $(32k4B1)$	y
$(0:1:-1:0:1:-1:-1:0:0:-1)$	32/2 $(32k4B1)$	y
$(1:1:0:0:2:0:1:1:1:1)$	32/2 $(32k4B1)$	y
$(0:0:0:1:1:-2:1:-2:1:0)$	8/1 $(8k4A1)$	n
$(0:0:1:1:1:-1:1:-1:1:2)$	8/1 $(8k4A1)$	n
$(0:2:-1:0:1:-1:-1:-1:0:-1)$	128/1 $(128k4A1)$	n
$(1:2:0:0:2:1:1:0:0:1)$	128/1 $(128k4A1)$	n
$(0:0:0:1:1:-1:1:0:1:1)$	32/1 $(32k4A1)$	y
$(0:1:0:0:2:0:0:1:1:-1)$	32/1 $(32k4A1)$	y
$(0:0:0:1:1:0:0:1:1:1)$	5/1 $(5k4A1)$	y
$(0:1:0:0:0:0:1:1:1:1)$	5/1 $(5k4A1)$	y
$(0:1:0:1:2:1:1:1:2:1)$	5/1 $(5k4A1)$	y
$(0:0:0:1:1:0:1:1:1:-1)$	8/1 $(8k4A1)$	n
$(0:1:-2:0:0:-2:0:-1:1:-1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:1:0:1:1:1:0)$	40/2 $(40k4B1)$	y
$(0:0:1:1:1:1:1:1:2:2)$	40/2 $(40k4B1)$	y
$(0:1:-1:0:0:-1:0:0:1:0)$	40/2 $(40k4B1)$	y
$(0:0:0:1:1:1:1:1:1:1)$	6/1 $(6k4A1)$	y
$(1:1:0:0:1:1:1:1:1:1)$	6/1 $(6k4A1)$	y
$(0:0:0:1:1:1:1:2:1:1)$	128/1 $(128k4A1)$	y
$(1:2:0:0:2:1:1:2:2:1)$	128/1 $(128k4A1)$	y
$(0:0:0:2:1:0:1:2:2:0)$	32/2 $(32k4B1)$	n
$(0:2:-2:0:1:-1:0:0:2:0)$	32/2 $(32k4B1)$	n
$(0:0:0:1:2:0:0:2:1:1)$	8/1 $(8k4A1)$	n
$(0:2:0:0:0:0:1:2:2:1)$	8/1 $(8k4A1)$	y
$(0:0:0:1:2:1:1:1:2:1)$	128/1 $(128k4A1)$	y
$(0:1:-1:0:-1:-1:-1:0:0:-1)$	128/1 $(128k4A1)$	n
$(1:1:0:0:0:0:1:1:1:1)$	128/1 $(128k4A1)$	n
$(0:0:0:2:0:-1:2:1:0:0)$	32/2 $(32k4B1)$	n
$(0:0:0:2:0:0:1:1:2:1)$	32/1 $(32k4A1)$	y
$(0:1:-1:0:1:0:0:0:1:1)$	32/1 $(32k4A1)$	y
$(0:0:1:1:0:0:1:1:0:0)$	128/1 $(128k4A1)$	n
$(0:0:1:-2:1:1:-2:1:1:1)$	6/1 $(6k4A1)$	y
$(0:0:1:-1:0:-1:-1:0:-2:0)$	8/1 $(8k4A1)$	y
$(0:1:-1:0:1:-1:-2:0:0:-2)$	8/1 $(8k4A1)$	y
$(0:0:1:-1:0:0:-1:1:0:0)$	32/1 $(32k4A1)$	y
$(0:1:0:1:1:1:1:1:2:2)$	32/1 $(32k4A1)$	y

parameter	weight four newform		rigid?
$(0:0:1:1:-2:-2:1:1:-2:-2)$	6/1	$(6k4A1)$	y
$(0:0:1:1:-1:0:0:1:1:-1)$	72/1	$(72k4C1)$	n
$(0:0:1:1:0:-2:-1:-1:0:2)$	96/4	$(96k4B1)$	n
$(0:0:1:1:0:-1:0:1:2:2)$	32/1	$(32k4A1)$	n
$(0:0:1:1:0:-1:1:0:2:2)$	32/1	$(32k4A1)$	n
$(0:1:-1:0:1:-1:2:0:2:0)$	32/1	$(32k4A1)$	n
$(0:1:-1:0:1:-1:-1:0:-1:0)$	96/4	$(96k4B1)$	n
$(0:0:1:1:0:-2:0:0:-2:2)$	96/4	$(96k4B1)$	n
$(0:0:1:1:0:-1:0:0:1:2)$	32/2	$(32k4B1)$	y
$(0:1:0:1:1:-1:1:-1:2:0)$	32/2	$(32k4B1)$	y
$(0:1:-1:-1:1:-1:-1:0:0:2)$	32/2	$(32k4B1)$	n
$(0:0:1:1:0:0:1:1:0:-2)$	8/1	$(8k4A1)$	n
$(0:0:1:1:0:0:1:1:0:2)$	32/1	$(32k4A1)$	y
$(0:1:0:1:1:-1:1:1:2:0)$	32/1	$(32k4A1)$	y
$(0:0:1:1:0:0:1:1:2:2)$	32/2	$(32k4B1)$	y
$(0:1:-1:0:1:-1:1:0:1:0)$	32/2	$(32k4B1)$	y
$(0:1:0:1:1:1:1:1:2:0)$	32/2	$(32k4B1)$	y
$(0:0:1:1:0:0:2:2:0:2)$	32/2	$(32k4B1)$	n
$(1:-1:-1:0:0:0:2:-2:-2:0)$	32/2	$(32k4B1)$	n
$(0:0:1:1:1:1:1:1:1:-2)$	6/1	$(6k4A1)$	y
$(0:0:1:1:1:1:1:1:1:1)$	24/1	$(24k4A1)$	n
$(0:0:1:1:2:1:2:2:1:2)$	8/1	$(8k4A1)$	n
$(0:1:-1:0:-1:-1:-1:0:1:0)$	8/1	$(8k4A1)$	y
$(0:0:2:-2:0:0:-2:1:-1:0)$	32/2	$(32k4B1)$	n
$(0:1:0:1:0:1:0:1:0:1)$	128/1	$(128k4A1)$	n
$(0:1:-1:0:0:0:-1:0:-1:-1)$	128/1	$(128k4A1)$	y
$(0:1:0:1:2:2:2:2:2:1)$	128/1	$(128k4A1)$	y
$(0:2:-1:0:2:-1:2:-1:2:-2)$	32/2	$(32k4B1)$	n
$(0:2:-1:0:2:1:2:1:2:-2)$	32/2	$(32k4B1)$	n
$(0:1:-2:0:1:1:-2:1:0:0)$	96/4	$(96k4B1)$	n
$(0:1:-2:0:1:-1:-1:-1:0:-2)$	8/1	$(8k4A1)$	n
$(1:1:0:0:2:0:1:1:2:2)$	8/1	$(8k4A1)$	n
$(0:1:-2:0:1:2:2:1:1:-2)$	32/2	$(32k4B1)$	n
$(0:1:-2:0:1:0:0:1:-1:-2)$	32/2	$(32k4B1)$	n
$(0:1:-2:0:2:-2:1:1:1:0)$	8/1	$(8k4A1)$	n
$(0:1:-1:-1:-1:-1:-1:0:0:2)$	128/1	$(128k4A1)$	n
$(0:1:-1:-1:1:1:1:0:0:2)$	32/1	$(32k4A1)$	y
$(0:1:-1:0:0:0:1:0:1:-1)$	32/1	$(32k4A1)$	y
$(0:1:0:0:2:0:1:0:1:-1)$	32/1	$(32k4A1)$	y
$(0:1:0:1:2:0:2:0:2:1)$	32/1	$(32k4A1)$	y
$(0:1:-1:0:1:-1:-1:-1:0:-1)$	5/1	$(5k4A1)$	y

parameter	weight four newform	rigid?
$(1:1:0:0:2:0:0:1:1:1)$	$5/1$ $(5k4A1)$	y
$(0:1:-1:0:1:-1:2:0:1:1)$	$8/1$ $(8k4A1)$	y
$(0:1:-1:0:1:0:0:0:-1:-1)$	$32/1$ $(32k4A1)$	y
$(0:1:0:1:1:2:2:1:2:1)$	$32/1$ $(32k4A1)$	y
$(0:2:0:0:2:0:1:1:2:1)$	$32/1$ $(32k4A1)$	y
$(0:1:-1:0:1:0:0:0:0:-1)$	$8/1$ $(8k4A1)$	y
$(0:1:0:0:1:0:1:0:1:1)$	$8/1$ $(8k4A1)$	y
$(0:1:0:1:1:1:2:1:2:1)$	$8/1$ $(8k4A1)$	y
$(0:1:-1:0:1:1:-1:0:0:1)$	$8/1$ $(8k4A1)$	y
$(1:1:-2:0:2:-1:0:1:1:-1)$	$8/1$ $(8k4A1)$	y
$(0:1:-1:0:1:1:0:0:-1:1)$	$8/1$ $(8k4A1)$	y
$(0:2:-2:0:2:0:0:0:1:-1)$	$8/1$ $(8k4A1)$	y
$(0:1:-1:0:1:1:1:0:0:-1)$	$32/1$ $(32k4A1)$	y
$(1:-1:0:0:0:-1:1:-1:0:-1)$	$32/1$ $(32k4A1)$	y
$(0:1:-1:1:1:0:0:0:2:0)$	$8/1$ $(8k4A1)$	y
$(0:1:0:1:1:0:0:2:2:2)$	$8/1$ $(8k4A1)$	y
$(0:1:0:1:1:0:2:2:2:2)$	$8/1$ $(8k4A1)$	y
$(0:1:0:0:0:-1:0:1:-1:-2)$	$8/1$ $(8k4A1)$	y
$(0:1:0:0:1:-1:0:0:0:-1)$	$12/1$ $(12k4A1)$	y
$(0:1:0:1:1:0:0:0:1:1)$	$12/1$ $(12k4A1)$	y
$(1:1:0:1:2:0:1:1:2:1)$	$12/1$ $(12k4A1)$	y
$(0:1:0:0:1:1:1:1:1:1)$	$12/1$ $(12k4A1)$	y
$(0:1:0:1:1:1:1:1:1:1)$	$12/1$ $(12k4A1)$	y
$(0:1:0:0:2:0:2:0:1:-2)$	$32/2$ $(32k4B1)$	n
$(0:2:-2:0:1:-1:1:0:2:-2)$	$32/2$ $(32k4B1)$	n
$(0:1:0:0:2:2:2:1:1:-1)$	$8/1$ $(8k4A1)$	n
$(0:2:-1:0:1:-1:1:-1:2:-1)$	$8/1$ $(8k4A1)$	y
$(0:1:0:1:-2:1:1:1:-2:1)$	$9/1$ $(9k4A1)$	n
$(1:1:0:0:-2:-2:1:1:1:1)$	$96/2$ $(96k4E1)$	y
$(0:1:0:1:-1:-1:-1:0:2:1)$	$128/1$ $(128k4A1)$	y
$(1:1:0:0:0:-1:1:1:0:-1)$	$128/1$ $(128k4A1)$	y
$(1:-1:0:0:0:-1:2:-1:0:-2)$	$32/2$ $(32k4B1)$	n
$(0:1:0:1:1:-2:2:-1:2:-1)$	$8/1$ $(8k4A1)$	y
$(0:1:0:1:1:-1:1:0:2:-1)$	$32/1$ $(32k4A1)$	y
$(0:1:0:1:1:0:1:1:-2:0)$	$6/1$ $(6k4A1)$	n
$(0:1:0:1:1:1:-2:1:-2:1)$	$6/1$ $(6k4A1)$	n
$(0:1:0:1:1:1:1:1:-2:1)$	$24/1$ $(24k4A1)$	y
$(0:1:0:1:1:1:2:1:-2:1)$	$10/1$ $(10k4A1)$	n
$(0:1:1:1:1:-2:1:-2:2:2)$	$24/1$ $(24k4A1)$	y
$(0:1:1:1:1:0:0:1:1:-1)$	$24/1$ $(24k4A1)$	n
$(0:1:1:1:1:1:1:-2:2:2)$	$12/1$ $(12k4A1)$	y

parameter	weight four newform	rigid?
$(0:1:1:1:1:1:1:2:2:-2)$	32/2 $(32k4B1)$	y
$(0:1:1:1:2:2:2:2:2:-2)$	128/1 $(128k4A1)$	y
$(0:2:-1:0:2:-2:-2:-1:0:-1)$	8/1 $(8k4A1)$	n
$(0:2:-1:0:2:-1:-1:-1:1:-1)$	8/1 $(8k4A1)$	n
$(1:1:0:0:2:-1:-1:1:1:1)$	8/1 $(8k4A1)$	n
$(1:1:1:1:-2:1:1:1:1:-2)$	12/1 $(12k4A1)$	n
$(1:-1:-1:-1:0:0:0:-2:-2:2)$	32/1 $(32k4A1)$	y
$(1:1:-1:-1:2:0:0:1:1:2)$	40/2 $(40k4B1)$	n
$(1:1:0:0:-2:0:1:1:0:2)$	6/1 $(6k4A1)$	n
$(1:1:0:0:-2:-2:1:1:-2:-2)$	6/1 $(6k4A1)$	n
$(1:1:0:0:-2:0:1:1:1:1)$	544/1	y
$(1:1:0:0:-2:1:1:1:1:1)$	12/1 $(12k4A1)$	y
$(1:1:0:0:2:1:2:2:1:2)$	8/1 $(8k4A1)$	y
$(1:1:0:1:-2:1:-2:1:-2:1)$	6/1 $(6k4A1)$	n
$(1:1:0:1:-2:1:2:1:2:1)$	8/1 $(8k4A1)$	y
$(1:1:0:1:2:0:-2:1:2:1)$	24/1 $(24k4A1)$	y
$(1:1:0:1:2:1:-2:1:2:1)$	8/1 $(8k4A1)$	y
$(1:1:1:1:-1:1:1:1:1:-2)$	24/1 $(24k4A1)$	n
$(1:2:1:1:2:2:2:2:2:-2)$	128/1 $(128k4A1)$	y

Sextic arrangement no. 6:

Equation for the arrangement of six planes:

$$xyzt(x+y+z)(x-y+2z)=0$$

Note: These examples are isomorphic over $\mathbb{Q}[\sqrt{2}]$.

parameter	weight four newform	rigid?
$(1:1:1:-2:-2:2:0:2:0:0)$	1568/1	n
$(1:1:1:-1:-2:2:0:2:0:0)$	288/1 $(288k4C1)$	n

Sextic arrangement no. 7:

Equation for the arrangement of six planes:

$$xyzt(x+y+z+t)(x-y+2z-2t)=0$$

parameter	weight four newform	rigid?
$(0:1:-1:0:-1:0:0:0:1:1)$	32/1 $(32k4A1)$	n

parameter	weight four newform	rigid?
$(1:1:-2:0:2:1:1:-1:1:0)$	32/2 $(32k4B1)$	n
$(0:0:1:-1:0:1:0:0:-1:0)$	6/1 $(6k4A1)$	y
$(1:-1:0:0:0:2:0:0:-2:0)$	6/1 $(6k4A1)$	y
$(0:0:1:1:-2:1:-2:-2:1:-2)$	32/1 $(32k4A1)$	n
$(0:0:1:1:0:0:1:1:0:-2)$	32/1 $(32k4A1)$	n
$(1:1:0:0:2:0:-2:-2:0:0)$	32/1 $(32k4A1)$	n

Sextic arrangement no. 8:

Equation for the arrangement of six planes:

$$xyzt(x+y+z)(x+2y-z+t)=0$$

parameter	weight four newform	rigid?
$(0:0:0:1:-1:0:0:-2:2:-1)$	6/1 $(6k4A1)$	y
$(0:0:1:1:0:1:0:-1:1:-2)$	6/1 $(6k4A1)$	y
$(1:0:0:1:1:-1:2:-2:2:-1)$	6/1 $(6k4A1)$	y
$(1:0:0:-1:-1:1:0:0:1:0)$	32/1 $(32k4A1)$	n
$(0:0:1:-2:1:-1:1:1:-2:1)$	8/1 $(8k4A1)$	n
$(0:0:2:-1:1:-2:1:2:-1:1)$	8/1 $(8k4A1)$	n
$(0:0:1:-1:-1:1:-1:1:-1:0)$	24/1 $(24k4A1)$	n
$(0:0:1:0:-1:-1:1:1:2:-1)$	32/1 $(32k4A1)$	n
$(0:0:1:0:-1:1:-1:1:0:-1)$	288/1 $(288k4C1)$	n
$(0:0:1:1:-1:1:-1:1:1:-2)$	24/1 $(24k4A1)$	y
$(2:0:-1:0:2:1:2:2:-2:-1)$	32/2 $(32k4B1)$	n
$(1:0:-1:0:0:0:0:0:1:1)$	6/1 $(6k4A1)$	y
$(1:-2:0:0:-1:-1:0:-1:-1:-1)$	6/1 $(6k4A1)$	y
$(1:0:1:0:2:2:0:2:-1:-1)$	6/1 $(6k4A1)$	y
$(1:0:-1:0:2:0:0:1:-1:1)$	6/1 $(6k4A1)$	y

Sextic arrangement no. 9:

Equation for the arrangement of six planes:

$$xyzt(x+y+z+t)(x+y-z-t)=0$$

parameter	weight four newform	rigid?
$(0:0:0:0:0:0:1:-1:0:-1)$	8/1 $(8k4A1)$	y
$(0:0:0:0:0:0:1:-1:0:0)$	32/1 $(32k4A1)$	n

parameter	weight four newform	rigid?
$(0:0:0:0:0:0:1:1:1:0)$	32/1 (32k4A1)	n
$(0:0:0:0:1:-1:0:0:1:1)$	32/1 (32k4A1)	y
$(0:0:0:0:1:-1:0:1:0:1)$	40/2 (40k4B1)	n
$(0:0:0:0:1:-1:1:0:0:1)$	40/2 (40k4B1)	n
$(0:0:0:0:1:0:0:0:0:-1)$	8/1 (8k4A1)	y
$(0:0:0:1:-2:0:-1:0:1:1)$	32/1 (32k4A1)	y
$(0:0:0:1:-1:-2:-1:2:1:1)$	544/1	n
$(0:0:0:1:-1:-1:-1:1:1:1)$	5/1 (5k4A1)	n
$(0:0:0:1:-1:0:-1:0:1:-1)$	32/1 (32k4A1)	y
$(0:0:0:1:-1:0:-1:0:1:1)$	32/2 (32k4B1)	n
$(0:1:-1:-1:1:-1:1:0:0:-2)$	32/1 (32k4A1)	n
$(0:0:0:1:0:1:0:1:1:1)$	8/1 (8k4A1)	y
$(0:0:0:1:1:0:-1:0:1:1)$	96/4 (96k4B1)	n
$(0:0:0:2:-1:0:-2:0:2:-2)$	32/2 (32k4B1)	y
$(0:0:1:1:-1:-1:1:1:-1:-1)$	24/1 (24k4A1)	n
$(0:0:1:1:1:-1:1:-1:1:1)$	24/1 (24k4A1)	n
$(0:0:1:-2:2:-1:-2:1:2:-1)$	6/1 (6k4A1)	y
$(0:0:1:1:-1:-1:1:1:-1:2)$	12/1 (12k4A1)	n
$(0:0:1:-1:-1:-1:-1:1:1:0)$	8/1 (8k4A1)	n
$(0:0:1:1:-1:0:0:0:0:-1)$	24/1 (24k4A1)	n
$(0:0:1:1:-1:-1:-1:1:1:-2)$	5/1 (5k4A1)	n
$(0:0:1:1:-1:-1:1:0:0:0)$	8/1 (8k4A1)	y
$(0:0:1:1:0:-1:0:0:1:-2)$	14/2 (14k4A1)	n
$(0:0:1:1:0:-1:0:0:1:2)$	8/1 (8k4A1)	y
$(0:0:1:1:1:-1:1:-1:1:2)$	8/1 (8k4A1)	y
$(0:0:1:2:-2:-1:2:1:-2:-1)$	14/2 (14k4A1)	y
$(0:1:-2:-2:1:-2:2:-1:1:-2)$	288/1 (288k4C1)	n
$(0:1:-2:0:1:-2:0:1:1:0)$	8/1 (8k4A1)	y
$(0:1:-2:1:-1:2:1:-1:-2:-1)$	6/1 (6k4A1)	y
$(0:1:-2:1:1:2:1:-1:-2:1)$	6/1 (6k4A1)	y
$(0:1:-1:-1:-1:-1:1:0:0:-2)$	8/1 (8k4A1)	y
$(0:1:-1:-1:0:0:1:0:2:-2)$	128/1 (128k4A1)	n
$(0:1:-1:-1:1:-1:1:0:0:0)$	128/1 (128k4A1)	y
$(0:1:-1:0:1:1:0:0:-1:-2)$	96/4 (96k4B1)	n
$(0:1:-1:0:-1:1:0:0:-1:0)$	128/1 (128k4A1)	n
$(0:1:-1:0:0:1:0:0:-1:0)$	6/1 (6k4A1)	y
$(0:1:-1:0:-1:1:0:0:-1:-1)$	32/1 (32k4A1)	y
$(0:1:-1:0:0:2:0:0:-2:0)$	12/1 (12k4A1)	n
$(0:1:-1:0:0:1:0:0:-1:-1)$	32/2 (32k4B1)	n
$(0:1:0:1:1:-1:-1:-1:0:1)$	32/1 (32k4A1)	y
$(0:1:-1:0:1:0:0:0:0:-1)$	8/1 (8k4A1)	y

parameter	weight four newform	rigid?
$(0:1:-1:0:2:0:0:0:0:-2)$	8/1 $(8k4A1)$	y
$(0:1:-1:1:1:1:1:0:-2:0)$	8/1 $(8k4A1)$	y
$(0:1:0:1:-1:0:1:1:-2:0)$	128/1 $(128k4A1)$	n
$(0:1:0:1:-1:0:1:1:-2:1)$	8/1 $(8k4A1)$	y
$(0:1:1:1:-1:-1:1:2:-2:2)$	6/1 $(6k4A1)$	y
$(0:1:1:1:1:-1:1:-2:2:0)$	128/1 $(128k4A1)$	y
$(0:1:1:1:1:-1:1:-2:2:2)$	8/1 $(8k4A1)$	y
$(0:1:1:1:1:-1:1:0:0:2)$	128/1 $(128k4A1)$	n
$(0:1:1:1:2:-2:2:-2:2:2)$	6/1 $(6k4A1)$	y
$(0:2:-1:-1:-2:-1:1:-1:1:-2)$	6/1 $(6k4A1)$	y
$(0:2:-1:-1:1:-1:1:-1:1:-2)$	12/1 $(12k4A1)$	n
$(1:1:1:1:-1:-2:-1:1:2:2)$	24/1 $(24k4A1)$	n
$(1:-1:-1:-1:0:0:0:-2:2:-2)$	32/2 $(32k4B1)$	y
$(1:1:1:1:1:-2:2:-1:1:1)$	72/1 $(72k4C1)$	n
$(1:1:1:1:1:-2:-1:2:1:1)$	72/1 $(72k4C1)$	n

Sextic arrangement no. 0 (cube):

Equation for the arrangement of six planes:

$$xyzt(x+y+z)(x+y+t)=0$$

parameter	weight four newform	rigid?
$(0:1:0:0:1:-2:0:0:1:2)$	32/2 $(32k4B1)$	n
$(0:0:0:0:0:0:1:-1:1:0)$	8/1 $(8k4A1)$	y
$(0:1:0:0:1:-1:1:0:1:0)$	8/1 $(8k4A1)$	y
$(0:1:0:0:1:0:0:0:0:-1)$	8/1 $(8k4A1)$	y
$(0:1:0:0:1:0:1:0:1:1)$	8/1 $(8k4A1)$	y
$(0:1:0:0:1:0:1:1:1:0)$	8/1 $(8k4A1)$	y
$(0:1:0:0:1:1:1:1:1:1)$	8/1 $(8k4A1)$	y
$(0:0:0:0:0:0:1:1:0:0)$	8/1 $(8k4A1)$	y
$(0:0:0:0:0:0:1:1:1:1)$	8/1 $(8k4A1)$	y
$(0:1:0:0:1:-1:0:0:0:-1)$	8/1 $(8k4A1)$	y
$(0:1:0:0:1:-1:0:0:1:0)$	8/1 $(8k4A1)$	y
$(0:1:0:0:1:0:0:0:1:1)$	8/1 $(8k4A1)$	y
$(1:1:0:0:2:0:1:1:0:0)$	8/1 $(8k4A1)$	y
$(0:0:0:0:0:0:1:1:0:1)$	8/1 $(8k4A1)$	y
$(0:1:0:0:1:0:0:1:1:1)$	8/1 $(8k4A1)$	y
$(0:0:0:0:0:0:2:1:1:1)$	32/1 $(32k4A1)$	y
$(0:1:0:0:1:-1:0:0:-1:-1)$	32/1 $(32k4A1)$	y
$(0:1:0:0:1:-1:0:0:1:1)$	32/1 $(32k4A1)$	y

parameter	weight four newform		rigid?
$(0:2:0:0:2:0:0:1:1:1)$	32/1	$(32k4A1)$	y
$(1:1:0:0:2:-1:1:0:0:-1)$	32/1	$(32k4A1)$	y
$(1:1:0:0:2:0:0:1:1:1)$	32/1	$(32k4A1)$	y
$(0:0:0:0:1:-2:0:-2:2:-2)$	32/2	$(32k4B1)$	n
$(0:0:0:1:0:0:-1:2:-1:-2)$	32/2	$(32k4B1)$	n
$(0:0:0:1:0:0:-1:2:1:0)$	32/2	$(32k4B1)$	n
$(0:0:0:0:1:-2:0:-2:1:-1)$	8/1	$(8k4A1)$	y
$(0:0:0:1:0:1:0:2:1:0)$	8/1	$(8k4A1)$	n
$(0:1:-1:0:1:0:0:0:-1:1)$	8/1	$(8k4A1)$	y
$(0:1:0:0:0:0:1:1:1:-1)$	8/1	$(8k4A1)$	n
$(0:1:0:0:2:-2:1:-1:1:-1)$	8/1	$(8k4A1)$	y
$(0:1:0:1:1:2:1:1:2:0)$	8/1	$(8k4A1)$	n
$(0:0:0:0:2:-1:-1:-1:-1:-2)$	96/4	$(96k4B1)$	n
$(0:0:0:0:1:-1:-1:0:0:-2)$	8/1	$(8k4A1)$	y
$(0:0:0:1:0:-2:2:0:1:-1)$	8/1	$(8k4A1)$	y
$(0:1:-1:0:1:-1:-1:0:1:-1)$	8/1	$(8k4A1)$	y
$(0:1:-1:0:1:1:-1:0:1:1)$	8/1	$(8k4A1)$	y
$(0:1:0:0:2:0:2:1:1:2)$	8/1	$(8k4A1)$	y
$(0:2:0:0:1:1:1:2:2:2)$	8/1	$(8k4A1)$	y
$(0:0:0:0:1:-1:0:-1:1:0)$	40/2	$(40k4B1)$	y
$(0:0:0:1:0:1:0:1:1:0)$	40/2	$(40k4B1)$	y
$(0:1:-1:0:1:-2:1:-1:1:0)$	40/2	$(40k4B1)$	y
$(0:1:-1:0:1:-1:0:-1:0:-1)$	40/2	$(40k4B1)$	y
$(0:1:-1:0:1:-1:1:-1:1:1)$	40/2	$(40k4B1)$	y
$(0:1:-1:0:1:0:0:0:0:1)$	40/2	$(40k4B1)$	y
$(0:0:0:0:1:-1:0:0:0:-1)$	32/2	$(32k4B1)$	y
$(0:0:0:0:1:0:0:0:1:1)$	32/2	$(32k4B1)$	y
$(0:0:0:1:0:-1:1:0:1:-1)$	32/2	$(32k4B1)$	y
$(0:0:0:1:0:-1:2:0:1:0)$	32/2	$(32k4B1)$	y
$(0:0:0:1:0:0:1:1:0:0)$	32/2	$(32k4B1)$	y
$(0:0:0:1:0:0:1:1:1:1)$	32/2	$(32k4B1)$	y
$(0:0:0:0:1:0:0:1:1:1)$	5/1	$(5k4A1)$	y
$(0:0:0:1:0:0:1:1:0:1)$	5/1	$(5k4A1)$	y
$(0:1:-1:0:1:0:0:1:1:1)$	5/1	$(5k4A1)$	y
$(0:1:0:0:0:0:1:1:0:1)$	5/1	$(5k4A1)$	y
$(0:1:0:1:1:-1:1:0:2:-1)$	5/1	$(5k4A1)$	y
$(1:-1:0:0:-1:0:0:-1:-1:-1)$	5/1	$(5k4A1)$	y
$(0:0:0:0:1:0:0:1:1:2)$	8/1	$(8k4A1)$	n
$(0:0:0:1:0:0:1:2:0:1)$	8/1	$(8k4A1)$	n
$(0:1:0:0:0:0:2:1:1:2)$	8/1	$(8k4A1)$	n
$(0:1:0:1:1:-1:1:1:2:-1)$	8/1	$(8k4A1)$	n
$(0:2:-1:0:2:0:0:1:2:1)$	8/1	$(8k4A1)$	n

parameter	weight four newform		rigid?
$(0:0:0:0:1:0:1:1:0:0)$	12/1	$(12k4A1)$	y
$(0:0:0:1:0:-1:0:-1:1:-1)$	12/1	$(12k4A1)$	y
$(0:1:0:0:0:-1:0:0:1:0)$	12/1	$(12k4A1)$	y
$(0:1:0:1:1:0:0:0:1:-1)$	12/1	$(12k4A1)$	y
$(0:1:0:1:1:0:1:0:2:1)$	12/1	$(12k4A1)$	y
$(1:1:0:0:1:0:1:1:0:0)$	12/1	$(12k4A1)$	y
$(0:0:0:0:1:0:1:2:0:1)$	32/1	$(32k4A1)$	y
$(0:0:0:1:0:-2:1:-1:0:-2)$	32/1	$(32k4A1)$	y
$(0:0:0:1:0:-1:0:0:-1:-2)$	32/1	$(32k4A1)$	y
$(0:0:0:1:0:-1:1:0:-1:-1)$	32/1	$(32k4A1)$	y
$(0:1:0:0:0:-1:0:1:-1:-1)$	32/1	$(32k4A1)$	y
$(0:1:0:0:0:-1:0:1:1:1)$	32/1	$(32k4A1)$	y
$(0:0:0:0:2:0:0:1:2:1)$	8/1	$(8k4A1)$	n
$(0:0:0:2:0:0:2:1:0:1)$	8/1	$(8k4A1)$	n
$(0:0:0:2:0:0:2:1:1:2)$	8/1	$(8k4A1)$	n
$(0:1:-2:0:1:-1:0:1:1:1)$	8/1	$(8k4A1)$	n
$(0:1:-2:0:1:0:0:1:1:2)$	8/1	$(8k4A1)$	n
$(0:1:0:0:-1:0:1:1:-1:1)$	8/1	$(8k4A1)$	n
$(0:0:0:1:-1:-1:1:0:1:-1)$	128/1	$(128k4A1)$	y
$(0:0:0:1:-1:0:1:1:1:1)$	128/1	$(128k4A1)$	y
$(0:0:0:1:-2:-1:1:0:-1:-1)$	128/1	$(128k4A1)$	y
$(0:0:0:2:-1:-1:2:0:0:-1)$	128/1	$(128k4A1)$	y
$(0:0:1:2:0:-1:2:0:1:-2)$	128/1	$(128k4A1)$	y
$(0:0:1:2:0:0:1:1:0:-2)$	128/1	$(128k4A1)$	y
$(0:1:0:1:-1:0:0:-1:2:-1)$	128/1	$(128k4A1)$	y
$(0:1:0:1:-1:0:0:1:2:1)$	128/1	$(128k4A1)$	y
$(0:0:0:1:-2:-1:2:0:-1:0)$	8/1	$(8k4A1)$	n
$(0:1:-2:0:2:-2:1:1:1:1)$	8/1	$(8k4A1)$	y
$(0:1:0:1:-1:0:-1:1:2:0)$	8/1	$(8k4A1)$	n
$(0:0:0:1:-1:-1:-2:0:0:-1)$	24/1	$(24k4A1)$	n
$(0:0:0:1:-1:-1:2:0:0:-1)$	40/3	$(40k4A1)$	y
$(0:0:1:1:0:0:1:1:0:-2)$	40/3	$(40k4A1)$	y
$(0:0:1:1:0:0:1:1:0:2)$	24/1	$(24k4A1)$	n
$(0:1:-1:0:2:-1:1:1:0:1)$	40/3	$(40k4A1)$	y
$(0:1:0:1:0:0:1:1:-1:1)$	24/1	$(24k4A1)$	n
$(0:0:0:1:-1:-1:1:-1:2:0)$	24/1	$(24k4A1)$	n
$(0:1:-1:-1:1:0:0:0:0:1)$	24/1	$(24k4A1)$	n
$(0:1:-1:0:2:-1:1:0:1:0)$	24/1	$(24k4A1)$	n
$(0:1:0:1:0:1:0:1:1:0)$	24/1	$(24k4A1)$	n
$(0:1:1:1:1:1:1:1:1:-1)$	24/1	$(24k4A1)$	n
$(0:1:1:1:1:1:2:1:2:1)$	24/1	$(24k4A1)$	n
$(0:0:0:1:-1:-1:1:0:0:-1)$	6/1	$(6k4A1)$	y

parameter	weight four newform		rigid?
$(0:0:1:1:0:0:1:1:0:-1)$	6/1	$(6k4A1)$	y
$(0:1:0:1:0:0:1:1:2:1)$	6/1	$(6k4A1)$	y
$(0:1:1:1:1:1:1:2:2:1)$	6/1	$(6k4A1)$	y
$(0:0:0:1:0:-2:-1:0:-1:2)$	96/4	$(96k4B1)$	n
$(0:0:0:1:0:2:0:2:2:1)$	32/2	$(32k4B1)$	n
$(0:0:0:1:0:-1:2:1:0:0)$	8/1	$(8k4A1)$	y
$(0:1:-1:0:1:0:0:0:2:2)$	8/1	$(8k4A1)$	y
$(0:1:0:0:0:0:1:2:1:2)$	8/1	$(8k4A1)$	y
$(0:1:0:0:2:0:1:2:1:2)$	8/1	$(8k4A1)$	y
$(0:1:0:1:1:0:2:2:2:2)$	8/1	$(8k4A1)$	y
$(1:-1:-1:0:0:0:1:-2:-1:0)$	8/1	$(8k4A1)$	y
$(0:0:0:1:0:0:-1:1:-1:-1)$	32/1	$(32k4A1)$	y
$(0:0:0:1:0:0:-1:1:0:0)$	32/1	$(32k4A1)$	y
$(0:1:-1:0:1:0:0:0:1:-1)$	32/1	$(32k4A1)$	y
$(0:1:-1:0:1:1:0:0:1:0)$	32/1	$(32k4A1)$	y
$(0:1:0:0:0:0:1:-1:1:1)$	32/1	$(32k4A1)$	y
$(0:1:0:0:2:1:2:1:1:1)$	32/1	$(32k4A1)$	y
$(0:0:0:1:1:-2:1:-2:1:-2)$	8/1	$(8k4A1)$	n
$(0:0:0:1:1:0:-1:0:-1:-2)$	8/1	$(8k4A1)$	n
$(0:0:0:1:1:0:1:0:1:2)$	8/1	$(8k4A1)$	n
$(0:0:0:1:1:0:1:2:1:0)$	8/1	$(8k4A1)$	n
$(0:0:1:-1:0:-1:-1:1:-1:0)$	8/1	$(8k4A1)$	n
$(0:2:-1:0:1:-1:0:-1:2:0)$	8/1	$(8k4A1)$	n
$(0:0:0:2:-2:-1:2:-1:2:0)$	32/1	$(32k4A1)$	n
$(0:1:-2:0:-1:-1:-1:-1:1:0)$	6/1	$(6k4A1)$	y
$(0:1:-2:0:-1:-1:1:-1:1:2)$	6/1	$(6k4A1)$	y
$(0:0:0:1:1:-1:1:-1:1:-1)$	5/1	$(5k4A1)$	y
$(0:0:0:1:1:0:0:0:0:-1)$	5/1	$(5k4A1)$	y
$(0:0:0:1:1:0:1:0:1:1)$	5/1	$(5k4A1)$	y
$(0:0:0:1:1:0:1:1:1:0)$	5/1	$(5k4A1)$	y
$(0:0:1:-1:0:0:-1:1:-1:0)$	5/1	$(5k4A1)$	y
$(0:1:-1:0:0:-1:0:-1:1:0)$	5/1	$(5k4A1)$	y
$(0:0:0:1:2:-1:2:2:1:0)$	6/1	$(6k4A1)$	y
$(0:1:-2:1:1:-2:2:1:2:1)$	6/1	$(6k4A1)$	y
$(0:1:-1:0:-1:-1:-2:0:1:0)$	6/1	$(6k4A1)$	y
$(0:0:0:1:2:0:0:0:1:-1)$	8/1	$(8k4A1)$	y
$(0:0:0:1:2:0:1:0:2:1)$	8/1	$(8k4A1)$	y
$(0:0:0:2:1:0:2:1:1:0)$	8/1	$(8k4A1)$	n
$(0:0:1:-2:0:0:-2:1:-2:-1)$	8/1	$(8k4A1)$	y
$(0:0:1:-2:0:1:-2:2:-2:1)$	8/1	$(8k4A1)$	y
$(0:1:-2:0:0:-2:0:-1:1:0)$	8/1	$(8k4A1)$	y
$(0:0:2:2:1:1:1:2:2:-2)$	24/1	$(24k4A1)$	y

parameter	weight four newform		rigid?
$(0:2:-2:-2:1:-1:-1:0:0:2)$	24/1	$(24k4A1)$	y
$(0:0:1:-2:1:1:-2:1:-1:1)$	14/2	$(14k4A1)$	n
$(0:0:1:-1:-2:0:0:1:-1:0)$	14/2	$(14k4A1)$	n
$(0:0:1:-1:-2:1:-1:2:-2:0)$	14/2	$(14k4A1)$	n
$(0:0:1:2:1:1:1:1:2:-1)$	14/2	$(14k4A1)$	n
$(0:1:-2:-1:0:-2:0:-1:0:1)$	14/2	$(14k4A1)$	n
$(0:1:-2:1:2:-2:2:-1:2:1)$	14/2	$(14k4A1)$	n
$(0:0:1:-2:2:0:0:1:-2:1)$	24/1	$(24k4A1)$	n
$(0:1:-2:-2:0:-2:0:-1:-1:2)$	24/1	$(24k4A1)$	n
$(0:0:1:1:-1:1:1:1:1:0)$	96/4	$(96k4B1)$	n
$(0:0:1:-1:-1:0:0:2:-1:1)$	20/1	$(20k4A1)$	y
$(0:0:1:1:1:1:2:2:1:2)$	20/1	$(20k4A1)$	y
$(0:1:-1:-1:0:-1:0:0:1:2)$	20/1	$(20k4A1)$	y
$(0:1:-1:1:2:-1:1:1:2:1)$	20/1	$(20k4A1)$	y
$(0:0:1:-1:-1:1:-1:1:-1:0)$	32/2	$(32k4B1)$	y
$(0:0:1:1:1:1:1:1:1:0)$	32/2	$(32k4B1)$	y
$(0:0:1:1:-1:0:0:1:1:1)$	9/1	$(9k4A1)$	y
$(0:1:1:1:0:0:1:-1:2:-1)$	9/1	$(9k4A1)$	y
$(0:0:1:1:1:1:1:1:1:-2)$	28/2	$(28k4A1)$	n
$(0:2:1:1:1:-1:-1:1:1:-2)$	168/1	$(168k4A1)$	n
$(0:1:-2:0:0:0:-1:-1:1:-1)$	6/1	$(6k4A1)$	y
$(0:1:-2:0:0:0:-1:1:1:1)$	6/1	$(6k4A1)$	y
$(0:1:-2:1:1:0:0:-1:2:-1)$	6/1	$(6k4A1)$	y
$(0:1:-2:1:1:0:0:1:2:1)$	6/1	$(6k4A1)$	y
$(0:2:-1:0:0:0:1:-1:2:2)$	6/1	$(6k4A1)$	y
$(0:2:-1:0:0:0:1:1:0:2)$	6/1	$(6k4A1)$	y
$(0:1:-2:0:1:-1:1:-1:1:2)$	8/1	$(8k4A1)$	y
$(0:1:0:0:-1:-1:1:1:1:0)$	8/1	$(8k4A1)$	n
$(0:1:-2:0:2:2:1:-1:1:-1)$	6/1	$(6k4A1)$	y
$(0:1:-1:0:0:0:-2:0:1:-1)$	6/1	$(6k4A1)$	y
$(0:1:-1:1:1:1:-1:0:2:0)$	6/1	$(6k4A1)$	y
$(0:1:0:1:2:-2:2:1:2:-1)$	6/1	$(6k4A1)$	y
$(0:2:-1:0:1:1:-1:1:2:1)$	6/1	$(6k4A1)$	y
$(0:1:-1:0:0:0:-1:0:1:0)$	12/1	$(12k4A1)$	y
$(0:1:-1:0:0:0:1:0:1:2)$	12/1	$(12k4A1)$	y
$(0:1:-1:1:1:-2:2:0:2:0)$	12/1	$(12k4A1)$	y
$(0:1:-1:1:1:0:0:0:2:0)$	12/1	$(12k4A1)$	y
$(0:1:0:1:2:-1:2:1:2:0)$	12/1	$(12k4A1)$	y
$(0:1:0:1:2:1:2:1:2:2)$	12/1	$(12k4A1)$	y
$(0:1:-1:1:-2:-2:2:0:2:0)$	24/1	$(24k4A1)$	n
$(0:1:0:0:1:-1:-1:0:0:-2)$	128/1	$(128k4A1)$	y
$(0:1:0:0:1:-1:-1:0:1:-1)$	128/1	$(128k4A1)$	y

parameter	weight four newform		rigid?
$(0:1:0:0:1:-2:2:-1:2:-2)$	32/2	$(32k4B1)$	n
$(0:1:0:0:1:-2:-1:-1:0:-1)$	128/1	$(128k4A1)$	y
$(0:2:0:0:2:-1:-1:0:0:-1)$	128/1	$(128k4A1)$	y
$(1:1:0:0:2:-1:-1:0:0:-1)$	128/1	$(128k4A1)$	y
$(1:1:0:0:2:-1:1:0:0:1)$	128/1	$(128k4A1)$	y
$(0:1:0:0:1:-1:-1:0:0:-1)$	6/1	$(6k4A1)$	y
$(1:1:0:0:2:0:1:1:0:1)$	6/1	$(6k4A1)$	y
$(0:1:0:0:1:-1:-1:1:1:-2)$	288/1	$(288k4C1)$	n
$(0:1:0:0:1:-1:1:0:1:-1)$	32/1	$(32k4A1)$	y
$(0:1:0:0:1:-1:1:0:2:0)$	32/1	$(32k4A1)$	y
$(0:1:0:0:1:1:1:0:1:1)$	32/1	$(32k4A1)$	y
$(1:-1:0:0:0:0:0:-1:0:-1)$	32/1	$(32k4A1)$	y
$(1:1:0:0:2:0:2:1:1:0)$	32/1	$(32k4A1)$	y
$(1:1:0:0:2:0:2:1:2:1)$	32/1	$(32k4A1)$	y
$(0:1:0:0:1:-1:1:1:1:0)$	32/1	$(32k4A1)$	y
$(0:1:0:0:1:1:1:1:1:2)$	32/1	$(32k4A1)$	y
$(0:1:0:0:1:-1:2:0:1:1)$	128/1	$(128k4A1)$	y
$(0:2:0:0:2:1:1:2:2:1)$	128/1	$(128k4A1)$	y
$(1:1:0:0:2:0:0:1:1:-1)$	128/1	$(128k4A1)$	y
$(0:2:0:0:2:-2:1:2:1:0)$	32/2	$(32k4B1)$	n
$(1:-1:0:0:0:-1:2:-1:-2:0)$	32/2	$(32k4B1)$	n
$(0:1:0:1:-1:-2:-1:1:2:2)$	6/1	$(6k4A1)$	y
$(0:1:0:1:-1:1:-1:1:2:-1)$	6/1	$(6k4A1)$	y
$(0:1:0:1:0:0:1:1:-2:1)$	10/1	$(10k4A1)$	y
$(0:1:0:1:1:-1:1:0:-2:-1)$	9/1	$(9k4A1)$	y
$(1:1:0:1:2:0:-2:1:2:1)$	20/1	$(20k4A1)$	y
$(1:1:0:1:2:0:-1:1:2:1)$	9/1	$(9k4A1)$	y
$(0:1:1:1:1:1:1:2:2:-2)$	96/2	$(96k4E1)$	y
$(1:-2:1:1:-1:2:2:-1:-1:-2)$	96/4	$(96k4B1)$	n
$(0:2:-1:-1:-1:-1:-1:1:1:2)$	24/1	$(24k4A1)$	y
$(0:2:-1:-1:2:-1:-1:1:1:2)$	6/1	$(6k4A1)$	y
$(0:2:-1:-1:2:2:2:1:1:2)$	24/1	$(24k4A1)$	n
$(0:2:-1:0:-2:-1:-2:-1:2:0)$	8/1	$(8k4A1)$	n
$(1:1:0:0:-2:0:1:1:0:0)$	6/1	$(6k4A1)$	n
$(1:1:0:0:-2:0:1:1:0:1)$	10/1	$(10k4A1)$	y
$(1:1:0:0:-2:0:1:1:1:0)$	40/3	$(40k4A1)$	n
$(1:1:0:0:-2:0:1:1:1:1)$	24/1	$(24k4A1)$	y
$(1:1:0:0:-2:1:1:1:1:1)$	8/1	$(8k4A1)$	y
$(1:1:0:0:-2:1:1:1:1:2)$	32/1	$(32k4A1)$	y
$(1:1:0:0:2:-1:-1:0:0:1)$	8/1	$(8k4A1)$	y
$(1:1:0:1:-2:1:2:1:2:1)$	40/2	$(40k4B1)$	y

parameter	weight four newform	rigid?
$(1:1:0:1:-1:1:-2:1:2:1)$	9/1 $(9k4A1)$	n
$(1:1:0:1:2:1:-2:1:2:1)$	12/1 $(12k4A1)$	y
$(1:1:1:1:-2:1:2:1:2:1)$	24/1 $(24k4A1)$	n
$(1:1:1:1:-2:1:1:1:1:-1)$	24/1 $(24k4A1)$	n
$(1:1:2:2:-2:2:2:2:2:0)$	32/1 $(32k4A1)$	n
$(1:1:2:2:0:2:2:2:2:0)$	32/2 $(32k4B1)$	n

Appendix C

Weight four newforms

The following table contains coefficients a_p for all primes $p \leq 97$ of weight four newforms with rational coefficients for $\Gamma_0(N)$ with $N \leq 2000$ (and for a few levels $N > 2000$). They have been computed with the help of W. Stein's package HECKE which is included in the MAGMA computer algebra system ([112]). As mentioned in 1.8.3, for some levels the data is missing due to lack of computer memory. These are

$$\begin{array}{llll} 1849 = 43 \cdot 43, & 1853 = 17 \cdot 109, & 1883 = 7 \cdot 269, & 1897 = 7 \cdot 271, \\ 1903 = 11 \cdot 173, & 1909 = 23 \cdot 83, & 1919 = 19 \cdot 101, & 1921 = 17 \cdot 113, \\ 1927 = 41 \cdot 47, & 1937 = 13 \cdot 149, & 1939 = 7 \cdot 277, & 1943 = 29 \cdot 67, \\ 1957 = 19 \cdot 103, & 1961 = 37 \cdot 53, & 1963 = 13 \cdot 151, & 1967 = 7 \cdot 281, \\ 1969 = 11 \cdot 179, & 1981 = 7 \cdot 283, & 1985 = 5 \cdot 397, & 1991 = 11 \cdot 181. \end{array}$$

The first column of the table contains my notation of weight four newforms for $\Gamma_0(N)$ (where N/k simply denotes the k-th newform for $\Gamma_0(N)$). I did not include Stein's notation in the table (since this would have required a lot of handwork) but only whenever a weight four newform occurs somewhere else in this thesis.

As explained in 1.7.3 some newforms are closely related by twisting. Their coefficients differ only in sign, depending on certain Legendre symbols. The second column of the table contains the twist of minimal level for the current newform if there is such a twist. The search for twists was performed with the help of a C++ program. The last 25 columns contain the coefficients of the newforms for the first 25 primes.

It is difficult to estimate the total computing time that was needed to produce the table. One level ~ 2000 affords around 6 hours on a 3 Gigahertz machine so the total time should be measured in months. However, the main problem is computer memory. To enlarge the table we would have to use significantly more memory than 2 Gigabytes (which is at present still rather expensive).

This is a simplified version of the MAGMA script that I used:

```
for i := 1 to 2000 do
    M := ModularForms(Gamma0(i),4);
    SetPrecision(M, 97);
    C := CuspidalSubspace(M);
    for j := 1 to NumberOfNewformClasses(C) do
        f := Newform(C,j);
        if BaseRing(Parent(f)) eq RationalField() then
            printf "%o %o ", Level(M), Coefficient(f,2);
            printf "%o %o ", Coefficient(f,3), Coefficient(f,5);
            printf "%o %o ", Coefficient(f,7), Coefficient(f,11);
            printf "%o %o ", Coefficient(f,13), Coefficient(f,17);
            printf "%o %o ", Coefficient(f,19), Coefficient(f,23);
            printf "%o %o ", Coefficient(f,29), Coefficient(f,31);
            printf "%o %o ", Coefficient(f,37), Coefficient(f,41);
            printf "%o %o ", Coefficient(f,43), Coefficient(f,47);
            printf "%o %o ", Coefficient(f,53), Coefficient(f,59);
            printf "%o %o ", Coefficient(f,61), Coefficient(f,67);
            printf "%o %o ", Coefficient(f,71), Coefficient(f,73);
            printf "%o %o ", Coefficient(f,79), Coefficient(f,83);
            printf "%o %o\n", Coefficient(f,89), Coefficient(f,97);
        end if;
    end for;
end for;
quit;
```

The actual script that I used restricted the considered levels to those given by theorem 1.4. I also had to restart MAGMA for single levels since there seem to be problems with garbage collection.

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
5/1		-4	2	-5	6	32	-38	26	100	-78	-50	-108	266	22	442	-514	2	500	-518	126	412	-878	600	282	-150	386
6/1		-2	-3	6	-16	12	38	-126	20	168	30	-88	254	42	-52	-96	198	-660	-538	884	792	218	-520	-492	810	1154
7/1		-1	-2	16	-7	-8	28	54	-110	48	-110	12	-246	182	128	324	-162	810	-488	244	-768	-702	440	-1302	730	294
8/1		0	-4	-2	24	-44	22	50	44	-56	198	-160	-162	-198	52	528	-242	-668	550	188	728	154	-656	236	714	-478
9/1		0	0	0	20	0	-70	0	56	0	0	308	110	0	-520	0	0	0	182	-880	0	1190	884	0	0	-1330
10/1		2	-8	5	-4	12	-58	66	-100	132	-90	152	-34	-438	32	-204	222	420	902	-1024	432	362	-160	72	810	1106
12/1		0	3	-18	8	36	-10	18	-100	72	-234	-16	-226	90	452	432	414	-684	422	332	-360	26	512	-1188	-630	-1054
13/1		-5	-7	-7	-13	-26	13	77	-126	-96	-82	196	-131	336	-201	-105	-432	-294	-56	478	9	98	1304	-308	-1190	70
14/1		-2	8	-14	-7	-28	18	74	80	-112	190	72	-346	162	-412	24	318	-200	-198	-716	392	538	240	-1072	810	1354
14/2		2	-2	-12	7	48	56	-114	2	-120	-54	236	146	126	-376	-12	174	138	380	-484	576	-1150	776	378	-390	-1330
15/1		3	-3	-5	20	-24	74	54	-124	-120	-78	200	-70	330	92	-24	450	24	-322	-196	-288	-430	-520	156	1026	-286
15/2		1	3	5	-24	52	22	-14	-20	-168	230	-288	-34	122	-188	256	-338	100	742	-84	-328	-38	-240	1212	330	866
16/1	8/1	0	4	-2	-24	44	22	50	-44	56	198	160	-162	-198	-52	-528	-242	668	550	-188	-728	154	656	-236	714	-478
17/1		-3	-8	6	-28	-24	-58	17	116	-60	30	-172	-58	-342	-148	288	318	252	110	-484	-708	362	-484	756	-774	-382
18/1	6/1	2	0	-6	-16	-12	38	126	20	-168	-30	-88	254	-42	-52	96	-198	660	-538	884	-792	218	-520	492	-810	1154
19/1		-3	-5	-12	11	-54	11	-93	19	183	-249	56	-250	240	-196	-168	435	195	-358	-961	-246	353	-34	234	-168	758
20/1		0	4	5	-16	-60	86	18	44	48	-186	176	254	186	-100	168	-498	-252	-58	-1036	168	506	272	948	-1014	-766
21/1		4	-3	-4	-7	62	-62	84	100	-42	-10	-48	-246	-248	68	324	258	120	622	904	-678	-642	740	468	200	-1266
21/2		-3	-3	-18	7	-36	-34	42	-124	0	102	-160	398	-318	-268	240	-498	-132	398	92	-720	-502	-1024	-204	354	-286
22/1		-2	4	14	-8	-11	-50	130	-108	-96	142	40	382	-118	220	520	238	-852	190	-12	-112	-6	304	820	202	-1406
22/2		-2	-7	-19	14	11	-72	-46	-20	-107	120	117	-201	-228	-242	-96	458	435	-668	439	-1113	-72	-70	358	895	409
22/3		2	1	-3	-10	11	-16	42	116	189	-120	-163	-409	468	110	144	90	-453	20	-97	-465	848	-742	438	-273	761
23/1		-2	-5	-6	-8	34	-57	-80	-70	23	245	103	-298	95	88	-357	-414	-408	822	926	335	-899	-1322	-36	-460	-964
24/1		0	3	14	-24	-28	-74	82	92	8	-138	80	30	282	4	240	-130	596	-218	-436	856	-998	-32	-1508	-246	866
25/1		1	7	0	6	-43	-28	91	-35	162	160	42	-314	-203	92	196	82	-280	-518	141	412	-763	510	777	-945	1246
25/2	5/1	4	-2	0	-6	32	38	-26	100	78	-50	-108	-266	22	-442	514	-2	500	-518	-126	412	878	600	-282	-150	-386
25/3	25/1	-1	-7	0	-6	-43	28	-91	-35	-162	160	42	314	-203	-92	-196	-82	-280	-518	-141	412	763	510	-777	-945	-1246
26/1		-2	3	11	19	-38	-13	-51	90	-52	-190	292	-441	312	373	-41	468	530	592	-206	-863	-322	-460	528	870	-346
26/2		2	-1	17	-35	2	13	-19	94	-72	246	-100	-11	-280	241	137	-232	-386	64	-670	55	-838	1016	420	-934	-1154
26/3		2	4	-18	20	-48	13	66	-16	168	6	20	254	-390	-124	-468	558	-96	-826	-160	-420	362	776	0	1626	-1294
27/1		3	0	15	-25	-15	20	72	2	114	30	101	-430	-30	110	-330	621	-660	-376	-250	-360	785	488	489	-450	-1105
27/2	27/1	-3	0	-15	-25	15	20	-72	2	-114	-30	101	-430	30	110	330	-621	660	-376	-250	360	785	488	-489	450	-1105
28/1		0	-10	-8	-7	-40	-12	-58	26	-64	-62	252	26	6	416	-396	-450	274	-576	-476	-448	-158	-936	530	-390	214
28/2		0	4	6	7	-12	-82	-30	68	216	246	-112	110	-246	-172	192	558	540	110	140	-840	-550	-208	516	-1398	1586
30/1		-2	3	5	32	-60	-34	42	-76	0	6	-232	134	234	-412	-360	222	660	-490	812	120	746	152	-804	-678	194
30/2		2	3	-5	-4	-48	2	-114	140	72	210	272	-334	-198	-268	216	-78	240	302	596	-768	-478	-640	-348	210	-1534
32/1		0	0	22	0	0	-18	-94	0	0	-130	0	214	-230	0	0	518	0	830	0	0	1098	0	0	-1670	594
32/2		0	8	-10	16	-40	-50	-30	40	48	-34	320	310	410	152	-416	-410	-200	30	776	400	-630	-1120	552	-326	-110
32/3	32/2	0	-8	-10	-16	40	-50	-30	-40	-48	-34	-320	310	410	-152	416	-410	200	30	-776	-400	-630	1120	-552	-326	-110
33/1		-1	-3	-4	-26	11	-32	74	-60	-182	-90	-8	-66	422	408	-506	348	-200	132	-1036	762	-542	-550	-132	570	14
33/2		-5	3	-14	-32	-11	-38	-2	72	68	-54	-152	174	94	-528	-340	-438	20	570	-460	-1092	562	-16	372	-966	-526
34/1		-2	-2	16	24	62	-62	-17	-20	-12	80	-208	-356	22	-312	24	-462	240	812	-216	732	178	700	-992	-390	-146
34/2		-2	-2	-18	-10	-6	74	17	-88	-114	-90	-310	86	90	368	-384	-258	240	302	-964	-390	722	-898	912	1446	-1438
35/1		1	-8	-5	7	12	-78	-94	40	32	-50	-248	-434	402	-68	536	22	-560	-278	-164	672	82	-1000	-448	-870	1026
36/1	12/1	0	0	18	8	-36	-10	-18	-100	-72	234	-16	-226	-90	452	-432	-414	684	422	332	360	26	512	1188	630	-1054
38/1		-2	-2	-9	-31	57	-52	69	19	-72	-150	32	-226	-258	-67	579	-432	-330	-13	-856	642	-487	-700	-12	-600	1424
39/1		0	-3	-12	2	-36	13	-78	74	-96	18	-214	-286	-384	524	300	558	576	74	38	-456	-682	704	-888	-1020	110
40/1		0	10	-5	-18	-16	-6	-6	-124	42	142	-188	202	54	66	38	738	564	-262	-554	140	882	-1160	642	-854	-478
40/2		0	-6	-5	-34	16	58	-70	4	-134	-242	100	-438	-138	178	22	162	-268	250	422	-852	306	-456	434	-726	1378
40/3		0	4	5	16	36	-42	-110	-116	16	198	240	-258	442	-292	392	142	-348	-570	692	168	-134	784	564	1034	-382
42/1		2	-3	18	7	-72	-34	6	92	-180	-114	56	-34	6	164	168	654	-492	-250	-124	36	1010	56	228	390	-70
42/2		2	3	2	-7	-8	-42	-2	-124	76	254	-72	398	462	212	-264	-162	-772	30	-764	-236	418	552	1036	30	-1190
44/1		0	-5	-7	-26	-11	52	46	-96	27	16	-293	-29	-472	-110	-224	754	825	-548	-123	1001	-1020	526	-158	-1217	-263
45/1		5	0	-5	-30	50	-20	-10	-44	120	-50	108	-40	400	280	-280	-610	50	-518	-180	700	-410	-516	660	-1500	-1630
45/2	45/1	-5	0	5	-30	-50	-20	10	-44	-120	50	108	-40	-400	280	280	610	-50	-518	-180	-700	-410	-516	-660	1500	-1630
45/3	15/2	-1	0	-5	-24	-52	22	14	-20	168	-230	-288	-34	-122	-188	-256	338	-100	742	-84	328	-38	-240	-1212	-330	866
45/4	15/1	-3	0	5	20	24	74	-54	-124	120	78	200	-70	-330	92	24	-450	-24	-322	-196	288	-430	-520	-156	-1026	-286
45/5	5/1	4	0	5	6	-32	-38	-26	100	78	50	-108	266	-22	442	514	-2	-500	-518	126	-412	-878	600	-282	150	386
46/1		-2	-1	-10	-12	-42	7	20	106	23	-227	67	74	-497	-88	215	314	176	-298	266	-981	-411	806	-952	-1332	-1328
46/2		2	-9	-20	2	-52	43	-50	-74	-23	-7	-273	-4	123	-152	75	86	-444	262	764	-21	681	426	902	-1272	-342
48/1	24/1	0	-3	14	24	28	-74	82	-92	-8	-138	-80	30	282	-4	-240	-130	-596	-218	436	-856	-998	32	1508	-246	866
48/2	12/1	0	-3	-18	-8	-36	-10	18	100	-72	-234	16	-226	90	-452	-432	414	684	422	-332	360	26	-512	1188	-630	-1054

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
48/3	6/1	0	3	6	16	-12	38	-126	-20	-168	30	88	254	42	52	96	198	660	-538	-884	-792	218	520	492	810	1154
49/1		2	7	7	0	-5	-14	-21	49	-159	58	147	219	350	-124	525	303	-105	-413	415	-432	-1113	-103	1092	-329	-882
49/2	7/1	-1	2	-16	0	-8	-28	-54	110	48	-110	-12	-246	-182	128	-324	-162	-810	488	244	-768	702	440	1302	-730	-294
49/3	49/1	2	-7	-7	0	-5	14	21	-49	-159	58	-147	219	-350	-124	-525	303	105	413	415	-432	1113	-103	-1092	329	882
49/4		-5	0	0	0	-68	0	0	0	-40	-166	0	450	0	-180	0	590	0	0	-740	688	0	-1384	0	0	0
50/1		-2	-7	0	34	27	28	-21	35	78	-120	182	-146	357	148	84	-702	-840	-238	-461	-708	133	650	903	735	-1106
50/2	10/1	-2	8	0	4	12	58	-66	-100	-132	-90	152	34	-438	-32	204	-222	420	902	1024	432	-362	-160	-72	810	-1106
50/3		-2	-2	0	-26	-28	-12	64	-60	58	90	-128	-236	242	-362	-226	108	-20	542	434	-1128	-632	-720	478	-490	-1456
50/4	50/3	2	2	0	26	-28	12	-64	-60	-58	90	-128	236	242	362	226	-108	-20	542	-434	-1128	632	-720	-478	-490	1456
50/5	50/1	2	7	0	-34	27	-28	21	35	-78	-120	182	146	357	-148	-84	702	-840	-238	461	-708	-133	650	-903	735	1106
51/1		-1	-3	16	34	-48	58	-17	20	58	0	-218	184	-138	148	-516	-162	-180	152	-956	-538	-462	390	1268	-770	494
51/2		1	-3	-10	-8	12	-26	17	-148	152	-66	-32	-266	-6	-92	-288	-546	420	350	940	424	378	288	748	-1558	530
51/3		-1	3	-20	-2	-48	-14	-17	92	-122	-36	-182	76	294	-428	-12	-234	-540	-820	700	794	-1038	858	1052	1102	710
52/1		0	-3	-13	-11	-2	-13	-51	150	-4	-118	-116	63	-288	-293	-335	-708	566	904	382	7	518	-100	-1440	1254	1262
53/1		0	1	-18	2	54	-43	-99	-61	207	-99	-160	-7	-414	-268	270	53	450	182	-556	693	-862	119	-333	1350	-187
54/1		-2	0	-3	29	57	20	72	-106	-174	210	47	2	6	218	-474	-81	-84	56	-142	-360	-1159	-160	-735	954	191
54/2		-2	0	-12	-7	-60	-79	108	11	132	-96	20	-169	-192	488	-204	-360	-156	83	47	-216	-511	-529	1128	-36	605
54/3	54/1	2	0	3	29	-57	20	-72	-106	174	-210	47	2	-6	218	474	81	84	56	-142	360	-1159	-160	735	-954	191
54/4	54/2	2	0	12	-7	60	-79	-108	11	-132	96	20	-169	192	488	204	360	156	83	47	216	-511	-529	-1128	36	605
55/1		1	-3	-5	-9	11	2	21	-85	22	-165	-83	1	-478	-8	126	-683	-290	257	776	-313	902	830	842	25	-1784
56/1		0	6	8	-7	56	-28	-90	74	-96	-222	-100	58	422	512	148	-642	-318	720	-412	448	994	-296	386	-6	-138
56/2		0	-2	-16	-7	24	-68	54	-46	176	-174	-116	74	-10	-480	-572	-162	-86	-904	660	1024	770	-904	682	-102	-218
57/1		-1	3	-12	-20	-4	-76	22	-19	82	242	-126	-180	-390	308	-522	-70	188	-706	104	-432	718	94	-1296	846	830
58/1		-2	7	5	-2	37	27	24	-88	-28	-29	-143	-360	386	381	-103	-431	288	-840	-180	706	716	931	1188	-642	486
58/2		2	-7	-15	-18	27	-57	-44	152	-152	-29	-173	-120	-314	339	-357	-59	-572	-420	660	726	1004	361	-168	58	-1206
60/1		0	-3	-5	-28	-24	-70	102	20	-72	306	-136	-214	-150	-292	-72	-414	-744	-418	188	480	434	1352	-612	-30	-286
60/2		0	-3	5	32	36	-10	-78	140	-192	6	-16	-34	-390	-52	408	-114	516	-58	-892	-120	-646	-1168	-732	-1590	194
62/1		-2	-2	1	-11	-18	-82	-6	25	58	180	31	-146	47	-12	-136	-232	715	-518	-436	387	678	660	-382	-800	-1631
62/2		2	-8	-3	-35	-46	20	8	97	28	-206	-31	-282	367	-562	-148	-84	-301	-236	60	699	-814	670	-650	1566	-615
63/1	7/1	1	0	-16	-7	8	28	-54	-110	-48	110	12	-246	-182	128	-324	162	-810	-488	244	768	-702	440	1302	-730	294
63/2	21/1	-4	0	4	-7	-62	-62	-84	100	42	10	-48	-246	248	68	-324	-258	-120	622	904	678	-642	740	-468	-200	-1266
63/3	21/2	3	0	18	7	36	-34	-42	-124	0	-102	-160	398	318	-268	-240	498	132	398	92	720	-502	-1024	204	-354	-286
64/1	8/1	0	4	2	24	44	-22	50	-44	-56	-198	-160	162	-198	-52	528	242	668	-550	-188	728	154	-656	-236	714	-478
64/2	32/2	0	8	10	-16	-40	50	-30	40	-48	34	-320	-310	410	152	416	410	-200	-30	776	-400	-630	1120	552	-326	-110
64/3	32/2	0	-8	10	16	40	50	-30	-40	48	34	320	-310	410	-152	-416	410	200	-30	-776	400	-630	-1120	-552	-326	-110
64/4	32/1	0	0	-22	0	0	18	-94	0	0	130	0	-214	-230	0	0	-518	0	-830	0	0	1098	0	0	-1670	594
64/5	8/1	0	-4	2	-24	-44	-22	50	44	56	-198	160	162	-198	52	-528	242	-668	-550	188	-728	154	656	236	714	-478
65/1		5	2	-5	-12	14	-13	98	-26	-114	58	306	86	-374	-314	620	362	266	634	612	-686	202	-516	48	-1230	350
66/1		-2	3	0	14	11	80	30	56	-126	-222	-16	-106	114	-52	246	-264	264	92	-796	426	-1174	842	852	-1062	-1282
66/2		2	-3	10	16	11	10	-10	-144	-84	218	-176	46	-26	-488	404	194	444	202	-84	-764	354	1312	-1252	-1222	-1358
68/1		0	-2	-8	-12	-10	-38	-17	4	120	56	164	-236	70	-144	48	-366	-504	-460	-768	72	-734	736	856	906	46
70/1		-2	-8	-5	-7	68	34	74	-128	-80	286	-24	294	66	-124	312	-34	168	170	564	616	250	-944	672	-1430	-1270
70/2		-2	-1	-5	7	-65	13	-73	-142	130	111	256	-266	-424	534	-269	-132	-224	-572	-108	560	586	57	252	-184	-605
70/3		-2	-3	5	-7	-17	-81	-91	102	-90	-129	116	314	-124	-434	497	-584	-332	220	384	-664	230	361	1172	40	-175
70/4		-2	4	5	7	60	38	42	-52	120	-234	-304	-106	-54	-196	336	438	-444	38	-988	-720	146	-808	612	1146	-70
70/5		2	7	-5	7	-33	-43	111	-70	42	-225	-88	-34	432	-178	411	-708	480	812	596	432	-358	425	972	960	-709
70/6		2	5	5	-7	-1	7	-51	30	-50	79	-212	-190	-308	422	121	664	628	-684	1056	744	726	-407	644	-880	-1351
71/1		1	1	-16	-1	24	7	72	-153	-213	232	149	-204	-432	71	273	-274	126	-134	-760	71	-457	112	-124	837	-1424
72/1		0	0	16	-12	64	58	32	-136	-128	-144	20	-18	-288	-200	384	496	-128	-458	-496	512	-602	1108	704	-960	206
72/2	24/1	0	0	-14	-24	28	-74	-82	92	-8	138	80	30	-282	4	-240	130	-596	-218	-436	-856	-998	-32	1508	246	866
72/3	72/1	0	0	-16	-12	-64	58	-32	-136	128	144	20	-18	288	-200	-384	-496	128	-458	-496	-512	-602	1108	-704	960	206
72/4	8/1	0	0	2	24	44	22	-50	44	56	-198	-160	-162	198	52	-528	242	668	550	188	-728	154	-656	-236	-714	-478
73/1		3	-8	6	-34	6	-34	90	-16	60	102	-214	-286	150	-322	-534	-474	786	-574	-16	192	73	-988	1242	-6	614
74/1		-2	-5	12	-7	-63	-28	6	-70	-6	-42	-292	37	351	32	357	57	432	-340	-1012	-609	539	818	1299	-390	1772
74/2		2	-5	-14	-19	5	6	-72	-44	182	10	-244	-37	-225	-2	221	-659	156	-620	416	-1125	-641	-484	1239	1304	-560
75/1	15/2	-1	-3	0	24	52	-22	14	-20	168	230	-288	34	122	188	-256	338	100	742	84	-328	38	-240	-1212	330	-866
75/2	15/1	-3	3	0	-20	-24	-74	-54	-124	120	-78	200	70	330	-92	24	-450	24	-322	196	-288	430	-520	-156	1026	286
77/1		3	4	12	7	11	38	-48	-70	12	126	-70	-358	-216	344	390	438	-552	830	-196	648	-16	1352	90	1146	-70
78/1		-2	-3	-16	28	34	-13	138	108	-52	-190	-176	342	240	-140	454	198	-154	34	-656	550	614	8	762	-444	1022
78/2		-2	3	-16	-8	-38	-13	-78	-72	-52	242	76	342	-336	76	94	-450	854	-110	-908	838	-970	-352	474	-1452	-562
78/3		-2	3	10	-8	40	13	130	-20	0	-18	-184	-74	-362	76	-452	382	464	358	-700	-748	1058	-976	-1008	-386	-614
78/4		2	-3	-20	-32	50	-13	-30	-120	-20	82	-44	-306	108	-356	-178	198	94	-62	-140	-778	62	-1096	-462	1224	614

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
78/5		2	-3	6	20	24	13	-30	-16	-72	-282	164	110	-126	164	-204	-738	120	614	848	132	218	-1096	552	210	-1726
78/6		2	3	4	4	2	-13	-6	-36	-20	-14	-152	-258	84	-188	254	366	550	-14	448	926	254	1328	186	-336	614
80/1	40/2	0	6	-5	34	-16	58	-70	-4	134	-242	-100	-438	-138	-178	-22	162	268	250	-422	852	306	456	-434	-726	1378
80/2	40/1	0	-10	-5	18	16	-6	-6	124	-42	142	188	202	54	-66	-38	738	-564	-262	554	-140	882	1160	-642	-854	-478
80/3	40/3	0	-4	5	-16	-36	-42	-110	116	-16	198	-240	-258	442	292	-392	142	348	-570	-692	-168	-134	-784	-564	1034	-382
80/4	5/1	0	-2	-5	-6	-32	-38	26	-100	78	-50	108	266	22	-442	514	2	-500	-518	-126	-412	-878	-600	-282	-150	386
80/5	20/1	0	-4	5	16	60	86	18	-44	-48	-186	-176	254	186	100	-168	-498	252	-58	1036	-168	506	-272	-948	-1014	-766
80/6	10/1	0	8	5	4	-12	-58	66	100	-132	-90	-152	-34	-438	-32	204	222	-420	902	1024	-432	362	160	-72	810	1106
82/1		2	-4	-18	-2	-52	28	14	-16	-36	-160	132	-294	-41	356	42	-548	252	-494	-616	738	-1010	-834	-1436	474	1598
82/2		2	10	-6	-10	-54	-82	42	134	48	30	-136	2	41	200	-30	390	-444	38	-610	-42	110	950	900	138	170
84/1		0	-3	6	7	36	62	114	-76	-24	54	-112	-178	378	-172	-192	-402	396	254	-1012	840	890	80	-108	-1638	1010
84/2		0	3	14	-7	4	54	-14	92	-152	-106	-144	158	-390	-508	-528	606	-364	678	844	-8	-422	384	-548	1194	-1502
85/1		3	-5	-5	-22	60	-31	17	-61	-78	69	-31	56	-6	-538	-465	723	-753	35	-322	-99	-1123	488	-852	1215	-601
85/2		3	-7	5	-22	-64	73	-17	-49	110	155	-197	-372	-262	258	-13	-653	-333	-355	814	47	-437	-384	-736	511	537
85/3		3	10	5	-22	-30	-46	17	104	42	-66	194	206	-126	-388	-540	78	432	-610	848	-174	362	398	828	630	-1486
86/1		-2	8	6	14	-43	-17	49	130	53	-180	-163	284	-323	-43	-56	-437	-420	552	-541	-18	1108	80	33	-1090	179
86/2		2	-4	-14	-14	-11	-9	9	-46	-19	216	-155	-76	5	-43	-392	579	-588	28	-621	-146	-192	-664	-1239	-1622	827
88/1		0	7	9	2	-11	0	-38	44	175	-264	159	-173	-220	-542	-264	682	421	308	177	365	-528	686	698	967	-1127
88/2		0	-1	-7	-6	-11	-40	-78	36	7	8	183	227	-36	322	-184	-6	-99	164	-695	-987	-248	-242	-1494	-905	-1031
89/1		-1	2	2	-4	-56	-16	-30	-50	-92	204	324	-20	270	86	0	534	-206	-672	-576	-352	-338	-336	630	89	-1506
89/2		-4	-7	11	8	-32	-4	39	-59	-83	66	-279	-350	-78	71	-258	597	-572	-420	-30	230	-497	-714	-420	89	1833
90/1		-2	0	-5	14	6	68	78	44	120	126	-244	-304	-480	104	600	-258	534	362	-268	-972	470	1244	396	-972	-46
90/2	10/1	-2	0	-5	-4	-12	-58	-66	-100	-132	90	152	-34	438	32	204	-222	-420	902	-1024	-432	362	-160	-72	-810	1106
90/3	30/2	-2	0	5	-4	48	2	114	140	-72	-210	272	-334	198	-268	-216	78	-240	302	596	768	-478	-640	348	-210	-1534
90/4	90/1	2	0	5	14	-6	68	-78	44	-120	-126	-244	-304	480	104	-600	258	-534	362	-268	972	470	1244	-396	972	-46
90/5	30/1	2	0	-5	32	60	-34	-42	-76	0	-6	-232	134	-234	-412	360	-222	-660	-490	812	-120	746	152	804	678	194
93/1		3	-3	-9	-34	33	65	-21	-97	-84	48	31	146	-378	182	-501	-402	102	209	-835	-105	542	1109	-597	-1638	-1483
95/1		0	4	-5	-22	-12	8	-66	19	-30	-6	-64	-16	54	182	594	396	-564	-706	-628	-984	14	-328	-294	918	-1564
95/2		3	-5	-5	-1	-24	-31	33	19	27	111	-94	-70	-510	-34	-192	-75	45	-28	371	384	-73	-1234	366	-1578	-538
95/3		3	7	5	11	-36	65	-87	19	-129	231	110	-142	-330	74	-336	501	633	-88	119	-204	407	1262	270	-30	1406
95/4		5	4	5	-32	-12	-42	114	19	160	214	-144	94	-6	-308	184	-274	276	-826	52	-344	-166	-688	996	1578	786
96/1		0	-3	10	-4	20	70	90	140	-192	-134	100	-170	-110	532	-56	-430	-20	270	-524	-80	330	1060	-1188	1274	-590
96/2		0	-3	-14	36	36	54	-22	-36	144	50	108	214	-446	-252	-72	-22	684	-466	180	-576	-54	972	684	346	-1134
96/3	96/2	0	3	-14	-36	-36	54	-22	36	-144	50	-108	214	-446	252	72	-22	-684	-466	-180	576	-54	-972	-684	346	-1134
96/4		0	-3	2	-12	-60	-42	10	-132	48	226	252	-362	-94	228	408	346	300	-466	-204	-1056	330	-612	-564	-1510	594
96/5	96/4	0	3	2	12	60	-42	10	132	-48	226	-252	-362	-94	-228	-408	346	-300	-466	204	1056	330	612	564	-1510	594
96/6	96/1	0	3	10	4	-20	70	90	-140	192	-134	-100	-170	-110	-532	56	-430	20	270	524	80	330	-1060	1188	1274	-590
98/1		-2	-1	7	0	35	66	59	137	-7	106	75	11	-498	260	-171	-417	-17	51	439	-784	295	-495	932	-873	-290
98/2	98/1	-2	1	-7	0	35	-66	-59	-137	-7	106	-75	11	498	260	171	-417	17	-51	439	-784	-295	-495	-932	873	290
98/3	14/1	-2	-8	14	0	-28	-18	-74	-80	-112	190	-72	-346	-162	-412	-24	318	200	198	-716	392	-538	240	1072	-810	-1354
98/4		2	-5	-9	0	-57	-70	51	5	69	114	23	-253	-42	-124	201	-393	219	-709	419	-96	-313	461	-588	-1017	-1834
98/5	14/2	2	2	12	0	48	-56	114	-2	-120	-54	-236	146	-126	-376	12	174	-138	-380	-484	576	1150	776	-378	390	1330
98/6	98/4	2	5	9	0	-57	70	-51	-5	69	114	-23	-253	42	-124	-201	-393	-219	709	419	-96	313	461	588	1017	1834
99/1	33/1	1	0	4	-26	-11	-32	-74	-60	182	90	-8	-66	-422	408	506	-348	200	132	-1036	-762	-542	-550	132	-570	14
99/2	33/2	5	0	14	-32	11	-38	2	72	-68	54	-152	174	-94	-528	340	438	-20	570	-460	1092	562	-16	-372	966	-526
100/1		0	-1	0	-26	45	-44	-117	-91	18	144	26	214	-459	460	468	-558	-72	-118	-251	108	-299	-898	-927	351	-386
100/2	20/1	0	-4	0	16	-60	-86	-18	44	-48	-186	176	-254	186	100	-168	498	-252	-58	1036	168	-506	272	-948	-1014	766
100/3	100/1	0	1	0	26	45	44	117	-91	-18	144	26	-214	-459	-460	-468	558	-72	-118	251	108	299	-898	927	351	386
102/1		-2	-3	-3	20	-51	-61	17	-43	-219	-150	290	56	15	83	426	-378	-210	-448	-124	900	-1078	722	-78	-144	-268
102/2		-2	3	-5	-32	27	-69	-17	-83	-117	94	198	-244	169	227	-382	686	450	-700	540	-276	-298	-182	282	-1468	-1140
102/3		2	-3	-12	-22	-48	2	-17	20	-54	84	62	44	-138	428	-516	174	-852	908	-508	-426	-574	110	-1308	798	-1690
102/4		2	-3	5	12	37	19	17	37	-3	-86	-142	-296	-121	3	402	174	270	-520	-780	84	-302	178	698	1512	-500
104/1		0	5	19	-3	-2	-13	77	-58	76	-6	-292	207	240	-317	-375	-692	214	-488	782	-1057	1174	892	704	6	830
104/2		0	1	-7	-21	6	13	-115	-46	144	-162	180	13	192	-33	383	288	442	-680	-722	-207	274	-936	-1204	-966	-138
105/1		0	-3	5	7	42	20	66	38	12	-258	146	434	-282	20	-72	336	-360	-682	812	810	-124	1136	156	-1038	1208
105/2		5	-3	5	7	12	30	-134	-92	112	-58	-224	-146	18	340	208	-754	380	718	412	-960	1066	896	436	-1038	-702
108/1		0	0	0	-37	0	-19	0	-163	0	0	308	323	0	-520	0	0	0	719	-127	0	-919	-1387	0	0	-523
108/2		0	0	-9	-1	-63	-28	-72	98	-126	126	-259	386	450	-34	54	693	-180	-280	-586	-504	161	440	-999	-882	-721
108/3		0	0	0	17	0	89	0	107	0	0	308	-433	0	-520	0	0	0	-901	1007	0	-271	503	0	0	1853
108/4	108/2	0	0	9	-1	63	-28	72	98	126	-126	-259	386	-450	-34	-54	-693	180	-280	-586	504	161	440	999	882	-721
110/1		-2	4	-5	-30	11	16	-112	-64	36	10	-48	-146	278	-330	476	150	732	-30	-848	240	-1128	788	-698	-458	134
110/2		-2	4	5	20	11	26	-42	116	96	270	32	-106	-462	-40	-504	-570	12	590	-388	-240	302	8	-48	282	-646

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
110/3		-2	-7	5	-35	11	26	101	127	-58	-27	-177	191	66	444	2	-669	386	-521	96	-427	1006	910	-818	601	-228
110/4		2	-4	-5	-22	-11	-20	-20	-8	-204	122	40	278	302	-330	60	-418	188	-670	-568	128	676	-876	-1130	822	-434
110/5		2	7	-5	11	11	2	-9	-85	-138	45	227	-19	-138	-88	-534	297	-450	287	-304	777	962	290	1422	-1455	116
110/6		2	-8	-5	26	11	92	-84	80	72	-30	-208	86	-378	542	216	-18	420	-718	-124	912	-268	-940	-498	150	446
110/7		2	1	5	23	-11	50	75	17	-174	-153	35	-277	-258	-220	210	-273	438	-475	992	-927	-934	974	-90	1377	-64
110/8		2	8	5	-12	-11	-34	-86	-4	148	134	-280	430	-6	-136	-28	-658	4	-90	96	816	-430	1296	-608	810	706
111/1		-1	-3	-4	-1	-13	73	99	-105	133	300	62	-37	-198	68	354	-7	220	322	-706	672	893	910	243	995	1234
111/2		1	3	-8	-13	-35	-35	-3	15	47	-12	-94	-37	54	-244	282	619	8	250	-478	-96	-955	-410	-579	37	-2
111/3		-4	3	2	-28	20	10	-78	-150	82	-222	-154	-37	-306	386	12	-46	658	250	-748	324	-130	-230	216	-118	898
112/1	56/2	0	2	-16	7	-24	-68	54	46	-176	-174	116	74	-10	480	572	-162	86	-904	-660	-1024	770	904	-682	-102	-218
112/2	56/1	0	-6	8	7	-56	-28	-90	-74	96	-222	100	58	422	-512	-148	-642	318	720	412	-448	994	296	-386	-6	-138
112/3	14/2	0	2	-12	-7	-48	56	-114	-2	120	-54	-236	146	126	376	12	174	-138	380	484	-576	-1150	-776	-378	-390	-1330
112/4	28/2	0	-4	6	-7	12	-82	-30	-68	-216	246	112	110	-246	172	-192	558	-540	110	-140	840	-550	208	-516	-1398	1586
112/5	7/1	0	2	16	7	8	28	54	110	-48	-110	-12	-246	182	-128	-324	-162	-810	-488	-244	768	-702	-440	1302	730	294
112/6	14/1	0	-8	-14	7	28	18	74	-80	112	190	-72	-346	162	412	-24	318	200	-198	716	-392	538	-240	1072	810	1354
112/7	28/1	0	10	-8	7	40	-12	-58	-26	64	-62	-252	26	6	-416	396	-450	-274	-576	476	448	-158	936	-530	-390	214
114/1		-2	-3	-19	9	-13	38	99	-19	68	130	262	-296	-8	73	-271	-502	540	587	684	992	-507	980	-492	810	-1046
114/2		-2	3	-7	-15	-49	14	-33	-19	-148	-278	94	160	400	73	173	170	-12	419	444	-952	-27	-556	-276	1386	130
114/3		-2	3	12	4	8	-24	62	19	194	102	18	-296	134	-60	-226	-362	-316	134	-240	-800	-578	1078	940	170	206
114/4		2	-3	-11	-15	-29	-82	27	-19	100	-118	70	232	8	-287	385	538	-300	-901	132	472	-1131	-52	276	-1302	-1310
115/1		1	4	-5	-32	40	-66	130	-88	23	-130	40	-334	-22	-272	24	258	612	-366	-496	248	826	-296	-1296	-646	-1438
115/2		2	-3	5	-2	-16	-47	-24	-56	-23	85	67	104	-53	-234	285	2	80	-764	236	-289	-225	24	684	-1370	-110
117/1	39/1	0	0	12	2	36	13	78	74	96	-18	-214	-286	384	524	-300	-558	-576	74	38	456	-682	704	888	1020	110
117/2	13/1	5	0	7	-13	26	13	-77	-126	96	82	196	-131	-336	-201	105	432	294	-56	478	-9	98	1304	308	1190	70
118/1		-2	5	-5	-33	-4	-30	-14	97	-134	1	-28	290	-5	192	-326	-537	59	472	856	168	-686	-919	362	-312	-514
118/2		2	-1	-13	-27	-8	42	2	-77	98	-295	-40	278	179	-132	-202	-345	-59	184	-356	-144	814	-181	-1250	-600	-790
118/3		2	-7	5	-15	-50	-66	14	-11	-172	287	26	128	167	-78	38	-147	-59	76	514	-1140	-848	359	-362	1212	836
119/1		-1	-6	-20	-7	60	-68	-17	-70	-176	-90	196	22	-138	328	-12	-234	-54	44	-596	200	1122	480	-838	778	1142
120/1		0	-3	-5	4	72	-6	38	52	152	-78	120	-150	362	-484	280	-670	696	222	-4	96	178	-632	-612	994	1634
120/2		0	-3	5	-16	-28	-26	-62	-68	-208	-58	160	270	282	76	-280	-210	196	742	836	-504	-1062	768	-1052	-726	-1406
120/3		0	3	5	8	20	22	-14	76	56	-154	160	-162	-390	388	-544	-210	-380	-794	-148	-840	858	144	316	1098	994
120/4		0	-3	-5	20	-56	-86	-106	4	136	-206	-152	282	-246	412	40	-126	56	-2	-388	-672	1170	408	668	66	-926
120/5		0	-3	5	0	4	54	114	44	96	134	-272	-98	-6	12	-200	654	36	-442	-188	-632	-390	688	1188	-694	-1726
120/6		0	3	-5	20	16	58	38	4	-80	82	-8	426	-246	-524	-464	-702	-592	574	-172	768	-558	408	164	-510	514
121/1		0	8	18	0	0	0	0	0	-108	0	340	-434	0	0	-36	-738	-720	0	-416	612	0	0	0	1674	-34
126/1		-2	0	22	-7	26	-54	74	116	-58	208	-252	50	126	164	-444	12	124	-162	-860	-238	-146	-984	656	-954	526
126/2		-2	0	-6	7	-30	2	-66	-52	-114	-72	-196	-286	378	164	228	348	348	-106	596	-630	-1042	-88	1440	-1374	-34
126/3	42/2	-2	0	-2	-7	8	-42	2	-124	-76	-254	-72	398	-462	212	264	162	772	30	-764	236	418	552	-1036	-30	-1190
126/4	14/2	-2	0	12	7	-48	56	114	2	120	54	236	146	-126	-376	12	-174	-138	380	-484	-576	-1150	776	-378	390	-1330
126/5	42/1	-2	0	-18	7	72	-34	-6	92	180	114	56	-34	-6	164	-168	-654	492	-250	-124	-36	1010	56	-228	-390	-70
126/6	126/1	2	0	-22	-7	-26	-54	-74	116	58	-208	-252	50	-126	164	444	-12	-124	-162	-860	238	-146	-984	-656	954	526
126/7	126/2	2	0	6	7	30	2	66	-52	114	72	-196	-286	-378	164	-228	-348	-348	-106	596	630	-1042	-88	-1440	1374	-34
126/8	14/1	2	0	14	-7	28	18	-74	80	112	-190	72	-346	-162	-412	-24	-318	200	-198	-716	-392	538	240	1072	-810	1354
127/1		-1	-8	-15	-25	-51	2	31	-123	-149	6	10	-348	-387	-80	266	347	-656	-158	-314	312	-646	-846	1352	1242	632
128/1		0	2	6	20	14	54	-66	162	172	-2	-128	158	202	-298	-408	-690	-322	-298	202	-700	-418	744	-678	-82	-1122
128/2	128/1	0	2	-6	-20	14	-54	-66	162	-172	2	128	-158	202	-298	408	690	-322	298	202	700	-418	-744	-678	-82	-1122
128/3	128/1	0	-2	6	-20	-14	54	-66	-162	-172	-2	128	158	202	298	408	-690	322	-298	-202	700	-418	-744	678	-82	-1122
128/4	128/1	0	-2	-6	20	-14	-54	-66	-162	172	2	-128	-158	202	298	-408	690	322	298	-202	-700	-418	744	678	-82	-1122
129/1		4	-3	11	9	57	43	-66	25	-112	75	32	-36	-268	-43	-611	148	780	-328	-246	902	-502	-380	753	50	-1391
129/2		-1	3	-2	6	-48	-62	-66	-92	106	-18	-196	0	502	-43	74	-40	744	752	36	-224	-1006	376	732	-1334	-242
130/1		-2	-2	5	8	6	13	114	38	150	114	-34	146	-30	122	336	-570	66	-502	728	582	-994	-988	-84	906	290
130/2		2	-4	-5	-8	-32	-13	-86	-56	68	-202	-56	66	490	460	-24	-294	-480	-338	676	120	-210	184	-660	-286	-1202
132/1		0	-3	0	2	-11	-88	-66	-40	6	-54	8	-106	354	-124	546	-408	552	404	-4	126	-166	-874	444	1002	-802
132/2		0	-3	-12	14	11	56	42	116	-30	198	-88	350	198	56	-594	-204	-312	620	356	-462	482	-238	492	954	-1426
132/3		0	-3	22	-20	11	22	110	48	72	-142	184	-194	-482	-80	392	-34	-108	382	84	-1040	-606	-1292	356	-406	1090
132/4		0	3	10	8	-11	18	46	40	44	186	-72	-114	174	-416	-156	-62	-348	-446	-956	-444	306	-664	-124	602	1522
133/1		4	8	6	-7	-68	8	14	-19	188	70	252	-186	192	488	-216	178	-500	-298	494	-618	-842	10	228	600	-976
134/1		2	10	6	-34	24	-46	-69	-79	99	183	-46	-277	420	-202	189	-522	639	-250	67	-552	-439	140	12	255	428
135/1		-1	0	-5	-6	47	-5	131	-56	-3	157	225	-70	-140	397	347	-4	-748	-338	492	-32	970	-1257	102	1488	974
135/2	135/1	1	0	5	-6	-47	-5	-131	-56	3	-157	225	-70	140	397	-347	4	748	-338	492	32	970	-1257	-102	-1488	974
135/3		-2	0	5	0	10	-80	7	-113	-81	-220	-189	170	-130	10	160	631	-560	229	750	890	-890	-27	429	-750	-1480
135/4	135/3	2	0	-5	0	-10	-80	-7	-113	81	220	-189	170	130	10	-160	-631	560	229	750	-890	-890	-27	-429	750	-1480

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
138/1		-2	-3	-10	32	-20	-26	-46	-92	23	-194	-120	-322	42	220	-192	-170	396	934	-988	-552	282	-888	-908	1242	-30
138/2		-2	3	2	-32	-48	22	42	-144	-23	174	-304	-318	74	192	392	-734	156	706	192	624	-406	696	-800	-102	-918
138/3		2	-3	-2	-34	2	-74	-68	88	-23	-178	240	-76	186	28	264	-598	492	352	-244	-984	1014	-438	-682	-1524	-198
140/1		0	1	-5	-7	-7	-23	-25	-62	-86	-29	-12	-150	204	-178	33	452	120	920	-300	520	370	-1013	-636	292	-1381
140/2		0	-5	-5	7	15	17	123	86	54	-177	212	74	-444	-46	471	-180	144	-376	356	-48	818	89	-780	1140	-169
140/3		0	8	-5	7	28	82	-46	8	-128	174	-152	-290	50	396	-296	-570	-272	-662	876	-880	-638	-600	624	698	754
140/4		0	-4	5	-7	68	22	-30	108	184	166	-32	-370	154	212	-512	-98	-860	390	60	840	-630	1312	-436	-598	914
140/5		0	9	5	-7	55	-69	113	-126	-102	-81	176	254	-184	-230	-187	-488	388	-728	-96	8	-994	337	188	-884	-451
140/6		0	-5	5	7	-15	-13	-27	-154	-186	3	-328	254	96	134	51	240	-396	-616	296	-48	-322	659	300	1020	-199
144/1	72/1	0	0	16	12	-64	58	32	136	128	-144	-20	-18	-288	200	-384	496	128	-458	496	-512	-602	-1108	-704	-960	206
144/2	72/1	0	0	-16	12	64	58	-32	136	-128	144	-20	-18	288	200	384	-496	-128	-458	496	512	-602	-1108	704	960	206
144/3	8/1	0	0	2	-24	-44	22	-50	-44	-56	-198	160	-162	198	-52	528	242	-668	550	-188	728	154	656	236	-714	-478
144/4	24/1	0	0	-14	24	-28	-74	-82	-92	8	138	-80	30	-282	-4	240	130	596	-218	436	856	-998	32	-1508	246	866
144/5	9/1	0	0	0	-20	0	-70	0	-56	0	0	-308	110	0	520	0	0	0	182	880	0	1190	-884	0	0	-1330
144/6	6/1	0	0	-6	16	12	38	126	-20	168	-30	88	254	-42	52	-96	-198	-660	-538	-884	792	218	520	-492	-810	1154
144/7	12/1	0	0	18	-8	36	-10	-18	100	72	234	16	-226	-90	-452	432	-414	-684	422	-332	-360	26	-512	-1188	630	-1054
145/1		1	-8	-5	-14	62	42	-114	-70	62	-29	142	146	162	352	-444	-238	840	2	-154	892	-38	1050	-778	1410	466
146/1		-2	4	-6	-8	-8	-62	10	28	48	-126	-280	-138	74	304	192	82	480	310	-204	480	73	-120	-168	954	818
147/1		-1	-3	12	0	20	-84	-96	12	-176	58	-264	258	0	156	-408	-722	492	-492	412	296	240	776	924	-744	-168
147/2		-3	-3	3	0	-15	64	-84	16	-84	-297	253	-316	-360	26	30	363	15	118	-370	-342	-362	467	-477	-906	-503
147/3		4	-3	-18	0	-50	36	-126	72	14	158	36	-162	270	-324	72	-22	-468	-792	232	-734	-180	236	-36	-234	-468
147/4	147/2	-3	3	-3	0	-15	-64	84	-16	-84	-297	-253	-316	360	26	-30	363	-15	-118	-370	-342	362	467	477	906	503
147/5	147/1	-1	3	-12	0	20	84	96	-12	-176	58	264	258	0	156	408	-722	-492	492	412	296	-240	776	-924	744	168
147/6	21/2	-3	3	18	0	-36	34	-42	124	0	102	160	398	318	-268	-240	-498	132	-398	92	-720	502	-1024	204	-354	286
147/7	21/1	4	3	4	0	62	62	-84	-100	-42	-10	48	-246	248	68	-324	258	-120	-622	904	-678	642	740	-468	-200	1266
147/8	147/3	4	3	18	0	-50	-36	126	-72	14	158	-36	-162	-270	-324	-72	-22	468	792	232	-734	180	236	36	234	468
150/1	30/2	-2	-3	0	4	-48	-2	114	140	-72	210	272	334	-198	268	-216	78	240	302	-596	-768	478	-640	348	210	1534
150/2		-2	-3	0	-1	42	-67	54	-115	-162	-210	-193	-286	12	263	414	-192	690	-733	299	-228	938	-160	-462	-240	-511
150/3		-2	3	0	-23	-30	-29	-78	149	-150	-234	-217	-146	-156	433	-30	552	-270	275	-803	660	646	992	846	-1488	319
150/4		-2	3	0	2	70	-54	22	24	100	216	208	254	-206	-292	320	402	-370	-550	-728	-540	-604	792	-404	-938	-56
150/5	30/1	2	-3	0	-32	-60	34	-42	-76	0	6	-232	-134	234	412	360	-222	660	-490	-812	120	-746	152	804	-678	-194
150/6	150/4	2	-3	0	-2	70	54	-22	24	-100	216	208	-254	-206	292	-320	-402	-370	-550	728	-540	604	792	404	-938	56
150/7	150/3	2	-3	0	23	-30	29	78	149	150	-234	-217	146	-156	-433	30	-552	-270	275	803	660	-646	992	-846	-1488	-319
150/8	150/2	2	3	0	1	42	67	-54	-115	162	-210	-193	286	12	-263	-414	192	690	-733	-299	-228	-938	-160	462	-240	511
150/9	6/1	2	3	0	16	12	-38	126	20	-168	30	-88	-254	42	52	96	-198	-660	-538	-884	792	-218	-520	492	810	-1154
153/1	51/2	-1	0	10	-8	-12	-26	-17	-148	-152	66	-32	-266	6	-92	288	546	-420	350	940	-424	378	288	-748	1558	530
153/2	17/1	3	0	-6	-28	24	-58	-17	116	60	-30	-172	-58	342	-148	-288	-318	-252	110	-484	708	362	-484	-756	774	-382
153/3	51/1	1	0	-16	34	48	58	17	20	-58	0	-218	184	138	148	516	162	180	152	-956	538	-462	390	-1268	770	494
153/4	51/3	1	0	20	-2	48	-14	17	92	122	36	-182	76	-294	-428	12	234	540	-820	700	-794	-1038	858	-1052	-1102	710
154/1		-2	-5	-1	-7	-11	-8	22	54	213	190	163	31	110	4	-80	-566	645	634	-729	431	-918	-254	904	901	-89
154/2		-2	0	2	-7	11	26	-46	-48	-128	-146	-128	-26	10	52	-544	318	-48	466	516	-392	754	0	624	-1590	1018
154/3		2	-2	18	7	-11	56	36	-28	180	-54	-334	386	-444	-316	-402	-486	-282	380	176	-324	800	-1144	468	-870	-1330
154/4		2	7	3	7	-11	-16	6	14	-51	54	95	-193	102	284	-72	-102	-63	-790	-433	135	-238	770	-1008	-639	11
154/5		2	-10	-14	7	-11	-16	108	116	68	122	-262	130	204	-396	166	442	702	196	-416	492	408	600	-1212	1146	-482
155/1		1	2	-5	16	2	-48	-94	-140	-68	300	31	296	-138	-318	-224	312	160	-128	716	912	182	-180	-1418	-1190	126
156/1		0	-3	-6	-4	36	13	66	56	96	222	260	-106	-90	44	168	30	348	-346	-256	-168	-814	200	1236	318	-502
156/2		0	3	-2	-32	-68	13	-14	4	72	102	-136	-386	250	-140	-296	526	332	-410	596	-880	506	-640	1380	1450	-446
158/1		-2	3	-9	9	-8	-47	84	-150	-2	180	-328	-346	-268	328	-351	498	365	492	784	897	378	79	-1102	-1255	1129
159/1		-5	-3	-21	-16	-45	-66	-123	-62	47	76	-17	-370	324	343	-394	-53	-379	-398	-234	56	-142	305	-434	-565	-581
159/2		-5	3	3	-16	-57	54	-39	82	-49	-260	-5	-250	-276	-137	206	-53	473	-830	366	248	-358	-1075	-602	47	1027
160/1		0	-2	-5	6	60	50	-30	40	178	166	20	10	-250	142	214	490	-800	250	-774	100	-230	-1320	982	874	-310
160/2	160/1	0	2	-5	-6	-60	50	-30	-40	-178	166	-20	10	-250	-142	-214	490	800	250	774	-100	-230	1320	-982	874	-310
162/1		-2	0	21	8	36	-49	21	-112	180	-135	308	-1	-42	20	84	-174	504	-385	272	-888	371	-652	84	21	-1246
162/2		-2	0	9	-31	15	-37	42	-28	-195	-111	-205	-166	261	-43	-177	-114	-159	191	-421	-156	182	1133	1083	1050	-901
162/3	162/2	2	0	-9	-31	-15	-37	-42	-28	195	111	-205	-166	-261	-43	177	114	159	191	-421	156	182	1133	-1083	-1050	-901
162/4	162/1	2	0	-21	8	-36	-49	-21	-112	-180	135	308	-1	42	20	-84	174	-504	-385	272	888	371	-652	-84	-21	-1246
165/1		0	-3	-5	2	-11	-22	72	122	72	96	-112	266	-96	-382	360	318	660	-430	380	168	218	-706	1068	-6	686
165/2		1	3	-5	36	11	2	66	140	-68	150	-128	-314	-118	172	-324	82	-740	122	-124	-988	2	1100	-868	-470	1186
166/1		2	-5	8	-31	3	-24	5	-144	-40	-71	-215	-111	438	-180	534	738	-79	-163	-578	-138	586	-412	-83	-1036	-1016
168/1		0	-3	-2	7	12	-66	-70	-92	16	-122	64	-306	50	20	-176	526	540	-818	-228	864	106	736	-588	146	-1214
168/2		0	3	-10	-7	-52	-10	-54	-52	48	-186	224	94	-478	-316	256	-66	420	342	668	-272	-86	1360	188	-366	1554
168/3		0	-3	4	-7	-26	2	-36	-76	-114	6	-256	-86	160	-220	308	258	264	606	-520	-286	-530	-44	1012	768	222

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
168/4		0	-3	-10	7	-12	30	34	148	152	-106	304	-114	202	116	224	-274	-660	382	12	-552	-614	880	-108	-86	1426
168/5		0	3	-2	-7	52	86	-30	-4	120	246	80	-290	-374	164	464	-162	180	-666	-628	296	-518	-1184	220	-774	-1086
168/6		0	3	-16	7	-18	-54	-128	52	-202	302	-200	-150	172	164	-460	-190	96	622	744	-54	742	-92	-228	-116	-554
169/1		4	2	17	20	-32	0	-13	30	78	197	-74	-227	-165	-156	-162	93	-864	145	862	654	215	-76	628	-266	238
169/2	169/1	-4	2	-17	-20	32	0	-13	-30	78	197	74	227	165	-156	162	93	864	145	-862	-654	-215	-76	-628	266	-238
169/3	13/1	5	-7	7	13	26	0	77	126	-96	-82	-196	131	-336	-201	105	-432	294	-56	-478	-9	-98	1304	308	1190	-70
169/4		3	-1	-9	15	-48	0	45	6	-162	-144	264	303	-192	97	111	-414	522	376	-36	357	-1098	-830	-438	-438	-852
169/5	169/4	-3	-1	9	-15	48	0	45	-6	-162	-144	-264	-303	192	97	-111	-414	-522	376	36	-357	1098	-830	438	438	852
170/1		-2	4	-5	-4	-12	-58	17	-52	84	-246	68	-358	-78	-412	408	750	-420	-190	596	324	1010	164	588	-486	-718
170/2		2	7	5	-10	24	41	-17	-103	-6	-45	5	-196	210	-58	-171	3	645	197	-46	-975	-637	272	-72	-609	-847
171/1		3	0	-6	-16	-18	-16	-24	19	60	186	-214	-196	282	20	240	-210	-240	-250	632	-168	-538	-142	126	-1470	434
171/2	171/1	-3	0	6	-16	18	-16	24	19	-60	-186	-214	-196	-282	20	-240	210	240	-250	632	168	-538	-142	-126	1470	434
171/3	57/1	1	0	12	-20	4	-76	-22	-19	-82	-242	-126	-180	390	308	522	70	-188	-706	104	432	718	94	1296	-846	830
171/4	19/1	3	0	12	11	54	11	93	19	-183	249	56	-250	-240	-196	168	-435	-195	-358	-961	246	353	-34	-234	168	758
174/1		-2	-3	-14	-21	37	-87	119	-50	48	-29	332	324	462	-132	-331	-222	250	-308	29	-928	488	190	-522	745	-1566
174/2		-2	-3	21	19	-38	-12	109	-65	108	-29	72	-311	377	-167	349	338	-155	802	-856	932	-222	110	168	-810	144
174/3		-2	-3	-8	19	-9	17	-7	-36	-182	29	-102	-166	-406	-80	-173	-68	222	106	681	-286	358	516	922	-1129	-1016
174/4		-2	3	0	-17	-23	-63	19	-8	42	-29	-198	-110	-514	-404	517	584	-182	430	365	-34	-54	236	258	213	156
174/5		2	-3	-10	7	-63	-7	-89	-78	-52	-29	192	200	166	-356	353	-154	258	520	-15	-764	244	186	-1018	553	1294
174/6		2	3	-18	-29	-49	-15	101	-110	84	29	132	-404	10	-224	-313	-374	-394	-56	-475	1120	420	-1018	1230	-45	270
175/1	35/1	-1	8	0	-7	12	78	94	40	-32	-50	-248	434	402	68	-536	-22	-560	-278	164	672	-82	-1000	448	-870	-1026
175/2	7/1	1	2	0	7	-8	-28	-54	-110	-48	-110	12	246	182	-128	-324	162	810	-488	-244	-768	702	440	1302	730	-294
176/1	88/2	0	1	-7	6	11	-40	-78	-36	-7	8	-183	227	-36	-322	184	-6	99	164	695	987	-248	242	1494	-905	-1031
176/2	88/1	0	-7	9	-2	11	0	-38	-44	-175	-264	-159	-173	-220	542	264	682	-421	308	-177	-365	-528	-686	-698	967	-1127
176/3	22/3	0	-1	-3	10	-11	-16	42	-116	-189	-120	163	-409	468	-110	-144	90	453	20	97	465	848	742	-438	-273	761
176/4	22/2	0	7	-19	-14	-11	-72	-46	20	107	120	-117	-201	-228	242	96	458	-435	-668	-439	1113	-72	70	-358	895	409
176/5	22/1	0	-4	14	8	11	-50	130	108	96	142	-40	382	-118	-220	-520	238	852	190	12	112	-6	-304	-820	202	-1406
176/6	44/1	0	5	-7	26	11	52	46	96	-27	16	293	-29	-472	110	224	754	-825	-548	123	-1001	-1020	-526	158	-1217	-263
180/1		0	0	-5	2	-30	-4	-90	-28	-120	-210	-4	200	-240	-136	120	30	450	-166	908	1020	-250	-916	1140	420	1538
180/2	180/1	0	0	5	2	30	-4	90	-28	120	210	-4	200	240	-136	-120	-30	-450	-166	908	-1020	-250	-916	-1140	-420	1538
180/3	20/1	0	0	-5	-16	60	86	-18	44	-48	186	176	254	-186	-100	-168	498	252	-58	-1036	-168	506	272	-948	1014	-766
180/4	60/2	0	0	-5	32	-36	-10	78	140	192	-6	-16	-34	390	-52	-408	114	-516	-58	-892	120	-646	-1168	732	1590	194
180/5	60/1	0	0	5	-28	24	-70	-102	20	72	-306	-136	-214	150	-292	72	414	744	-418	188	-480	434	1352	612	30	-286
181/1		-3	7	-18	20	-33	20	-102	-34	3	-216	-133	-376	264	263	-93	-462	-279	155	-52	477	-943	992	1122	744	-1222
182/1		-2	7	0	7	39	13	24	38	39	-96	227	425	-105	344	99	-540	114	-565	-385	-156	-673	749	-1044	-690	317
182/2		2	-2	-5	-7	-36	-13	26	-47	-99	-61	-23	-50	70	-19	191	195	264	310	-190	-166	873	-1191	259	-635	133
182/3		2	5	16	7	-15	-13	-44	-138	111	-12	215	55	-133	-180	471	-260	110	-271	-799	912	747	-883	-924	142	-1407
182/4		2	-8	3	7	-54	13	-96	-151	33	183	-331	-88	-42	353	-465	195	552	470	254	132	-943	-727	-1197	753	1037
183/1		1	3	3	-14	-46	-20	-31	-4	-147	30	204	-349	100	97	104	-6	492	-61	20	353	-401	498	-439	1045	-161
184/1		0	8	-4	-4	26	70	94	54	-23	-86	-144	-172	-42	386	-80	-108	164	-400	398	-320	-810	-204	102	1018	-1370
184/2		0	-4	22	8	-20	22	98	-12	23	-10	192	-106	186	332	-544	390	-716	110	836	-280	-486	288	180	650	-1262
186/1		-2	-3	3	-7	0	2	120	-115	-138	-168	31	-376	-159	-448	264	564	-135	416	-268	-579	92	-430	342	522	1001
186/2		-2	3	-11	9	-30	-16	-60	-11	-16	-130	-31	-266	-273	-22	-188	-156	-9	312	324	647	730	538	518	714	113
186/3		-2	3	-11	-22	63	15	95	-11	108	56	31	230	378	102	-157	-466	270	591	-513	647	-262	-175	-443	714	485
186/4		-2	3	15	17	24	2	-48	-115	30	264	31	-160	-51	128	480	132	309	-280	-604	-159	-652	-838	-690	-534	329
186/5		2	-3	-7	-3	-18	-52	-60	-119	-20	178	-31	58	-285	230	164	180	351	288	324	-65	-386	-758	814	594	-1615
186/6		2	3	-1	-6	39	89	27	-23	-68	64	-31	-206	-138	218	-379	630	-366	-279	123	121	-674	-965	493	570	-1003
186/7		2	3	-21	-19	-12	-34	-72	-7	-30	-84	31	380	9	-268	-480	276	309	-712	116	-783	1040	386	54	-1446	1625
187/1		1	4	6	-24	-11	-58	17	28	-24	222	-112	-394	410	-204	240	-386	564	-530	108	-392	-406	1008	236	1354	-1774
189/1		3	0	-12	7	12	-61	-117	2	-75	3	263	218	-246	515	318	-459	-255	-862	479	-117	-430	-646	-348	-585	-376
189/2		0	0	21	7	-21	2	-42	119	147	210	65	-97	-399	92	252	-672	-504	632	650	567	-448	-484	462	-1407	488
189/3	189/2	0	0	-21	7	21	2	42	119	-147	-210	65	-97	399	92	-252	672	504	632	650	-567	-448	-484	-462	1407	488
189/4	189/1	-3	0	12	7	-12	-61	117	2	75	-3	263	218	246	515	-318	459	255	-862	479	117	-430	-646	348	585	-376
190/1		-2	2	5	-12	-20	-4	-34	-19	40	-150	-200	-156	-218	248	-180	72	-48	-134	334	-520	438	980	-156	670	1124
190/2		-2	-2	5	8	44	0	-74	19	84	266	136	424	470	-236	-240	36	736	650	-830	-216	254	-1220	-688	102	-1280
190/3		2	-4	5	-20	-44	42	-86	19	-164	-162	-312	226	34	-432	580	506	364	518	924	320	-542	-1208	-1120	-1022	1166
192/1	96/1	0	-3	-10	4	20	-70	90	140	192	134	-100	170	-110	532	56	430	-20	-270	-524	80	330	-1060	-1188	1274	-590
192/2	24/1	0	-3	-14	-24	28	74	82	-92	8	138	80	-30	282	-4	240	130	-596	218	436	856	-998	-32	1508	-246	866
192/3	12/1	0	-3	18	8	-36	10	18	100	72	234	-16	226	90	-452	432	-414	684	-422	-332	-360	26	512	1188	-630	-1054
192/4	6/1	0	3	-6	-16	-12	-38	-126	-20	168	-30	-88	-254	42	52	-96	-198	660	538	-884	792	218	-520	492	810	1154
192/5	96/1	0	3	-10	-4	-20	-70	90	-140	-192	134	100	170	-110	-532	-56	430	20	-270	524	-80	330	1060	1188	1274	-590
192/6	96/4	0	-3	-2	12	-60	42	10	-132	-48	-226	-252	362	-94	228	-408	-346	300	466	-204	1056	330	612	-564	-1510	594

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
192/7	6/1	0	-3	-6	16	12	-38	-126	20	-168	-30	88	-254	42	-52	96	-198	-660	538	884	-792	218	520	-492	810	1154
192/8	96/2	0	-3	14	-36	36	-54	-22	-36	-144	-50	-108	-214	-446	-252	72	22	684	466	180	576	-54	-972	684	346	-1134
192/9	96/4	0	3	-2	-12	60	42	10	132	48	-226	252	362	-94	-228	408	-346	-300	466	204	-1056	330	-612	564	-1510	594
192/10	96/2	0	3	14	36	-36	-54	-22	36	144	-50	108	-214	-446	252	-72	22	-684	466	-180	-576	-54	972	-684	346	-1134
192/11	24/1	0	3	-14	24	-28	74	82	92	-8	138	-80	-30	282	4	-240	130	596	218	-436	-856	-998	32	-1508	-246	866
192/12	12/1	0	3	18	-8	36	10	18	-100	-72	234	16	226	90	452	-432	-414	-684	-422	332	360	26	-512	-1188	-630	-1054
195/1		-3	-3	-5	2	24	13	24	-70	90	-120	-196	-214	-54	-196	120	18	-312	-322	-376	240	1136	-808	1092	-618	-880
195/2		-5	3	-5	8	-56	-13	58	24	36	-242	-64	-254	-414	-164	-40	82	-744	494	-508	384	462	-816	-92	1210	-530
195/3		4	3	5	18	10	-13	46	-14	-36	-22	42	-46	-226	-224	-50	-290	130	70	-138	-586	-758	1068	378	1374	-1822
195/4		-3	3	5	-16	-36	13	-30	68	-120	-186	8	-226	-342	-76	-552	-738	780	-154	596	1056	-22	-112	-684	90	-334
196/1		0	4	20	0	44	44	-72	-100	-120	218	280	-30	-120	220	-88	110	-580	-380	-980	-112	640	-488	-660	-320	-248
196/2	28/2	0	-4	-6	0	-12	82	30	-68	216	246	112	110	246	-172	-192	558	-540	-110	140	-840	550	-208	-516	1398	-1586
196/3	196/1	0	-4	-20	0	44	-44	72	100	-120	218	-280	-30	120	220	88	110	580	380	-980	-112	-640	-488	660	320	248
196/4	28/1	0	10	8	0	-40	12	58	-26	-64	-62	-252	26	-6	416	396	-450	-274	576	-476	-448	158	-936	-530	390	-214
198/1		2	0	-8	-22	-11	-54	-26	-38	-64	294	36	-390	138	-242	132	388	-732	430	520	420	-594	506	380	-256	418
198/2	22/2	2	0	19	14	-11	-72	46	-20	107	-120	117	-201	228	-242	96	-458	-435	-668	439	1113	-72	-70	-358	-895	409
198/3	22/1	2	0	-14	-8	11	-50	-130	-108	96	-142	40	382	118	220	-520	-238	852	190	-12	112	-6	304	-820	-202	-1406
198/4	66/1	2	0	0	14	-11	80	-30	56	126	222	-16	-106	-114	-52	-246	264	-264	92	-796	-426	-1174	842	-852	1062	-1282
198/5	66/2	-2	0	-10	16	-11	10	10	-144	84	-218	-176	46	26	-488	-404	-194	-444	202	-84	764	354	1312	1252	1222	-1358
198/6	22/3	-2	0	3	-10	-11	-16	-42	116	-189	120	-163	-409	-468	110	-144	-90	453	20	-97	465	848	-742	-438	273	761
198/7	198/1	-2	0	8	-22	11	-54	26	-38	64	-294	36	-390	-138	-242	-132	-388	732	430	520	-420	-594	506	-380	256	418
200/1		0	-1	0	6	-19	-12	75	-91	-174	-272	-230	182	117	-372	52	402	312	170	-763	-52	981	1054	-351	799	-962
200/2		0	5	0	2	39	84	-61	151	-58	192	-18	-138	229	-164	-212	578	-336	858	-209	-780	-403	-230	-1293	-1369	382
200/3	200/2	0	-5	0	-2	39	-84	61	151	58	192	-18	138	229	164	212	-578	-336	858	209	-780	403	-230	1293	-1369	-382
200/4	8/1	0	4	0	-24	-44	-22	-50	44	56	198	-160	162	-198	-52	-528	242	-668	550	-188	728	-154	-656	-236	714	478
200/5	40/1	0	-10	0	18	-16	6	6	-124	-42	142	-188	-202	54	-66	-38	-738	564	-262	554	140	-882	-1160	-642	-854	478
200/6		0	9	0	26	-59	28	5	109	-194	-32	10	-198	117	388	-68	-18	392	-710	-253	-612	-549	414	-121	-81	-1502
200/7	200/6	0	-9	0	-26	-59	-28	-5	109	194	-32	10	198	117	-388	68	18	392	-710	253	-612	549	414	121	-81	1502
200/8	40/3	0	-4	0	-16	36	42	110	-116	-16	198	240	258	442	292	-392	-142	-348	-570	-692	168	134	784	-564	1034	382
200/9	40/2	0	6	0	34	16	-58	70	4	134	-242	100	438	-138	-178	-22	-162	-268	250	-422	-852	-306	-456	-434	-726	-1378
200/10	200/1	0	0	0	-6	-19	12	-75	-91	174	-272	-230	-182	117	372	-52	-402	312	170	763	-52	-981	1054	351	799	962
201/1		-4	-3	-19	13	26	26	-96	124	153	-188	-229	-271	-225	121	272	-503	351	436	67	-792	-97	-848	865	430	-270
202/1		2	-2	3	-30	22	-51	-13	-71	-41	204	97	-434	-240	-440	497	122	590	-728	862	627	280	-335	-328	994	674
202/2		2	-8	18	-13	-12	-16	117	143	42	-9	-16	440	-84	-283	354	-273	-612	83	974	-1062	272	50	-1056	720	-1531
203/1		4	8	2	7	2	-26	-80	128	0	29	160	-274	-36	246	-244	114	-420	188	624	1120	-352	438	-676	-336	-216
203/2		0	-4	14	7	-28	70	-14	140	72	29	208	254	186	-444	-160	270	-684	86	-708	280	506	480	-1060	810	1314
204/1		0	3	-3	-16	-57	-25	17	-13	-93	-6	110	248	-333	-115	-294	-318	-30	668	-220	540	1214	-442	-438	60	1568
205/1		-1	2	5	8	-54	68	-10	-150	-64	-56	-336	66	-41	188	-536	172	-24	-262	442	652	-54	-104	1236	370	1294
205/2		0	2	-5	26	-18	2	-134	-30	-188	-190	192	-174	41	332	566	-718	180	-418	286	62	-378	1150	432	-1030	-254
207/1	23/1	2	0	6	-8	-34	-57	80	-70	-23	-245	103	-298	-95	88	357	414	408	822	926	-335	-899	-1322	36	460	-964
208/1	104/1	0	-5	19	3	2	-13	77	58	-76	-6	292	207	240	317	375	-692	-214	-488	-782	1057	1174	-892	-704	6	830
208/2	13/1	0	7	-7	13	26	13	77	126	96	-82	-196	-131	336	201	105	-432	294	-56	-478	-9	98	-1304	308	-1190	70
208/3	26/1	0	-3	11	-19	38	-13	-51	-90	52	-190	-292	-441	312	-373	41	468	-530	592	206	863	-322	460	-528	870	-346
208/4	104/2	0	-1	-7	21	-6	13	-115	46	-144	-162	-180	13	192	33	-383	288	-442	-680	722	207	274	936	1204	-966	-138
208/5	26/3	0	-4	-18	-20	48	13	66	16	-168	6	-20	254	-390	124	468	558	96	-826	160	420	362	-776	0	1626	-1294
208/6	52/1	0	3	-13	11	2	-13	-51	-150	4	-118	116	63	-288	293	335	-708	-566	904	-382	-7	518	100	1440	1254	1262
208/7	26/2	0	0	17	35	-2	13	-19	-94	72	246	100	-11	-280	-241	-137	-232	386	64	670	-55	-838	-1016	-420	-934	-1154
210/1		-2	-3	-5	7	12	2	-18	56	-156	-186	-52	-178	-138	-412	-456	-198	348	110	-196	-936	542	992	-276	630	110
210/2		-2	-3	5	-7	-44	54	98	-60	-144	-210	-208	-226	-502	484	-232	-530	-764	814	60	848	-958	-152	308	-1094	554
210/3		-2	-3	5	7	12	-58	42	-4	24	294	128	-58	282	428	384	-138	468	-250	-556	624	-958	632	84	810	-790
210/4		-2	3	5	-7	28	54	-46	12	0	6	296	134	146	556	-448	46	748	-50	-156	-1024	-310	856	-628	-590	-1390
210/5		-2	3	-5	7	0	26	18	92	0	-6	-4	410	174	248	420	102	-588	650	152	-168	-610	-1048	-684	-834	110
210/6		-2	3	-5	-7	28	-86	-66	-48	140	-34	-284	-346	-274	-4	-448	-94	308	510	-156	336	-1170	16	772	1630	110
210/7		2	3	-5	-7	56	54	94	36	-84	-258	-40	-178	-146	148	-200	-130	188	94	-444	532	770	-536	-1076	-1090	1274
210/8		2	-3	5	-7	16	58	34	64	-16	62	60	150	474	-292	240	-662	-324	-514	-372	-412	-770	-560	-852	1466	-178
210/9		2	-3	-5	7	24	14	54	44	156	174	-88	-34	-138	164	-216	318	-204	-442	-316	-252	98	-1000	516	-522	-310
210/10		2	-3	-5	-7	-4	-42	-86	-96	-96	-78	80	50	-26	-32	-20	-382	356	-134	888	868	-70	400	-1052	-634	1202
213/1		5	-3	13	-7	51	-50	18	-27	174	17	-139	84	-135	-451	-264	346	381	-383	-772	-71	-607	-2	1426	-966	1090
216/1		0	0	4	3	28	-11	44	29	172	192	116	-69	384	328	156	-392	412	-425	257	-1000	-359	877	-328	-1572	-1483
216/2		0	0	-1	-9	17	-44	-56	-94	50	30	-139	-174	-318	-242	630	-547	236	328	614	-296	433	-56	1225	-1506	1391
216/3	216/1	0	0	-4	3	-28	-11	-44	29	-172	-192	116	-69	-384	328	-156	392	-412	-425	257	1000	-359	877	328	1572	-1483
216/4	216/2	0	0	0	-9	-17	-44	56	-94	-50	-30	-139	-174	318	-242	-630	547	-236	328	614	296	433	-56	-1225	1506	1391

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
217/1		-1	-8	4	7	66	-78	78	-106	-28	88	-31	152	-18	-506	-484	364	770	-222	-220	-512	-646	-380	-832	-1402	414
220/1		0	5	-5	-19	-11	-62	19	-131	138	-79	217	-91	158	120	-546	-439	290	-373	728	-709	850	-1194	58	753	1228
220/2		0	8	5	24	-11	-22	22	-28	-44	110	-40	-362	210	260	-460	662	-68	606	-312	360	-1042	-552	268	-966	-1334
220/3		0	-5	5	11	-11	-22	9	89	138	201	77	119	-102	260	294	51	270	-733	728	-849	830	-214	138	633	-892
221/1		5	4	6	-20	70	-13	-17	96	-98	-48	-312	262	360	-460	-168	666	780	-392	-24	-320	-748	90	1280	782	-524
222/1		2	3	-16	-24	8	-78	12	-16	-198	-72	280	37	-30	244	56	-654	38	526	-516	-552	-842	588	368	1136	726
222/2		2	-3	0	-16	48	50	60	20	162	-264	332	37	330	368	-504	354	222	-322	-532	-888	-922	1328	-696	-1488	806
222/3		-2	-3	-2	0	28	-42	90	-28	-48	-42	-152	37	-342	-500	-224	-426	628	262	-60	504	-1190	552	4	-110	-846
224/1		0	-2	0	7	20	-20	-50	10	-72	-134	-180	-270	-250	92	-236	150	570	-200	176	-640	250	-640	882	1074	270
224/2	224/1	0	2	0	-7	-20	-20	-50	-10	72	-134	180	-270	-250	-92	236	150	-570	-200	-176	640	250	640	-882	1074	270
225/1	25/1	-1	0	0	6	43	-28	-91	-35	-162	-160	42	-314	203	92	-196	-82	280	-518	141	-412	-763	510	-777	945	1246
225/2	45/1	-5	0	0	30	50	20	10	-44	-120	-50	108	40	400	-280	280	610	50	-518	180	700	410	-516	-660	-1500	1630
225/3	15/1	3	0	0	-20	24	-74	54	-124	-120	78	200	70	-330	-92	-24	450	-24	-322	196	288	430	-520	156	-1026	286
225/4	5/1	-4	0	0	-6	-32	38	26	100	-78	50	-108	-266	-22	-442	-514	2	-500	-518	-126	-412	878	600	282	150	-386
225/5	45/1	5	0	0	30	-50	20	-10	-44	120	50	108	40	-400	-280	-280	-610	-50	-518	180	-700	410	-516	660	1500	1630
225/6	9/1	0	0	0	-20	0	70	0	56	0	0	308	-110	0	520	0	0	0	182	880	0	-1190	884	0	0	1330
225/7	15/2	0	0	0	24	-52	-22	-14	-20	-168	-230	-288	34	-122	188	256	-338	-100	742	84	328	38	-240	1212	-330	-866
225/8	25/1	0	0	0	-6	43	28	91	-35	162	-160	42	314	203	-92	196	82	280	-518	-141	-412	763	510	777	945	-1246
228/1		0	-3	4	-12	40	-40	-66	-19	-98	-130	262	-296	-442	-164	-542	334	60	614	0	400	318	1154	-636	-630	1006
228/2		0	-3	-7	21	-37	26	-33	-19	-76	-218	-266	-32	64	133	305	-766	-72	-805	264	92	285	1088	420	426	-314
228/3		0	3	-3	-17	-19	-30	-97	19	-28	126	-126	64	80	-453	107	-326	56	47	-168	1060	-659	592	892	-310	-874
230/1		2	-1	5	-32	-30	19	-60	-58	23	85	-65	-34	143	-332	-561	-422	392	-246	894	-737	1041	1114	-936	824	-868
230/2		2	0	-5	-18	-32	-47	20	36	-23	-27	-33	56	-157	18	65	-14	-744	552	-156	699	-609	-644	512	-102	578
230/3		-2	7	5	20	6	47	-132	146	23	-99	-253	-118	495	272	639	-342	240	-370	698	-357	-259	542	-1248	-828	992
230/4		-2	4	-5	3	-2	-38	-45	-74	23	283	-303	79	-407	-328	360	-561	101	-268	-69	-641	994	-884	503	1608	1082
230/5		-2	-5	-5	12	22	19	96	-98	23	-227	-285	-398	271	-100	-285	18	-352	-478	330	835	-1127	322	572	-504	1712
231/1		3	3	-14	-7	-11	2	-74	0	-148	26	112	-98	-10	208	460	258	-204	178	-924	-748	-230	-456	-228	-198	562
231/2		5	3	-6	7	-11	70	126	-80	-200	134	-244	-314	278	-372	-84	182	-756	694	820	160	-2	40	760	-102	-862
231/3		-2	3	0	-7	-11	7	-14	-45	-88	-69	22	57	-380	48	-385	-672	-469	-342	-139	132	145	1244	522	822	272
231/4		2	-3	11	-7	11	-5	-118	-105	-68	-195	214	33	-376	-168	61	24	625	-558	173	168	973	-1072	1458	-198	-352
231/5		-3	-3	-4	-7	11	50	-28	30	112	130	-146	-302	4	-548	86	-246	120	-638	-132	-692	-152	768	1098	-1158	1618
234/1	26/1	2	0	-11	19	38	-13	51	90	52	190	292	-441	-312	373	41	-468	-530	592	-206	863	-322	-460	-528	-870	-346
234/2	78/3	2	0	-10	-8	-40	13	-130	-20	0	18	-184	-74	362	76	452	-382	-464	358	-700	748	1058	-976	1008	386	-614
234/3		2	0	-2	-26	-52	-13	-48	18	52	-224	310	-18	-330	328	-616	324	-188	-110	118	656	-178	836	-60	-870	1238
234/4	78/2	2	0	16	-8	38	-13	78	-72	52	-242	76	342	336	76	-94	450	-854	-110	-908	-838	-970	-352	-474	1452	-562
234/5	78/1	2	0	16	28	-34	-13	-138	108	52	190	-176	342	-240	-140	-454	-198	154	34	-656	-550	614	8	-762	444	1022
234/6	234/3	-2	0	2	-26	52	-13	48	18	-52	224	310	-18	330	328	616	-324	188	-110	118	-656	-178	836	60	870	1238
234/7	26/2	-2	0	-17	-35	-2	13	19	94	72	-246	-100	-11	280	241	-137	232	386	64	-670	-55	-838	1016	-420	934	-1154
234/8	78/4	-2	0	20	-32	-50	-13	30	-120	20	-82	-44	-306	-108	-356	178	-198	-94	-62	-140	778	62	-1096	462	-1224	614
234/9	26/3	-2	0	18	20	48	13	-66	-16	-168	-6	20	254	390	-124	468	-558	96	-826	-160	420	362	776	0	-1626	-1294
234/10	78/6	-2	0	-4	4	-2	-13	6	-36	20	14	-152	-258	-84	-188	-254	-366	-550	-14	448	-926	254	1328	-186	336	614
234/11	78/5	-2	0	-6	20	-24	13	30	-16	72	282	164	110	126	164	204	738	-120	614	848	-132	218	-1096	-552	-210	-1726
236/1		0	2	2	-3	-59	-33	47	40	-40	-4	-124	-157	221	291	-526	132	-59	82	-524	-15	538	947	-575	546	-34
240/1	30/2	0	-3	-5	4	48	2	-114	-140	-72	210	-272	-334	-198	268	-216	-78	-240	302	-596	768	-478	640	348	210	-1534
240/2	120/6	0	-3	-5	-20	-16	58	38	-4	80	82	8	426	-246	524	464	-702	592	574	172	-768	-558	-408	-164	-510	514
240/3	30/1	0	-3	5	-32	60	-34	42	76	0	6	232	134	234	412	360	222	-660	-490	-812	-120	746	-152	804	-678	194
240/4	15/2	0	-3	5	24	-52	22	-14	20	168	230	288	-34	122	188	-256	-338	-100	742	84	328	-38	240	-1212	330	866
240/5	120/3	0	-3	5	-8	-20	22	-14	-76	-56	-154	-160	-162	-390	-388	544	-210	380	-794	148	840	858	-144	-316	1098	994
240/6	60/2	0	3	5	-32	-36	-10	-78	-140	192	6	16	-34	-390	52	-408	-114	-516	-58	892	120	-646	1168	732	-1590	194
240/7	120/2	0	3	5	16	28	-26	-62	68	208	-58	-160	270	282	-76	280	-210	-196	742	-836	504	-1062	-768	1052	-726	-1406
240/8	120/5	0	3	5	0	-4	54	114	-44	-96	134	272	-98	-6	-12	200	654	-36	-442	188	632	-390	-688	-1188	-694	-1726
240/9	60/1	0	3	-5	28	24	-70	102	-20	72	306	136	-214	-150	292	72	-414	744	-418	-188	-480	434	-1352	612	-30	-286
240/10	120/1	0	3	-5	-4	-72	-6	38	-52	-152	-78	-120	-150	362	484	-280	-670	-696	222	4	-96	178	632	612	994	1634
240/11	120/4	0	3	-5	-20	56	-86	-106	-4	-136	-206	152	282	-246	-412	-40	-126	-56	-2	388	672	1170	-408	-668	66	-926
240/12	15/1	0	3	-5	-20	24	74	54	124	120	-78	-200	-70	330	-92	24	450	-24	-322	196	288	-430	520	-156	1026	-286
242/1		-2	5	-15	36	0	-12	84	60	105	-120	205	115	420	168	-180	270	-429	600	-65	-237	-12	840	-288	255	-1375
242/2		-2	4	3	-8	0	-83	-123	112	36	21	128	107	201	-308	-492	-345	204	-470	-760	900	742	-92	864	-645	299
242/3	22/3	-2	0	-3	10	0	16	-42	-116	189	120	-163	-409	-468	-110	144	90	-453	-20	-97	-465	-848	742	-438	-273	761
242/4	22/2	2	-7	-19	-14	0	72	46	20	-107	-120	117	-201	228	242	-96	458	435	668	439	-1113	72	70	-358	895	409
242/5	242/1	2	5	-15	-36	0	12	-84	-60	105	120	205	115	-420	-168	-180	270	-429	-600	-65	-237	12	-840	288	255	-1375
242/6	242/2	2	4	3	8	0	83	123	-112	36	-21	128	107	-201	308	-492	-345	204	470	-760	900	-742	92	-864	-645	299
242/7	22/1	2	4	14	8	0	50	-130	108	-96	-142	40	382	118	-220	520	238	-852	-190	-12	-112	6	-304	-820	202	-1406

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
243/1		3	0	-3	-10	12	8	-126	-91	-93	183	170	-172	474	-172	-159	603	564	-430	-439	-351	-727	1232	498	-1044	1151
243/2	243/1	-3	0	3	-10	-12	8	126	-91	93	-183	170	-172	-474	-172	159	-603	-564	-430	-439	351	-727	1232	-498	1044	1151
243/3		0	0	0	17	0	-19	0	-163	0	0	-289	-433	0	449	0	0	0	182	-880	0	1190	-1387	0	0	-523
243/4		0	0	0	-37	0	89	0	107	0	0	-19	323	0	71	0	0	0	182	-880	0	1190	503	0	0	1853
244/1		0	2	5	-7	-61	-47	16	112	-33	-98	-56	68	53	-144	84	6	-335	-61	793	120	-819	763	-468	1116	-1726
245/1	5/1	-4	-2	5	0	32	38	-26	-100	-78	-50	108	266	-22	442	514	2	-500	518	126	412	878	600	-282	150	-386
245/2		3	-2	5	0	-45	59	-54	-121	69	-162	-88	-259	195	-286	45	597	-360	392	-280	48	668	782	768	-1194	902
245/3	245/2	3	2	-5	0	-45	-59	54	121	69	-162	88	-259	-195	-286	-45	597	360	-392	-280	48	-668	782	-768	1194	-902
245/4		0	6	-5	0	-44	6	-24	-114	-52	146	-276	-210	444	492	-612	50	294	450	-668	-308	12	596	-966	-408	-1200
245/5	35/1	0	8	5	0	12	78	94	-40	32	-50	248	-434	-402	-68	-536	22	560	278	-164	672	-82	-1000	448	870	-1026
245/6	245/4	0	-6	5	0	-44	-6	24	114	-52	146	276	-210	-444	492	612	50	-294	-450	-668	-308	-12	596	966	408	1200
246/1		2	3	-14	-28	0	16	-107	-138	-32	99	-35	149	41	-339	511	-58	-136	-335	682	389	-323	10	-834	526	-330
247/1		-3	-5	7	-27	60	13	21	-19	202	36	-96	-155	-254	-405	-567	-382	746	-206	-448	457	-312	688	44	1352	-1674
252/1	28/1	0	0	8	-7	40	-12	58	26	64	62	252	26	-6	416	396	450	-274	-576	-476	448	-158	-936	-530	390	214
252/2	84/2	0	0	-14	-7	-4	54	14	92	152	106	-144	158	390	-508	528	-606	364	678	844	8	-422	384	548	-1194	-1502
252/3	28/2	0	0	-6	7	12	-82	30	68	-216	-246	-112	110	246	-172	-192	-558	-540	110	140	840	-550	-208	-516	1398	1586
252/4	84/1	0	0	-6	7	-36	62	-114	-76	24	-54	-112	-178	-378	-172	192	402	-396	254	-1012	-840	890	80	108	1638	1010
253/1		2	7	-8	-24	-11	-11	-94	110	23	-257	207	170	369	-402	471	-628	-556	168	-118	847	-849	-124	-1388	202	-1348
254/1		2	0	-1	-21	7	-50	-73	47	-33	66	26	364	-387	-204	-338	405	-484	82	586	-540	562	-1094	1272	966	160
255/1		-2	-3	5	-17	41	8	-17	-9	0	-75	68	-217	-287	-32	-423	-343	2	-20	-46	112	-647	646	-296	486	-418
255/2		-4	-3	-5	-8	-38	74	17	72	132	-246	158	14	-286	-62	-318	-446	-200	-350	770	-946	-962	838	338	942	-1630
256/1		0	10	0	0	-18	0	90	106	0	0	0	0	-522	-290	0	0	846	0	-70	0	430	0	-1350	-1026	-1910
256/2	256/1	0	-10	0	0	18	0	90	-106	0	0	0	0	-522	290	0	0	-846	0	70	0	430	0	1350	-1026	-1910
256/3		0	-8	-12	32	8	20	-98	88	-32	-172	-256	-92	102	296	-320	-76	-408	-636	-552	416	138	-64	-392	-582	238
256/4	256/3	0	-8	12	-32	8	-20	-98	88	32	172	256	92	102	296	320	76	-408	636	-552	-416	138	64	-392	-582	238
256/5	256/3	0	8	12	32	-8	-20	-98	-88	-32	172	-256	92	102	-296	-320	76	408	636	552	416	138	-64	392	-582	238
256/6	256/3	0	8	-12	-32	-8	20	-98	-88	32	-172	256	-92	102	-296	320	-76	408	-636	552	-416	138	64	392	-582	238
256/7		0	0	-4	0	0	92	94	0	0	284	0	396	230	0	0	572	0	-468	0	0	1098	0	0	-1670	-594
256/8	256/7	0	0	4	0	0	-92	94	0	0	-284	0	-396	230	0	0	-572	0	468	0	0	1098	0	0	-1670	-594
258/1		2	3	-9	-25	-69	-31	0	17	132	-237	38	326	-72	43	201	-84	-612	-496	-502	-288	-160	170	-561	654	449
259/1		3	-2	2	7	20	-74	76	-136	-16	86	-238	-37	86	-468	264	-298	420	838	-244	-152	514	204	314	680	-712
260/1		0	-2	5	-4	18	13	-54	-70	-66	-78	-46	-358	-438	98	-300	78	-114	-166	788	-198	-58	-340	1080	-6	-142
264/1		0	3	-6	-14	11	6	-108	-98	-32	-8	-40	50	-8	-486	40	710	-604	322	-476	216	502	-862	592	354	446
264/2		0	3	-6	-8	-11	-30	-18	-56	-100	26	-136	-178	110	288	116	-398	196	-782	292	180	-398	56	548	282	-142
264/3		0	-3	12	22	11	-48	-54	100	58	262	248	-130	-26	216	22	620	-424	340	-620	810	-1118	-214	988	-6	590
264/4		0	-3	-18	-28	11	-18	-34	80	128	162	-312	-290	-146	256	432	-490	836	230	900	520	-798	-484	-812	74	-1790
270/1		2	0	5	-34	-48	-70	-27	119	-51	-30	-133	218	156	-88	-516	-639	-654	461	182	900	704	-1375	915	-1116	-16
270/2		2	0	5	8	18	8	15	23	63	156	-85	74	246	-190	288	-177	792	-907	-322	-270	254	-1123	-771	-198	-1192
270/3		2	0	-5	-13	-30	-61	12	-49	18	-186	-160	-91	378	-268	144	570	204	-877	-187	-606	431	1151	102	984	-265
270/4		2	0	-5	14	-3	47	39	32	99	-51	83	314	108	299	-531	-564	-12	230	-268	-120	1106	-739	-1086	120	-1642
270/5		2	0	-5	-22	-12	38	-105	-157	-117	66	-25	314	-504	380	-252	3	-318	293	-322	-120	44	917	309	1272	1328
270/6		2	0	-5	-4	42	20	93	59	9	120	47	-262	126	-178	144	741	-444	221	-538	690	-1126	665	75	-1086	1544
270/7	270/2	-2	0	-5	8	-18	8	-15	23	-63	-156	-85	74	-246	-190	-288	177	-792	-907	-322	270	254	-1123	771	198	-1192
270/8	270/1	-2	0	-5	-34	48	-70	27	119	51	30	-133	218	-156	-88	516	639	654	461	182	-900	704	-1375	-915	1116	-16
270/9	270/3	-2	0	5	-13	30	-61	-12	-49	-18	186	-160	-91	-378	-268	-144	-570	-204	-877	-187	606	431	1151	-102	-984	-265
270/10	270/6	-2	0	5	-4	-42	20	-93	59	-9	-120	47	-262	-126	-178	-144	-741	444	221	-538	-690	-1126	665	-75	1086	1544
270/11	270/5	-2	0	5	-22	12	38	105	-157	117	-66	-25	314	504	380	252	-3	318	293	-322	120	44	917	-309	-1272	1328
270/12	270/4	-2	0	5	14	3	47	-39	32	-99	51	83	314	-108	299	531	564	12	230	-268	120	1106	-739	1086	-120	-1642
272/1	17/1	0	8	6	28	24	-58	17	-116	60	30	172	-58	-342	148	-288	318	-252	110	484	708	362	484	-756	-774	-382
272/2	68/1	0	2	-8	12	10	-38	-17	-4	-120	56	-164	-236	70	144	-48	-366	504	-460	768	-72	-734	-736	-856	906	46
272/3	34/1	0	2	16	-24	-62	-62	-17	20	12	80	208	-356	22	312	-24	-462	-240	812	216	-732	178	-700	992	-390	-146
272/4	34/2	0	2	-18	10	6	74	17	88	114	-90	310	86	90	-368	384	-258	-240	302	964	390	722	898	-912	1446	-1438
273/1		-4	-3	0	-7	-6	-13	-4	-52	6	14	-48	-190	180	356	536	210	244	470	240	854	-82	-876	504	-660	1318
273/2		-1	3	-5	7	-1	13	19	-117	-141	-131	-128	55	0	-201	-96	510	-156	-845	-470	324	-373	-526	266	-250	322
273/3		-1	3	9	-7	-57	-13	-37	107	-183	191	-240	-379	-84	-313	296	-414	40	65	-1086	-208	635	-582	798	-726	1498
275/1	55/1	-1	3	0	9	11	-2	-21	-85	-22	-165	-83	-1	-478	8	-126	683	-290	257	-776	-313	-902	830	-842	25	1784
276/1		0	3	8	34	36	-62	-60	30	-23	234	140	-174	194	-42	-400	76	252	-566	-6	264	-286	486	-980	1632	-1626
276/2		0	3	2	-22	-14	-50	-52	-20	23	-74	24	104	-30	112	-288	-386	-204	-308	152	-720	486	462	742	180	786
278/1		-2	-4	9	-5	7	-5	-12	148	-100	139	-287	-370	-2	122	-540	-582	-94	-418	895	-519	-218	319	141	-889	-1590
279/1	93/1	-3	0	9	-34	-33	65	21	-97	84	-48	31	146	378	182	501	402	-102	209	-835	105	542	1109	597	1638	-1483
280/1		0	-1	5	7	-39	-17	-15	74	-14	-237	-180	-318	-348	-22	-193	-208	452	340	-408	528	-554	539	164	-576	-827
280/2		0	7	5	7	9	23	41	34	-6	131	4	26	-260	-190	167	-368	324	-164	200	784	-410	1211	-1132	-72	-707

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
280/3		0	-4	5	7	20	-10	-14	12	104	-122	224	158	378	404	112	270	324	-186	156	-360	-102	-912	1068	-1590	866
280/4		0	5	-5	-7	-39	-19	-37	-18	-90	99	-32	46	-248	178	429	-652	40	-36	-348	72	-1190	699	-116	-704	223
282/1		-2	-3	3	11	15	-28	60	-94	45	75	200	149	222	380	-47	594	846	650	-160	114	-340	-373	1122	-582	-811
282/2		-2	3	7	-9	-45	-70	-38	30	-99	211	-2	-275	394	-182	-47	80	-776	-130	-290	234	-506	-1221	468	1082	449
282/3		-2	3	-8	-12	60	2	-110	-126	84	40	-14	-254	-164	-422	-47	-502	628	26	-386	-720	574	156	660	-1018	-1570
282/4		-2	3	-3	-33	-31	62	58	130	151	-23	250	-43	-282	342	47	-412	324	518	734	-322	22	707	-1096	254	-767
282/5		2	3	-11	-25	-15	-26	-54	-6	-89	-31	-70	-171	390	262	47	12	196	-442	-386	70	22	171	-672	-42	-607
285/1		-3	-3	5	32	-12	-10	-30	19	-48	150	224	254	-54	-196	-504	78	132	230	740	-120	122	1184	612	1050	-1006
287/1		-1	-2	-12	-7	-36	-28	-114	-54	104	30	180	202	-41	-264	436	-582	-674	-236	48	968	82	-120	-266	-278	-266
288/1		0	0	4	0	0	18	104	0	0	284	0	214	472	0	0	572	0	830	0	0	-1098	0	0	-176	-594
288/2	32/1	0	0	-22	0	0	-18	94	0	0	130	0	214	230	0	0	-518	0	830	0	0	1098	0	0	1670	594
288/3	288/1	0	0	-4	0	0	18	-104	0	0	-284	0	214	-472	0	0	-572	0	830	0	0	-1098	0	0	176	-594
288/4	96/2	0	0	14	36	-36	54	22	-36	-144	-50	108	214	446	-252	72	22	-684	-466	180	576	-54	972	-684	-346	-1134
288/5	96/2	0	0	14	-36	36	54	22	36	144	-50	-108	214	446	252	-72	22	684	-466	-180	-576	-54	-972	684	-346	-1134
288/6	32/2	0	0	10	16	40	-50	30	40	-48	34	320	310	-410	152	416	410	200	30	776	-400	-630	-1120	-552	326	-110
288/7	32/2	0	0	10	-16	-40	-50	30	-40	48	34	-320	310	-410	-152	-416	410	-200	30	-776	400	-630	1120	552	326	-110
288/8	96/1	0	0	-10	-4	-20	70	-90	140	192	134	100	-170	110	532	56	430	20	270	-524	80	330	1060	1188	-1274	-590
288/9	96/1	0	0	-10	4	20	70	-90	-140	-192	134	-100	-170	110	-532	-56	430	-20	270	524	-80	330	-1060	-1188	-1274	-590
288/10	96/4	0	0	-2	-12	60	-42	-10	-132	-48	-226	252	-362	94	228	-408	-346	-300	-466	-204	1056	330	-612	564	1510	594
288/11	96/4	0	0	-2	12	-60	-42	-10	132	48	-226	-252	-362	94	-228	408	-346	300	-466	204	-1056	330	612	-564	1510	594
289/1	17/1	-3	8	-6	28	24	-58	0	116	60	-30	172	58	342	-148	288	318	252	-110	-484	708	-362	484	756	-774	382
290/1		2	2	-5	-24	12	-18	-44	-100	-68	-29	172	156	22	-18	-114	-518	100	282	-204	-768	32	-20	1332	-530	-804
290/2		2	-2	-5	12	-48	-2	-24	-48	8	-29	-328	-280	-154	-206	-102	546	548	50	-480	816	124	296	-408	1558	304
290/3		2	0	5	-20	-52	-42	-22	28	36	29	24	-266	-38	88	188	-194	-460	-314	896	-416	-606	992	-24	-774	1626
290/4		-2	2	-5	-32	12	22	104	12	212	-29	52	0	-354	526	-58	314	-692	730	-140	696	-404	116	-172	-42	176
294/1	42/1	2	3	-18	0	-72	34	-6	-92	-180	-114	-56	-34	-6	164	-168	654	492	250	-124	36	-1010	56	-228	-390	70
294/2	42/2	2	-3	-2	0	-8	42	2	124	76	254	72	398	-462	212	264	-162	772	-30	-764	-236	-418	552	-1036	-30	1190
294/3		-2	-3	6	0	-30	-53	84	97	84	-180	-179	-145	-126	-325	366	-768	264	-818	-523	-342	43	-1171	810	600	-386
294/4		-2	-3	-15	0	-9	-88	-84	104	-84	51	185	44	-168	326	-138	639	159	722	-166	1086	218	-583	-597	-1038	-169
294/5		-2	-3	-8	0	40	-4	84	-148	84	58	136	-222	-420	-164	-488	478	-548	-692	-908	-524	-440	1216	684	-604	832
294/6	294/5	-2	3	8	0	40	4	-84	148	84	58	-136	-222	420	-164	488	478	548	692	-908	-524	440	1216	-684	604	-832
294/7	294/4	-2	3	15	0	-9	88	84	-104	-84	51	-185	44	168	326	138	639	-159	-722	-166	1086	-218	-583	597	1038	169
294/8	294/3	-2	3	-6	0	-30	53	-84	-97	84	-180	179	-145	126	-325	-366	-768	-264	818	-523	-342	-43	-1171	-810	-600	386
294/9	6/1	-2	3	-6	0	12	-38	126	-20	168	30	88	254	-42	-52	96	198	660	538	884	792	-218	-520	492	-810	-1154
297/1		4	0	-8	-11	-11	-92	-65	156	173	-177	94	-171	-47	-135	29	492	695	-684	2	528	-20	-1129	-300	30	491
297/2	297/1	-4	0	8	-11	11	-92	65	156	-173	177	94	-171	47	-135	-29	-492	-695	-684	2	-528	-20	-1129	300	-30	491
297/3		-1	0	-13	28	11	-5	29	12	-173	-234	-98	168	-361	210	160	-633	646	-831	305	-372	-866	-271	561	-654	-1831
297/4		-1	0	11	-20	11	67	-115	-36	-149	246	142	-360	-457	-318	-272	207	-506	297	-319	-1092	-338	905	-543	-414	905
297/5	297/4	0	0	-11	-20	-11	67	115	-36	149	-246	142	-360	457	-318	272	-207	506	297	-319	1092	-338	905	543	414	905
297/6	297/3	0	0	13	28	-11	-5	-29	12	173	234	-98	168	361	210	-160	633	-646	-831	305	372	-866	-271	-561	654	-1831
300/1		0	-3	0	-13	6	5	-78	65	138	66	299	-214	360	203	78	636	786	467	-217	-360	-286	272	498	0	-511
300/2		0	-3	0	22	-14	-30	62	-120	188	96	184	406	130	148	448	-414	266	-838	248	1020	484	-48	548	-650	-1816
300/3	12/1	0	-3	0	-8	36	10	-18	-100	-72	-234	-16	226	90	-452	-432	-414	-684	422	-332	-360	-26	512	1188	-630	1054
300/4		0	-3	0	7	-54	55	-18	-25	18	-54	-271	-314	-360	163	-522	36	126	47	343	-1080	1054	-568	-1422	1440	439
300/5	300/1	0	3	0	13	6	-5	78	65	-138	66	299	214	360	-203	-78	-636	786	467	217	-360	286	272	-498	0	511
300/6	60/1	0	3	0	28	-24	70	-102	20	72	306	-136	214	-150	292	72	414	-744	-418	-188	480	-434	1352	612	-30	286
300/7	60/2	0	3	0	-32	36	10	78	140	192	6	-16	34	-390	52	-408	114	516	-58	892	-120	646	-1168	732	-1590	-194
300/8	300/4	0	3	0	-7	-54	-55	18	-25	-18	-54	-271	314	-360	-163	522	-36	126	47	-343	-1080	-1054	-568	1422	1440	-439
300/9	300/2	0	3	0	-22	-14	30	-62	-120	-188	96	184	-406	130	-148	-448	414	266	-838	-248	1020	-484	-48	-548	-650	1816
301/1		3	10	6	7	60	-88	-78	74	-120	186	-70	254	-126	43	-222	-450	96	182	-916	-264	92	128	-132	168	182
303/1		-3	3	-8	-5	-22	82	3	-17	32	-275	-114	322	16	169	-160	-667	-706	-785	812	-1042	-814	558	-1472	592	593
304/1	38/1	0	2	-9	31	-57	-52	69	-19	72	-150	-32	-226	-258	67	-579	-432	330	-13	856	-642	-487	700	12	-600	1424
304/2	19/1	0	5	-12	-11	54	11	-93	-19	-183	-249	-56	-250	240	196	168	435	-195	-358	961	246	353	34	-234	-168	758
306/1	34/1	2	0	-16	24	-62	-62	17	-20	12	-80	-208	-356	-22	-312	-24	462	-240	812	-216	-732	178	700	992	390	-146
306/2	34/2	2	0	18	-10	6	74	-17	-88	114	90	-310	86	-90	368	384	258	-240	302	-964	390	722	-898	-912	-1446	-1438
306/3	102/2	2	0	5	-32	-27	-69	17	-83	117	-94	198	-244	-169	227	382	-686	-450	-700	540	276	-298	-182	-282	1468	-1140
306/4	102/1	2	0	3	20	51	-61	-17	-43	219	150	290	56	-15	83	-426	378	210	-448	-124	-900	-1078	722	78	144	-268
306/5		2	0	-9	-10	15	-25	-17	-151	-57	-234	-4	-22	333	-289	240	312	282	374	-604	480	-412	-466	-192	462	1244
306/6	306/5	-2	0	9	-10	-15	-25	17	-151	57	234	-4	-22	-333	-289	-240	-312	-282	374	-604	-480	-412	-466	192	-462	1244
306/7	102/4	-2	0	-5	12	-37	19	-17	37	3	86	-142	-296	121	3	-402	-174	-270	-520	-780	-84	-302	178	-698	-1512	-500
306/8	102/3	-2	0	12	-22	48	2	17	20	54	-84	62	44	138	428	516	-174	852	908	-508	426	-574	110	1308	-798	-1690
308/1		0	4	-12	7	11	-10	24	-94	-180	30	-94	-214	-48	8	30	54	360	-178	-292	312	728	-1288	66	-390	-1510

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
308/2		0	-7	-1	7	11	12	2	82	7	-102	-171	-357	-114	-344	96	-430	-201	-2	313	-579	-438	494	748	457	-1037
310/1		2	-10	-5	0	60	16	-112	-124	-146	-62	-31	140	474	94	-156	-28	-348	-646	-404	-912	-380	-992	-1066	-354	414
310/2		2	-1	5	-18	-30	-2	-25	-51	-148	64	31	-69	-11	25	-50	75	-483	-596	-604	-65	593	-328	933	838	350
310/3		-2	7	-5	-4	-4	-2	79	133	176	186	-31	191	-35	-391	242	-237	33	360	398	1031	173	-522	537	-240	-826
310/4		-2	-2	-5	-24	-22	24	86	-68	-104	-72	-31	232	70	158	432	440	24	-428	20	864	218	252	-398	-302	222
310/5		-2	-2	-5	20	44	-20	-68	-68	182	-6	-31	-208	202	422	608	264	684	474	680	-16	-376	-1200	1230	6	1190
314/1		2	-8	0	5	-10	27	-93	20	55	-168	-286	-381	0	-115	320	-532	69	520	-226	378	-276	-56	-304	-187	1030
315/1	105/2	-5	0	-5	7	-12	30	134	-92	-112	58	-224	-146	-18	340	-208	754	-380	718	412	960	1066	896	-436	1038	-702
315/2	35/1	-1	0	5	7	-12	-78	94	40	-32	50	-248	-434	-402	-68	-536	-22	560	-278	-164	-672	82	-1000	448	870	1026
315/3		-3	0	-5	7	-60	38	84	110	-120	-162	236	-376	126	-34	6	-582	-492	-880	-826	666	-826	-592	-792	-1002	1442
315/4	315/3	3	0	5	7	60	38	-84	110	120	162	236	-376	-126	-34	-6	582	492	-880	-826	-666	-826	-592	792	1002	1442
315/5	105/1	0	0	-5	7	-42	20	-66	38	-12	258	146	434	282	20	72	-336	360	-682	812	-810	-124	1136	-156	1038	1208
316/1		0	6	6	0	4	34	54	144	-8	180	128	-232	446	-86	-192	-348	-838	780	904	312	-234	79	-316	566	-806
320/1	10/1	0	8	-5	-4	-12	58	66	100	132	90	152	34	-438	-32	-204	-222	-420	-902	1024	432	362	-160	-72	810	1106
320/2	10/1	0	-8	-5	4	12	58	66	-100	-132	90	-152	34	-438	32	204	-222	420	-902	-1024	-432	362	160	72	810	1106
320/3	40/2	0	-6	5	34	16	-58	-70	4	134	242	-100	438	-138	178	-22	-162	-268	-250	422	852	306	456	434	-726	1378
320/4	40/1	0	10	5	18	-16	6	-6	-124	-42	-142	188	-202	54	66	-38	-738	564	262	-554	-140	882	1160	642	-854	-478
320/5	40/1	0	-10	5	-18	16	6	-6	124	42	-142	-188	-202	54	-66	38	-738	-564	262	554	140	882	-1160	-642	-854	-478
320/6	40/2	0	6	5	-34	-16	-58	-70	-4	-134	242	100	438	-138	-178	22	-162	268	-250	-422	-852	306	-456	-434	-726	1378
320/7	20/1	0	4	-5	16	-60	-86	18	44	-48	186	-176	-254	186	-100	-168	498	-252	58	-1036	-168	506	-272	948	-1014	-766
320/8	40/3	0	4	-5	-16	36	42	-110	-116	-16	-198	-240	258	442	-292	-392	-142	-348	570	692	-168	-134	-784	564	1034	-382
320/9	160/1	0	2	5	6	-60	-50	-30	-40	178	-166	20	-10	-250	-142	214	-490	800	-250	774	100	-230	-1320	-982	874	-310
320/10	5/1	0	2	5	-6	32	38	26	100	78	50	108	-266	22	442	514	-2	500	518	126	-412	-878	-600	282	-150	386
320/11	40/3	0	-4	-5	16	-36	42	-110	116	16	-198	240	258	442	292	392	-142	348	570	-692	168	-134	784	-564	1034	-382
320/12	20/1	0	-4	-5	-16	60	-86	18	-44	48	186	176	-254	186	100	168	498	252	58	1036	168	506	272	-948	-1014	-766
320/13	160/1	0	-2	5	-6	60	-50	-30	40	-178	-166	-20	-10	-250	142	-214	-490	-800	-250	-774	-100	-230	1320	982	874	-310
320/14	5/1	0	-2	5	6	-32	38	26	-100	-78	50	-108	-266	22	-442	-514	-2	-500	518	-126	412	-878	600	-282	-150	386
324/1		0	0	3	-4	-24	-25	-21	-52	168	-177	-124	-265	426	-160	-540	-258	528	-505	-244	204	-397	200	-540	-453	290
324/2	324/1	0	0	-3	-4	24	-25	21	-52	-168	177	-124	-265	-426	-160	540	258	-528	-505	-244	-204	-397	200	540	453	290
325/1	65/1	-5	-2	0	12	14	13	-98	-26	114	58	306	-86	-374	314	-620	-362	266	634	-612	-686	-202	-516	-48	-1230	-350
325/2	13/1	5	7	0	13	-26	-13	-77	-126	96	-82	196	131	336	201	105	432	-294	-56	-478	9	-98	1304	308	-1190	-70
325/3		-3	-4	0	28	2	13	-44	-94	18	118	-100	-126	474	200	-448	754	-446	-638	868	536	58	232	108	1038	774
325/4	325/3	3	4	0	-28	2	-13	44	-94	-18	118	-100	126	474	-200	448	-754	-446	-638	-868	536	-58	232	-108	1038	-774
330/1		2	3	-5	10	-11	44	124	-56	100	42	-120	86	222	54	76	-162	-68	-734	-552	-320	292	676	422	-490	174
330/2		2	3	-5	-34	11	-88	36	-100	12	-90	-208	86	-438	362	516	102	-420	-118	416	-408	-808	-160	-18	-930	1406
330/3		2	-3	-5	2	-11	-28	-36	-64	12	-126	-280	-298	54	62	-444	366	108	146	848	48	-628	-676	342	-570	-178
330/4		2	-3	5	-24	11	-30	-110	56	-144	-182	24	-234	-26	-68	224	-146	-116	-818	-4	176	-826	532	1008	1098	42
330/5		-2	3	-5	-6	-11	48	-52	-76	-132	134	-192	30	-334	334	-572	-10	-220	-302	392	-704	-184	16	-1446	-114	-642
330/6		-2	3	-5	-6	11	-40	80	56	44	178	-16	-146	414	158	-44	166	44	402	744	1056	1136	-468	182	678	-1082
330/7		-2	3	5	-16	-11	38	18	44	168	54	8	-130	-174	164	528	510	780	-82	92	336	-574	56	1044	426	1298
330/8		-2	3	5	-16	11	-50	-70	-44	-96	-122	184	134	-86	-12	-264	-194	-716	182	-436	-104	-134	-648	-628	-102	418
330/9		-2	-3	5	20	11	26	6	-28	-48	-162	128	86	66	344	312	486	-84	494	716	-432	206	440	192	-294	1082
330/10		-2	-3	-5	2	11	-16	96	-112	180	-102	-208	110	-90	-10	-180	-618	-36	-286	-928	48	-520	-412	-618	-234	422
333/1	111/3	4	0	-2	-28	-20	10	78	-150	-82	222	-154	-37	306	386	-12	46	-658	250	-748	-324	-130	-230	-216	118	898
333/2	111/2	-1	0	8	-13	35	-35	3	15	-47	12	-94	-37	-54	-244	-282	-619	-8	250	-478	96	-955	-410	579	-37	-2
333/3	111/1	0	0	4	-1	13	73	-99	-105	-133	-300	62	-37	198	68	-354	7	-220	322	-706	-672	893	910	-243	-995	1234
336/1	21/2	0	3	-18	-7	36	-34	42	124	0	102	160	398	-318	268	-240	-498	132	398	-92	720	-502	1024	204	354	-286
336/2	168/3	0	3	4	7	26	2	-36	76	114	6	256	-86	160	220	-308	258	-264	606	520	286	-530	44	-1012	768	222
336/3	21/1	0	3	-4	7	-62	-62	84	-100	42	-10	48	-246	-248	-68	-324	258	-120	622	-904	678	-642	-740	-468	200	-1266
336/4	168/1	0	3	-2	-7	-12	-66	-70	92	-16	-122	-64	-306	50	-20	176	526	-540	-818	228	-864	106	-736	588	146	-1214
336/5	42/1	0	3	18	-7	72	-34	6	-92	180	-114	-56	-34	6	-164	-168	654	492	-250	124	-36	1010	-56	-228	390	-70
336/6	168/4	0	3	-10	-7	12	30	34	-148	-152	-106	-304	-114	202	-116	-224	-274	660	382	-12	552	-614	-880	108	-86	1426
336/7	84/1	0	3	6	-7	-36	62	114	76	24	54	112	-178	378	172	192	-402	-396	254	1012	-840	890	-80	108	-1638	1010
336/8	168/5	0	-3	-2	7	-52	86	-30	4	-120	246	-80	-290	-374	-164	-464	-162	-180	-666	628	-296	-518	1184	-220	-774	-1086
336/9	84/2	0	-3	14	7	-4	54	-14	-92	152	-106	144	158	-390	508	528	606	364	678	-844	8	-422	-384	548	1194	-1502
336/10	168/2	0	-3	-10	7	52	-10	-54	52	-48	-186	-224	94	-478	316	-256	-66	-420	342	-668	272	-86	-1360	-188	-366	1554
336/11	168/6	0	-3	-16	-7	18	-54	-128	-52	202	302	200	-150	172	-164	460	-190	-96	622	-744	54	742	92	228	-116	-554
336/12	42/2	0	-3	2	7	8	-42	-2	124	-76	254	72	398	462	-212	264	-162	772	30	764	236	418	-552	-1036	30	-1190
338/1		2	-3	-2	5	-13	0	27	-75	-187	-13	104	-423	-195	199	-388	618	-491	175	-817	-79	-230	764	732	1041	97
338/2	26/1	2	3	-11	-19	38	0	-51	-90	-52	-190	-292	441	-312	373	41	468	-530	592	206	863	322	-460	-528	-870	346
338/3	338/1	-2	-3	2	-5	13	0	27	75	-187	-13	-104	423	195	199	388	618	491	175	817	79	230	764	-732	-1041	-97
338/4	26/2	-2	-1	-17	35	-2	0	-19	-94	-72	246	100	11	280	241	-137	-232	386	64	670	-55	838	1016	-420	934	1154

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
338/5	26/3	-2	4	18	-20	48	0	66	16	168	6	-20	-254	390	-124	468	558	96	-826	160	420	-362	776	0	-1626	1294
339/1		-4	3	17	-15	-24	-7	-123	-108	-69	-30	233	2	-374	90	288	-446	-585	-39	308	315	-280	730	-1378	-458	-818
340/1		0	2	5	2	-30	-62	17	-56	-110	206	114	-194	-430	4	-68	206	496	-290	8	-798	314	366	-1276	86	-1006
340/2		0	-5	5	2	12	-13	17	35	30	-249	-229	-124	-66	-262	-75	-543	-225	-535	386	231	-547	-376	768	-537	1367
341/1		5	-4	-6	-19	-11	12	-105	-10	-11	-48	31	47	122	-289	216	50	621	454	63	-750	502	-1222	-781	-478	321
342/1	114/2	2	0	7	-15	49	14	33	-19	148	278	94	160	-400	73	-173	-170	12	419	444	952	-27	-556	276	-1386	130
342/2	114/1	2	0	19	9	13	38	-99	-19	-68	-130	262	-296	8	73	271	502	-540	587	684	-992	-507	980	492	-810	-1046
342/3	38/1	2	0	9	-31	-57	-52	-69	19	72	150	32	-226	258	-67	-579	432	330	-13	-856	-642	-487	-700	12	600	1424
342/4	114/3	2	0	-12	4	-8	-24	-62	19	-194	-102	18	-296	-134	-60	226	362	316	134	-240	800	-578	1078	-940	-170	206
342/5	114/4	-2	0	11	-15	29	-82	-27	-19	-100	118	70	232	-8	-287	-385	-538	300	-901	132	-472	-1131	-52	-276	1302	-1310
345/1		-3	3	5	26	54	2	-72	68	-23	-102	-16	344	162	-280	-360	114	-768	704	560	408	998	-550	966	-804	-310
345/2		-5	3	5	-6	46	-50	-116	-152	23	206	-120	-28	-118	292	-344	-326	748	-584	-684	152	118	870	-1278	228	-790
345/3		0	-3	-5	-16	-48	-46	-30	-46	-23	30	116	68	54	380	420	-642	186	-34	-124	1026	-646	-610	-612	642	476
345/4		0	3	-5	16	52	-38	-54	40	23	170	232	386	482	132	-144	82	100	-398	-124	-428	-78	-960	-1488	470	1126
348/1		0	3	-9	21	-66	-72	-25	137	-112	29	88	-375	-397	87	-327	182	449	-510	-864	144	-370	1174	-528	986	360
350/1	70/6	-2	-5	0	7	-1	-7	51	30	50	79	-212	190	-308	-422	-121	-664	628	-684	-1056	744	-726	-407	-644	-880	1351
350/2	70/5	-2	-7	0	-7	-33	43	-111	-70	-42	-225	-88	34	432	178	-411	708	480	812	-596	432	358	425	-972	960	709
350/3		-2	4	0	7	5	-82	12	-42	-175	0	226	19	16	-281	-334	398	106	48	-483	-15	-1044	-1253	-758	86	-710
350/4		-2	3	0	-7	37	18	-121	-45	-72	210	-148	-136	227	-32	-346	-452	-140	-578	-801	-478	-247	610	653	-1115	614
350/5		-2	10	0	7	9	-52	96	-10	75	189	-232	305	-438	353	-486	-354	-672	206	599	-471	614	743	996	180	-184
350/6		-2	-8	0	-7	-7	-26	44	142	115	0	6	-411	-444	221	-258	626	-162	-820	519	61	-1160	-809	-678	370	-310
350/7		-2	7	0	-7	-37	-51	-41	-108	70	-249	-134	334	206	376	287	6	-2	-940	-106	456	-650	-1239	-428	-220	1055
350/8		-2	-1	0	7	-35	58	107	23	-200	-174	76	184	431	144	526	108	76	118	687	530	-299	402	897	-799	1510
350/9	14/2	-2	2	0	-7	48	-56	114	2	120	-54	236	-146	126	376	12	-174	138	380	484	576	1150	776	-378	-390	1330
350/10		-2	2	0	-7	-27	64	24	62	105	141	-124	439	-354	211	102	306	348	410	349	-339	70	731	-528	960	-1340
350/11	70/3	2	3	0	7	-17	81	91	102	90	-129	116	-314	-124	434	-497	584	-332	220	-384	-664	-230	361	-1172	40	175
350/12	350/7	2	-7	0	7	-37	51	41	-108	-70	-249	-134	-334	206	-376	-287	-6	-2	-940	106	456	650	-1239	428	-220	-1055
350/13	350/4	2	-3	0	7	37	-18	121	-45	72	210	-148	136	227	32	346	452	-140	-578	801	-478	247	610	-653	-1115	-614
350/14	14/1	2	-8	0	7	-28	-18	-74	80	112	190	72	346	162	412	-24	-318	-200	-198	716	392	-538	240	1072	810	-1354
350/15	350/10	2	-2	0	7	-27	-64	-24	62	-105	141	-124	-439	-354	-211	-102	-306	348	410	-349	-339	-70	731	528	960	1340
350/16	350/5	2	-10	0	-7	9	52	-96	-10	-75	189	-232	-305	-438	-353	486	354	-672	206	-599	-471	-614	743	-996	180	184
350/17	70/1	2	8	0	7	68	-34	-74	-128	80	286	-24	-294	66	124	-312	34	168	170	-564	616	-250	-944	-672	-1430	1270
350/18	350/6	2	8	0	7	-7	26	-44	142	-115	0	6	411	-444	-221	258	-626	-162	-820	-519	61	1160	-809	678	370	310
350/19	70/4	2	-4	0	-7	60	-38	-42	-52	-120	-234	-304	106	-54	196	-336	-438	-444	38	988	-720	-146	-808	-612	1146	70
350/20	350/3	2	-4	0	-7	5	82	-12	-42	175	0	226	-19	16	281	334	-398	106	48	483	-15	1044	-1253	758	86	710
350/21	70/2	2	0	0	-7	-65	-13	73	-142	-130	111	256	266	-424	-534	269	132	-224	-572	108	560	-586	57	-252	-184	605
350/22	350/8	2	0	0	-7	-35	-58	-107	23	200	-174	76	-184	431	-144	-526	-108	76	118	-687	530	299	402	-897	-799	-1510
351/1		-1	0	4	11	-55	13	46	-90	201	157	-47	-359	-378	-453	-384	-633	663	-134	628	342	86	-526	-1003	-331	-1406
351/2	351/1	0	0	-4	11	55	13	-46	-90	-201	-157	-47	-359	378	-453	384	633	-663	-134	628	-342	86	-526	1003	331	-1406
357/1		-1	-3	-6	7	18	-8	-17	86	80	-44	112	-256	-270	-380	56	58	194	-530	-296	100	286	-380	-1086	1298	-166
357/2		-5	-3	-1	-7	45	-83	17	-22	134	210	112	331	-228	307	-504	-555	540	-118	-719	40	-855	-429	-1433	35	701
358/1		-2	-8	7	-24	60	-93	33	107	202	207	-288	-2	-204	83	-149	-376	-539	-2	-423	168	-682	-134	119	351	-1666
360/1	40/3	0	0	-5	16	-36	-42	110	-116	-16	-198	240	-258	-442	-292	-392	-142	348	-570	692	-168	-134	784	-564	-1034	-382
360/2		0	0	-5	-18	-34	12	102	164	-48	-146	100	328	288	120	-16	126	-642	602	436	-652	1062	388	444	820	-766
360/3		0	0	-5	34	18	12	-106	-44	56	270	204	120	80	536	-536	542	-174	186	332	-132	-602	-548	-492	-1052	482
360/4		0	0	-5	2	34	-68	38	4	-152	46	-260	-312	-48	-200	-104	414	2	-38	-244	-708	-378	-852	-844	1380	514
360/5	120/2	0	0	-5	-16	28	-26	62	-68	208	58	160	270	-282	76	280	210	-196	742	836	504	-1062	768	1052	726	-1406
360/6	120/3	0	0	-5	8	-20	22	14	76	-56	154	160	-162	390	388	544	210	380	-794	-148	840	858	144	-316	-1098	994
360/7	120/5	0	0	-5	0	-4	54	-114	44	-96	-134	-272	-98	6	12	200	-654	-36	-442	-188	632	-390	688	-1188	694	-1726
360/8	120/1	0	0	5	4	-72	-6	-38	52	-152	78	120	-150	-362	-484	-280	670	-696	222	-4	-96	178	-632	612	-994	1634
360/9	360/4	0	0	5	2	-34	-68	-38	4	152	-46	-260	-312	48	-200	104	-414	-2	-38	-244	708	-378	-852	844	-1380	514
360/10	40/2	0	0	5	-34	-16	58	70	4	134	242	100	-438	138	178	-22	-162	268	250	422	852	306	-456	-434	726	1378
360/11	360/3	0	0	5	34	-18	12	106	-44	-56	-270	204	120	-80	536	536	-542	174	186	332	132	-602	-548	492	1052	482
360/12	40/1	0	0	5	-18	16	-6	6	-124	-42	-142	-188	202	-54	66	-38	-738	-564	-262	-554	-140	882	-1160	-642	854	-478
360/13	360/2	0	0	5	-18	34	12	-102	164	48	146	100	328	-288	120	16	-126	642	602	436	652	1062	388	-444	-820	-766
360/14	120/6	0	0	5	20	-16	58	-38	4	80	-82	-8	426	246	-524	464	702	592	574	-172	-768	-558	408	-164	510	514
360/15	120/4	0	0	5	20	56	-86	106	4	-136	206	-152	282	246	412	-40	126	-56	-2	-388	672	1170	408	-668	-66	-926
361/1	19/1	3	5	-12	11	-54	-11	-93	0	183	249	-56	250	-240	-196	-168	-435	-195	-358	961	246	353	34	234	168	-758
361/2		0	0	14	-36	40	0	14	0	212	0	0	0	0	128	364	0	0	630	0	0	1078	0	-112	0	0
363/1		3	3	-12	12	0	-66	-114	42	18	186	-308	-146	42	-366	618	-408	-132	630	-452	-282	684	1272	-432	954	326
363/2		4	-3	-13	26	0	-73	31	-108	-86	-207	208	45	247	-450	-500	-441	598	378	494	-594	1034	352	360	-351	1079
363/3	363/2	-4	-3	-13	-26	0	73	-31	108	-86	207	208	45	-247	450	-500	-441	598	-378	494	-594	-1034	-352	-360	-351	1079

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
363/4		-1	-3	7	-4	0	-43	41	72	104	273	-272	-165	-403	-120	-220	-741	-112	858	284	-624	-586	-308	0	-321	179
363/5	363/1	-3	3	-12	-12	0	66	114	-42	18	-186	-308	-146	-42	366	618	-408	-132	-630	-452	-282	-684	-1272	432	954	326
363/6	33/2	5	3	-14	32	0	38	2	-72	68	54	-152	174	-94	528	-340	-438	20	-570	-460	-1092	-562	16	-372	-966	-526
363/7	363/4	0	-3	7	4	0	43	-41	-72	104	-273	-272	-165	403	120	-220	-741	-112	-858	284	-624	586	308	0	-321	179
363/8	33/1	0	-3	-4	26	0	32	-74	60	-182	90	-8	-66	-422	-408	-506	348	-200	-132	-1036	762	542	550	132	570	14
364/1		0	7	-8	-7	-57	-13	44	-110	21	-28	-71	43	-113	212	-175	-348	546	529	527	-448	63	135	-1340	-866	-1163
364/2		0	4	-19	7	38	-13	120	-7	-9	171	313	160	354	-197	67	-617	40	-90	490	-540	275	-233	1291	627	-89
364/3		0	0	17	-7	-50	-13	-108	-9	-19	23	-173	156	426	-59	-311	-165	-856	-626	334	472	-253	-1051	405	915	719
365/1		-3	10	5	-16	48	-88	9	137	57	213	-97	-97	201	-421	192	690	186	-304	146	420	73	-196	-612	-399	1280
365/2		-3	-7	-5	-1	-39	-16	-130	-136	-50	-123	-254	41	-215	162	99	-676	691	-304	601	-803	-73	-593	-312	1169	1586
366/1		2	3	-3	-33	-19	-29	-62	-4	-9	160	-276	-32	-93	284	-334	428	123	61	385	-120	-535	-923	-1074	-476	1446
366/2		2	3	-18	12	-44	-74	-2	-44	16	-270	44	-2	-78	184	-344	-702	628	61	80	520	570	812	-44	-1106	-1054
368/1	46/2	0	9	-20	-2	52	43	-50	74	23	-7	273	-4	123	152	-75	86	444	262	-764	21	681	-426	-902	-1272	-342
368/2	184/1	0	-8	-4	4	-26	70	94	-54	23	-86	144	-172	-42	-386	80	-108	-164	-400	-398	320	-810	204	-102	1018	-1370
368/3	23/1	0	5	-6	8	-34	-57	-80	70	-23	245	-103	-298	95	-88	357	-414	408	822	-926	-335	-899	1322	36	-460	-964
368/4	184/2	0	4	22	-8	20	22	98	12	-23	-10	-192	-106	186	-332	544	390	716	110	-836	280	-486	-288	-180	650	-1262
368/5	46/1	0	0	-10	12	42	7	20	-106	-23	-227	-67	74	-497	88	-215	314	-176	-298	-266	981	-411	-806	952	-1332	-1328
369/1		-3	0	-13	-28	-8	-91	-91	-89	68	-64	-173	188	-41	-290	-496	446	183	180	-239	921	-503	-74	47	369	-620
369/2	369/1	3	0	13	-28	8	-91	91	-89	-68	64	-173	188	41	-290	496	-446	-183	180	-239	-921	-503	-74	-47	-369	-620
370/1		2	6	5	3	5	-16	115	110	6	-111	-79	-37	171	361	-428	-527	112	-323	-464	-366	712	176	-180	446	-1407
370/2		2	-2	5	2	-72	2	-66	38	-36	-90	-70	37	-438	272	-198	-354	-498	542	2	408	-358	722	-174	-102	-574
370/3		2	0	5	-25	9	-76	-24	-40	-72	60	26	37	267	-382	267	171	396	-898	-676	-21	-691	-394	309	-918	-766
374/1		-2	4	-8	25	-11	-72	17	-42	102	-23	-224	-58	333	-22	-327	293	347	-866	-991	42	-875	-840	-478	-95	662
378/1		2	0	20	-7	28	3	115	-106	-149	11	81	2	-6	-139	474	-531	-817	498	793	853	490	-330	-404	831	-1424
378/2		2	0	-1	-7	-44	-66	7	-4	-86	176	162	-199	-363	-451	-9	174	587	-156	-560	532	-854	-747	613	1266	64
378/3		2	0	-9	7	-45	-16	-66	11	-27	12	-169	209	-291	-394	-174	-228	-474	-232	992	153	686	1046	-708	-195	-88
378/4		2	0	-15	7	42	-88	45	-106	-114	-66	-304	-187	-69	29	-471	414	597	218	-628	-288	1190	-295	1311	-1206	-1186
378/5		2	0	-7	-7	28	30	-47	164	94	200	162	137	-141	293	-471	306	-331	-204	928	-740	706	-195	-485	-114	-344
378/6		2	0	-7	-7	-17	12	-38	-43	-131	-160	45	-331	111	230	-282	-396	-214	768	388	-551	274	390	-440	-105	304
378/7	378/3	-2	0	9	7	45	-16	66	11	27	-12	-169	209	291	-394	174	228	474	-232	992	-153	686	1046	708	195	-88
378/8	378/1	-2	0	-20	-7	-28	3	-115	-106	149	-11	81	2	6	-139	-474	531	817	498	793	-853	490	-330	404	-831	-1424
378/9	378/4	-2	0	15	7	-42	-88	-45	-106	114	66	-304	-187	69	29	471	-414	-597	218	-628	288	1190	-295	-1311	1206	-1186
378/10	378/2	-2	0	0	-7	44	-66	-7	-4	86	-176	162	-199	363	-451	9	-174	-587	-156	-560	-532	-854	-747	-613	-1266	64
378/11	378/5	-2	0	7	-7	-28	30	47	164	-94	-200	162	137	141	293	471	-306	331	-204	928	740	706	-195	485	114	-344
378/12	378/6	-2	0	7	-7	17	12	38	-43	131	160	45	-331	-111	230	282	396	214	768	388	551	274	390	440	105	304
380/1		0	0	-5	19	20	-77	-11	-19	79	-303	214	-250	-230	-402	48	-417	99	332	-319	-1088	-373	102	934	498	-1386
384/1		0	-3	4	-10	-4	26	14	8	-148	72	-18	262	-378	-432	-148	360	-428	-442	-692	-540	-1018	-386	108	-382	298
384/2	384/1	0	-3	-4	10	-4	-26	14	8	148	-72	18	-262	-378	-432	148	-360	-428	442	-692	540	-1018	386	108	-382	298
384/3		0	-3	8	10	68	-46	-74	16	20	228	162	262	30	264	-124	-204	340	950	-436	780	518	1010	852	-686	-806
384/4	384/3	0	-3	-8	-10	68	46	-74	16	-20	-228	-162	-262	30	264	124	204	340	-950	-436	-780	518	-1010	852	-686	-806
384/5	384/3	0	3	8	-10	-68	-46	-74	-16	-20	228	-162	262	30	-264	124	-204	-340	950	436	-780	518	-1010	-852	-686	-806
384/6	384/3	0	3	-8	10	-68	46	-74	-16	20	-228	162	-262	30	-264	-124	204	-340	-950	436	780	518	1010	-852	-686	-806
384/7	384/1	0	3	4	10	4	26	14	-8	148	72	18	262	-378	432	148	360	428	-442	692	540	-1018	386	-108	-382	298
384/8	384/1	0	3	-4	-10	4	-26	14	-8	-148	-72	-18	-262	-378	432	-148	-360	428	442	692	-540	-1018	-386	-108	-382	298
385/1		3	4	-5	-7	11	-46	106	-140	-128	210	-252	-78	442	-356	-72	466	316	-682	224	-528	-142	148	1112	-254	1694
385/2		0	-2	5	7	-11	-52	48	68	-66	66	-340	242	-54	524	390	522	744	830	170	-636	296	1160	-684	-642	-562
385/3		0	-2	5	7	11	80	-84	68	-198	-198	56	-286	78	260	-402	-534	-180	-622	-1018	-900	956	-424	636	-378	758
385/4		0	10	-5	7	11	54	86	-98	-82	4	112	196	120	148	464	-488	-368	-614	-836	948	-554	50	-484	-690	-1368
385/5		0	2	-5	7	11	22	6	70	182	-20	32	76	352	132	-624	592	720	442	-164	452	-698	-950	-628	30	656
387/1	129/1	-4	0	-11	9	-57	43	66	25	112	-75	32	-36	268	-43	611	-148	-780	-328	-246	-902	-502	-380	-753	-50	-1391
387/2	129/2	0	0	2	6	48	-62	66	-92	-106	18	-196	0	-502	-43	-74	40	-744	752	36	224	-1006	376	-732	1334	-242
390/1		2	3	-5	-28	-36	13	42	-112	-168	-210	-76	278	150	-460	-264	582	-204	614	-304	1080	-934	128	348	-834	-1582
390/2		2	-3	5	-12	-48	13	-62	-32	-8	-58	-124	-162	74	-396	-164	270	-416	70	448	-1092	10	328	-144	-502	1042
390/3		2	-3	-5	-25	-21	13	123	146	99	-246	182	-295	9	452	390	315	-24	-727	596	771	326	-889	-96	795	983
390/4		2	-3	-5	8	-40	-13	10	0	-180	22	-144	34	-502	-76	-168	-422	104	-82	-540	512	622	104	348	-286	494
390/5		2	-3	-5	8	12	13	-42	-52	132	282	116	398	174	-76	456	150	-156	230	-592	408	-730	728	36	-1482	1742
390/6		-2	-3	-5	-14	-36	-13	68	-158	46	-8	-176	62	30	252	-120	758	252	398	884	-80	-660	568	1084	1250	84
390/7		-2	-3	5	5	-35	-13	23	-30	63	-190	330	43	-473	-232	270	-193	-200	-679	-12	-899	154	215	-1308	-1019	-427
390/8		-2	3	5	-13	-15	13	-75	-130	45	-138	-34	-379	243	416	378	-3	-816	-607	-700	57	-1162	-1	672	969	-949
390/9		-2	3	-5	2	0	13	-60	50	210	-228	116	386	378	-4	-312	-198	624	638	200	-408	1148	824	1332	54	-244
390/10		-2	3	-5	-15	39	-13	-15	54	-143	-122	-246	-225	469	-484	234	33	0	-831	772	-793	-998	-681	-772	-465	-79
390/11		-2	3	-5	24	0	13	50	28	-208	190	248	-186	-194	348	260	462	-520	-506	772	780	-62	736	1464	406	922

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
392/1	56/1	0	-6	-8	0	56	28	90	-74	-96	-222	100	58	-422	512	-148	-642	318	-720	-412	448	-994	-296	-386	6	138
392/2		0	-4	12	0	12	-76	8	100	-56	-166	232	-414	-72	-452	-424	-18	-444	284	524	-1008	-896	-40	-1388	-448	824
392/3	56/2	0	2	16	0	24	68	-54	46	176	-174	116	74	10	-480	572	-162	86	904	660	1024	-770	-904	-682	102	218
392/4	392/2	0	4	-12	0	12	76	-8	-100	-56	-166	-232	-414	72	-452	424	-18	444	-284	524	-1008	896	-40	1388	448	-824
392/5	8/1	0	4	2	0	-44	-22	-50	-44	-56	198	160	-162	198	52	-528	-242	668	-550	188	728	-154	-656	-236	-714	478
396/1		0	0	-12	26	-11	-34	-126	110	180	18	-292	-238	-426	146	-528	-408	-324	-550	824	-552	-850	866	660	-768	-286
396/2	44/1	0	0	7	-26	11	52	-46	-96	-27	-16	-293	-29	472	-110	224	-754	-825	-548	-123	-1001	-1020	526	158	1217	-263
396/3	132/3	0	0	-22	-20	-11	22	-110	48	-72	142	184	-194	482	-80	-392	34	108	382	84	1040	-606	-1292	-356	406	1090
396/4	132/4	0	0	-10	8	11	18	-46	40	-44	-186	-72	-114	-174	-416	156	62	348	-446	-956	444	306	-664	124	-602	1522
396/5	132/1	0	0	0	2	11	-88	66	-40	-6	54	8	-106	-354	-124	-546	408	-552	404	-4	-126	-166	-874	-444	-1002	-802
396/6	132/2	0	0	12	14	-11	56	-42	116	30	-198	-88	350	-198	56	594	204	312	620	356	462	482	-238	-492	-954	-1426
396/7	396/1	0	0	12	26	11	-34	126	110	-180	-18	-292	-238	426	146	528	408	324	-550	824	552	-850	866	-660	768	-286
399/1		3	-3	-8	-7	18	68	4	-19	118	166	304	350	378	-456	-304	-394	844	-418	130	-404	58	-178	828	-870	948
399/2		-1	3	-12	7	34	0	-16	19	6	-138	-240	-218	-238	-376	352	6	-268	814	-694	100	-422	-666	756	-446	-728
400/1	40/2	0	-6	0	-34	-16	-58	70	-4	-134	-242	-100	438	-138	178	22	-162	268	250	422	852	-306	456	434	-726	-1378
400/2	200/6	0	9	0	26	59	-28	-5	-109	-194	-32	-10	198	117	388	-68	18	-392	-710	-253	612	549	-414	-121	-81	1502
400/3	10/1	0	-8	0	-4	-12	58	-66	100	132	-90	-152	34	-438	32	-204	-222	-420	902	-1024	-432	-362	160	72	810	-1106
400/4	200/2	0	-5	0	-2	-39	84	-61	-151	58	192	18	-138	229	164	212	578	336	858	209	780	-403	230	1293	-1369	382
400/5	40/1	0	10	0	-18	16	6	6	124	42	142	188	-202	54	66	38	-738	-564	-262	-554	-140	-882	1160	642	-854	478
400/6	50/3	0	-2	0	-26	28	12	-64	60	58	90	128	236	242	-362	-226	-108	20	542	434	1128	632	720	478	-490	1456
400/7	200/6	0	-9	0	-26	59	28	5	-109	194	-32	-10	-198	117	-388	68	-18	-392	-710	253	612	-549	-414	121	-81	-1502
400/8	200/2	0	5	0	2	-39	-84	61	-151	-58	192	18	138	229	-164	-212	-578	336	858	-209	780	403	230	-1293	-1369	-382
400/9	8/1	0	-4	0	24	44	-22	-50	-44	-56	198	160	162	-198	52	528	242	668	550	188	-728	-154	656	236	714	478
400/10	50/1	0	7	0	-34	-27	28	-21	-35	-78	-120	-182	-146	357	-148	-84	-702	840	-238	461	708	133	-650	-903	735	-1106
400/11	25/1	0	7	0	6	43	28	-91	35	162	160	-42	314	-203	92	196	-82	280	-518	141	-412	763	-510	777	-945	-1246
400/12	50/3	0	2	0	26	28	-12	64	60	-58	90	128	-236	242	362	226	108	20	542	-434	1128	-632	720	-478	-490	-1456
400/13	5/1	0	2	0	6	-32	38	-26	-100	-78	-50	108	-266	22	442	-514	-2	-500	-518	126	-412	878	-600	282	-150	-386
400/14	25/1	0	-7	0	-6	43	-28	91	35	-162	160	-42	-314	-203	-92	-196	82	280	-518	-141	-412	-763	-510	-777	-945	1246
400/15	50/1	0	-7	0	34	-27	-28	21	-35	78	-120	-182	146	357	148	84	702	840	-238	-461	708	-133	-650	903	735	1106
400/16	20/1	0	4	0	-16	60	-86	-18	-44	48	-186	-176	-254	186	-100	168	498	252	-58	-1036	-168	-506	-272	948	-1014	766
400/17	40/3	0	4	0	16	-36	42	110	116	16	198	-240	258	442	-292	392	-142	348	-570	692	-168	134	-784	564	1034	382
400/18	100/1	0	-1	0	-26	-45	44	117	91	18	144	-26	-214	-459	460	468	558	72	-118	-251	-108	299	898	-927	351	386
400/19	200/1	0	-1	0	6	19	12	-75	91	-174	-272	230	-182	117	-372	52	-402	-312	170	-763	52	-981	-1054	-351	799	962
400/20	200/1	0	0	0	-6	19	-12	75	91	174	-272	230	182	117	372	-52	402	-312	170	763	52	981	-1054	351	799	-962
400/21	100/1	0	0	0	26	-45	-44	-117	91	-18	144	-26	214	-459	-460	-468	-558	72	-118	251	-108	-299	898	927	351	-386
402/1		-2	-3	14	20	68	18	42	76	-132	-22	-244	142	-406	316	-204	558	-380	578	-67	-260	282	916	1140	-1350	-1286
405/1		-5	0	5	9	-8	43	-122	-59	-213	224	-36	206	413	-392	-311	-377	337	40	348	62	-1214	-294	534	-810	-928
405/2	405/1	5	0	-5	9	8	43	122	-59	213	-224	-36	206	-413	-392	311	377	-337	40	348	-62	-1214	-294	-534	810	-928
408/1		0	-3	6	-24	44	6	17	-20	-152	270	-272	-250	186	260	-320	-770	-348	-210	-148	-360	-646	-1168	-788	-1238	882
408/2		0	3	-7	4	-21	-25	17	-69	15	58	-298	72	-369	-59	-138	262	50	-568	124	100	-158	710	214	-1016	-1780
410/1		-2	2	5	-2	16	8	-20	-120	-164	54	64	146	-41	28	294	-668	116	-822	362	-868	-14	-1324	-264	190	-96
414/1	138/3	-2	0	2	-34	-2	-74	68	88	23	178	240	-76	-186	28	-264	598	-492	352	-244	984	1014	-438	682	1524	-198
414/2	46/2	-2	0	20	2	52	43	50	-74	23	7	-273	-4	-123	-152	-75	-86	444	262	764	21	681	426	-902	1272	-342
414/3	138/2	2	0	-2	-32	48	22	-42	-144	23	-174	-304	-318	-74	192	-392	734	-156	706	192	-624	-406	696	800	102	-918
414/4	46/1	2	0	10	-12	42	7	-20	106	-23	227	67	74	497	-88	-215	-314	-176	-298	266	981	-411	806	952	1332	-1328
414/5	138/1	2	0	10	32	20	-26	46	-92	-23	194	-120	-322	-42	220	192	170	-396	934	-988	552	282	-888	908	-1242	-30
416/1		0	-5	-3	5	30	13	-19	70	20	-30	-100	-111	-180	85	-295	-132	-230	-220	-670	55	-602	-360	-540	-270	-606
416/2		0	-1	-1	-5	-10	-13	93	82	192	-106	-172	379	-148	329	631	160	478	300	722	-335	90	788	-96	-866	-998
416/3	416/2	0	1	-1	5	10	-13	93	-82	-192	-106	172	379	-148	-329	-631	160	-478	300	-722	335	90	-788	96	-866	-998
416/4	416/1	0	5	-3	-5	-30	13	-19	-70	-20	-30	100	-111	-180	-85	295	-132	230	-220	670	-55	-602	360	540	-270	-606
418/1		-2	8	-14	-6	11	-2	14	19	208	130	262	204	342	-352	624	-232	-360	-348	-96	-118	498	740	428	850	374
420/1		0	-3	-5	-7	32	42	-38	-36	96	-198	-220	-46	-290	-152	124	62	68	-614	-456	-416	-826	-272	508	110	-874
420/2		0	-3	5	7	-36	-34	-6	-28	192	-186	176	-418	-30	-412	-432	-306	-564	-322	716	-48	-1078	-496	468	1314	-1438
420/3		0	3	-5	7	-36	-34	-30	-16	-48	-126	8	74	-138	-352	-396	-78	-60	-70	-664	156	410	344	444	-1002	290
420/4		0	3	5	-7	-44	-42	-94	-36	24	54	-112	-322	-22	292	272	-578	-44	-26	12	-280	410	-320	-1252	-38	1250
420/5		0	3	5	7	-16	-14	130	104	-88	54	28	-266	202	348	104	402	-100	310	-324	-644	-290	744	1044	298	-290
420/6		0	3	5	7	36	38	-78	-52	120	54	80	254	-6	-172	0	-66	420	-106	92	1176	698	-1024	-516	714	-862
425/1	85/3	-3	-10	0	22	-30	46	-17	104	-42	-66	194	-206	-126	388	540	-78	432	-610	-848	-174	-362	398	-828	630	1486
425/2	85/1	-3	5	0	22	60	31	-17	-61	78	69	-31	-56	-6	538	465	-723	-753	35	322	-99	1123	488	852	1215	601
425/3	85/2	-3	7	0	22	-64	-73	17	-49	-110	155	-197	372	-262	-258	13	653	-333	-355	-814	47	437	-384	736	511	-537
425/4	17/1	3	8	0	28	-24	58	-17	116	60	30	-172	58	-342	148	-288	-318	252	110	484	-708	-362	-484	-756	-774	382
427/1		0	-1	4	-7	63	-46	-135	3	53	218	196	92	-100	401	381	76	-176	-61	913	-79	760	89	751	-115	1124

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
429/1		-1	-3	-19	19	11	-13	44	-90	163	255	-68	-426	-13	63	124	-642	-605	-363	-91	582	283	830	1278	480	-1306
430/1		-2	0	5	22	-43	47	-111	-62	-43	-44	45	220	-115	-43	-24	-533	108	-304	-525	-546	-116	-192	-927	1302	-1613
430/2		2	-8	5	-21	16	93	-60	-47	-208	49	-147	-24	-135	43	-474	-750	452	-69	-65	-418	-1021	235	-264	212	-1106
430/3		2	-8	5	22	-27	7	-103	82	93	92	-147	148	381	-43	472	755	108	-112	107	614	-548	880	553	-906	-805
432/1	27/1	0	0	-15	25	-15	20	-72	-2	114	-30	-101	-430	30	-110	-330	-621	-660	-376	250	-360	785	-488	489	450	-1105
432/2	54/2	0	0	-12	7	60	-79	108	-11	-132	-96	-20	-169	-192	-488	204	-360	156	83	-47	216	-511	529	-1128	-36	605
432/3	108/2	0	0	-9	1	63	-28	-72	-98	126	126	259	386	450	34	-54	693	180	-280	586	504	161	-440	999	-882	-721
432/4	216/1	0	0	-4	-3	28	-11	-44	-29	172	-192	-116	-69	-384	-328	156	392	412	-425	-257	-1000	-359	-877	-328	1572	-1483
432/5	54/1	0	0	-3	-29	-57	20	72	106	174	210	-47	2	6	-218	474	-81	84	56	142	360	-1159	160	735	954	191
432/6	216/2	0	0	-1	9	-17	-44	-56	94	-50	30	139	-174	-318	242	-630	-547	-236	328	-614	296	433	56	-1225	-1506	1391
432/7	108/3	0	0	0	-17	0	89	0	-107	0	0	-308	-433	0	520	0	0	0	-901	-1007	0	-271	-503	0	0	1853
432/8	108/1	0	0	0	37	0	-19	0	163	0	0	-308	323	0	520	0	0	0	719	127	0	-919	1387	0	0	-523
432/9	216/2	0	0	1	9	17	-44	56	94	50	-30	139	-174	318	242	630	547	236	328	-614	-296	433	56	1225	1506	1391
432/10	54/1	0	0	3	-29	57	20	-72	106	-174	-210	-47	2	-6	-218	-474	81	-84	56	142	-360	-1159	160	-735	-954	191
432/11	216/1	0	0	4	-3	-28	-11	44	-29	-172	192	-116	-69	384	-328	-156	-392	-412	-425	-257	1000	-359	-877	328	-1572	-1483
432/12	108/2	0	0	9	1	-63	-28	72	-98	-126	-126	259	386	-450	34	54	-693	-180	-280	586	-504	161	-440	-999	882	-721
432/13	54/2	0	0	12	7	-60	-79	-108	-11	132	96	-20	-169	192	-488	-204	360	-156	83	-47	-216	-511	529	1128	36	605
432/14	27/1	0	0	15	25	15	20	72	-2	-114	30	-101	-430	-30	-110	330	621	660	-376	250	360	785	-488	-489	-450	-1105
434/1		-2	-9	-3	-7	28	-68	-46	-102	-75	65	-31	-314	-64	354	-121	-350	-672	492	857	520	-193	911	-271	1251	992
435/1		-2	-3	5	29	-15	3	121	-40	-116	29	-116	36	-170	230	231	456	576	342	-269	302	-372	-348	-512	1525	-560
435/2		-1	3	5	4	-36	-22	-2	-56	-40	29	152	34	-250	-412	-120	-762	-188	-54	-244	600	6	-640	664	150	-1690
435/3		5	-3	5	16	-44	78	18	-28	184	29	-224	254	-78	-260	312	574	180	-610	-340	296	394	-960	-908	-990	1234
438/1		2	-3	12	16	-10	40	-94	160	-24	108	-268	90	154	430	-36	56	618	454	444	-144	73	-480	906	-714	-1186
440/1		0	-5	5	1	11	18	-113	55	190	-69	-255	51	-314	-484	470	-545	-102	129	-664	-1029	-758	634	-654	-511	1736
440/2		0	-4	5	8	11	-58	114	-4	-152	-138	208	-226	-294	276	-240	-370	-716	-650	124	232	-454	-144	-692	-1206	-1438
440/3		0	0	5	16	-11	-70	-10	-12	-84	30	-72	310	18	-388	-516	-298	204	-210	-432	-440	46	-616	740	-6	490
440/4		0	6	5	-32	11	-48	-36	-44	58	-278	-112	194	-314	396	-410	170	404	250	-26	-468	-164	-664	1348	534	-1498
441/1	147/3	-4	0	-18	0	50	-36	-126	-72	-14	-158	-36	-162	270	-324	72	22	-468	792	232	734	180	236	-36	-234	468
441/2	21/1	-4	0	-4	0	-62	62	84	-100	42	10	48	-246	-248	68	324	-258	120	-622	904	678	642	740	468	200	1266
441/3	147/3	-4	0	18	0	50	36	126	72	-14	-158	36	-162	-270	-324	-72	22	468	-792	232	734	-180	236	36	234	-468
441/4	49/1	-2	0	-7	0	5	-14	21	49	159	-58	147	219	-350	-124	-525	-303	105	-413	415	432	-1113	-103	-1092	329	-882
441/5	49/1	-2	0	7	0	5	14	-21	-49	159	-58	-147	219	350	-124	525	-303	-105	413	415	432	1113	-103	1092	-329	882
441/6	9/1	0	0	0	0	0	70	0	-56	0	0	-308	110	0	-520	0	0	0	-182	-880	0	-1190	884	0	0	1330
441/7	147/1	1	0	-12	0	-20	-84	96	12	176	-58	-264	258	0	156	408	722	-492	-492	412	-296	240	776	-924	744	-168
441/8	147/1	1	0	12	0	-20	84	-96	-12	176	-58	264	258	0	156	-408	722	492	492	412	-296	-240	776	924	-744	168
441/9	7/1	1	0	16	0	8	-28	54	110	-48	110	-12	-246	182	128	324	162	810	488	244	768	702	440	-1302	730	-294
441/10	21/2	3	0	-18	0	36	34	42	124	0	-102	160	398	-318	-268	240	498	-132	-398	92	720	502	-1024	-204	354	286
441/11	147/2	3	0	-3	0	15	64	84	16	84	297	253	-316	360	26	-30	-363	-15	118	-370	342	-362	467	477	906	-503
441/12	147/2	3	0	3	0	15	-64	-84	-16	84	297	-253	-316	-360	26	30	-363	15	-118	-370	342	362	467	-477	-906	503
441/13	49/4	5	0	0	0	68	0	0	0	40	166	0	450	0	-180	0	-590	0	0	-740	-688	0	-1384	0	0	0
442/1		-2	-8	16	-22	10	-13	17	-120	148	282	10	-356	316	188	-456	194	-64	-754	-536	-858	-692	840	-860	614	-1520
442/2		-2	-4	4	26	-38	-13	17	-64	200	-134	82	392	-248	-348	-160	-526	-744	102	-304	-618	1064	-460	-228	646	-1116
442/3		2	8	12	-10	-22	-13	17	148	112	-10	22	-144	68	-164	-52	-78	-556	-182	-236	-542	-628	228	-952	1214	1528
444/1		0	-3	-4	-25	67	57	27	-17	-107	-4	-274	-37	-342	52	82	17	-420	610	110	-960	205	-1330	51	-533	178
445/1		-1	2	-5	12	8	64	-94	46	-12	-164	276	-100	-338	-266	-352	-202	818	-560	0	-320	-338	-624	-810	89	542
448/1	28/1	0	-10	8	7	-40	12	-58	26	64	62	-252	-26	6	416	396	450	274	576	-476	448	-158	936	530	-390	214
448/2	14/1	0	-8	14	-7	28	-18	74	-80	-112	-190	72	346	162	412	24	-318	200	198	716	392	538	240	1072	810	1354
448/3	56/1	0	-6	-8	-7	-56	28	-90	-74	-96	222	-100	-58	422	-512	148	642	318	-720	412	448	994	-296	-386	-6	-138
448/4	28/2	0	-4	-6	7	12	82	-30	-68	216	-246	-112	-110	-246	172	192	-558	-540	-110	-140	-840	-550	-208	-516	-1398	1586
448/5	7/1	0	-2	-16	7	-8	-28	54	-110	-48	110	-12	246	182	128	-324	162	810	488	244	768	-702	-440	-1302	730	294
448/6	224/1	0	-2	0	-7	20	20	-50	10	72	134	180	270	-250	92	236	-150	570	200	176	640	250	640	882	1074	270
448/7	14/2	0	-2	12	-7	48	-56	-114	2	120	54	-236	-146	126	-376	12	-174	138	-380	-484	-576	-1150	-776	378	-390	-1330
448/8	56/2	0	-2	16	7	24	68	54	-46	-176	174	116	-74	-10	-480	572	162	-86	904	660	-1024	770	904	682	-102	-218
448/9	7/1	0	2	-16	-7	8	-28	54	110	48	110	12	246	182	-128	324	162	-810	488	-244	-768	-702	440	1302	730	294
448/10	224/1	0	2	0	7	-20	20	-50	-10	-72	134	-180	270	-250	-92	-236	-150	-570	200	-176	-640	250	-640	-882	1074	270
448/11	14/2	0	2	12	7	-48	-56	-114	-2	-120	54	236	-146	126	376	-12	-174	-138	-380	484	576	-1150	776	-378	-390	-1330
448/12	56/2	0	2	16	-7	-24	68	54	46	176	174	-116	-74	-10	480	-572	162	86	904	-660	1024	770	-904	-682	-102	-218
448/13	28/2	0	4	-6	-7	-12	82	-30	68	-216	-246	112	-110	-246	-172	-192	-558	540	-110	140	840	-550	208	516	-1398	1586
448/14	56/1	0	6	-8	7	56	28	-90	74	96	222	100	-58	422	512	-148	642	-318	-720	-412	-448	994	296	386	-6	-138
448/15	14/1	0	8	14	7	-28	-18	74	80	112	-190	-72	346	162	-412	-24	-318	-200	198	-716	-392	538	-240	-1072	810	1354
448/16	28/1	0	10	8	-7	40	12	-58	-26	-64	62	252	-26	6	-416	-396	450	-274	576	476	-448	-158	-936	-530	-390	214
450/1	50/1	-2	0	0	-34	-27	-28	-21	35	78	120	182	146	-357	-148	84	-702	840	-238	461	708	-133	650	903	-735	1106

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
450/2	30/1	-2	0	0	-32	60	34	42	-76	0	-6	-232	-134	-234	412	-360	222	-660	-490	-812	-120	-746	152	-804	678	-194
450/3	90/1	-2	0	0	-14	-6	-68	78	44	120	-126	-244	304	480	-104	600	-258	-534	362	268	972	-470	1244	396	972	46
450/4		-2	0	0	-11	36	-17	-12	-91	60	276	191	-254	60	49	-600	612	744	167	457	588	970	164	696	1248	1099
450/5	150/4	-2	0	0	-2	-70	54	22	24	100	-216	208	-254	206	292	320	402	370	-550	728	540	604	792	-404	938	56
450/6	150/2	-2	0	0	1	-42	67	54	-115	-162	210	-193	286	-12	-263	414	-192	-690	-733	-299	228	-938	-160	-462	240	511
450/7	450/4	-2	0	0	11	-36	17	-12	-91	60	-276	191	254	-60	-49	-600	612	-744	167	-457	-588	-970	164	696	-1248	-1099
450/8	6/1	-2	0	0	16	-12	-38	-126	20	168	-30	-88	-254	-42	52	-96	198	660	-538	-884	-792	-218	-520	-492	-810	-1154
450/9	150/3	-2	0	0	23	30	29	-78	149	-150	234	-217	146	156	-433	-30	552	270	275	803	-660	-646	992	846	1488	-319
450/10	50/3	-2	0	0	26	28	12	64	-60	58	-90	-128	236	-242	362	-226	108	20	542	-434	1128	632	-720	478	490	1456
450/11	50/3	2	0	0	-26	28	-12	-64	-60	-58	-90	-128	-236	-242	-362	226	-108	20	542	434	1128	-632	-720	-478	490	-1456
450/12	150/3	2	0	0	-23	30	-29	78	149	150	234	-217	-146	156	433	30	-552	270	275	-803	-660	646	992	-846	1488	319
450/13	90/1	2	0	0	-14	6	-68	-78	44	-120	126	-244	304	-480	-104	-600	258	534	362	268	-972	-470	1244	-396	-972	46
450/14	450/4	2	0	0	-11	-36	-17	12	-91	-60	-276	191	-254	-60	49	600	-612	-744	167	457	-588	970	164	-696	-1248	1099
450/15	150/2	2	0	0	-1	-42	-67	-54	-115	162	210	-193	-286	-12	263	-414	192	-690	-733	299	228	938	-160	462	240	-511
450/16	150/4	2	0	0	2	-70	-54	-22	24	-100	-216	208	254	206	-292	-320	-402	370	-550	-728	540	-604	792	404	938	-56
450/17	10/1	2	0	0	4	-12	58	66	-100	132	90	152	34	438	-32	-204	222	-420	902	1024	-432	-362	-160	72	-810	-1106
450/18	30/2	2	0	0	4	48	-2	-114	140	72	-210	272	334	198	268	216	-78	-240	302	-596	768	478	-640	-348	-210	1534
450/19	450/4	2	0	0	11	36	17	12	-91	-60	276	191	254	60	-49	600	-612	744	167	-457	588	-970	164	-696	1248	-1099
450/20	50/1	2	0	0	34	-27	28	21	35	-78	120	182	-146	-357	148	-84	702	840	-238	-461	708	133	650	-903	-735	-1106
453/1		-3	3	-19	-32	-7	-57	-66	-90	-195	-267	21	190	9	462	-220	-360	-676	-166	101	789	-188	-770	952	-1335	1058
455/1		-3	-5	5	-7	-25	-13	46	74	-55	82	325	-181	201	298	-421	-432	24	-75	-911	464	-285	-335	306	-782	-1739
455/2		-3	1	5	7	-9	13	-6	-70	105	-258	263	371	111	-538	-411	60	156	-529	173	-372	-727	-1051	-198	-930	-745
455/3		-3	8	5	-7	40	13	-6	-56	140	134	-52	14	370	64	-96	-458	856	-634	-716	412	1002	328	124	50	1810
455/4		1	-4	5	-7	-12	13	-94	-4	-24	6	-160	350	362	452	-384	-546	148	870	396	744	474	-176	188	-646	1090
462/1		-2	-3	-21	7	-11	65	-54	65	132	39	-178	-439	96	272	-375	612	-507	758	-1087	0	-673	-700	1218	-1350	-808
462/2		-2	-3	-14	-7	11	38	54	40	8	-170	92	294	-258	-52	-76	-322	260	22	-436	-368	-2	-200	-952	-70	-1086
462/3		-2	-3	11	-7	11	-37	-46	15	-92	205	142	-431	-8	448	149	-672	-615	322	-411	-968	-227	0	-1302	-870	-1736
462/4		-2	3	-7	7	11	-67	30	-7	28	121	-310	-71	-180	-108	71	128	-429	22	-803	468	-117	-96	-1122	-1146	-92
462/5		2	-3	-4	-7	-11	62	-120	118	-188	62	-322	-198	48	32	-326	-482	400	70	-124	-712	304	-1016	430	442	-966
462/6		2	-3	1	-7	-11	-43	100	-87	-58	-223	88	37	128	-458	-341	-342	-105	190	-579	128	-161	-396	-420	-798	1414
462/7		2	-3	3	7	-11	41	6	-43	120	111	266	-79	216	284	213	-216	393	350	821	-264	-865	-484	1158	330	980
462/8		2	3	-17	7	-11	-21	-104	-161	194	9	-180	-363	-108	-386	333	-122	537	-950	-83	180	177	-220	1112	-394	826
462/9		2	3	-13	-7	11	-67	8	21	-194	-221	88	-347	292	-458	221	-642	273	-530	561	604	703	552	-144	750	-1370
464/1	58/1	0	-7	5	2	-37	27	24	88	28	-29	143	-360	386	-381	103	-431	-288	-840	180	-706	716	-931	-1188	-642	486
464/2	58/2	0	7	-15	18	-27	-57	-44	-152	152	-29	173	-120	-314	-339	357	-59	572	-420	-660	-726	1004	-361	168	58	-1206
465/1		-4	3	5	22	-20	-72	-80	40	-112	-258	31	-364	-114	468	206	-384	-40	586	398	-804	8	-600	-172	-838	-1026
465/2		-3	-3	-5	-24	-38	-18	-44	-136	-62	-296	-31	-278	-210	476	-276	10	-632	-732	316	984	-12	1196	24	-738	238
465/3		-1	3	5	7	-41	-18	-8	-95	-25	264	31	-220	-96	-147	-490	117	-586	-230	-334	-1059	-187	1077	-520	1361	54
468/1	156/2	0	0	2	-32	68	13	14	4	-72	-102	-136	-386	-250	-140	296	-526	-332	-410	596	880	506	-640	-1380	-1450	-446
468/2	156/1	0	0	6	-4	-36	13	-66	56	-96	-222	260	-106	90	44	-168	-30	-348	-346	-256	168	-814	200	-1236	-318	-502
468/3	52/1	0	0	13	-11	2	-13	51	150	4	118	-116	63	288	-293	335	708	-566	904	382	-7	518	-100	1440	-1254	1262
470/1		-2	1	5	-19	-45	-61	120	119	78	-18	131	188	78	62	47	714	492	-241	-466	-12	605	-1252	-87	-1554	1034
470/2		2	-5	5	-25	-3	23	-24	137	162	-294	-91	344	306	158	-47	186	648	-553	1034	228	-7	776	987	-1410	-298
474/1		-2	3	18	9	-35	7	111	66	133	-279	-220	-76	2	463	432	-42	-418	-372	622	438	-297	79	167	932	-1625
474/2		2	3	-6	-27	23	-53	45	-102	35	-177	-268	200	-386	-497	168	-486	718	444	838	42	831	79	-827	868	-1481
475/1	95/4	-5	-4	0	32	-12	42	-114	19	-160	214	-144	-94	-6	308	-184	274	276	-826	-52	-344	166	-688	-996	1578	-786
475/2	95/3	-3	-7	0	-11	-36	-65	87	19	129	231	110	142	-330	-74	336	-501	633	-88	-119	-204	-407	1262	-270	-30	-1406
475/3	95/2	-3	5	0	1	-24	31	-33	19	-27	111	-94	70	-510	34	192	75	45	-28	-371	384	73	-1234	-366	-1578	538
475/4	95/1	0	-4	0	22	-12	-8	66	19	30	-6	-64	16	54	-182	-594	-396	-564	-706	628	-984	-14	-328	294	918	1564
475/5	19/1	3	5	0	-11	-54	-11	93	19	-183	-249	56	250	240	196	168	-435	195	-358	961	-246	-353	-34	-234	-168	-758
477/1		-1	0	18	-31	57	-54	-78	43	19	71	-218	-238	387	-356	4	53	-356	-731	-711	-17	-928	-34	-1072	1324	-1559
477/2	53/1	0	0	18	2	-54	-43	99	-61	-207	99	-160	-7	414	-268	-270	-53	-450	182	-556	-693	-862	119	333	-1350	-187
477/3	477/1	1	0	-18	-31	-57	-54	78	43	-19	-71	-218	-238	-387	-356	-4	-53	356	-731	-711	17	-928	-34	1072	-1324	-1559
477/4	159/2	5	0	-3	-16	57	54	39	82	49	260	-5	-250	276	-137	-206	53	-473	-830	366	-248	-358	-1075	602	-47	1027
477/5	159/1	5	0	21	-16	45	-66	123	-62	-47	-76	-17	-370	-324	343	394	53	379	-398	-234	-56	-142	305	434	565	-581
480/1		0	-3	-5	-12	20	-58	-70	92	-112	66	108	-58	66	388	408	474	540	14	276	96	-790	-308	1036	1210	1426
480/2		0	-3	-5	8	-4	-6	-2	16	60	-142	176	-214	-278	68	-116	-350	-684	-394	-108	96	-398	-136	-436	-750	82
480/3		0	-3	5	-32	-64	-6	38	116	120	-122	-164	146	-238	148	184	470	216	806	732	-264	-638	-596	884	930	322
480/4		0	-3	5	-4	40	-90	-70	40	108	166	-40	-130	-310	-268	-556	-370	240	-130	876	-840	250	-880	-188	-726	-1550
480/5		0	-3	5	12	24	38	-6	-104	-100	230	56	190	202	148	-124	206	128	190	204	440	1210	-816	1412	-214	1202
480/6		0	-3	5	16	24	-14	-18	36	104	-250	-28	-54	354	228	408	262	-64	374	300	1016	274	788	-396	786	-1086
480/7	480/2	0	3	-5	-8	4	-6	-2	-16	-60	-142	-176	-214	-278	-68	116	-350	684	-394	108	-96	-398	136	436	-750	82

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
480/8	480/1	0	3	-5	12	-20	-58	-70	-92	112	66	-108	-58	66	-388	-408	474	-540	14	-276	-96	-790	308	-1036	1210	1426
480/9	480/6	0	3	5	-16	-24	-14	-18	-36	-104	-250	28	-54	354	-228	-408	262	64	374	-300	-1016	274	-788	396	786	-1086
480/10	480/5	0	3	5	-12	-24	38	-6	104	100	230	-56	190	202	-148	124	206	-128	190	-204	-440	1210	816	-1412	-214	1202
480/11	480/4	0	3	5	4	-40	-90	-70	-40	-108	166	40	-130	-310	268	556	-370	-240	-130	-876	840	250	880	188	-726	-1550
480/12	480/3	0	3	5	32	64	-6	38	-116	-120	-122	164	146	-238	-148	-184	470	-216	806	-732	264	-638	596	-884	930	322
482/1		-2	-2	7	-23	20	-38	80	-149	207	-15	112	-176	453	-175	184	-715	-430	319	820	835	1034	632	708	968	-509
484/1	44/1	0	-5	-7	26	0	-52	-46	96	27	-16	-293	-29	472	110	-224	754	825	548	-123	1001	1020	-526	158	-1217	-263
486/1		-2	0	-12	-19	66	-73	-54	-145	42	102	179	89	-372	215	318	-198	24	326	20	1098	218	665	1002	1296	-1495
486/2		-2	0	-6	5	-18	29	12	29	0	-198	71	383	282	-163	-546	-642	342	-70	-232	-114	-430	389	-960	-618	-1015
486/3		-2	0	3	-22	-36	-16	30	11	99	297	62	-76	318	-388	129	-363	684	578	551	-879	1001	-736	714	1020	911
486/4		-2	0	6	8	30	-55	90	89	78	-96	-55	-46	132	143	-60	738	546	-241	-493	594	245	1061	-1374	900	1709
486/5		-2	0	18	11	6	77	96	-85	12	222	-271	-241	18	-325	-42	282	-726	146	920	798	-502	635	1092	-1434	545
486/6	486/5	2	0	-18	11	-6	77	-96	-85	-12	-222	-271	-241	-18	-325	42	-282	726	146	920	-798	-502	635	-1092	1434	545
486/7	486/4	2	0	-6	8	-30	-55	-90	89	-78	96	-55	-46	-132	143	60	-738	-546	-241	-493	-594	245	1061	1374	-900	1709
486/8	486/3	2	0	-3	-22	36	-16	-30	11	-99	-297	62	-76	-318	-388	-129	363	-684	578	551	879	1001	-736	-714	-1020	911
486/9	486/2	2	0	6	5	18	29	-12	29	0	198	71	383	-282	-163	546	642	-342	-70	-232	114	-430	389	960	618	-1015
486/10	486/1	2	0	12	-19	-66	-73	54	-145	-42	-102	179	89	372	215	-318	198	-24	326	20	-1098	218	665	-1002	-1296	-1495
489/1		-3	3	-17	-1	18	-47	54	144	27	67	-243	-98	-81	-149	584	-198	302	-270	-746	-922	-218	820	355	-1600	-1193
490/1		-2	-10	5	0	53	-25	-14	95	1	-206	-108	-57	-243	434	231	263	-24	-116	-204	484	692	466	-228	362	-854
490/2	70/4	-2	-4	-5	0	60	-38	-42	52	120	-234	304	-106	54	-196	-336	438	444	-38	-988	-720	-146	-808	-612	-1146	70
490/3		-2	-1	-5	0	-2	-8	-52	26	67	69	-332	196	353	-369	88	582	-350	-467	291	770	628	1170	525	89	-290
490/4	70/2	-2	1	5	0	-65	-13	73	142	130	111	-256	-266	424	534	269	-132	224	572	-108	560	-586	57	-252	184	605
490/5	490/3	-2	1	5	0	-2	8	52	-26	67	69	332	196	-353	-369	-88	582	350	467	291	770	-628	1170	-525	-89	290
490/6	70/3	-2	3	-5	0	-17	81	91	-102	-90	-129	-116	314	124	-434	-497	-584	332	-220	384	-664	-230	361	-1172	-40	175
490/7	70/1	-2	8	5	0	68	-34	-74	128	-80	286	24	294	-66	-124	-312	-34	-168	-170	564	616	-250	-944	-672	1430	1270
490/8	490/1	-2	10	-5	0	53	25	14	-95	1	-206	108	-57	243	434	-231	263	24	116	-204	484	-692	466	228	-362	854
490/9	70/5	2	-7	5	0	-33	43	-111	70	42	-225	88	-34	-432	-178	-411	-708	-480	-812	596	432	358	425	-972	-960	709
490/10	70/6	2	-5	-5	0	-1	-7	51	-30	-50	79	212	-190	308	422	-121	664	-628	684	1056	744	-726	-407	-644	880	1351
490/11		2	-1	5	0	-30	-44	24	-2	-183	-279	40	-76	423	305	-456	-198	462	-281	-499	-534	-800	-790	597	-1017	1330
490/12		2	-1	5	0	-9	-51	-81	-86	48	211	-254	-20	-74	-318	167	-170	-854	580	-58	152	-702	-419	-124	768	-1085
490/13	490/11	2	1	-5	0	-30	44	-24	2	-183	-279	-40	-76	-423	305	456	-198	-462	281	-499	-534	800	-790	-597	1017	-1330
490/14	490/12	2	1	-5	0	-9	51	81	86	48	211	254	-20	74	-318	-167	-170	854	-580	-58	152	702	-419	124	-768	1085
490/15	10/1	2	8	-5	0	12	58	-66	100	132	-90	-152	-34	438	32	204	222	-420	-902	-1024	432	-362	-160	-72	-810	-1106
492/1		0	-3	5	-26	34	-85	97	-79	186	-168	271	-2	41	268	84	378	337	-358	279	837	705	-384	1293	-347	694
492/2		0	3	-12	10	-41	58	-53	56	-162	1	-15	-363	41	-91	-195	-670	-192	193	-646	891	-35	-426	728	294	188
495/1	55/1	-1	0	5	-9	-11	2	-21	-85	-22	165	-83	1	478	-8	-126	683	290	257	776	313	902	830	-842	-25	-1784
495/2	165/2	-1	0	5	36	-11	2	-66	140	68	-150	-128	-314	118	172	324	-82	740	122	-124	988	2	1100	868	470	1186
495/3	165/1	0	0	5	2	11	-22	-72	122	-72	-96	-112	266	96	-382	-360	-318	-660	-430	380	-168	218	-706	-1068	6	686
496/1	62/1	0	2	1	11	18	-82	-6	-25	-58	180	-31	-146	47	12	136	-232	-715	-518	436	-387	678	-660	382	-800	-1631
496/2	62/2	0	8	-3	35	46	20	8	-97	-28	-206	31	-282	367	562	148	-84	301	-236	-60	-699	-814	-670	650	1566	-615
502/1		-2	4	9	-27	54	-30	-25	-30	27	2	-219	-366	429	-50	-336	526	-346	-336	-601	634	-293	-571	-167	-690	218
503/1		5	7	-4	-3	35	-17	24	62	189	258	-332	-412	-220	347	-85	60	196	-517	-521	-78	-546	-8	423	552	-1010
504/1	168/1	0	0	2	7	-12	-66	70	-92	-16	122	64	-306	-50	20	176	-526	-540	-818	-228	-864	106	736	588	-146	-1214
504/2	168/5	0	0	2	-7	-52	86	30	-4	-120	-246	80	-290	374	164	-464	162	-180	-666	-628	-296	-518	-1184	-220	774	-1086
504/3	168/3	0	0	-4	-7	26	2	36	-76	114	-6	-256	-86	-160	-220	-308	-258	-264	606	-520	286	-530	-44	-1012	-768	222
504/4	56/1	0	0	-8	-7	-56	-28	90	74	96	222	-100	58	-422	512	-148	642	318	720	-412	-448	994	-296	-386	6	-138
504/5	168/4	0	0	10	7	12	30	-34	148	-152	106	304	-114	-202	116	-224	274	660	382	12	552	-614	880	108	86	1426
504/6	168/2	0	0	10	-7	52	-10	54	-52	-48	186	224	94	478	-316	-256	66	-420	342	668	272	-86	1360	-188	366	1554
504/7	168/6	0	0	16	7	18	-54	128	52	202	-302	-200	-150	-172	164	460	190	-96	622	744	54	742	-92	228	116	-554
504/8	56/2	0	0	16	-7	-24	-68	-54	-46	-176	174	-116	74	10	-480	572	162	86	-904	660	-1024	770	-904	-682	102	-218
505/1		-3	-2	5	-3	58	-78	-49	19	202	87	-20	-236	102	127	146	221	464	-125	-974	-84	-890	-128	-670	-1130	-1585
506/1		-2	4	5	-17	-11	67	-5	-54	23	-137	-203	139	368	22	-605	-446	516	262	-489	-661	-816	151	-422	328	412
507/1	39/1	0	-3	12	-2	36	0	-78	-74	-96	18	214	286	384	524	-300	558	-576	74	-38	456	682	704	888	1020	-110
507/2		1	3	-7	10	22	0	37	-30	-162	-113	-196	-13	-285	-246	462	-537	-576	-635	-202	1086	805	884	-518	-194	1202
507/3	507/2	-1	3	7	-10	-22	0	37	30	-162	-113	196	13	285	-246	-462	-537	576	-635	202	-1086	-805	884	518	194	-1202
507/4		3	3	-9	2	30	0	-111	-46	-6	-105	-100	17	-231	-514	-162	639	600	233	926	-930	-253	-1324	810	498	1358
507/5	507/4	-3	3	9	-2	-30	0	-111	46	-6	-105	100	-17	231	-514	162	639	-600	233	-926	930	253	-1324	-810	-498	-1358
510/1		2	3	5	-25	-61	-54	-17	-93	-66	75	30	19	135	-228	599	363	-480	-278	-416	420	813	-18	-852	-504	1018
510/2		2	3	-5	-25	1	-26	17	-49	-110	-35	-302	-369	145	96	-575	-127	408	50	316	100	985	18	-68	896	-78
510/3		-2	3	5	3	27	-34	-17	127	58	-81	58	-69	379	52	423	-329	660	-490	-580	4	717	-602	1052	772	890
510/4		-2	3	5	-6	0	56	-17	-116	112	270	220	66	-44	-182	-216	418	-618	878	-598	1066	276	1000	656	1438	-1432
510/5		-2	3	5	-23	17	-12	17	-65	44	-291	-188	321	-503	-148	311	-449	198	368	218	-56	-761	-530	-1112	-942	370

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
510/6		-2	3	-5	11	49	10	17	-109	22	169	-98	-57	253	216	409	-283	324	-178	904	100	-503	666	172	404	1314
510/7		-2	3	-5	-28	-68	-42	17	60	204	234	-228	346	162	372	136	-114	-612	-542	-812	204	-126	-660	588	378	-350
510/8		-2	-3	5	11	-15	-28	-17	-109	108	129	-16	-193	-195	248	153	-51	-42	-64	-34	-1056	-439	-1330	504	-198	-1138
515/1		3	-8	5	-34	3	-25	30	-22	-72	72	20	104	153	62	-99	540	576	-610	-898	-114	-709	1139	-717	876	836
516/1		0	3	5	-1	-39	-67	0	-11	-180	-55	190	-86	176	43	207	172	-844	-324	-146	72	16	758	701	-238	-527
518/1		-2	4	18	7	36	-22	126	-160	168	294	-28	37	-6	-28	-600	-258	816	-862	116	576	122	464	84	-810	374
519/1		-1	3	6	-12	30	-60	5	-59	92	161	-39	-324	42	-334	44	72	-440	71	-562	-525	495	-1304	-443	-702	938
519/2		2	3	9	-24	9	18	14	-116	32	-208	-261	-363	-30	-211	104	387	436	-820	407	765	-693	58	514	-162	-1384
520/1		0	-2	-5	30	-12	-13	-46	72	-98	126	56	-158	-162	-98	-362	-166	-576	-870	-738	-928	234	128	-1038	730	1250
520/2		0	-10	5	-12	-62	13	58	122	-26	114	338	-342	-230	282	140	-418	-306	-38	-372	-742	-554	812	-864	1146	-1390
522/1	174/4	2	0	0	-17	23	-63	-19	-8	-42	29	-198	-110	514	-404	-517	-584	182	430	365	34	-54	236	-258	-213	156
522/2	58/1	2	0	-5	-2	-37	27	-24	-88	28	29	-143	-360	-386	381	103	431	-288	-840	-180	-706	716	931	-1188	642	486
522/3	174/3	2	0	8	19	9	17	7	-36	182	-29	-102	-166	406	-80	173	68	-222	106	681	286	358	516	-922	1129	-1016
522/4	174/1	2	0	14	-21	-37	-87	-119	-50	-48	29	332	324	-462	-132	331	222	-250	-308	29	928	488	190	522	-745	-1566
522/5	174/2	2	0	-21	19	38	-12	-109	-65	-108	29	72	-311	-377	-167	-349	-338	155	802	-856	-932	-222	110	-168	810	144
522/6	174/5	-2	0	10	7	63	-7	89	-78	52	29	192	200	-166	-356	-353	154	-258	520	-15	764	244	186	1018	-553	1294
522/7	58/2	-2	0	15	-18	-27	-57	44	152	152	29	-173	-120	314	339	357	59	572	-420	660	-726	1004	361	168	-58	-1206
522/8	174/6	-2	0	18	-29	49	-15	-101	-110	-84	-29	132	-404	-10	-224	313	374	394	-56	-475	-1120	420	-1018	-1230	45	270
525/1	105/1	0	3	0	-7	42	-20	-66	38	-12	-258	146	-434	-282	-20	72	-336	-360	-682	-812	810	124	1136	-156	-1038	-1208
525/2		2	-3	0	7	-21	-24	22	16	25	167	10	133	-168	97	400	182	488	28	967	-285	838	-469	406	324	114
525/3	525/2	-2	3	0	-7	-21	24	-22	16	-25	167	10	-133	-168	-97	-400	-182	488	28	-967	-285	-838	-469	-406	324	-114
525/4		3	3	0	-7	-6	-41	-27	-4	-75	-123	-205	262	57	-407	60	-327	33	-427	628	300	-98	686	-1401	714	-494
525/5	21/2	3	3	0	-7	-36	34	-42	-124	0	102	-160	-398	-318	268	-240	498	-132	398	-92	-720	502	-1024	204	354	286
525/6	525/4	-3	-3	0	7	-6	41	27	-4	75	-123	-205	-262	57	407	-60	327	33	-427	-628	300	98	686	1401	714	494
525/7	21/1	-4	3	0	7	62	62	-84	100	42	-10	-48	246	-248	-68	-324	-258	120	622	-904	-678	642	740	-468	200	1266
525/8	105/2	-5	3	0	-7	12	-30	134	-92	-112	-58	-224	146	18	-340	-208	754	380	718	-412	-960	-1066	896	-436	-1038	702
528/1	132/1	0	3	0	-2	11	-88	-66	40	-6	-54	-8	-106	354	124	-546	-408	-552	404	4	-126	-166	874	-444	1002	-802
528/2	33/1	0	3	-4	26	-11	-32	74	60	182	-90	8	-66	422	-408	506	348	200	132	1036	-762	-542	550	132	570	14
528/3	66/2	0	3	10	-16	-11	10	-10	144	84	218	176	46	-26	488	-404	194	-444	202	84	764	354	-1312	1252	-1222	-1358
528/4	264/3	0	3	12	-22	-11	-48	-54	-100	-58	262	-248	-130	-26	-216	-22	620	424	340	620	-810	-1118	214	-988	-6	590
528/5	132/2	0	3	-12	-14	-11	56	42	-116	30	198	88	350	198	-56	594	-204	312	620	-356	462	482	238	-492	954	-1426
528/6	264/4	0	3	-18	28	-11	-18	-34	-80	-128	162	312	-290	-146	-256	-432	-490	-836	230	-900	-520	-798	484	812	74	-1790
528/7	132/3	0	3	22	20	-11	22	110	-48	-72	-142	-184	-194	-482	80	-392	-34	108	382	-84	1040	-606	1292	-356	-406	1090
528/8	66/1	0	-3	0	-14	-11	80	30	-56	126	-222	16	-106	114	52	-246	-264	-264	92	796	-426	-1174	-842	-852	-1062	-1282
528/9	264/2	0	-3	-6	8	11	-30	-18	56	100	26	136	-178	110	-288	-116	-398	-196	-782	-292	-180	-398	-56	-548	282	-142
528/10	264/1	0	-3	-6	14	-11	6	-108	98	32	-8	40	50	-8	486	-40	710	604	322	476	-216	502	862	-592	354	446
528/11	132/4	0	-3	10	-8	11	18	46	-40	-44	186	72	-114	174	416	156	-62	348	-446	956	444	306	664	124	602	1522
528/12	33/2	0	-3	-14	32	11	-38	-2	-72	-68	-54	152	174	94	528	340	-438	-20	570	460	1092	562	16	-372	-966	-526
529/1	23/1	-2	-5	6	8	-34	-57	80	70	0	245	103	298	95	-88	-357	414	-408	-822	-926	335	-899	1322	36	460	964
529/2		3	4	0	0	0	-74	0	0	0	282	-344	0	426	0	48	0	-396	0	0	1176	-1226	0	0	0	0
530/1		-2	-4	5	6	40	-2	106	-86	-84	-54	82	-122	-78	-74	218	53	-212	-58	852	342	1034	542	1268	-402	866
530/2		-2	-5	-5	6	20	-31	63	-53	95	-157	128	341	-120	250	-162	53	-188	388	-452	-421	-790	-1295	189	506	-1417
537/1		-1	3	-12	11	-26	83	-110	30	-6	186	130	-124	63	-398	-437	-141	-677	155	-780	720	22	-407	-367	-1108	-1118
539/1		3	3	2	0	11	-73	62	-84	124	-203	-224	412	-176	400	-586	-234	531	367	105	-878	-236	351	-342	366	-1001
539/2	539/1	3	-3	-2	0	11	73	-62	84	124	-203	224	412	176	400	586	-234	-531	-367	105	-878	236	351	342	-366	1001
539/3	77/1	3	-4	-12	0	11	-38	48	70	12	126	70	-358	216	344	-390	438	552	-830	-196	648	16	1352	-90	-1146	70
540/1		0	0	5	17	-30	-61	-120	-43	90	-90	8	317	-30	-220	-180	-630	-840	599	107	-210	-421	353	-1350	1020	-997
540/2		0	0	5	-22	9	17	75	-4	-183	-129	-187	-34	-264	443	-609	228	-60	-454	-244	-444	398	-349	-1038	-852	914
540/3	540/1	0	0	-5	17	30	-61	120	-43	-90	90	8	317	30	-220	180	630	840	599	107	210	-421	353	1350	-1020	-997
540/4	540/2	0	0	-5	-22	-9	17	-75	-4	183	129	-187	-34	264	443	609	-228	60	-454	-244	444	398	-349	1038	852	914
544/1		0	4	8	-14	8	-46	-17	-116	94	-112	-50	-20	62	-68	60	162	724	-388	-172	1090	-1062	-114	68	-666	-1322
544/2	544/1	0	-4	8	14	-8	-46	-17	116	-94	-112	50	-20	62	68	-60	162	-724	-388	172	-1090	-1062	114	-68	-666	-1322
544/3		0	6	18	-2	26	-22	17	44	78	50	170	58	130	-68	192	-690	-388	226	344	90	-966	1078	36	-298	-1006
544/4	544/3	0	-6	18	2	-26	-22	17	-44	-78	50	-170	58	130	68	-192	-690	388	226	-344	-90	-966	-1078	-36	-298	-1006
545/1		-1	2	-5	3	2	58	89	16	-216	250	-102	-412	124	148	-514	356	-262	-254	-717	-368	1012	-675	-762	-117	1142
546/1		2	3	9	-7	62	-13	-16	79	-155	51	243	412	-406	-103	429	-169	320	-614	258	-264	-121	-967	-679	1059	-21
546/2		2	3	-12	7	-50	-13	-58	-40	-64	-110	124	-50	84	-12	-82	-442	-618	-278	20	-390	-2	-680	322	968	1022
546/3		2	3	-14	-7	8	13	-98	-28	-52	-2	-168	-146	-514	-236	-216	-66	-84	446	292	100	450	392	-292	-402	314
546/4		2	-3	-9	7	-18	13	60	-43	9	-249	-79	-412	222	-295	411	-237	-384	-466	-1042	-288	-691	1001	39	-339	713
546/5		-2	-3	12	7	-22	-13	-2	-88	-80	-22	-92	118	324	84	-134	-194	210	-470	-292	-66	-506	-776	-778	-920	-490
549/1	183/1	-1	0	-3	-14	46	-20	31	-4	147	-30	204	-349	-100	97	-104	6	-492	-61	20	-353	-401	498	439	-1045	-161
550/1		2	2	0	16	11	37	36	5	87	45	167	196	72	-233	336	-78	-720	482	166	1137	-308	-160	-813	1155	-299

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
550/2	22/1	2	-4	0	8	-11	50	-130	-108	96	142	40	-382	-118	-220	-520	-238	-852	190	12	-112	6	304	-820	202	1406
550/3	110/2	2	-4	0	-20	11	-26	42	116	-96	270	32	106	-462	40	504	570	12	590	388	-240	-302	8	48	282	646
550/4	110/1	2	-4	0	30	11	-16	112	-64	-36	10	-48	146	278	330	-476	-150	732	-30	848	240	1128	788	698	-458	-134
550/5	22/2	2	7	0	-14	11	72	46	-20	107	120	117	201	-228	242	96	-458	435	-668	-439	-1113	72	-70	-358	895	-409
550/6	110/3	2	7	0	35	11	-26	-101	127	58	-27	-177	-191	66	-444	-2	669	386	-521	-96	-427	-1006	910	818	601	228
550/7		2	-9	0	-5	11	-36	17	41	44	285	-323	-29	208	-430	-336	-725	-648	-565	748	-265	-602	8	708	137	-44
550/8	22/3	-2	-1	0	10	11	16	-42	116	-189	-120	-163	409	468	-110	-144	-90	-453	20	97	-465	-848	-742	-438	-273	-761
550/9	110/7	-2	-1	0	-23	-11	-50	-75	17	174	-153	35	277	-258	220	-210	273	438	-475	-992	-927	934	974	90	1377	64
550/10	550/1	-2	-2	0	-16	11	-37	-36	5	-87	45	167	-196	72	233	-336	78	-720	482	-166	1137	308	-160	813	1155	299
550/11	110/4	-2	4	0	22	-11	20	20	-8	204	122	40	-278	302	330	-60	418	188	-670	568	128	-676	-876	1130	822	434
550/12	110/5	-2	-7	0	-11	11	-2	9	-85	138	45	227	19	-138	88	534	-297	-450	287	304	777	-962	290	-1422	-1455	-116
550/13	110/6	-2	8	0	-26	11	-92	84	80	-72	-30	-208	-86	-378	-542	-216	18	420	-718	124	912	268	-940	498	150	-446
550/14	110/8	-2	-8	0	12	-11	34	86	-4	-148	134	-280	-430	-6	136	28	658	4	-90	-96	816	430	1296	608	810	-706
550/15	550/7	-2	9	0	5	11	36	-17	41	-44	285	-323	29	208	430	336	725	-648	-565	-748	-265	602	8	-708	137	44
552/1		0	3	8	-22	-4	-14	-116	30	-23	-38	60	-310	-366	326	-464	348	44	434	-406	472	-222	-642	756	-728	-1370
552/2		0	3	-14	2	58	-50	-76	60	23	-106	-24	-256	-126	-304	32	-642	436	-460	-232	224	-282	-426	702	764	-686
553/1		-1	4	4	7	8	-44	90	-138	-64	256	284	-404	70	-510	24	292	-884	98	-664	216	-1022	-79	-646	-70	-1458
555/1		1	3	5	25	3	22	35	-118	28	83	153	-37	73	155	-136	201	198	497	700	1044	1040	-828	-1434	1310	131
555/2		1	-3	5	25	-57	-26	107	-46	124	-157	-75	-37	-443	-265	512	-459	-570	-259	-428	-1020	-232	-468	-114	134	-889
558/1	62/1	2	0	-1	-11	18	-82	6	25	-58	-180	31	-146	-47	-12	136	232	-715	-518	-436	-387	678	660	382	800	-1631
558/2	186/1	2	0	-3	-7	0	2	-120	-115	138	168	31	-376	159	-448	-264	-564	135	416	-268	579	92	-430	-342	-522	1001
558/3	186/2	2	0	11	9	30	-16	60	-11	16	130	-31	-266	273	-22	188	156	9	312	324	-647	730	538	-518	-714	113
558/4	186/3	2	0	11	-22	-63	15	-95	-11	-108	-56	31	230	-378	102	157	466	-270	591	-513	-647	-262	-175	443	-714	485
558/5	186/4	2	0	-15	17	-24	2	48	-115	-30	-264	31	-160	51	128	-480	-132	-309	-280	-604	159	-652	-838	690	534	329
558/6	186/6	-2	0	1	-6	-39	89	-27	-23	68	-64	-31	-206	138	218	379	-630	366	-279	123	-121	-674	-965	-493	-570	-1003
558/7	62/2	-2	0	3	-35	46	20	-8	97	-28	206	-31	-282	-367	-562	148	84	301	-236	60	-699	-814	670	650	-1566	-615
558/8	186/5	-2	0	7	-3	18	-52	60	-119	20	-178	-31	58	285	230	-164	-180	-351	288	324	65	-386	-758	-814	-594	-1615
558/9	186/7	-2	0	21	-19	12	-34	72	-7	30	84	31	380	-9	-268	480	-276	-309	-712	116	783	1040	386	-54	1446	1625
560/1	280/1	0	1	5	-7	39	-17	-15	-74	14	-237	180	-318	-348	22	193	-208	-452	340	408	-528	-554	-539	-164	-576	-827
560/2	70/2	0	1	-5	-7	65	13	-73	142	-130	111	-256	-266	-424	-534	269	-132	224	-572	108	-560	586	-57	-252	-184	-605
560/3	140/1	0	-1	-5	7	7	-23	-25	62	86	-29	12	-150	204	178	-33	452	-120	920	300	-520	370	1013	636	292	-1381
560/4	70/3	0	3	5	7	17	-81	-91	-102	90	-129	-116	314	-124	434	-497	-584	332	220	-384	664	230	-361	-1172	40	-175
560/5	140/4	0	4	5	7	-68	22	-30	-108	-184	166	32	-370	154	-212	512	-98	860	390	-60	-840	-630	-1312	436	-598	914
560/6	280/3	0	4	5	-7	-20	-10	-14	-12	-104	-122	-224	158	378	-404	-112	270	-324	-186	-156	360	-102	912	-1068	-1590	866
560/7	70/4	0	-4	5	-7	-60	38	42	52	-120	-234	304	-106	-54	196	-336	438	444	38	988	720	146	808	-612	1146	-70
560/8	140/6	0	5	5	-7	15	-13	-27	154	186	3	328	254	96	-134	-51	240	396	-616	-296	48	-322	-659	-300	1020	-199
560/9	140/2	0	5	-5	-7	-15	17	123	-86	-54	-177	-212	74	-444	46	-471	-180	-144	-376	-356	48	818	-89	780	1140	-169
560/10	70/6	0	-5	5	7	1	7	-51	-30	50	79	212	-190	-308	-422	-121	664	-628	-684	-1056	-744	726	407	-644	-880	-1351
560/11	280/4	0	-5	-5	7	39	-19	-37	18	90	99	32	46	-248	-178	-429	-652	-40	-36	348	-72	-1190	-699	116	-704	223
560/12	280/2	0	-7	5	-7	-9	23	41	-34	6	131	-4	26	-260	190	-167	-368	-324	-164	-200	-784	-410	-1211	1132	-72	-707
560/13	70/5	0	-7	-5	-7	33	-43	111	70	-42	-225	88	-34	432	178	-411	-708	-480	812	-596	-432	-358	-425	-972	960	-709
560/14	70/1	0	8	-5	7	-68	34	74	128	80	286	24	294	66	124	-312	-34	-168	170	-564	-616	250	944	-672	-1430	-1270
560/15	35/1	0	8	-5	-7	-12	-78	-94	-40	-32	-50	248	-434	402	68	-536	22	560	-278	164	-672	82	1000	448	-870	1026
560/16	140/3	0	-8	-5	-7	-28	82	-46	-8	128	174	152	-290	50	-396	296	-570	272	-662	-876	880	-638	600	-624	698	754
560/17	140/5	0	-9	5	7	-55	-69	113	126	102	-81	-176	254	-184	230	187	-488	-388	-728	96	-8	-994	-337	-188	-884	-451
561/1		1	-3	10	12	11	-26	17	-28	-28	-6	68	354	-86	308	-128	414	860	850	340	-996	218	828	468	1322	-230
561/2		-3	-3	18	8	-11	-58	17	-88	-48	30	-112	-34	450	-88	-156	-546	-264	362	572	-648	254	-952	-828	498	-46
564/1		0	-3	-12	36	-16	-58	-38	106	148	-260	70	2	-132	-30	47	194	276	794	614	872	382	524	596	494	46
564/2		0	-3	-21	3	55	-4	56	-2	-43	131	-308	125	6	-552	-47	-62	594	-550	908	-362	-968	-229	1066	-1346	805
567/1		1	0	14	-7	47	-86	9	-131	12	260	-54	-246	-383	-169	-96	-300	-429	-380	-155	-72	117	-526	-576	278	-201
567/2	567/1	-1	0	-14	-7	-47	-86	-9	-131	-12	-260	-54	-246	383	-169	96	300	429	-380	-155	72	117	-526	576	-278	-201
568/1		0	7	6	31	30	-35	-6	85	103	-122	65	-186	-222	69	-59	-142	186	-110	922	71	-1177	-582	268	145	-638
570/1		2	3	5	-34	-18	-48	-34	-19	-128	-80	112	-124	-208	42	-144	-378	440	-118	496	72	-738	920	832	440	-864
570/2		2	3	-5	-8	-20	-82	-18	19	-88	-186	-248	262	246	288	-168	-302	72	-546	-804	240	602	-800	-116	766	790
570/3		2	3	-5	-24	32	2	106	-19	152	90	52	306	62	-268	456	-318	300	502	-644	-608	-198	260	-1248	110	-574
570/4		2	-3	5	-4	-12	-46	-102	19	-84	222	8	-214	-126	-160	36	-318	-516	-346	-700	-480	338	248	720	-30	614
570/5		2	-3	-5	-34	28	-6	8	19	-204	262	298	346	-296	340	-204	462	194	-46	-20	1080	-922	-1382	10	180	514
570/6		-2	-3	5	-2	-16	-10	36	19	124	-174	-74	94	-240	-276	540	146	606	450	180	-456	14	550	1442	212	-830
570/7		-2	-3	5	26	54	32	78	19	12	204	-256	-340	-156	326	-132	90	-360	-838	-16	888	854	-640	-84	828	1424
572/1		0	7	-15	-1	11	13	-81	-16	108	-216	-40	-391	408	-421	-399	-582	450	-856	752	759	-544	578	924	-240	-736
572/2		0	10	13	-11	-11	13	66	-136	217	-79	-38	-54	63	435	276	-266	735	-903	-853	-344	-1055	316	-348	-332	-688
574/1		2	-1	5	7	-32	-38	-123	156	-106	-91	-191	-20	41	497	14	-479	-466	599	952	-603	-1034	-975	-684	-1413	863

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
575/1		1	-6	0	-7	35	89	0	-113	23	-35	250	-314	-437	-207	144	628	-358	-206	544	-862	-9	159	-471	1174	-28
575/2		1	-10	0	23	-43	-79	-96	-103	-23	-27	110	-42	171	-265	-280	-428	638	842	-80	-1050	87	17	63	126	668
575/3	115/1	-1	-4	0	32	40	66	-130	-88	-23	-130	40	334	-22	272	-24	-258	612	-366	496	248	-826	-296	1296	-646	1438
575/4	575/1	-1	6	0	7	35	-89	0	-113	-23	-35	250	314	-437	207	-144	-628	-358	-206	-544	-862	9	159	471	1174	28
575/5	575/2	-1	10	0	-23	-43	79	96	-103	23	-27	110	42	171	265	280	428	638	842	80	-1050	-87	17	-63	126	-668
575/6	23/1	2	5	0	8	34	57	80	-70	-23	245	103	298	95	-88	357	414	-408	822	-926	335	899	-1322	36	-460	964
575/7	115/2	-2	3	0	2	-16	47	24	-56	23	85	67	-104	-53	234	-285	-2	80	-764	-236	-289	225	24	-684	-1370	110
576/1	9/1	0	0	0	20	0	70	0	-56	0	0	308	-110	0	520	0	0	0	-182	880	0	1190	884	0	0	-1330
576/2	9/1	0	0	0	-20	0	70	0	56	0	0	-308	-110	0	-520	0	0	0	-182	-880	0	1190	-884	0	0	-1330
576/3	96/4	0	0	2	12	60	42	-10	-132	48	226	-252	362	94	228	408	346	-300	466	-204	-1056	330	612	564	1510	594
576/4	96/4	0	0	2	-12	-60	42	-10	132	-48	226	252	362	94	-228	-408	346	300	466	204	1056	330	-612	-564	1510	594
576/5	8/1	0	0	-2	24	-44	-22	-50	-44	56	198	-160	162	198	-52	-528	-242	-668	-550	-188	-728	154	-656	236	-714	-478
576/6	8/1	0	0	-2	-24	44	-22	-50	44	-56	198	160	162	198	52	528	-242	668	-550	188	728	154	656	-236	-714	-478
576/7	288/1	0	0	4	0	0	-18	-104	0	0	284	0	-214	-472	0	0	572	0	-830	0	0	-1098	0	0	176	-594
576/8	288/1	0	0	-4	0	0	-18	104	0	0	-284	0	-214	472	0	0	-572	0	-830	0	0	-1098	0	0	-176	-594
576/9	6/1	0	0	6	16	-12	-38	126	20	168	30	88	-254	-42	-52	-96	198	660	538	884	792	218	520	492	-810	1154
576/10	6/1	0	0	6	-16	12	-38	126	-20	-168	30	-88	-254	-42	52	96	198	-660	538	-884	-792	218	-520	-492	-810	1154
576/11	96/1	0	0	10	4	-20	-70	-90	140	-192	-134	-100	170	110	532	-56	-430	20	-270	-524	-80	330	-1060	1188	-1274	-590
576/12	96/1	0	0	10	-4	20	-70	-90	-140	192	-134	100	170	110	-532	56	-430	-20	-270	524	80	330	1060	-1188	-1274	-590
576/13	32/2	0	0	-10	16	-40	50	30	-40	-48	-34	320	-310	-410	-152	416	-410	-200	-30	-776	-400	-630	-1120	552	326	-110
576/14	32/2	0	0	-10	-16	40	50	30	40	48	-34	-320	-310	-410	152	-416	-410	200	-30	776	400	-630	1120	-552	326	-110
576/15	24/1	0	0	14	24	28	74	-82	92	8	-138	-80	-30	-282	4	240	-130	-596	218	-436	856	-998	32	1508	246	866
576/16	24/1	0	0	14	-24	-28	74	-82	-92	-8	-138	80	-30	-282	-4	-240	-130	596	218	436	-856	-998	-32	-1508	246	866
576/17	96/2	0	0	-14	36	36	-54	22	36	-144	50	108	-214	446	252	72	-22	684	466	-180	576	-54	972	684	-346	-1134
576/18	96/2	0	0	-14	-36	-36	-54	22	-36	144	50	-108	-214	446	-252	-72	-22	-684	466	180	-576	-54	-972	-684	-346	-1134
576/19	72/1	0	0	16	12	-64	-58	-32	-136	-128	-144	-20	18	288	-200	384	496	128	458	-496	512	-602	-1108	-704	960	206
576/20	72/1	0	0	16	-12	64	-58	-32	136	128	-144	20	18	288	200	-384	496	-128	458	496	-512	-602	1108	704	960	206
576/21	72/1	0	0	-16	12	64	-58	32	-136	128	144	-20	18	-288	-200	-384	-496	-128	458	-496	-512	-602	-1108	704	-960	206
576/22	72/1	0	0	-16	-12	-64	-58	32	136	-128	144	20	18	-288	200	384	-496	128	458	496	512	-602	1108	-704	-960	206
576/23	12/1	0	0	-18	8	36	10	-18	100	-72	-234	-16	226	-90	-452	-432	414	-684	-422	-332	360	26	512	-1188	630	-1054
576/24	12/1	0	0	-18	-8	-36	10	-18	-100	72	-234	16	226	-90	452	432	414	684	-422	332	-360	26	-512	1188	630	-1054
576/25	32/1	0	0	22	0	0	18	94	0	0	-130	0	-214	230	0	0	518	0	-830	0	0	1098	0	0	1670	594
578/1		2	2	8	-34	30	-42	0	-60	-42	-144	170	-176	240	-508	-136	-318	-300	320	-676	650	952	894	1132	-350	-1176
578/2	578/1	2	-2	-8	34	-30	-42	0	-60	42	144	-170	176	-240	-508	-136	-318	-300	-320	-676	-650	-952	-894	1132	-350	1176
578/3	34/1	-2	2	-16	-24	-62	-62	0	-20	12	-80	208	356	-22	-312	24	-462	240	-812	-216	-732	-178	-700	-992	-390	146
578/4	34/2	-2	2	18	10	6	74	0	-88	114	90	310	-86	-90	368	-384	-258	240	-302	-964	390	-722	898	912	1446	1438
581/1		-5	-4	-16	7	-39	50	-110	5	-54	-80	-327	-198	-56	-326	-286	-665	-504	-793	-202	693	-921	405	83	951	126
585/1	195/1	3	0	5	2	-24	13	-24	-70	-90	120	-196	-214	54	-196	-120	-18	312	-322	-376	-240	1136	-808	-1092	618	-880
585/2	195/4	3	0	-5	-16	36	13	30	68	120	186	8	-226	342	-76	552	738	-780	-154	596	-1056	-22	-112	684	-90	-334
585/3	195/3	-4	0	-5	18	-10	-13	-46	-14	36	22	42	-46	226	-224	50	290	-130	70	-138	586	-758	1068	-378	-1374	-1822
585/4	195/2	5	0	5	8	56	-13	-58	24	-36	242	-64	-254	414	-164	40	-82	744	494	-508	-384	462	-816	92	-1210	-530
585/5	65/1	-5	0	5	-12	-14	-13	-98	-26	114	-58	306	86	374	-314	-620	-362	-266	634	612	686	202	-516	-48	1230	350
588/1		0	3	-4	0	-20	4	-24	-44	72	-38	-184	-30	216	-164	-520	-146	-460	-628	556	592	-1024	-104	324	-896	920
588/2	84/1	0	3	-6	0	36	-62	-114	76	-24	54	112	-178	-378	-172	192	-402	-396	-254	-1012	840	-890	80	108	1638	-1010
588/3	588/1	0	-3	4	0	-20	-4	24	44	72	-38	184	-30	-216	-164	520	-146	460	628	556	592	1024	-104	-324	896	-920
588/4	84/2	0	-3	-14	0	4	-54	14	-92	-152	-106	144	158	390	-508	528	606	364	-678	844	-8	422	384	548	-1194	1502
588/5	12/1	0	-3	18	0	36	10	-18	100	72	-234	16	-226	-90	452	-432	414	684	-422	332	-360	-26	512	1188	630	1054
592/1	74/1	0	5	12	7	63	-28	6	70	6	-42	292	37	351	-32	-357	57	-432	-340	1012	609	539	-818	-1299	-390	1772
592/2	74/2	0	5	-14	19	-5	6	-72	44	-182	10	244	-37	-225	2	-221	-659	-156	-620	-416	1125	-641	484	-1239	1304	-560
594/1		2	0	-8	-11	11	37	8	-81	-12	-52	-140	-305	-440	-128	-64	-220	48	247	-129	496	93	-533	512	1640	-899
594/2	594/1	-2	0	8	-11	-11	37	-8	-81	12	52	-140	-305	440	-128	64	220	-48	247	-129	-496	93	-533	-512	-1640	-899
595/1		1	-4	5	7	20	38	17	-100	136	-170	16	-82	-198	-188	-448	174	-204	-26	-1044	680	538	-1280	-1332	-1142	-862
595/2		-1	-4	-5	7	-14	8	-17	86	-40	284	64	-40	-174	340	-512	282	-54	118	-552	-908	-674	-740	-30	-310	602
595/3		-5	-7	5	7	62	-55	17	-7	-218	-173	-119	434	258	40	-37	-279	225	523	-768	-25	571	316	-174	-1043	113
598/1		-2	5	-9	-11	30	-13	-13	40	-23	188	240	127	2	473	247	306	124	-644	218	1137	1128	-208	-882	-918	-256
600/1	120/5	0	3	0	0	4	-54	-114	44	-96	134	-272	98	-6	-12	200	-654	36	-442	188	-632	390	688	-1188	-694	1726
600/2		0	3	0	-4	-28	16	-108	32	28	-238	-180	40	422	-276	-60	-220	-804	-358	884	-64	152	-932	1292	-1146	-824
600/3	120/1	0	3	0	-4	72	6	-38	52	-152	-78	120	150	362	484	-280	670	696	222	4	96	-178	-632	612	994	-1634
600/4		0	3	0	5	14	1	46	19	-46	14	133	258	84	-167	410	456	-194	-17	653	828	570	-552	142	-1104	841
600/5		0	3	0	10	-14	-82	18	-136	-140	112	72	26	-446	396	-144	158	-342	314	-152	-932	-548	-512	284	-810	1304
600/6		0	3	0	10	-46	-34	66	104	164	224	-72	-22	194	108	-480	286	426	698	328	188	-740	1168	412	1206	-1384
600/7	120/2	0	3	0	16	-28	26	62	-68	208	-58	160	-270	282	-76	280	210	196	742	-836	-504	1062	768	1052	-726	1406

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
600/8		0	3	0	-19	22	1	-58	-53	58	22	-35	-270	-468	-431	-230	0	446	127	-811	36	522	1368	-1138	144	-1079
600/9	120/4	0	3	0	-20	-56	86	106	4	-136	-206	-152	-282	-246	-412	-40	126	56	-2	388	-672	-1170	408	-668	66	926
600/10	600/2	0	-3	0	4	-28	-16	108	32	-28	-238	-180	-40	422	276	60	220	-804	-358	-884	-64	-152	-932	-1292	-1146	824
600/11	600/4	0	-3	0	-5	14	-1	-46	19	46	14	133	-258	84	167	-410	-456	-194	-17	-653	828	-570	-552	-142	-1104	-841
600/12	120/3	0	-3	0	-8	20	-22	14	76	-56	-154	160	162	-390	-388	544	210	-380	-794	148	-840	-858	144	-316	1098	-994
600/13	600/5	0	-3	0	-10	-14	82	-18	-136	140	112	72	-26	-446	-396	144	-158	-342	314	152	-932	548	-512	-284	-810	-1304
600/14	600/6	0	-3	0	-10	-46	34	-66	104	-164	224	-72	22	194	-108	480	-286	426	698	-328	188	740	1168	-412	1206	1384
600/15	600/8	0	-3	0	19	22	-1	58	-53	-58	22	-35	270	-468	431	230	0	446	127	811	36	-522	1368	1138	144	1079
600/16	120/6	0	-3	0	-20	16	-58	-38	4	80	82	-8	-426	-246	524	464	702	-592	574	172	768	558	408	-164	-510	-514
600/17	24/1	0	-3	0	24	-28	74	-82	92	-8	-138	80	-30	282	-4	-240	130	596	-218	436	856	998	-32	1508	-246	-866
602/1		2	-9	2	-7	-57	14	96	-131	-78	191	89	-295	364	43	492	6	348	870	69	121	-86	1336	-336	-145	1260
603/1	201/1	4	0	19	13	-26	26	96	124	-153	188	-229	-271	225	121	-272	503	-351	436	67	792	-97	-848	-865	-430	-270
605/1		1	-5	5	29	0	-88	21	105	160	-165	-85	-15	-270	12	-370	-615	396	-835	-540	-187	-58	-620	828	-1535	-90
605/2	55/1	-1	-3	-5	9	0	-2	-21	85	22	165	-83	1	478	8	126	-683	-290	-257	776	-313	-902	-830	-842	25	-1784
605/3	605/1	-1	-5	5	-29	0	88	-21	-105	160	165	-85	-15	270	-12	-370	-615	396	835	-540	-187	58	620	-828	-1535	-90
605/4	5/1	4	2	-5	-6	0	38	-26	-100	-78	50	-108	266	-22	-442	-514	2	500	518	126	412	878	-600	-282	-150	386
606/1		2	3	-4	-17	-50	70	-93	-143	88	263	-282	178	-412	-473	-572	391	190	493	44	-278	626	6	-52	152	-559
608/1		0	1	-8	17	70	-61	83	-19	-115	279	72	-34	108	192	392	131	609	338	461	-750	1177	22	810	-476	1426
608/2	608/1	0	-1	-8	-17	-70	-61	83	19	115	279	-72	-34	108	-192	-392	131	-609	338	-461	750	1177	-22	-810	-476	1426
609/1		-3	3	12	7	-60	32	24	-160	-144	29	-106	-286	180	260	-126	-246	-174	302	-136	-1008	992	512	-1218	540	-100
609/2		5	3	4	7	16	58	-46	-64	-42	-29	-56	26	306	318	112	-186	120	368	-348	266	-844	-1026	-296	294	-60
610/1		-2	4	5	-10	52	-74	60	-140	90	-6	-180	-64	-378	-314	284	204	-36	-61	1034	624	-1114	-116	-996	-122	1274
610/2		-2	10	5	20	-18	38	114	-124	-72	-66	2	-322	-306	272	582	306	-150	61	-844	-1110	-862	-862	1182	-558	146
612/1	204/1	0	0	3	-16	57	-25	-17	-13	93	6	110	248	333	-115	294	318	30	668	-220	-540	1214	-442	438	-60	1568
612/2	68/1	0	0	8	-12	10	-38	17	4	-120	-56	164	-236	-70	-144	-48	366	504	-460	-768	-72	-734	736	-856	-906	46
612/3		0	0	17	6	-17	43	17	67	51	34	-124	106	119	-387	204	204	306	242	-732	0	256	466	476	1326	712
612/4	612/3	0	0	-17	6	17	43	-17	67	-51	-34	-124	106	-119	-387	-204	-204	-306	242	-732	0	256	466	-476	-1326	712
615/1		1	3	5	11	-18	-68	36	-15	-73	-210	-213	81	41	247	-284	-273	235	-28	-944	-228	-978	620	-988	-225	666
615/2		-3	3	5	16	-12	-26	-86	-84	-96	62	96	-162	41	-100	-72	142	4	-362	-596	256	-102	-392	1012	-838	-150
615/3		-5	3	5	-10	-6	-2	84	-54	-40	-180	-240	-126	41	-20	382	-102	40	-262	-356	792	594	-4	692	-906	-1272
616/1		0	9	15	-7	11	0	6	-14	79	-146	221	-37	70	-204	496	266	-553	-734	-287	-1083	802	854	-992	-351	-301
618/1		2	3	-8	12	-57	-38	-94	-127	-27	117	233	341	-424	182	-50	396	-420	-464	-542	-276	-176	462	-1108	-1297	1055
623/1		-1	5	-19	7	-8	-70	-51	121	-197	30	-135	412	-42	65	366	-267	460	618	90	842	565	-792	924	89	-21
623/2		-1	7	7	7	54	44	-55	-5	43	-46	129	-160	-90	431	180	279	804	218	-746	658	157	-26	840	89	-461
624/1	78/5	0	3	6	-20	-24	13	-30	16	72	-282	-164	110	-126	-164	204	-738	-120	614	-848	-132	218	1096	-552	210	-1726
624/2	156/1	0	3	-6	4	-36	13	66	-56	-96	222	-260	-106	-90	-44	-168	30	-348	-346	256	168	-814	-200	-1236	318	-502
624/3	39/1	0	3	-12	-2	36	13	-78	-74	96	18	214	-286	-384	-524	-300	558	-576	74	-38	456	-682	-704	888	-1020	110
624/4	78/1	0	3	-16	-28	-34	-13	138	-108	52	-190	176	342	240	140	-454	198	154	34	656	-550	614	-8	-762	-444	1022
624/5	78/4	0	3	-20	32	-50	-13	-30	120	20	82	44	-306	108	356	178	198	-94	-62	140	778	62	1096	462	1224	614
624/6	156/2	0	-3	-2	32	68	13	-14	-4	-72	102	136	-386	250	140	296	526	-332	-410	-596	880	506	640	-1380	1450	-446
624/7	78/6	0	-3	4	-4	-2	-13	-6	36	20	-14	152	-258	84	188	-254	366	-550	-14	-448	-926	254	-1328	-186	-336	614
624/8	78/3	0	-3	10	8	-40	13	130	20	0	-18	184	-74	-362	-76	452	382	-464	358	700	748	1058	976	1008	-386	-614
624/9	78/2	0	-3	-16	8	38	-13	-78	72	52	242	-76	342	-336	-76	-94	-450	-854	-110	908	-838	-970	352	-474	-1452	-562
630/1	210/5	2	0	5	7	0	26	-18	92	0	6	-4	410	-174	248	-420	-102	588	650	152	168	-610	-1048	684	834	110
630/2	210/1	2	0	5	7	-12	2	18	56	156	186	-52	-178	138	-412	456	198	-348	110	-196	936	542	992	276	-630	110
630/3		2	0	5	7	-34	-86	-134	-124	-22	168	148	130	46	-204	44	-372	404	-842	180	-218	-314	-744	864	-554	934
630/4	70/2	2	0	5	7	65	13	73	-142	-130	-111	256	-266	424	534	269	132	224	-572	-108	-560	586	57	-252	184	-605
630/5	210/6	2	0	5	-7	-28	-86	66	-48	-140	34	-284	-346	274	-4	448	94	-308	510	-156	-336	-1170	16	-772	-1630	110
630/6	70/1	2	0	5	-7	-68	34	-74	-128	80	-286	-24	294	-66	-124	-312	34	-168	170	564	-616	250	-944	-672	1430	-1270
630/7	210/3	2	0	-5	7	-12	-58	-42	-4	-24	-294	128	-58	-282	428	-384	138	-468	-250	-556	-624	-958	632	-84	-810	-790
630/8		2	0	-5	7	-24	74	-24	-34	168	-162	128	380	126	-34	294	318	444	-592	110	-198	866	776	-576	-354	614
630/9		2	0	-5	7	30	-34	30	128	-210	216	128	2	234	236	132	-168	12	758	164	-306	866	-304	720	186	1370
630/10	70/4	2	0	-5	7	-60	38	-42	-52	-120	234	-304	-106	54	-196	-336	-438	444	38	-988	720	146	-808	-612	-1146	-70
630/11		2	0	-5	-7	-10	18	46	-132	-126	48	44	-334	-182	-524	-308	332	-172	94	-156	-1010	-58	-8	880	626	-1066
630/12	70/3	2	0	-5	-7	17	-81	91	102	90	129	116	314	124	-434	-497	584	332	220	384	664	230	361	-1172	-40	-175
630/13	210/4	2	0	-5	-7	-28	54	46	12	0	-6	296	134	-146	556	448	-46	-748	-50	-156	1024	-310	856	628	590	-1390
630/14	210/2	2	0	-5	-7	44	54	-98	-60	144	210	-208	-226	502	484	232	530	764	814	60	-848	-958	-152	-308	1094	554
630/15	630/8	-2	0	5	7	24	74	24	-34	-168	162	128	380	-126	-34	-294	-318	-444	-592	110	198	866	776	576	354	614
630/16	210/9	-2	0	5	7	-24	14	-54	44	-156	-174	-88	-34	138	164	216	-318	204	-442	-316	252	98	-1000	-516	522	-310
630/17	630/9	-2	0	5	7	-30	-34	-30	128	210	-216	128	2	-234	236	-132	168	-12	758	164	306	866	-304	-720	-186	1370
630/18	70/5	-2	0	5	7	33	-43	-111	-70	-42	225	-88	-34	-432	-178	-411	708	-480	812	596	-432	-358	425	-972	-960	-709
630/19	210/10	-2	0	5	-7	4	-42	86	-96	96	78	80	50	26	-32	20	382	-356	-134	888	-868	-70	400	1052	634	1202

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
630/20	630/11	-2	0	5	-7	10	18	-46	-132	126	-48	44	-334	182	-524	308	-332	172	94	-156	1010	-58	-8	-880	-626	-1066
630/21	210/7	-2	0	5	-7	-56	54	-94	36	84	258	-40	-178	146	148	200	130	-188	94	-444	-532	770	-536	1076	1090	1274
630/22	630/3	-2	0	-5	7	34	-86	134	-124	22	-168	148	130	-46	-204	-44	372	-404	-842	180	218	-314	-744	-864	554	934
630/23	70/6	-2	0	-5	-7	1	7	51	30	50	-79	-212	-190	308	422	-121	-664	-628	-684	1056	-744	726	-407	-644	880	-1351
630/24	210/8	-2	0	-5	-7	-16	58	-34	64	16	-62	60	150	-474	-292	-240	662	324	-514	-372	412	-770	-560	852	-1466	-178
632/1		0	-7	7	19	-44	-31	-36	102	154	-204	104	-46	208	-304	187	418	-113	-488	-516	-821	-566	-79	-478	-543	-1319
633/1		-4	-3	3	30	3	-23	-114	19	104	-246	88	321	-266	93	-147	-642	20	-330	-526	585	-330	137	1068	-756	950
637/1	13/1	-5	7	7	0	-26	-13	-77	126	-96	-82	-196	-131	-336	-201	105	-432	294	56	478	9	-98	1304	308	1190	-70
638/1		-2	-5	-11	0	-11	2	112	38	-71	29	19	-199	472	292	392	234	115	26	-481	-901	-354	-38	454	-1199	-1601
639/1	71/1	-1	0	16	-1	-24	7	-72	-153	213	-232	149	-204	432	71	-273	274	-126	-134	-760	-71	-457	112	124	-837	-1424
639/2	213/1	-5	0	-13	-7	-51	-50	-18	-27	-174	-17	-139	84	135	-451	264	-346	-381	-383	-772	71	-607	-2	-1426	966	1090
640/1		0	8	5	2	22	10	10	110	-154	222	-92	-34	398	268	-10	-582	746	-226	-172	928	570	64	-864	-874	306
640/2	640/1	0	8	-5	-2	22	-10	10	110	154	-222	92	34	398	268	10	582	746	226	-172	-928	570	-64	-864	-874	306
640/3	640/1	0	-8	5	-2	-22	10	10	-110	154	222	92	-34	398	-268	10	-582	-746	-226	172	-928	570	-64	864	-874	306
640/4	640/1	0	-8	-5	2	-22	-10	10	-110	-154	-222	-92	34	398	-268	-10	582	-746	226	172	928	570	64	864	-874	306
642/1		2	3	-7	-8	-30	-13	-9	-11	-100	132	-230	-231	-362	414	-4	-158	-84	283	-868	597	-124	-1036	1126	-1536	1074
642/2		2	3	9	-34	-4	-77	-121	-11	-64	4	170	249	20	-236	-380	-108	-340	151	670	945	-208	-416	364	534	312
642/3		-2	-3	7	26	-52	10	-32	14	-123	-134	-122	20	105	363	75	338	-215	-326	-439	-150	-50	-869	-1038	-783	1710
648/1		0	0	5	36	-64	-65	-59	-28	-160	57	164	-321	246	-8	-84	-478	32	415	-220	-884	-77	-80	-1268	-123	1346
648/2	648/1	0	0	-5	36	64	-65	59	-28	160	-57	164	-321	-246	-8	84	478	-32	415	-220	884	-77	-80	1268	123	1346
650/1	130/1	2	2	0	-8	6	-13	-114	38	-150	114	-34	-146	-30	-122	-336	570	66	-502	-728	582	994	-988	84	906	-290
650/2		2	-3	0	12	6	-13	-14	-152	-125	169	296	-126	-110	-387	-356	-435	26	-437	182	-1098	-556	757	-306	-524	-610
650/3	26/1	2	-3	0	-19	-38	13	51	90	52	-190	292	441	312	-373	41	-468	530	592	206	-863	322	-460	-528	870	346
650/4		2	7	0	22	6	-13	-14	28	125	209	-14	-316	0	-57	84	-425	756	-217	62	-988	294	517	224	-14	-420
650/5	26/2	-2	1	0	35	2	-13	19	94	72	246	-100	11	-280	-241	-137	232	-386	64	670	55	838	1016	-420	-934	1154
650/6	650/2	-2	3	0	-12	6	13	14	-152	125	169	296	126	-110	387	356	435	26	-437	-182	-1098	556	757	306	-524	610
650/7	130/2	-2	4	0	8	-32	13	86	-56	-68	-202	-56	-66	490	-460	24	294	-480	-338	-676	120	210	184	660	-286	1202
650/8	26/3	-2	-4	0	-20	-48	-13	-66	-16	-168	6	20	-254	-390	124	468	-558	-96	-826	160	-420	-362	776	0	1626	1294
650/9	650/4	-2	-7	0	-22	6	13	14	28	-125	209	-14	316	0	57	-84	425	756	-217	-62	-988	-294	517	-224	-14	420
654/1		-2	-3	-6	-19	-12	20	75	86	111	-78	-169	-58	315	389	333	-342	-327	293	-862	6	-1063	932	822	-636	-1393
654/2		-2	-3	-14	-11	12	-12	79	-10	43	210	167	-66	-313	-267	329	538	205	-523	634	-378	273	-1340	478	-780	1159
654/3		-2	-3	20	-19	60	-50	31	8	47	220	-37	-278	327	-483	441	-54	-403	497	-572	-1058	313	994	1036	378	1171
655/1		5	2	-5	33	50	16	8	-53	-96	175	-72	-391	427	424	440	-592	194	-680	783	-380	112	-282	-1023	-1239	944
656/1	82/1	0	4	-18	2	52	28	14	16	36	-160	-132	-294	-41	-356	-42	-548	-252	-494	616	-738	-1010	834	1436	474	1598
656/2	82/2	0	-10	-6	10	54	-82	42	-134	-48	30	136	2	41	-200	30	390	444	38	610	42	110	-950	-900	138	170
657/1	73/1	-3	0	-6	-34	-6	-34	-90	-16	-60	-102	-214	-286	-150	-322	534	474	-786	-574	-16	-192	73	-988	-1242	6	614
658/1		-2	4	16	-7	18	-72	42	-136	-166	-118	-236	-170	488	-422	47	18	556	10	806	1020	-192	-1120	1212	-714	-566
660/1		0	-3	5	0	11	-42	-14	-52	96	-26	-144	126	58	364	-328	-50	-284	-794	-316	-280	-358	784	324	-1398	-894
663/1		4	3	-10	-10	18	-13	-17	-74	-132	210	-230	-46	-114	36	446	-754	-50	-226	582	-370	826	272	162	-186	-790
665/1		1	-1	-5	-7	-44	-64	11	-19	53	34	-24	-63	45	-439	-24	-167	-350	-376	578	189	313	1324	168	474	-346
665/2		1	-8	-5	7	-33	27	41	-19	107	55	142	106	-303	-158	-604	472	745	-413	-749	-1023	322	-145	-283	1350	-1704
665/3		1	9	-5	7	12	24	-9	19	83	154	-112	-43	-23	323	-8	549	-186	504	958	315	-275	292	232	-54	-130
665/4		3	-5	5	-7	16	4	-47	19	97	246	-144	-85	85	-179	348	155	270	712	226	-21	387	516	276	1202	-698
665/5		-4	7	-5	7	2	-8	-79	-19	117	-120	-58	-69	-263	-403	376	207	-220	-638	236	-253	-853	1120	822	1050	1496
666/1	222/3	2	0	2	0	-28	-42	-90	-28	48	42	-152	37	342	-500	224	426	-628	262	-60	-504	-1190	552	-4	110	-846
666/2		2	0	6	-33	41	-89	87	45	-11	-308	-252	-37	-182	-252	-288	715	-546	-250	-64	-24	-339	-970	1317	291	-328
666/3	74/1	2	0	-12	-7	63	-28	-6	-70	6	42	-292	37	-351	32	-357	-57	-432	-340	-1012	609	539	818	-1299	390	1772
666/4	222/2	-2	0	0	-16	-48	50	-60	20	-162	264	332	37	-330	368	504	-354	-222	-322	-532	888	-922	1328	696	1488	806
666/5	666/2	-2	0	-6	-33	-41	-89	-87	45	11	308	-252	-37	182	-252	288	-715	546	-250	-64	24	-339	-970	-1317	-291	-328
666/6	74/2	-2	0	14	-19	-5	6	72	-44	-182	-10	-244	-37	225	-2	-221	659	-156	-620	416	1125	-641	-484	-1239	-1304	-560
666/7	222/1	-2	0	16	-24	-8	-78	-12	-16	198	72	280	37	30	244	-56	654	-38	526	-516	552	-842	588	-368	-1136	726
670/1		-2	-6	-5	31	-29	-44	36	98	-208	150	-150	153	40	324	228	450	-544	-327	67	-715	-226	-142	-673	-1587	655
670/2		-2	7	-5	31	-4	-30	134	-76	154	-43	24	376	-186	-97	80	663	-225	-244	-67	384	-288	-310	1230	-851	741
672/1		0	3	6	-7	4	-46	-82	-84	-44	70	152	-146	94	-488	32	-562	476	34	520	36	-654	608	-284	-954	-1694
672/2		0	3	-18	-7	44	58	-130	92	84	-250	-72	-354	334	-416	-464	-450	-516	58	-656	-940	178	1072	660	1254	210
672/3	672/1	0	-3	6	7	-4	-46	-82	84	44	70	-152	-146	94	488	-32	-562	-476	34	-520	-36	-654	-608	284	-954	-1694
672/4	672/2	0	-3	-18	7	-44	58	-130	-92	-84	-250	72	-354	334	416	464	-450	516	58	656	940	178	-1072	-660	1254	210
675/1		0	0	0	17	0	-70	0	107	0	0	-289	323	0	71	0	0	0	-901	-880	0	-919	-1387	0	0	1853
675/2	675/1	0	0	0	-17	0	70	0	107	0	0	-289	-323	0	-71	0	0	0	-901	880	0	919	-1387	0	0	-1853
675/3		0	0	0	37	0	70	0	-163	0	0	-19	433	0	-449	0	0	0	719	880	0	271	503	0	0	523
675/4	675/3	0	0	0	-37	0	-70	0	-163	0	0	-19	-433	0	449	0	0	0	719	-880	0	-271	503	0	0	-523
675/5	135/1	1	0	0	6	47	5	-131	-56	3	157	225	70	-140	-397	-347	4	-748	-338	-492	-32	-970	-1257	-102	1488	-974

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
675/6	135/1	-1	0	0	6	-47	5	131	-56	-3	-157	225	70	140	-397	347	-4	748	-338	-492	32	-970	-1257	102	-1488	-974
675/7	135/3	2	0	0	0	10	80	-7	-113	81	-220	-189	-170	-130	-10	-160	-631	-560	229	-750	890	890	-27	-429	-750	1480
675/8	135/3	-2	0	0	0	-10	80	7	-113	-81	220	-189	-170	130	-10	160	631	560	229	-750	-890	890	-27	429	750	1480
675/9	27/1	3	0	0	25	15	-20	72	2	114	-30	101	430	30	-110	-330	621	660	-376	250	360	-785	488	489	450	1105
675/10	27/1	-3	0	0	25	-15	-20	-72	2	-114	30	101	430	-30	-110	330	-621	-660	-376	250	-360	-785	488	-489	-450	1105
676/1	52/1	0	-3	13	11	2	0	-51	-150	-4	-118	116	-63	288	-293	335	-708	-566	904	-382	-7	-518	-100	1440	-1254	-1262
678/1		2	3	1	-33	-14	13	-19	16	-127	154	-37	-212	-474	-324	-492	-394	-231	441	452	665	-806	200	-490	1562	1642
678/2		2	3	-9	11	-50	-53	17	-66	-163	138	-57	-32	396	-146	-288	246	607	-365	-658	-603	-916	344	-212	126	-1646
680/1		0	2	5	2	-14	-30	17	72	178	238	34	286	242	52	-52	-306	240	478	56	722	218	-930	-332	982	1234
680/2		0	-2	-5	-22	-18	-70	17	-128	194	210	-222	-438	26	220	164	-434	728	426	-680	394	-638	-42	-324	-266	898
680/3		0	-8	-5	12	-34	-30	17	32	168	-62	274	-314	282	162	-22	354	-480	98	-374	-958	-762	-830	978	-578	-1006
681/1		-1	3	-8	-36	-64	-89	-57	79	-108	93	88	-29	-325	-351	273	-331	-738	698	-566	899	-387	-560	-231	946	-1705
682/1		2	8	-10	-31	11	-72	7	-38	-63	76	31	-115	6	-135	-416	-218	219	-162	-447	1138	-874	-658	893	42	1097
684/1	228/3	0	0	3	-17	19	-30	97	19	28	-126	-126	64	-80	-453	-107	326	-56	47	-168	-1060	-659	592	-892	310	-874
684/2	228/1	0	0	-4	-12	-40	-40	66	-19	98	130	262	-296	442	-164	542	-334	-60	614	0	-400	318	1154	636	630	1006
684/3	228/2	0	0	7	21	37	26	33	-19	76	218	-266	-32	-64	133	-305	766	72	-805	264	-92	285	1088	-420	-426	-314
684/4		0	0	18	-32	46	-72	-8	19	148	282	210	-188	-326	84	472	254	208	-586	456	8	358	106	782	178	-1534
684/5	684/4	0	0	-18	-32	-46	-72	8	19	-148	-282	210	-188	326	84	-472	-254	-208	-586	456	-8	358	106	-782	-178	-1534
688/1	86/2	0	4	-14	14	11	-9	9	46	19	216	155	-76	5	43	392	579	588	28	621	146	-192	664	1239	-1622	827
688/2	86/1	0	-8	6	-14	43	-17	49	-130	-53	-180	163	284	-323	43	56	-437	420	552	541	18	1108	-80	-33	-1090	179
690/1		2	3	5	-7	-60	-64	-129	-52	-23	-99	-115	137	327	500	-258	555	471	614	-307	-627	-1072	692	903	-528	-250
690/2		2	3	-5	-5	-58	10	27	-154	23	-205	-103	143	-447	264	128	-521	565	-492	371	-65	530	-740	-457	-144	-38
690/3		2	3	-5	-20	32	-30	-98	-84	23	-110	-48	-62	378	-556	-72	-366	-720	-2	-844	800	-30	1040	148	-574	1602
690/4		-2	3	-5	-5	-46	-66	79	-30	23	79	225	-41	237	104	276	579	345	-104	-61	-253	-498	356	283	176	930
690/5		-2	3	-5	-16	42	44	-42	-52	23	-108	-160	146	6	302	276	678	312	182	-226	726	206	-568	492	-264	-412
690/6		-2	-3	5	16	4	-26	-30	-100	-23	94	-232	230	-150	-156	544	-34	-388	174	-484	440	-550	376	652	-1350	-542
690/7		-2	-3	5	-18	-70	-86	-56	108	23	186	-120	-232	-398	120	88	-190	696	504	432	-72	102	-218	82	-828	650
690/8		-2	-3	5	-19	-24	44	75	-16	-23	-123	-43	-43	207	236	-30	519	39	-190	-295	-603	668	-1276	-573	456	-1186
693/1	231/3	2	0	-1	-7	11	7	14	-45	88	69	22	57	380	48	385	672	469	-342	-139	-132	145	1244	-522	-822	272
693/2	231/4	-2	0	-11	-7	-11	-5	118	-105	68	195	214	33	376	-168	-61	-24	-625	-558	173	-168	973	-1072	-1458	198	-352
693/3	231/5	3	0	4	-7	-11	50	28	30	-112	-130	-146	-302	-4	-548	-86	246	-120	-638	-132	692	-152	768	-1098	1158	1618
693/4	77/1	-3	0	-12	7	-11	38	48	-70	-12	-126	-70	-358	216	344	-390	-438	552	830	-196	-648	-16	1352	-90	-1146	-70
693/5	231/1	-3	0	14	-7	11	2	74	0	148	-26	112	-98	10	208	-460	-258	204	178	-924	748	-230	-456	228	198	562
693/6	231/2	-5	0	6	7	11	70	-126	-80	200	-134	-244	-314	-278	-372	84	-182	756	694	820	-160	-2	40	-760	102	-862
700/1		0	1	0	-7	-37	-38	35	73	64	226	108	360	279	32	-222	-508	420	-610	825	190	-275	742	-1041	1417	-106
700/2	140/1	0	-1	0	7	-7	23	25	-62	86	-29	-12	150	204	178	-33	-452	120	920	300	520	-370	-1013	636	292	1381
700/3	700/1	0	-1	0	7	-37	38	-35	73	-64	226	108	-360	279	-32	222	508	420	-610	-825	190	275	742	1041	1417	106
700/4	140/4	0	4	0	7	68	-22	30	108	-184	166	-32	370	154	-212	512	98	-860	390	-60	840	630	1312	436	-598	-914
700/5	28/2	0	-4	0	-7	-12	82	30	68	-216	246	-112	-110	-246	172	-192	-558	540	110	-140	-840	550	-208	-516	-1398	-1586
700/6		0	5	0	7	-65	-13	113	16	-186	-57	-258	134	-414	284	-419	130	334	-56	-534	952	-502	-371	-1220	880	241
700/7	140/2	0	5	0	-7	15	-17	-123	86	-54	-177	212	-74	-444	46	-471	180	144	-376	-356	-48	-818	89	780	1140	169
700/8	140/6	0	5	0	-7	-15	13	27	-154	186	3	-328	-254	96	-134	-51	-240	-396	-616	-296	-48	322	659	-300	1020	199
700/9	700/6	0	-5	0	-7	-65	13	-113	16	186	-57	-258	-134	-414	-284	419	-130	334	-56	534	952	502	-371	1220	880	-241
700/10		0	7	0	-7	-7	3	61	48	58	219	298	-170	50	484	131	210	-782	488	494	-240	58	-1065	1036	608	-1339
700/11	700/10	0	-7	0	7	-7	-3	-61	48	-58	219	298	170	50	-484	-131	-210	-782	488	-494	-240	-58	-1065	-1036	608	1339
700/12	140/3	0	-8	0	-7	28	-82	46	8	128	174	-152	290	50	-396	296	570	-272	-662	-876	-880	638	-600	-624	698	-754
700/13	140/5	0	-9	0	7	55	69	-113	-126	102	-81	176	-254	-184	230	187	488	388	-728	96	8	994	337	-188	-884	451
700/14	28/1	0	10	0	7	-40	12	58	26	64	-62	252	-26	6	-416	396	450	274	-576	476	-448	158	-936	-530	-390	-214
704/1	22/3	0	1	3	10	11	16	42	116	-189	120	163	409	468	110	-144	-90	-453	-20	-97	465	848	742	438	-273	761
704/2	88/2	0	1	7	-6	11	40	-78	-36	7	-8	183	-227	-36	-322	-184	6	99	-164	695	-987	-248	-242	1494	-905	-1031
704/3	22/3	0	-1	3	-10	-11	16	42	-116	189	120	-163	409	468	-110	144	-90	453	-20	97	-465	848	-742	-438	-273	761
704/4	88/2	0	-1	7	6	-11	40	-78	36	-7	-8	-183	-227	-36	322	184	6	-99	-164	-695	987	-248	242	-1494	-905	-1031
704/5	22/1	0	4	-14	8	-11	50	130	-108	96	-142	-40	-382	-118	220	-520	-238	-852	-190	-12	112	-6	-304	820	202	-1406
704/6	22/1	0	-4	-14	-8	11	50	130	108	-96	-142	40	-382	-118	-220	520	-238	852	-190	12	-112	-6	304	-820	202	-1406
704/7	44/1	0	5	7	-26	11	-52	46	96	27	-16	-293	29	-472	110	-224	-754	-825	548	123	1001	-1020	526	158	-1217	-263
704/8	44/1	0	-5	7	26	-11	-52	46	-96	-27	-16	293	29	-472	-110	224	-754	825	548	-123	-1001	-1020	-526	-158	-1217	-263
704/9	88/1	0	7	-9	-2	-11	0	-38	44	-175	264	-159	173	-220	-542	264	-682	421	-308	177	-365	-528	-686	698	967	-1127
704/10	22/2	0	7	19	14	-11	72	-46	20	-107	-120	117	201	-228	242	-96	-458	-435	668	-439	-1113	-72	-70	-358	895	409
704/11	88/1	0	-7	-9	2	11	0	-38	-44	175	264	159	173	-220	542	-264	-682	-421	-308	-177	365	-528	686	-698	967	-1127
704/12	22/2	0	-7	19	-14	11	72	-46	-20	107	-120	-117	201	-228	-242	96	-458	435	668	439	1113	-72	70	358	895	409
705/1		3	3	-5	-13	42	11	123	-73	177	255	-232	164	-183	470	47	543	549	-235	344	27	-850	806	84	-1110	-520
705/2		-4	-3	5	7	60	-45	-117	-13	37	-169	252	-174	-347	330	-47	-601	453	629	754	-129	134	-64	-716	1280	-684

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
705/3		5	-3	-5	-11	-18	93	-39	71	199	251	24	72	145	6	-47	-55	285	-703	148	279	386	518	1276	122	-1020
708/1		0	3	-4	15	1	-45	-31	124	170	158	152	-205	83	81	68	288	-59	58	556	585	-44	365	1213	420	164
710/1		2	1	5	-1	-30	23	-90	-145	-81	78	-43	158	234	-85	-291	-498	390	398	-538	71	11	290	828	633	-538
710/2		2	5	-5	11	62	-37	46	-25	155	158	117	-218	-182	239	161	702	-70	502	-26	71	-73	-570	-356	1249	-174
710/3		-2	1	-5	-7	18	71	-60	-31	213	132	-55	32	-312	-433	-477	66	360	578	-268	71	-439	-574	648	-231	-1270
710/4		-2	2	5	23	28	-49	-22	-115	-128	-71	-114	-308	-46	470	-461	207	-362	754	772	-71	-616	-735	-418	1631	-1377
710/5		-2	-8	-5	2	-12	-28	-24	-28	-18	162	320	-70	294	-244	462	420	-204	86	422	71	-1006	560	-552	690	-436
714/1		2	3	-5	7	-45	-77	-17	-27	-77	-50	-18	196	-7	-317	338	78	586	-236	-244	-772	-482	-366	-654	-4	-60
714/2		2	-3	-2	7	-42	16	17	-98	60	-84	216	36	-354	-36	-512	-406	758	-190	-384	-984	-790	-284	438	-1570	-530
714/3		2	-3	-2	-7	28	30	17	-56	-80	-126	272	134	402	412	272	294	100	706	-244	304	638	-760	-444	138	1150
714/4		2	-3	-17	7	-7	41	17	147	-135	-174	-164	-274	-99	-261	28	-376	-192	30	496	-224	710	-1164	-842	-1040	600
714/5		-2	3	2	-7	-16	6	-17	16	-4	234	-280	134	-330	-428	40	-498	284	-14	764	-748	38	-640	276	126	-1274
714/6		-2	3	7	-7	-21	-9	-17	1	51	-266	-30	-376	85	227	-10	122	-666	-184	-996	972	398	190	-894	596	756
715/1		1	7	-5	-19	11	13	-49	125	152	105	37	-49	-8	362	436	17	150	-333	356	447	-138	1000	1042	-1005	656
715/2		-1	-1	-5	3	-11	13	137	-53	96	31	-123	-225	-36	442	40	753	-702	-347	956	287	-798	-376	-630	-1317	-1148
720/1	180/1	0	0	5	-2	-30	-4	90	28	-120	210	4	200	240	136	120	-30	450	-166	-908	1020	-250	916	1140	-420	1538
720/2	360/4	0	0	5	-2	34	-68	-38	-4	-152	-46	260	-312	48	200	-104	-414	2	-38	244	-708	-378	852	-844	-1380	514
720/3	30/2	0	0	5	4	-48	2	114	-140	72	-210	-272	-334	198	268	216	78	240	302	-596	-768	-478	640	-348	-210	-1534
720/4	120/1	0	0	5	-4	72	-6	-38	-52	152	78	-120	-150	-362	484	280	670	696	222	4	96	178	632	-612	-994	1634
720/5	5/1	0	0	5	-6	32	-38	-26	-100	-78	50	108	266	-22	-442	-514	-2	500	-518	-126	412	-878	-600	282	150	386
720/6	90/1	0	0	5	-14	6	68	-78	-44	120	-126	244	-304	480	-104	600	258	534	362	268	-972	470	-1244	396	972	-46
720/7	40/1	0	0	5	18	-16	-6	6	124	42	-142	188	202	-54	-66	38	-738	564	-262	554	140	882	1160	642	854	-478
720/8	360/2	0	0	5	18	-34	12	-102	-164	-48	146	-100	328	-288	-120	-16	-126	-642	602	-436	-652	1062	-388	444	-820	-766
720/9	120/6	0	0	5	-20	16	58	-38	-4	-80	-82	8	426	246	524	-464	702	-592	574	172	768	-558	-408	164	510	514
720/10	15/1	0	0	5	-20	-24	74	-54	124	-120	78	-200	-70	-330	-92	-24	-450	24	-322	196	-288	-430	520	156	-1026	-286
720/11	120/4	0	0	5	-20	-56	-86	106	-4	136	206	152	282	246	-412	40	126	56	-2	388	-672	1170	-408	668	-66	-926
720/12	60/1	0	0	5	28	-24	-70	-102	-20	-72	-306	136	-214	150	292	-72	414	-744	-418	-188	480	434	-1352	-612	30	-286
720/13	45/1	0	0	5	30	50	-20	10	44	120	50	-108	-40	-400	-280	-280	610	50	-518	180	700	-410	516	660	1500	-1630
720/14	40/2	0	0	5	34	16	58	70	-4	-134	242	-100	-438	138	-178	22	-162	-268	250	-422	-852	306	456	434	726	1378
720/15	360/3	0	0	5	-34	18	12	106	44	56	-270	-204	120	-80	-536	-536	-542	-174	186	-332	-132	-602	548	-492	1052	482
720/16	120/5	0	0	-5	0	4	54	-114	-44	96	-134	272	-98	6	-12	-200	-654	36	-442	188	-632	-390	-688	1188	694	-1726
720/17	180/1	0	0	-5	-2	30	-4	-90	28	120	-210	4	200	-240	136	-120	30	-450	-166	-908	-1020	-250	916	-1140	420	1538
720/18	360/4	0	0	-5	-2	-34	-68	38	-4	152	46	260	-312	-48	200	104	414	-2	-38	244	708	-378	852	844	1380	514
720/19	10/1	0	0	-5	4	12	-58	-66	100	132	90	-152	-34	438	-32	-204	-222	420	902	1024	432	362	160	72	-810	1106
720/20	120/3	0	0	-5	-8	20	22	14	-76	56	154	-160	-162	390	-388	-544	210	-380	-794	148	-840	858	-144	316	-1098	994
720/21	90/1	0	0	-5	-14	-6	68	78	-44	-120	126	244	-304	-480	-104	-600	-258	-534	362	268	972	470	-1244	-396	-972	-46
720/22	120/2	0	0	-5	16	-28	-26	62	68	-208	58	-160	270	-282	-76	-280	210	196	742	-836	-504	-1062	-768	-1052	726	-1406
720/23	20/1	0	0	-5	16	-60	86	-18	-44	48	186	-176	254	-186	100	168	498	-252	-58	1036	168	506	-272	948	1014	-766
720/24	40/3	0	0	-5	-16	36	-42	110	116	16	-198	-240	-258	-442	292	392	-142	-348	-570	-692	168	-134	-784	564	-1034	-382
720/25	360/2	0	0	-5	18	34	12	102	-164	48	-146	-100	328	288	-120	16	126	642	602	-436	652	1062	-388	-444	820	-766
720/26	15/2	0	0	-5	24	52	22	14	20	-168	-230	288	-34	-122	188	256	338	100	742	84	-328	-38	240	1212	-330	866
720/27	45/1	0	0	-5	30	-50	-20	-10	44	-120	-50	-108	-40	400	-280	280	-610	-50	-518	180	-700	-410	516	-660	-1500	-1630
720/28	60/2	0	0	-5	-32	36	-10	78	-140	-192	-6	16	-34	390	52	408	114	516	-58	892	-120	-646	1168	-732	1590	194
720/29	30/1	0	0	-5	-32	-60	-34	-42	76	0	-6	232	134	-234	412	-360	-222	660	-490	-812	120	746	-152	-804	678	194
720/30	360/3	0	0	-5	-34	-18	12	-106	44	-56	270	-204	120	80	-536	536	542	174	186	-332	132	-602	548	492	-1052	482
722/1	38/1	2	2	-9	-31	57	52	69	0	-72	150	-32	226	258	-67	579	432	330	-13	856	-642	-487	700	-12	600	-1424
722/2		2	5	3	-32	4	-69	19	0	67	51	-132	-14	-413	129	-617	383	-599	-217	-225	701	1015	349	-592	-1349	-613
722/3		2	-5	-12	8	9	26	114	0	-78	-204	98	-334	177	-316	-492	678	-579	-352	755	6	-145	-316	-567	-114	-943
722/4	722/3	-2	5	-12	8	9	-26	114	0	-78	204	-98	334	-177	-316	-492	-678	579	-352	-755	-6	-145	316	-567	114	943
722/5	722/2	-2	-5	3	-32	4	69	19	0	67	-51	132	14	413	129	-617	-383	599	-217	225	-701	1015	-349	-592	1349	613
725/1	145/1	-1	8	0	14	62	-42	114	-70	-62	-29	142	-146	162	-352	444	238	840	2	154	892	38	1050	778	1410	-466
726/1	66/1	2	3	0	-14	0	-80	-30	-56	-126	222	-16	-106	-114	52	246	-264	264	-92	-796	426	1174	-842	-852	-1062	-1282
726/2		2	3	-5	-16	0	-21	-101	88	44	-237	-72	-141	-297	52	12	175	396	-650	-560	-300	-966	932	-664	203	1627
726/3		2	3	6	6	0	12	42	66	0	-6	16	178	462	294	-516	186	264	-936	628	-300	684	294	-972	-270	-430
726/4		2	-3	0	-11	0	34	-36	37	6	42	113	311	-18	412	18	750	546	25	-535	300	499	343	1386	-1392	53
726/5	6/1	2	-3	6	16	0	-38	126	-20	168	-30	-88	254	-42	52	-96	198	-660	538	884	792	-218	520	492	810	1154
726/6	726/2	-2	3	-5	16	0	21	101	-88	44	237	-72	-141	297	-52	12	175	396	650	-560	-300	966	-932	664	203	1627
726/7	726/3	-2	3	6	-6	0	-12	-42	-66	0	6	16	178	-462	-294	-516	186	264	936	628	-300	-684	-294	972	-270	-430
726/8	726/4	-2	-3	0	11	0	-34	36	-37	6	-42	113	311	18	-412	18	750	546	-25	-535	300	-499	-343	-1386	-1392	53
726/9	66/2	-2	-3	10	-16	0	-10	10	144	-84	-218	-176	46	26	488	404	194	444	-202	-84	-764	-354	-1312	1252	-1222	-1358
730/1		2	2	-5	-4	-8	52	101	75	-183	235	-13	261	417	477	116	702	-470	-388	-314	-188	73	-440	332	425	-1104
730/2		2	-5	5	17	-33	2	-66	-58	-24	-171	80	353	45	-364	-219	-504	-87	-280	107	-1089	73	-295	-18	-483	-1084

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
732/1		0	3	3	-6	22	-80	-101	124	-5	-234	-100	145	-204	319	-44	78	100	61	-844	-1025	-9	-790	373	679	815
735/1	105/1	0	3	-5	0	42	-20	-66	-38	12	-258	-146	434	282	20	72	336	360	682	812	810	124	1136	-156	1038	-1208
735/2		1	3	5	0	10	-6	-84	-48	56	-232	6	-48	150	-426	18	-58	-348	-882	-182	-524	690	-1024	-384	246	-1122
735/3	735/2	1	-3	-5	0	10	6	84	48	56	-232	-6	-48	-150	-426	-18	-58	348	882	-182	-524	-690	-1024	384	-246	1122
735/4	15/2	1	-3	-5	0	52	-22	14	20	-168	230	288	-34	-122	-188	-256	-338	-100	-742	-84	-328	38	-240	-1212	-330	-866
735/5	15/1	3	3	5	0	-24	-74	-54	124	-120	-78	-200	-70	-330	92	24	450	-24	322	-196	-288	430	-520	-156	-1026	286
735/6		3	3	5	0	-45	31	-96	-149	-141	48	178	371	-225	344	-375	-663	60	-392	-280	258	-578	152	432	234	-1352
735/7	735/6	3	-3	-5	0	-45	-31	96	149	-141	48	-178	371	225	344	375	-663	-60	392	-280	258	578	152	-432	-234	1352
735/8		-4	3	5	0	-10	24	-54	12	-134	118	-144	-378	-330	204	-312	142	-108	168	448	146	360	236	-324	-1194	1728
735/9	735/8	-4	-3	-5	0	-10	-24	54	-12	-134	118	144	-378	330	204	312	142	108	-168	448	146	-360	236	324	1194	-1728
735/10	105/2	5	3	-5	0	12	-30	134	92	112	-58	224	-146	-18	340	-208	-754	-380	-718	412	-960	-1066	896	-436	1038	702
738/1	82/2	-2	0	6	-10	54	-82	-42	134	-48	-30	-136	2	-41	200	30	-390	444	38	-610	42	110	950	-900	-138	170
738/2	246/1	-2	0	14	-28	-1	16	107	-138	32	-99	-35	149	-41	-339	-511	58	136	-335	682	-389	-323	10	834	-526	-330
738/3	82/1	-2	0	18	-2	52	28	-14	-16	36	160	132	-294	41	356	-42	548	-252	-494	-616	-738	-1010	-834	1436	-474	1598
740/1		0	-8	5	4	-20	-10	-62	8	192	-154	124	37	186	92	476	-258	-176	-458	336	-232	-470	-676	-608	-102	-30
741/1		5	3	20	-9	-33	-13	-6	19	93	210	238	-198	-368	-23	-470	-362	-54	-792	-259	1029	748	145	-468	1500	-1586
742/1		-2	2	20	-7	-4	50	-122	-62	-204	-178	46	-446	-420	52	504	53	-372	-56	-260	1088	532	180	822	358	-10
742/2		-2	-8	15	7	53	-82	42	-131	-32	-255	111	-47	455	-524	608	-53	-82	-747	-416	252	-682	786	907	644	-702
744/1		0	-3	14	-24	-4	46	-30	-116	168	254	-31	22	-70	-212	256	-122	660	750	-804	792	-934	576	-44	970	1346
754/1		-2	-8	10	-32	2	13	74	-98	72	-29	-258	320	336	376	282	114	408	-810	-720	1116	176	-1324	688	168	-1104
756/1		0	0	9	7	6	-28	-99	-130	-102	102	140	101	-165	-91	-111	-66	-675	-394	212	48	674	953	-825	-1398	-322
756/2	756/1	0	0	-9	7	-6	-28	99	-130	102	-102	140	101	165	-91	111	66	675	-394	212	-48	674	953	825	1398	-322
756/3		0	0	13	-7	14	-72	5	-34	-82	-182	-72	-175	219	131	-309	-66	139	138	844	-856	-422	-165	233	-1014	874
756/4	756/3	0	0	-13	-7	-14	-72	-5	-34	82	182	-72	-175	-219	131	309	66	-139	138	844	856	-422	-165	-233	1014	874
759/1		-5	3	-3	1	11	-49	75	-16	23	265	-119	-343	-346	110	-175	442	-684	636	629	-399	-54	1205	482	332	-746
760/1		0	-8	-5	-30	20	-12	54	19	114	178	-296	-164	438	-162	74	288	-324	590	-728	464	-906	712	-102	-1202	-616
762/1		-2	-3	15	11	39	-52	-99	29	-183	-222	-304	164	159	2	-510	261	654	236	-646	180	-682	272	264	108	-466
765/1	255/1	2	0	-5	-17	-41	8	17	-9	0	75	68	-217	287	-32	423	343	-2	-20	-46	-112	-647	646	296	-486	-418
765/2	85/1	-3	0	5	-22	-60	-31	-17	-61	78	-69	-31	56	6	-538	465	-723	753	35	-322	99	-1123	488	852	-1215	-601
765/3	85/3	-3	0	-5	-22	30	-46	-17	104	-42	66	194	206	126	-388	540	-78	-432	-610	848	174	362	398	-828	-630	-1486
765/4	85/2	-3	0	-5	-22	64	73	17	-49	-110	-155	-197	-372	262	258	13	653	333	-355	814	-47	-437	-384	736	-511	537
765/5	255/2	4	0	5	-8	38	74	-17	72	-132	246	158	14	286	-62	318	446	200	-350	770	946	-962	838	-338	-942	-1630
768/1		0	3	8	12	-12	-20	62	108	-72	128	204	228	22	-204	600	-256	-828	84	348	456	-822	1356	108	938	1278
768/2	768/1	0	3	-8	-12	-12	20	62	108	72	-128	-204	-228	22	-204	-600	256	-828	-84	348	-456	-822	-1356	108	938	1278
768/3	768/1	0	-3	8	-12	12	-20	62	-108	72	128	-204	228	22	204	-600	-256	828	84	-348	-456	-822	-1356	-108	938	1278
768/4	768/1	0	-3	-8	12	12	20	62	-108	-72	-128	204	-228	22	204	600	256	828	-84	-348	456	-822	1356	-108	938	1278
770/1		2	4	-5	7	-11	26	42	68	72	210	188	266	-150	68	240	186	-612	-250	392	408	-238	-1180	360	498	326
770/2		2	10	-5	7	-11	68	-72	44	12	-30	-202	-46	-120	-244	-390	666	690	704	-136	-1164	908	896	-1044	-54	-106
770/3		-2	4	-5	-7	11	-10	70	40	88	-42	108	-42	-242	-356	-252	-578	-620	-394	764	-384	-322	-824	680	-614	-1582
770/4		-2	-4	5	7	-11	6	-6	84	-112	-34	0	286	-350	172	-528	654	-380	-538	-156	608	-798	-640	1044	1386	338
770/5		-2	-4	-5	-7	11	-2	30	56	72	-122	-140	-42	-90	428	380	478	-340	142	-724	-144	-218	344	-8	570	802
774/1	86/1	2	0	-6	14	43	-17	-49	130	-53	180	-163	284	323	-43	56	437	420	552	-541	18	1108	80	-33	1090	179
774/2	258/1	-2	0	9	-25	69	-31	0	17	-132	237	38	326	72	43	-201	84	612	-496	-502	288	-160	170	561	-654	449
774/3	86/2	-2	0	14	-14	11	-9	-9	-46	19	-216	-155	-76	-5	-43	392	-579	588	28	-621	146	-192	-664	1239	1622	827
775/1		1	-8	0	-19	37	-33	26	10	22	90	31	331	57	-183	301	-633	230	-233	-824	387	-1118	180	217	530	-1134
775/2	155/1	-1	-2	0	-16	2	48	94	-140	68	300	31	-296	-138	318	224	-312	160	-128	-716	912	-182	-180	1418	-1190	-126
775/3	775/1	-1	8	0	19	37	33	-26	10	-22	90	31	-331	57	183	-301	633	230	-233	824	387	1118	180	-217	530	1134
777/1		1	3	14	7	-12	-58	-78	-116	-8	38	-240	37	-230	484	-304	206	-252	-394	-196	-392	1178	448	-772	-1494	546
780/1		0	3	5	24	60	13	58	28	-80	-274	-88	174	458	-252	-440	78	-212	-698	340	432	82	-368	996	890	250
780/2		0	-3	-5	-4	8	-13	46	12	48	-182	168	298	-22	-244	-552	658	824	-802	-228	-496	442	-1384	-1332	-1534	-598
780/3		0	-3	-5	7	-3	-13	-31	34	103	82	-206	-109	165	240	306	-211	-408	485	-316	-815	-966	-1175	340	215	-279
781/1		-1	-7	6	-21	11	-77	-26	5	-207	90	-233	-356	-178	-387	-161	-582	180	-118	954	71	-427	860	-1392	685	-656
784/1	49/4	0	0	0	0	68	0	0	0	40	-166	0	450	0	180	0	590	0	0	740	-688	0	1384	0	0	0
784/2	98/1	0	1	7	0	-35	66	59	-137	7	106	-75	11	-498	-260	171	-417	17	51	-439	784	295	495	-932	-873	-290
784/3	98/1	0	-1	-7	0	-35	-66	-59	137	7	106	75	11	498	-260	-171	-417	-17	-51	-439	784	-295	495	932	873	290
784/4	14/2	0	-2	12	0	-48	-56	114	2	120	-54	236	146	-126	376	-12	174	138	-380	484	-576	1150	-776	378	390	1330
784/5	56/2	0	-2	16	0	-24	68	-54	-46	-176	-174	-116	74	10	480	-572	-162	-86	904	-660	-1024	-770	904	682	102	218
784/6	7/1	0	-2	-16	0	8	-28	-54	-110	-48	-110	12	-246	-182	-128	324	-162	810	488	-244	768	702	-440	-1302	-730	-294
784/7	28/2	0	4	-6	0	12	82	30	68	-216	246	-112	110	246	172	192	558	540	-110	-140	840	550	208	516	1398	-1586
784/8	392/2	0	4	12	0	-12	-76	8	-100	56	-166	-232	-414	-72	452	424	-18	444	284	-524	1008	-896	40	1388	-448	824
784/9	196/1	0	4	-20	0	-44	-44	72	-100	120	218	280	-30	120	-220	-88	110	-580	380	980	112	-640	488	-660	320	248
784/10	8/1	0	-4	2	0	44	-22	-50	44	56	198	-160	-162	198	-52	528	-242	-668	-550	-188	-728	-154	656	236	-714	478

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
784/11	392/2	0	-4	-12	0	-12	76	-8	100	56	-166	232	-414	72	452	-424	-18	-444	-284	-524	1008	896	40	-1388	448	-824
784/12	196/1	0	-4	20	0	-44	44	-72	100	120	218	-280	-30	-120	-220	88	110	580	-380	980	112	640	488	660	-320	-248
784/13	98/4	0	5	-9	0	57	-70	51	-5	-69	114	-23	-253	-42	124	-201	-393	-219	-709	-419	96	-313	-461	588	-1017	-1834
784/14	98/4	0	-5	9	0	57	70	-51	5	-69	114	23	-253	42	124	201	-393	219	709	-419	96	313	-461	-588	1017	1834
784/15	56/1	0	6	-8	0	-56	28	90	74	96	-222	-100	58	-422	-512	148	-642	-318	-720	412	-448	-994	296	386	6	138
784/16	49/1	0	7	-7	0	5	14	21	49	159	58	147	219	-350	124	525	303	-105	413	-415	432	1113	103	1092	329	882
784/17	49/1	0	-7	7	0	5	-14	-21	-49	159	58	-147	219	350	124	-525	303	105	-413	-415	432	-1113	103	-1092	-329	-882
784/18	14/1	0	8	14	0	28	-18	-74	80	112	190	72	-346	-162	412	24	318	-200	198	716	-392	-538	-240	-1072	-810	-1354
784/19	28/1	0	-10	8	0	40	12	58	26	64	-62	252	26	-6	-416	-396	-450	274	576	476	448	158	936	530	390	-214
785/1		0	-4	-5	23	6	-37	-15	-54	97	214	168	79	-318	293	280	-308	-537	-936	962	-288	-572	360	-444	-915	-1704
786/1		2	3	-7	-15	39	-44	-41	-48	-125	-72	122	381	-410	-78	-561	225	-357	326	-115	-517	-848	-506	114	144	-1454
786/2		-2	-3	2	-22	-45	-69	-85	55	-28	81	85	-80	84	-234	-220	-156	173	429	-264	-568	-794	520	686	242	-936
792/1	264/2	0	0	6	-8	11	-30	18	-56	100	-26	-136	-178	-110	288	-116	398	-196	-782	292	-180	-398	56	-548	-282	-142
792/2	264/1	0	0	6	-14	-11	6	108	-98	32	8	-40	50	8	-486	-40	-710	604	322	-476	-216	502	-862	-592	-354	446
792/3	88/2	0	0	7	-6	11	-40	78	36	-7	-8	183	227	36	322	184	6	99	164	-695	987	-248	-242	1494	905	-1031
792/4	88/1	0	0	-9	2	11	0	38	44	-175	264	159	-173	220	-542	264	-682	-421	308	177	-365	-528	686	-698	-967	-1127
792/5	264/3	0	0	-12	22	-11	-48	54	100	-58	-262	248	-130	26	216	-22	-620	424	340	-620	-810	-1118	-214	-988	6	590
792/6	264/4	0	0	18	-28	-11	-18	34	80	-128	-162	-312	-290	146	256	-432	490	-836	230	900	-520	-798	-484	812	-74	-1790
798/1		2	-3	0	7	-42	20	-96	19	102	90	-196	-214	378	-376	216	-750	-252	182	-286	264	-358	-862	-384	42	-1240
798/2		2	-3	-10	7	8	-50	114	19	-148	-30	304	-274	-202	-116	-324	-550	628	-58	-756	-216	-278	-952	-1184	1542	-870
798/3		-2	-3	-12	7	-60	74	-36	19	-192	12	-160	254	114	-412	-330	360	-12	-610	-1024	990	-322	704	318	90	1154
800/1	32/1	0	0	0	0	0	18	94	0	0	-130	0	-214	-230	0	0	-518	0	830	0	0	-1098	0	0	-1670	-594
800/2		0	0	0	0	0	92	-104	0	0	130	0	-396	230	0	0	-572	0	-830	0	0	-592	0	0	1670	-1816
800/3	800/2	0	0	0	0	0	-92	104	0	0	130	0	396	230	0	0	572	0	-830	0	0	592	0	0	1670	1816
800/4	160/1	0	2	0	-6	60	-50	30	40	-178	166	20	-10	-250	-142	-214	-490	-800	250	774	100	230	-1320	-982	874	310
800/5	160/1	0	-2	0	6	-60	-50	30	-40	178	166	-20	-10	-250	142	214	-490	800	250	-774	-100	230	1320	982	874	310
800/6		0	5	0	-10	15	8	-21	-105	-10	-20	230	-54	-195	-300	-480	322	-560	-730	255	40	317	830	75	-705	-1434
800/7	800/6	0	5	0	-10	-15	-8	21	105	-10	-20	-230	54	-195	-300	-480	-322	560	-730	255	-40	-317	-830	75	-705	1434
800/8	800/6	0	-5	0	10	15	-8	21	-105	10	-20	230	54	-195	300	480	-322	-560	-730	-255	40	-317	830	-75	-705	1434
800/9	800/6	0	-5	0	10	-15	8	-21	105	10	-20	-230	-54	-195	300	480	322	560	-730	-255	-40	317	-830	-75	-705	-1434
800/10	32/2	0	8	0	16	40	50	30	-40	48	-34	-320	-310	410	152	-416	410	200	30	776	-400	630	1120	552	-326	110
800/11	32/2	0	-8	0	-16	-40	50	30	40	-48	-34	320	-310	410	-152	416	410	-200	30	-776	400	630	-1120	-552	-326	110
801/1	89/1	1	0	-2	-4	56	-16	30	-50	92	-204	324	-20	-270	86	0	-534	206	-672	-576	352	-338	-336	-630	-89	-1506
801/2	89/2	4	0	-11	8	32	-4	-39	-59	83	-66	-279	-350	78	71	258	-597	572	-420	-30	-230	-497	-714	420	-89	1833
804/1		0	3	8	8	62	-36	-99	121	189	65	140	-161	-352	-146	315	570	421	-358	-67	-776	1209	616	-564	1125	1156
805/1		1	4	5	-7	28	-42	58	20	23	182	-56	230	26	268	600	-186	-420	582	596	248	-1094	352	372	434	986
805/2		5	-8	-5	7	4	-58	118	-132	-23	198	-236	-146	-326	-476	-236	-282	224	130	228	120	-886	-800	84	414	-626
810/1		2	0	-5	2	-9	-16	-6	-67	30	-45	-247	-124	3	80	36	-486	249	-10	-322	-453	-346	-352	-204	-729	716
810/2		2	0	-5	-16	57	-64	99	-49	-198	-66	146	-28	-411	-223	-132	-654	-33	458	-385	-642	-247	-106	-324	414	-1885
810/3		2	0	-5	-28	45	32	-84	149	-90	9	-259	-262	-111	-466	-606	-132	135	-826	-538	357	-52	-724	6	1617	1094
810/4	810/1	-2	0	5	2	9	-16	6	-67	-30	45	-247	-124	-3	80	-36	486	-249	-10	-322	453	-346	-352	204	729	716
810/5	810/2	-2	0	5	-16	-57	-64	-99	-49	198	66	146	-28	411	-223	132	654	33	458	-385	642	-247	-106	324	-414	-1885
810/6	810/3	-2	0	5	-28	-45	32	84	149	90	-9	-259	-262	111	-466	606	132	-135	-826	-538	-357	-52	-724	-6	-1617	1094
812/1		0	-2	-6	7	-60	38	-36	62	-96	-29	-22	362	-216	128	210	-258	180	596	-556	-744	-364	1076	624	900	-700
815/1		-2	4	5	-9	-16	-13	61	96	89	-206	40	-353	341	122	-138	164	-542	634	-865	-633	-706	252	-486	-20	1778
816/1	102/1	0	3	-3	-20	51	-61	17	43	219	-150	-290	56	15	-83	-426	-378	210	-448	124	-900	-1078	-722	78	-144	-268
816/2	102/4	0	3	5	-12	-37	19	17	-37	3	-86	142	-296	-121	-3	-402	174	-270	-520	780	-84	-302	-178	-698	1512	-500
816/3	408/1	0	3	6	24	-44	6	17	20	152	270	272	-250	186	-260	320	-770	348	-210	148	360	-646	1168	788	-1238	882
816/4	51/2	0	3	-10	8	-12	-26	17	148	-152	-66	32	-266	-6	92	288	-546	-420	350	-940	-424	378	-288	-748	-1558	530
816/5	102/3	0	3	-12	22	48	2	-17	-20	54	84	-62	44	-138	-428	516	174	852	908	508	426	-574	-110	1308	798	-1690
816/6	51/1	0	3	16	-34	48	58	-17	-20	-58	0	218	184	-138	-148	516	-162	180	152	956	538	-462	-390	-1268	-770	494
816/7	204/1	0	-3	-3	16	57	-25	17	13	93	-6	-110	248	-333	115	294	-318	30	668	220	-540	1214	442	438	60	1568
816/8	102/2	0	-3	-5	32	-27	-69	-17	83	117	94	-198	-244	169	-227	382	686	-450	-700	-540	276	-298	182	-282	-1468	-1140
816/9	408/2	0	-3	-7	-4	21	-25	17	69	-15	58	298	72	-369	59	138	262	-50	-568	-124	-100	-158	-710	-214	-1016	-1780
816/10	51/3	0	-3	-20	2	48	-14	-17	-92	122	-36	182	76	294	428	12	-234	540	-820	-700	-794	-1038	-858	-1052	1102	710
819/1	273/2	1	0	5	7	1	13	-19	-117	141	131	-128	55	0	-201	96	-510	156	-845	-470	-324	-373	-526	-266	250	322
819/2	273/3	1	0	-9	-7	57	-13	37	107	183	-191	-240	-379	84	-313	-296	414	-40	65	-1086	208	635	-582	-798	726	1498
819/3	273/1	4	0	0	-7	6	-13	4	-52	-6	-14	-48	-190	-180	356	-536	-210	-244	470	240	-854	-82	-876	-504	660	1318
822/1		2	3	9	-19	-40	-26	-124	-109	-69	115	-2	-245	30	510	305	330	-582	-213	380	-976	989	982	159	-203	1830
822/2		2	3	-9	15	-30	-72	-58	-21	33	-99	-188	-23	-2	120	495	-82	152	-143	44	-296	-827	-1318	719	-391	-1012
822/3		-2	3	7	-1	-16	26	-60	-151	55	-43	30	-45	-174	-142	-571	302	414	-317	-184	256	413	778	499	-1485	-186
825/1	165/1	0	3	0	-2	-11	22	-72	122	-72	96	-112	-266	-96	382	-360	-318	660	-430	-380	168	-218	-706	-1068	-6	-686

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
825/2	33/1	1	3	0	26	11	32	-74	-60	182	-90	-8	66	422	-408	506	-348	-200	132	1036	762	542	-550	132	570	-14
825/3	165/2	-1	-3	0	-36	11	-2	-66	140	68	150	-128	314	-118	-172	324	-82	-740	122	124	-988	-2	1100	868	-470	-1186
825/4		3	3	0	7	11	16	-21	125	81	186	-58	253	63	100	219	192	249	-64	-272	-645	112	509	1254	756	-839
825/5	825/4	-3	-3	0	-7	11	-16	21	125	-81	186	-58	-253	63	-100	-219	-192	249	-64	272	-645	-112	509	-1254	756	839
825/6		4	-3	0	-21	11	68	-21	125	-137	-150	292	349	497	208	369	-542	235	482	734	587	518	-1045	608	-770	-1541
825/7	825/6	-4	3	0	21	11	-68	21	125	137	-150	292	-349	497	-208	-369	542	235	482	-734	587	-518	-1045	-608	-770	1541
825/8		5	3	0	3	-11	32	33	47	113	-54	178	19	139	-308	195	152	-625	320	200	-947	-448	-721	142	404	79
825/9	33/2	5	-3	0	32	-11	38	2	72	-68	-54	-152	-174	94	528	340	438	20	570	460	-1092	-562	-16	-372	-966	526
825/10	825/8	-5	-3	0	-3	-11	-32	-33	47	-113	-54	178	-19	139	308	-195	-152	-625	320	-200	-947	448	-721	-142	404	-79
828/1	276/2	0	0	-2	-22	14	-50	52	-20	-23	74	24	104	30	112	288	386	204	-308	152	720	486	462	-742	-180	786
828/2	276/1	0	0	-8	34	-36	-62	60	30	23	-234	140	-174	-194	-42	400	-76	-252	-566	-6	-264	-286	486	980	-1632	-1626
832/1	416/2	0	1	1	-5	10	13	93	-82	192	106	-172	-379	-148	-329	631	-160	-478	-300	-722	-335	90	788	96	-866	-998
832/2	104/2	0	1	7	21	6	-13	-115	-46	-144	162	-180	-13	192	-33	-383	-288	442	680	-722	207	274	936	-1204	-966	-138
832/3	26/2	0	1	-17	-35	-2	-13	-19	-94	-72	-246	-100	11	-280	-241	137	232	386	-64	670	55	-838	1016	-420	-934	-1154
832/4	416/2	0	-1	1	5	-10	13	93	82	-192	106	172	-379	-148	329	-631	-160	478	-300	722	335	90	-788	-96	-866	-998
832/5	104/2	0	-1	7	-21	-6	-13	-115	46	144	162	180	-13	192	33	383	-288	-442	680	722	-207	274	-936	1204	-966	-138
832/6	26/2	0	-1	-17	35	2	-13	-19	94	72	-246	100	11	-280	241	-137	232	-386	-64	-670	-55	-838	-1016	420	-934	-1154
832/7	26/1	0	3	-11	-19	-38	13	-51	90	52	190	-292	441	312	373	41	-468	530	-592	-206	863	-322	460	528	870	-346
832/8	52/1	0	3	13	-11	2	13	-51	-150	-4	118	-116	-63	-288	293	-335	708	-566	-904	-382	7	518	-100	1440	1254	1262
832/9	26/1	0	-3	-11	19	38	13	-51	-90	-52	190	292	441	312	-373	-41	-468	-530	-592	206	-863	-322	-460	-528	870	-346
832/10	52/1	0	-3	13	11	-2	13	-51	150	4	118	116	-63	-288	-293	335	708	566	-904	382	-7	518	100	-1440	1254	1262
832/11	26/3	0	4	18	-20	-48	-13	66	-16	-168	-6	-20	-254	-390	-124	468	-558	-96	826	-160	420	362	-776	0	1626	-1294
832/12	26/3	0	-4	18	20	48	-13	66	16	168	-6	20	-254	-390	124	-468	-558	96	826	160	-420	362	776	0	1626	-1294
832/13	416/1	0	5	3	5	-30	-13	-19	-70	20	30	-100	111	-180	-85	-295	132	230	220	670	55	-602	-360	540	-270	-606
832/14	104/1	0	5	-19	3	-2	13	77	-58	-76	6	292	-207	240	-317	375	692	214	488	782	1057	1174	-892	704	6	830
832/15	416/1	0	-5	3	-5	30	-13	-19	70	-20	30	100	111	-180	85	295	132	-230	220	-670	-55	-602	360	-540	-270	-606
832/16	104/1	0	-5	-19	-3	2	13	77	58	76	6	-292	-207	240	317	-375	692	-214	488	-782	-1057	1174	892	-704	6	830
832/17	13/1	0	7	7	-13	26	-13	77	126	-96	82	196	131	336	201	-105	432	294	56	-478	9	98	1304	308	-1190	70
832/18	13/1	0	-7	7	13	-26	-13	77	-126	96	82	-196	131	336	-201	105	432	-294	56	478	-9	98	-1304	-308	-1190	70
833/1	119/1	-1	6	20	0	60	68	17	70	-176	-90	-196	22	138	328	12	-234	54	-44	-596	200	-1122	480	838	-778	-1142
833/2	17/1	-3	8	-6	0	-24	58	-17	-116	-60	30	172	-58	342	-148	-288	318	-252	-110	-484	-708	-362	-484	-756	774	382
834/1		2	3	-15	-11	17	73	-18	-146	-116	-169	151	-226	26	50	-96	-372	-254	-670	-761	-531	268	1225	615	-1019	48
840/1		0	3	5	7	-16	-62	-14	-56	-136	-154	-116	6	-150	-20	152	-78	124	166	140	204	-210	-984	628	138	-1202
840/2		0	3	5	-7	20	54	82	-116	88	-186	-128	190	250	36	384	-82	-124	390	524	344	186	272	-388	714	-510
840/3		0	3	5	-7	22	-44	-110	-22	-36	-122	-186	306	-330	20	-64	504	-560	-418	-452	-146	-236	536	-92	-574	184
840/4		0	3	5	-7	-44	22	66	-132	-168	54	144	-354	-22	-156	-240	-354	-76	-154	-628	8	1018	96	348	218	-1598
840/5		0	-3	-5	-7	0	-54	74	-20	-160	-246	84	306	-370	-88	460	686	-684	186	-904	912	-26	-320	732	1150	1526
840/6		0	-3	-5	-7	-58	4	-42	-78	72	102	-90	-390	442	-204	-120	-300	-104	302	836	-74	148	-552	1428	454	424
845/1	5/1	4	2	5	-6	-32	0	26	-100	-78	-50	108	-266	-22	442	514	2	-500	-518	-126	-412	878	600	-282	150	-386
845/2	65/1	-5	2	5	12	-14	0	98	26	-114	58	-306	-86	374	-314	-620	362	-266	634	-612	686	-202	-516	-48	1230	-350
846/1	282/4	2	0	3	-33	31	62	-58	130	-151	23	250	-43	282	342	-47	412	-324	518	734	322	22	707	1096	-254	-767
846/2	282/1	2	0	-3	11	-15	-28	-60	-94	-45	-75	200	149	-222	380	47	-594	-846	650	-160	-114	-340	-373	-1122	582	-811
846/3	282/2	2	0	-7	-9	45	-70	38	30	99	-211	-2	-275	-394	-182	47	-80	776	-130	-290	-234	-506	-1221	-468	-1082	449
846/4	282/3	2	0	8	-12	-60	2	110	-126	-84	-40	-14	-254	164	-422	47	502	-628	26	-386	720	574	156	-660	1018	-1570
846/5	282/5	-2	0	11	-25	15	-26	54	-6	89	31	-70	-171	-390	262	-47	-12	-196	-442	-386	-70	22	171	672	42	-607
847/1	7/1	1	-2	16	7	0	-28	-54	110	48	110	12	-246	-182	-128	324	-162	810	488	244	-768	702	-440	1302	730	294
847/2	77/1	-3	4	12	-7	0	-38	48	70	12	-126	-70	-358	216	-344	390	438	-552	-830	-196	648	16	-1352	-90	1146	-70
848/1	53/1	0	-1	-18	-2	-54	-43	-99	61	-207	-99	160	-7	-414	268	-270	53	-450	182	556	-693	-862	-119	333	1350	-187
850/1	34/2	2	2	0	10	-6	-74	-17	-88	114	-90	-310	-86	90	-368	384	258	240	302	964	-390	-722	-898	-912	1446	1438
850/2	34/1	2	2	0	-24	62	62	17	-20	12	80	-208	356	22	312	-24	462	240	812	216	732	-178	700	992	-390	146
850/3	170/1	2	-4	0	4	-12	58	-17	-52	-84	-246	68	358	-78	412	-408	-750	-420	-190	-596	324	-1010	164	-588	-486	718
850/4	170/2	-2	-7	0	10	24	-41	17	-103	6	-45	5	196	210	58	171	-3	645	197	46	-975	637	272	72	-609	847
855/1	95/1	0	0	5	-22	12	8	66	19	30	6	-64	-16	-54	182	-594	-396	564	-706	-628	984	14	-328	294	-918	-1564
855/2		3	0	5	-34	18	56	66	19	-120	-66	236	236	228	20	240	330	-330	-430	308	948	1118	-52	-1044	960	326
855/3	285/1	3	0	-5	32	12	-10	30	19	48	-150	224	254	54	-196	504	-78	-132	230	740	120	122	1184	-612	-1050	-1006
855/4	95/2	-3	0	5	-1	24	-31	-33	19	-27	-111	-94	-70	510	-34	192	75	-45	-28	371	-384	-73	-1234	-366	1578	-538
855/5	95/3	-3	0	-5	11	36	65	87	19	129	-231	110	-142	330	74	336	-501	-633	-88	119	204	407	1262	-270	30	1406
855/6	855/2	-3	0	-5	-34	-18	56	-66	19	120	66	236	236	-228	20	-240	-330	330	-430	308	-948	1118	-52	1044	-960	326
855/7	95/4	-5	0	-5	-32	12	-42	-114	19	-160	-214	-144	94	6	-308	-184	274	-276	-826	52	344	-166	-688	-996	-1578	786
858/1		-2	3	13	27	-11	13	-24	54	73	57	40	254	25	227	-412	234	-611	-305	-697	322	29	274	226	168	-514
861/1		0	-3	-17	7	-22	-45	48	88	169	-55	48	-371	-41	112	273	21	790	-428	-365	-192	382	199	842	-1436	-581
861/2		3	3	8	7	-47	-36	-54	-77	-145	-2	105	238	41	223	-279	60	-511	349	-50	228	184	13	-68	-274	-1871

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
864/1		0	0	19	13	-65	-56	108	58	66	118	-145	190	430	530	74	-295	-628	360	146	-388	753	1136	-153	-850	391
864/2	864/1	0	0	19	-13	65	-56	108	-58	-66	118	145	190	430	-530	-74	-295	628	360	-146	388	753	-1136	153	-850	391
864/3	864/1	0	0	-19	13	65	-56	-108	58	-66	-118	-145	190	-430	530	-74	295	628	360	146	388	753	1136	153	850	391
864/4	864/1	0	0	-19	-13	-65	-56	-108	-58	66	-118	145	190	-430	-530	74	295	-628	360	-146	-388	753	-1136	-153	850	391
867/1	51/2	1	3	10	8	-12	-26	0	-148	-152	66	32	266	6	-92	-288	-546	420	-350	940	-424	-378	-288	748	-1558	-530
867/2	51/1	-1	3	-16	-34	48	58	0	20	-58	0	218	-184	138	148	-516	-162	-180	-152	-956	538	462	-390	1268	-770	-494
867/3	51/3	-1	-3	20	2	48	-14	0	92	122	36	182	-76	-294	-428	-12	-234	-540	820	700	-794	1038	-858	1052	1102	-710
867/4		-3	3	-9	4	9	2	0	-40	-174	225	103	160	-78	452	282	555	-93	-638	-766	624	-929	-191	-1140	576	349
867/5	867/4	-3	-3	9	-4	-9	2	0	-40	174	-225	-103	-160	78	452	282	555	-93	638	-766	-624	929	191	-1140	576	-349
867/6		4	3	-2	-31	72	-47	0	-61	-62	-132	47	125	-288	-59	-264	-600	-408	937	403	-34	-642	-1248	22	674	1315
867/7	867/6	4	-3	2	31	-72	-47	0	-61	62	132	-47	-125	288	-59	-264	-600	-408	-937	403	34	642	1248	22	674	-1315
870/1		2	3	5	-19	7	-73	-49	-160	12	-29	-168	-104	282	-198	381	32	240	-138	131	-58	-508	-280	-908	1335	-1744
870/2		2	3	5	-24	-68	52	-54	10	-158	-29	-88	266	-388	-128	-504	162	180	-868	-474	-288	162	560	512	1100	1286
870/3		2	3	-5	7	-37	-75	29	-14	-192	29	-256	216	-22	-268	183	-166	-222	456	-31	-504	600	1230	-150	-1121	-610
870/4		2	3	-5	-14	26	-12	-118	-56	-108	29	-102	-134	-176	460	8	478	-628	-524	-556	784	-1038	-1234	18	-624	1686
870/5		2	3	-5	26	57	-58	-24	50	57	-29	92	-49	357	137	-354	-483	-600	932	-1054	942	-163	380	327	1530	-529
870/6		2	-3	5	2	6	68	-54	68	-108	-29	110	146	-300	260	360	450	-168	860	-100	576	-970	866	978	-180	674
870/7		-2	3	-5	15	35	49	117	-50	-16	29	-304	104	322	-368	75	318	-306	524	201	180	-456	314	766	1459	-574
870/8		-2	-3	-5	30	35	-38	-84	-146	137	29	-256	-67	463	19	-30	669	156	-796	-870	-990	-225	296	-761	-458	557
876/1		0	-3	4	-2	-20	-2	128	-76	-144	96	250	6	294	-106	-488	12	-236	-38	744	-520	-73	-120	600	-1518	-1186
880/1	440/3	0	0	5	-16	11	-70	-10	12	84	30	72	310	18	388	516	-298	-204	-210	432	440	46	616	-740	-6	490
880/2	110/7	0	-1	5	-23	11	50	75	-17	174	-153	-35	-277	-258	220	-210	-273	-438	-475	-992	927	-934	-974	90	1377	-64
880/3	55/1	0	3	-5	9	-11	2	21	85	-22	-165	83	1	-478	8	-126	-683	290	257	-776	313	902	-830	-842	25	-1784
880/4	440/2	0	4	5	-8	-11	-58	114	4	152	-138	-208	-226	-294	-276	240	-370	716	-650	-124	-232	-454	144	692	-1206	-1438
880/5	110/4	0	4	-5	22	11	-20	-20	8	204	122	-40	278	302	330	-60	-418	-188	-670	568	-128	676	876	1130	822	-434
880/6	110/2	0	-4	5	-20	-11	26	-42	-116	-96	270	-32	-106	-462	40	504	-570	-12	590	388	240	302	-8	48	282	-646
880/7	110/1	0	-4	-5	30	-11	16	-112	64	-36	10	48	-146	278	330	-476	150	-732	-30	848	-240	-1128	-788	698	-458	134
880/8	440/1	0	5	5	-1	-11	18	-113	-55	-190	-69	255	51	-314	484	-470	-545	102	129	664	1029	-758	-634	654	-511	1736
880/9	220/3	0	5	5	-11	11	-22	9	-89	-138	201	-77	119	-102	-260	-294	51	-270	-733	-728	849	830	214	-138	633	-892
880/10	220/1	0	-5	-5	19	11	-62	19	131	-138	-79	-217	-91	158	-120	546	-439	-290	-373	-728	709	850	1194	-58	753	1228
880/11	440/4	0	-6	5	32	-11	-48	-36	44	-58	-278	112	194	-314	-396	410	170	-404	250	26	468	-164	664	-1348	534	-1498
880/12	110/3	0	7	5	35	-11	26	101	-127	58	-27	177	191	66	-444	-2	-669	-386	-521	-96	427	1006	-910	818	601	-228
880/13	110/5	0	-7	-5	-11	-11	2	-9	85	138	45	-227	-19	-138	88	534	297	450	287	304	-777	962	-290	-1422	-1455	116
880/14	110/6	0	8	-5	-26	-11	92	-84	-80	-72	-30	208	86	-378	-542	-216	-18	-420	-718	124	-912	-268	940	498	150	446
880/15	110/8	0	-8	5	12	11	-34	-86	4	-148	134	280	430	-6	136	28	-658	-4	-90	-96	-816	-430	-1296	608	810	706
880/16	220/2	0	-8	5	-24	11	-22	22	28	44	110	40	-362	210	-260	460	662	68	606	312	-360	-1042	552	-268	-966	-1334
882/1	6/1	2	0	6	0	-12	-38	-126	-20	-168	-30	88	254	42	-52	-96	-198	-660	538	884	-792	-218	-520	-492	810	-1154
882/2	294/3	2	0	6	0	30	53	84	-97	-84	180	179	-145	-126	-325	366	768	264	818	-523	342	-43	-1171	810	600	386
882/3	126/2	2	0	-6	0	30	-2	-66	52	114	72	196	-286	378	164	228	-348	348	106	596	630	1042	-88	1440	-1374	34
882/4	294/3	2	0	-6	0	30	-53	-84	97	-84	180	-179	-145	126	-325	-366	768	-264	-818	-523	342	43	-1171	-810	-600	-386
882/5	98/1	2	0	7	0	-35	-66	59	-137	7	-106	-75	11	-498	260	-171	417	-17	-51	439	784	-295	-495	932	-873	290
882/6	98/1	2	0	-7	0	-35	66	-59	137	7	-106	75	11	498	260	171	417	17	51	439	784	295	-495	-932	873	-290
882/7	294/5	2	0	8	0	-40	-4	-84	-148	-84	-58	136	-222	420	-164	488	-478	548	-692	-908	524	-440	1216	-684	604	832
882/8	294/5	2	0	-8	0	-40	4	84	148	-84	-58	-136	-222	-420	-164	-488	-478	-548	692	-908	524	440	1216	684	-604	-832
882/9	14/1	2	0	-14	0	28	-18	74	-80	112	-190	-72	-346	162	-412	24	-318	-200	198	-716	-392	-538	240	-1072	810	-1354
882/10	294/4	2	0	15	0	9	-88	84	104	84	-51	185	44	168	326	138	-639	-159	722	-166	-1086	218	-583	597	1038	-169
882/11	294/4	2	0	-15	0	9	88	-84	-104	84	-51	-185	44	-168	326	-138	-639	159	-722	-166	-1086	-218	-583	-597	-1038	169
882/12	126/1	2	0	22	0	-26	54	74	-116	58	-208	252	50	126	164	-444	-12	124	162	-860	238	146	-984	656	-954	-526
882/13	42/2	-2	0	2	0	8	42	-2	124	-76	-254	72	398	462	212	-264	162	-772	-30	-764	236	-418	552	1036	30	1190
882/14	126/2	-2	0	6	0	-30	-2	66	52	-114	-72	196	-286	-378	164	-228	348	-348	106	596	-630	1042	-88	-1440	1374	34
882/15	98/4	-2	0	9	0	57	-70	-51	5	-69	-114	23	-253	42	-124	-201	393	-219	-709	419	96	-313	461	588	1017	-1834
882/16	98/4	-2	0	-9	0	57	70	51	-5	-69	-114	-23	-253	-42	-124	201	393	219	709	419	96	313	461	-588	-1017	1834
882/17	14/2	-2	0	-12	0	-48	-56	-114	-2	120	54	-236	146	126	-376	-12	-174	138	-380	-484	-576	1150	776	378	-390	1330
882/18	42/1	-2	0	18	0	72	34	6	-92	180	114	-56	-34	6	164	168	-654	-492	250	-124	-36	-1010	56	228	390	70
882/19	126/1	-2	0	-22	0	26	54	-74	-116	-58	208	252	50	-126	164	444	12	-124	162	-860	-238	146	-984	-656	954	-526
884/1		0	2	-10	26	20	-13	-17	-52	-8	152	-14	-34	-280	-132	352	106	-372	288	-676	-894	44	-964	-516	-82	-1284
884/2		0	2	12	4	-2	-13	-17	-140	36	-288	-36	208	-38	-88	-440	282	376	244	-368	-168	198	1060	-120	-126	-1878
885/1		-3	3	-5	-27	-71	-65	-63	-50	-40	178	-306	-201	-303	493	-228	-312	59	134	-204	47	-210	-567	19	-568	-560
891/1		1	0	7	22	-11	-77	91	126	-16	-135	70	69	-266	552	476	-510	266	-189	482	624	-791	-316	1302	1449	1358
891/2	891/1	-1	0	-7	22	11	-77	-91	126	16	135	70	69	266	552	-476	510	-266	-189	482	-624	-791	-316	-1302	-1449	1358
891/3		4	0	19	-26	-11	-56	-104	-96	-40	-18	49	75	-296	372	149	417	17	90	1073	285	-962	596	498	-1230	-331
891/4	891/3	-4	0	-19	-26	11	-56	104	-96	40	18	49	75	296	372	-149	-417	-17	90	1073	-285	-962	596	-498	1230	-331

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
892/1		0	-7	-2	-14	-45	-72	1	78	-189	-259	-106	-271	-189	48	326	359	-171	-618	-389	-672	-1169	232	78	-177	1524
897/1		1	3	6	-8	12	13	-126	116	23	-154	-32	22	-70	12	-48	-602	212	-706	-124	120	-198	-1200	852	-806	546
900/1	180/1	0	0	0	-2	30	4	-90	-28	-120	210	-4	-200	240	136	120	30	-450	-166	-908	-1020	250	-916	1140	-420	-1538
900/2	180/1	0	0	0	-2	-30	4	90	-28	120	-210	-4	-200	-240	136	-120	-30	450	-166	-908	1020	250	-916	-1140	420	-1538
900/3	300/4	0	0	0	7	54	55	18	-25	-18	54	-271	-314	360	163	522	-36	-126	47	343	1080	1054	-568	1422	-1440	439
900/4	300/4	0	0	0	-7	54	-55	-18	-25	18	54	-271	314	360	-163	-522	36	-126	47	-343	1080	-1054	-568	-1422	-1440	-439
900/5	12/1	0	0	0	-8	-36	10	18	-100	72	234	-16	226	-90	-452	432	414	684	422	-332	360	-26	512	-1188	630	1054
900/6	300/1	0	0	0	13	-6	-5	-78	65	138	-66	299	214	-360	-203	78	636	-786	467	217	360	286	272	498	0	511
900/7	300/1	0	0	0	-13	-6	5	78	65	-138	-66	299	-214	-360	203	-78	-636	-786	467	-217	360	-286	272	-498	0	-511
900/8	20/1	0	0	0	16	60	-86	18	44	48	186	176	-254	-186	100	168	-498	252	-58	1036	-168	-506	272	948	1014	766
900/9		0	0	0	17	0	-19	0	107	0	0	-19	110	0	449	0	0	0	-901	-127	0	1190	884	0	0	1853
900/10	900/9	0	0	0	-17	0	19	0	107	0	0	-19	-110	0	-449	0	0	0	-901	127	0	-1190	884	0	0	-1853
900/11	300/2	0	0	0	22	14	-30	-62	-120	-188	-96	184	406	-130	148	-448	414	-266	-838	248	-1020	484	-48	-548	650	-1816
900/12	300/2	0	0	0	-22	14	30	62	-120	188	-96	184	-406	-130	-148	448	-414	-266	-838	-248	-1020	-484	-48	548	650	1816
900/13	100/1	0	0	0	26	-45	44	-117	-91	18	-144	26	-214	459	-460	468	-558	72	-118	251	-108	299	-898	-927	-351	386
900/14	100/1	0	0	0	-26	-45	-44	117	-91	-18	-144	26	214	459	460	-468	558	72	-118	-251	-108	-299	-898	927	-351	-386
900/15	60/1	0	0	0	28	24	70	102	20	-72	-306	-136	214	150	292	-72	-414	744	-418	-188	-480	-434	1352	-612	30	286
900/16	60/2	0	0	0	-32	-36	10	-78	140	-192	-6	-16	34	390	52	408	-114	-516	-58	892	120	646	-1168	-732	1590	-194
900/17		0	0	0	37	0	-89	0	-163	0	0	-289	-110	0	-71	0	0	0	719	-1007	0	-1190	884	0	0	523
900/18	900/17	0	0	0	-37	0	89	0	-163	0	0	-289	110	0	71	0	0	0	719	1007	0	1190	884	0	0	-523
902/1		2	-4	5	21	11	74	-9	7	33	-45	-29	281	-41	-58	-556	372	735	840	396	-688	-366	-880	1232	980	-426
902/2		-2	8	21	-31	11	78	-11	-85	63	145	-83	-151	41	278	524	228	625	-468	44	-588	-62	-480	628	960	-1826
903/1		1	-3	11	-7	14	-23	39	-29	33	26	75	-279	373	43	-138	33	-45	-662	-887	378	-966	-292	1284	-1084	-1692
906/1		2	3	-15	10	-45	91	-76	-152	-33	-191	39	176	-243	162	384	-330	-440	110	-621	-109	-362	-392	-488	-707	-1386
909/1	303/1	3	0	8	-5	22	82	-3	-17	-32	275	-114	322	-16	169	160	667	706	-785	812	1042	-814	558	1472	-592	593
910/1		2	-2	5	-7	-36	-13	-44	58	146	254	292	230	0	-264	-264	580	54	-250	20	-166	558	-176	-196	520	1218
910/2		2	-2	-5	7	12	13	120	-142	-174	-90	20	326	-252	344	-336	-636	-906	-646	-700	522	-322	-1384	1296	-12	1154
910/3		2	5	5	-7	-1	-13	138	30	139	-110	-23	377	259	422	499	-92	-128	-495	-267	-768	-219	727	-490	-1258	539
910/4		2	-8	5	7	-30	-13	44	-44	-124	150	-86	270	176	-510	552	-716	76	-410	-736	-988	390	240	-124	-1200	-1110
910/5		-2	8	-5	-7	14	13	-112	-32	-96	-162	178	-6	208	-74	-504	-300	-584	-18	-16	1044	-198	-1152	1092	-984	-1738
912/1	228/1	0	3	4	12	-40	-40	-66	19	98	-130	-262	-296	-442	164	542	334	-60	614	0	-400	318	-1154	636	-630	1006
912/2	228/2	0	3	-7	-21	37	26	-33	19	76	-218	266	-32	64	-133	-305	-766	72	-805	-264	-92	285	-1088	-420	426	-314
912/3	114/4	0	3	-11	15	29	-82	27	19	-100	-118	-70	232	8	287	-385	538	300	-901	-132	-472	-1131	52	-276	-1302	-1310
912/4	114/1	0	3	-19	-9	13	38	99	19	-68	130	-262	-296	-8	-73	271	-502	-540	587	-684	-992	-507	-980	492	810	-1046
912/5	228/3	0	-3	-3	17	19	-30	-97	-19	28	126	126	64	80	453	-107	-326	-56	47	168	-1060	-659	-592	-892	-310	-874
912/6	114/2	0	-3	-7	15	49	14	-33	19	148	-278	-94	160	400	-73	-173	170	12	419	-444	952	-27	556	276	1386	130
912/7	114/3	0	-3	12	-4	-8	-24	62	-19	-194	102	-18	-296	134	60	226	-362	316	134	240	800	-578	-1078	-940	170	206
912/8	57/1	0	-3	-12	20	4	-76	22	19	-82	242	126	-180	-390	-308	522	-70	-188	-706	-104	432	718	-94	1296	846	830
918/1		2	0	9	-28	36	-49	-17	-109	57	135	-280	212	-165	-517	-168	-12	42	-484	173	159	-448	-106	-24	-198	-724
918/2	918/1	-2	0	-9	-28	-36	-49	17	-109	-57	-135	-280	212	165	-517	168	12	-42	-484	173	-159	-448	-106	24	198	-724
924/1		0	-3	-2	-7	11	-34	42	-164	200	118	-88	30	-462	356	-184	-274	188	190	476	-392	1042	112	812	1034	114
928/1		0	7	-13	-16	45	61	-102	68	-194	-29	-149	400	280	-263	-509	-605	578	-718	260	-738	652	917	-678	-1008	-1764
928/2	928/1	0	-7	-13	16	-45	61	-102	-68	194	-29	149	400	280	263	509	-605	-578	-718	-260	738	652	-917	678	-1008	-1764
930/1		2	3	-5	-18	0	28	10	-120	-152	-28	31	-204	-94	88	296	-714	-390	-134	-442	-574	-1072	1160	-12	212	-226
930/2		2	3	-5	26	-48	-88	-54	-160	-48	-120	31	-304	162	272	96	582	30	-478	-334	1062	212	-640	972	120	86
930/3		-2	-3	5	-1	63	74	72	23	-15	-84	31	440	-276	83	270	-171	834	-802	14	-333	-715	-979	816	-441	-430
931/1	19/1	-3	5	12	0	-54	-11	93	-19	183	-249	-56	-250	-240	-196	168	435	-195	358	-961	-246	-353	-34	-234	168	-758
931/2	133/1	4	-8	-6	0	-68	-8	-14	19	188	70	-252	-186	-192	488	216	178	500	298	494	-618	842	10	-228	-600	976
933/1		3	3	3	5	-22	-74	126	-101	93	-34	-142	-441	353	-215	-424	-510	354	221	-630	783	938	903	-611	38	-1180
935/1		-3	1	5	-4	11	11	17	35	-132	111	35	164	120	-358	129	-465	57	101	776	-117	263	1184	222	-1641	-901
936/1	104/2	0	0	7	-21	-6	13	115	-46	-144	162	180	13	-192	-33	-383	-288	-442	-680	-722	207	274	-936	1204	966	-138
936/2	104/1	0	0	-19	-3	2	-13	-77	-58	-76	6	-292	207	-240	-317	375	692	-214	-488	782	1057	1174	892	-704	-6	830
938/1		2	1	9	7	-24	-61	-48	-16	-57	-123	-169	-43	219	20	-582	54	450	722	67	1155	290	-178	-1326	1224	290
938/2		2	-5	-9	7	-6	49	6	6	19	43	-101	-137	355	158	-216	-482	-536	-890	67	-477	606	-1320	-258	1150	-362
940/1		0	5	5	-5	-1	-43	-130	131	160	-274	-135	-244	-324	436	47	-216	-450	397	204	548	-523	-1194	-635	94	12
944/1	118/2	0	1	-13	27	8	42	2	77	-98	-295	40	278	179	132	202	-345	59	184	356	144	814	181	1250	-600	-790
944/2	236/1	0	-2	2	3	59	-33	47	-40	40	-4	124	-157	221	-291	526	132	59	82	524	15	538	-947	575	546	-34
944/3	118/1	0	-5	-5	33	4	-30	-14	-97	134	1	28	290	-5	-192	326	-537	-59	472	-856	-168	-686	919	-362	-312	-514
944/4	118/3	0	7	5	15	50	-66	14	11	172	287	-26	128	167	78	-38	-147	59	76	-514	1140	-848	-359	362	1212	836
945/1		4	0	-5	7	51	-33	26	79	194	-248	-80	232	351	529	53	529	-326	160	-515	600	697	572	-1381	1551	-810
945/2	945/1	-4	0	5	7	-51	-33	-26	79	-194	248	-80	232	-351	529	-53	-529	326	160	-515	-600	697	572	1381	-1551	-810
945/3		5	0	5	7	66	-87	91	-92	157	221	91	16	-18	-263	118	-223	-889	250	-83	-195	-554	176	1444	69	270

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
945/4	945/3	-5	0	-5	7	-66	-87	-91	-92	-157	-221	91	16	18	-263	-118	223	889	250	-83	195	-554	176	-1444	-69	270
946/1		2	-5	8	0	-11	34	-54	-94	126	113	-330	-394	280	-43	610	-465	116	-23	-344	488	-903	-101	-611	-764	-727
946/2		-2	-2	10	-27	11	-2	54	59	153	193	-177	-214	-190	-43	209	339	-161	13	-695	-848	-606	-614	1052	1070	-1627
950/1	190/2	2	2	0	-8	44	0	74	19	-84	266	136	-424	470	236	240	-36	736	650	830	-216	-254	-1220	688	102	1280
950/2	38/1	2	2	0	31	57	52	-69	19	72	-150	32	226	-258	67	-579	432	-330	-13	856	642	487	-700	12	-600	-1424
950/3	190/1	2	-2	0	12	-20	4	34	-19	-40	-150	-200	156	-218	-248	180	-72	-48	-134	-334	-520	-438	980	156	670	-1124
950/4	190/3	-2	4	0	20	-44	-42	86	19	164	-162	-312	-226	34	432	-580	-506	364	518	-924	320	542	-1208	1120	-1022	-1166
957/1		1	3	-17	-29	-11	-32	-89	-87	-106	29	-326	147	-413	-33	-1	-162	581	-378	410	1080	370	-610	-1020	882	-286
960/1	30/2	0	3	5	4	-48	-2	-114	140	-72	-210	-272	334	-198	-268	-216	78	240	-302	596	768	-478	640	-348	210	-1534
960/2	120/1	0	3	5	4	-72	6	38	-52	152	78	120	150	362	484	280	670	-696	-222	4	96	178	-632	612	994	1634
960/3	480/2	0	3	5	8	4	6	-2	-16	60	142	176	214	-278	-68	-116	350	684	394	108	96	-398	-136	436	-750	82
960/4	480/1	0	3	5	-12	-20	58	-70	-92	-112	-66	108	58	66	-388	408	-474	-540	-14	-276	96	-790	-308	-1036	1210	1426
960/5	15/1	0	3	5	20	24	-74	54	124	-120	78	200	70	330	-92	-24	-450	-24	322	196	-288	-430	-520	-156	1026	-286
960/6	120/4	0	3	5	20	56	86	-106	-4	136	206	-152	-282	-246	-412	40	126	-56	2	388	-672	1170	408	-668	66	-926
960/7	120/6	0	3	5	-20	16	-58	38	4	80	-82	8	-426	-246	-524	464	702	-592	-574	-172	-768	-558	-408	164	-510	514
960/8	60/1	0	3	5	-28	24	70	102	-20	-72	-306	-136	214	-150	292	-72	414	744	418	-188	480	434	1352	612	-30	-286
960/9	120/5	0	3	-5	0	-4	-54	114	-44	96	-134	-272	98	-6	-12	-200	-654	-36	442	188	-632	-390	688	-1188	-694	-1726
960/10	480/4	0	3	-5	-4	-40	90	-70	-40	108	-166	-40	130	-310	268	-556	370	-240	130	-876	-840	250	-880	188	-726	-1550
960/11	120/3	0	3	-5	-8	20	-22	-14	76	-56	154	-160	162	-390	388	544	210	-380	794	-148	840	858	-144	316	1098	994
960/12	480/5	0	3	-5	12	-24	-38	-6	104	-100	-230	56	-190	202	-148	-124	-206	-128	-190	-204	440	1210	-816	-1412	-214	1202
960/13	480/6	0	3	-5	16	-24	14	-18	-36	104	250	-28	54	354	-228	408	-262	64	-374	-300	1016	274	788	396	786	-1086
960/14	120/2	0	3	-5	-16	28	26	-62	68	-208	58	160	-270	282	-76	-280	210	-196	-742	-836	-504	-1062	768	1052	-726	-1406
960/15	15/2	0	3	-5	24	52	-22	-14	-20	168	-230	288	34	122	-188	-256	338	100	-742	-84	328	-38	240	1212	330	866
960/16	60/2	0	3	-5	32	-36	10	-78	-140	-192	-6	-16	34	-390	52	408	114	-516	58	892	-120	-646	-1168	732	-1590	194
960/17	30/1	0	3	-5	-32	-60	34	42	-76	0	-6	232	-134	234	-412	360	-222	660	490	812	-120	746	-152	-804	-678	194
960/18	480/3	0	3	-5	-32	64	6	38	-116	120	122	-164	-146	-238	-148	184	-470	-216	-806	-732	-264	-638	-596	-884	930	322
960/19	30/2	0	-3	5	-4	48	-2	-114	-140	72	-210	272	334	-198	268	216	78	-240	-302	-596	-768	-478	-640	348	210	-1534
960/20	120/1	0	-3	5	-4	72	6	38	52	-152	78	-120	150	362	-484	-280	670	696	-222	-4	-96	178	632	-612	994	1634
960/21	480/2	0	-3	5	-8	-4	6	-2	16	-60	142	-176	214	-278	68	116	350	-684	394	-108	-96	-398	136	-436	-750	82
960/22	480/1	0	-3	5	12	20	58	-70	92	112	-66	-108	58	66	388	-408	-474	540	-14	276	-96	-790	308	1036	1210	1426
960/23	120/6	0	-3	5	20	-16	-58	38	-4	-80	-82	-8	-426	-246	524	-464	702	592	-574	172	768	-558	408	-164	-510	514
960/24	15/1	0	-3	5	-20	-24	-74	54	-124	120	78	-200	70	330	92	24	-450	24	322	-196	288	-430	520	156	1026	-286
960/25	120/4	0	-3	5	-20	-56	86	-106	4	-136	206	152	-282	-246	412	-40	126	56	2	-388	672	1170	-408	668	66	-926
960/26	60/1	0	-3	5	28	-24	70	102	20	72	-306	136	214	-150	-292	72	414	-744	418	188	-480	434	-1352	-612	-30	-286
960/27	120/5	0	-3	-5	0	4	-54	114	44	-96	-134	272	98	-6	12	200	-654	36	442	-188	632	-390	-688	1188	-694	-1726
960/28	480/4	0	-3	-5	4	40	90	-70	40	-108	-166	40	130	-310	-268	556	370	240	130	876	840	250	880	-188	-726	-1550
960/29	120/3	0	-3	-5	8	-20	-22	-14	-76	56	154	160	162	-390	-388	-544	210	380	794	148	-840	858	144	-316	1098	994
960/30	480/5	0	-3	-5	-12	24	-38	-6	-104	100	-230	-56	-190	202	148	124	-206	128	-190	204	-440	1210	816	1412	-214	1202
960/31	120/2	0	-3	-5	16	-28	26	-62	-68	208	58	-160	-270	282	76	280	210	196	-742	836	504	-1062	-768	-1052	-726	-1406
960/32	480/6	0	-3	-5	-16	24	14	-18	36	-104	250	28	54	354	228	-408	-262	-64	-374	300	-1016	274	-788	-396	786	-1086
960/33	15/2	0	-3	-5	-24	-52	-22	-14	20	-168	-230	-288	34	122	188	256	338	-100	-742	84	-328	-38	-240	-1212	330	866
960/34	30/1	0	-3	-5	32	60	34	42	76	0	-6	-232	-134	234	412	-360	-222	-660	490	-812	120	746	152	804	-678	194
960/35	480/3	0	-3	-5	32	-64	6	38	116	-120	122	164	-146	-238	148	-184	-470	216	-806	732	264	-638	596	884	930	322
960/36	60/2	0	-3	-5	-32	36	10	-78	140	192	-6	16	34	-390	-52	-408	114	516	58	-892	120	-646	1168	-732	-1590	194
961/1		1	0	2	16	0	0	0	156	0	0	0	0	-278	0	-616	0	-740	0	684	1000	0	0	0	0	-1906
962/1		-2	-5	12	-29	3	-13	138	40	-116	-86	170	37	-243	-122	159	409	-184	738	-44	293	-539	-238	-923	446	1024
966/1		2	-3	-9	7	-12	11	96	-40	-23	-231	-94	47	-27	479	423	516	882	842	-844	654	-496	260	-156	-414	1343
966/2		-2	3	-6	7	48	38	114	56	-23	-162	-16	-46	-342	248	-24	426	-852	338	488	336	362	1184	-336	-78	746
968/1	88/2	0	-1	-7	6	0	40	78	-36	7	-8	183	227	36	-322	-184	-6	-99	-164	-695	-987	248	242	1494	-905	-1031
968/2		0	-2	13	10	0	-27	27	-38	150	285	-198	57	227	64	390	-267	280	-50	-546	772	-178	1058	378	-1185	-733
968/3	968/2	0	-2	13	-10	0	27	-27	38	150	-285	-198	57	-227	-64	390	-267	280	50	-546	772	178	-1058	-378	-1185	-733
968/4	8/1	0	-4	-2	-24	0	-22	-50	-44	-56	-198	-160	-162	198	-52	528	-242	-668	-550	188	728	-154	656	-236	714	-478
968/5	88/1	0	7	9	-2	0	0	38	-44	175	264	159	-173	220	542	-264	682	421	-308	177	365	528	-686	-698	967	-1127
969/1		-4	-3	19	-1	28	-76	-17	-19	180	180	-152	306	-170	144	-457	105	310	840	-93	83	-4	210	1183	333	-1429
969/2		5	-3	2	0	-48	-90	17	-19	8	-146	68	14	14	356	-408	222	564	34	-340	-1020	850	1260	-172	98	-26
969/3		-5	-3	-22	-8	-24	34	17	19	-112	-294	-164	-206	78	-548	576	-558	-84	278	-572	-176	-822	540	-1232	-658	386
970/1		2	-4	5	33	-37	5	-102	-100	-57	-222	-201	365	-252	502	-440	128	-570	-373	-761	524	448	-1091	572	457	97
972/1		0	0	0	17	0	-70	0	56	0	0	-19	-433	0	71	0	0	0	719	-127	0	-919	884	0	0	-1330
972/2		0	0	0	20	0	-19	0	-163	0	0	-19	110	0	71	0	0	0	-901	1007	0	-271	-1387	0	0	-523
972/3		0	0	0	20	0	89	0	107	0	0	-289	110	0	449	0	0	0	719	-127	0	-919	503	0	0	1853
972/4		0	0	0	-37	0	-70	0	56	0	0	-289	323	0	449	0	0	0	-901	1007	0	-271	884	0	0	-1330
975/1	39/1	0	3	0	-2	-36	-13	78	74	96	18	-214	286	-384	-524	-300	-558	576	74	-38	-456	682	704	888	-1020	-110

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
975/2		0	3	0	-17	39	-13	93	-76	-174	-57	281	286	-264	166	-315	207	-519	-511	-803	624	-698	554	-507	360	-1370
975/3	975/2	0	-3	0	17	39	13	-93	-76	174	-57	281	-286	-264	-166	315	-207	-519	-511	803	624	698	554	507	360	1370
975/4	195/1	3	3	0	-2	24	-13	-24	-70	-90	-120	-196	214	-54	196	-120	-18	-312	-322	376	240	-1136	-808	-1092	-618	880
975/5		3	-3	0	-4	34	-13	100	-22	150	14	-292	-354	-102	-344	-448	238	-190	-254	764	1096	-418	-1352	-156	-930	114
975/6	195/4	3	-3	0	16	-36	-13	30	68	120	-186	8	226	-342	76	552	738	780	-154	-596	1056	22	-112	684	90	334
975/7	975/5	-3	3	0	4	34	13	-100	-22	-150	14	-292	354	-102	344	448	-238	-190	-254	-764	1096	418	-1352	156	-930	-114
975/8	195/3	-4	-3	0	-18	10	13	-46	-14	36	-22	42	46	-226	224	50	290	130	70	138	-586	758	1068	-378	1374	1822
975/9	195/2	5	-3	0	-8	-56	13	-58	24	-36	-242	-64	254	-414	164	40	-82	-744	494	508	384	-462	-816	92	1210	530
976/1	244/1	0	-2	5	7	61	-47	16	-112	33	-98	56	68	53	144	-84	6	335	-61	-793	-120	-819	-763	468	1116	-1726
980/1		0	1	-5	0	-21	-9	-123	50	180	-197	170	-80	470	270	313	-290	-370	80	470	-712	-330	457	820	1020	1433
980/2	140/1	0	-1	5	0	-7	23	25	62	-86	-29	12	-150	-204	-178	-33	452	-120	-920	-300	520	-370	-1013	636	-292	1381
980/3	980/1	0	-1	5	0	-21	9	123	-50	180	-197	-170	-80	-470	270	-313	-290	370	-80	470	-712	330	457	-820	-1020	-1433
980/4		0	2	5	0	27	-41	-6	49	-81	66	-188	23	-51	-202	-327	-201	24	196	-964	-1140	868	638	-36	810	-1550
980/5		0	2	-5	0	-69	-11	102	-47	-51	222	-152	221	333	290	555	261	-360	-824	776	-720	580	-226	816	-918	-470
980/6	980/5	0	-2	5	0	-69	11	-102	47	-51	222	152	221	-333	290	-555	261	360	824	776	-720	-580	-226	-816	918	470
980/7	980/4	0	-2	-5	0	27	41	6	-49	-81	66	188	23	51	-202	327	-201	-24	-196	-964	-1140	-868	638	36	-810	1550
980/8	140/4	0	4	-5	0	68	-22	30	-108	184	166	32	-370	-154	212	512	-98	860	-390	60	840	630	1312	436	598	-914
980/9	20/1	0	-4	-5	0	-60	-86	-18	-44	48	-186	-176	254	-186	-100	-168	-498	252	58	-1036	168	-506	272	-948	1014	766
980/10	140/2	0	5	5	0	15	-17	-123	-86	54	-177	-212	74	444	-46	-471	-180	-144	376	356	-48	-818	89	780	-1140	169
980/11	140/6	0	5	-5	0	-15	13	27	154	-186	3	328	254	-96	134	-51	240	396	616	296	-48	322	659	-300	-1020	199
980/12	140/3	0	-8	5	0	28	-82	46	-8	-128	174	152	-290	-50	396	296	-570	272	662	876	-880	638	-600	-624	-698	-754
980/13	140/5	0	-9	-5	0	55	69	-113	126	-102	-81	-176	254	184	-230	187	-488	-388	728	-96	8	994	337	-188	884	451
984/1		0	3	-16	18	-1	82	-119	-16	110	-225	167	81	-41	-65	225	-322	764	61	830	535	349	538	436	810	260
987/1		-3	3	2	-7	46	44	-78	-152	-148	122	314	-406	480	-52	-47	-110	-144	426	-716	448	-926	524	-904	-642	1630
987/2		-3	-3	10	-7	8	2	-30	100	-188	-98	248	-82	318	-472	47	-274	420	30	1000	-784	142	-496	844	1410	-974
990/1	330/10	2	0	5	2	-11	-16	-96	-112	-180	102	-208	110	90	-10	180	618	36	-286	-928	-48	-520	-412	618	234	422
990/2	330/5	2	0	5	-6	11	48	52	-76	132	-134	-192	30	334	334	572	10	220	-302	392	704	-184	16	1446	114	-642
990/3	330/6	2	0	5	-6	-11	-40	-80	56	-44	-178	-16	-146	-414	158	44	-166	-44	402	744	-1056	1136	-468	-182	-678	-1082
990/4	110/1	2	0	5	-30	-11	16	112	-64	-36	-10	-48	-146	-278	-330	-476	-150	-732	-30	-848	-240	-1128	788	698	458	134
990/5		2	0	-5	-8	-11	80	-74	-98	22	-90	-168	362	-66	-294	286	-474	-728	892	-444	-842	-1082	-1196	764	-1024	-642
990/6	330/7	2	0	-5	-16	11	38	-18	44	-168	-54	8	-130	174	164	-528	-510	-780	-82	92	-336	-574	56	-1044	-426	1298
990/7	330/8	2	0	-5	-16	-11	-50	70	-44	96	122	184	134	86	-12	264	194	716	182	-436	104	-134	-648	628	102	418
990/8	330/9	2	0	-5	20	-11	26	-6	-28	48	162	128	86	-66	344	-312	-486	84	494	716	432	206	440	-192	294	1082
990/9	110/2	2	0	-5	20	-11	26	42	116	-96	-270	32	-106	462	-40	504	570	-12	590	-388	240	302	8	48	-282	-646
990/10	110/3	2	0	-5	-35	-11	26	-101	127	58	27	-177	191	-66	444	-2	669	-386	-521	96	427	1006	910	818	-601	-228
990/11	330/3	-2	0	5	2	11	-28	36	-64	-12	126	-280	-298	-54	62	444	-366	-108	146	848	-48	-628	-676	-342	570	-178
990/12	990/5	-2	0	5	-8	11	80	74	-98	-22	90	-168	362	66	-294	-286	474	728	892	-444	842	-1082	-1196	-764	1024	-642
990/13	330/1	-2	0	5	10	11	44	-124	-56	-100	-42	-120	86	-222	54	-76	162	68	-734	-552	320	292	676	-422	490	174
990/14	110/5	-2	0	5	11	-11	2	9	-85	138	-45	227	-19	138	-88	534	-297	450	287	-304	-777	962	290	-1422	1455	116
990/15	110/4	-2	0	5	-22	11	-20	20	-8	204	-122	40	278	-302	-330	-60	418	-188	-670	-568	-128	676	-876	1130	-822	-434
990/16	110/6	-2	0	5	26	-11	92	84	80	-72	30	-208	86	378	542	-216	18	-420	-718	-124	-912	-268	-940	498	-150	446
990/17	330/2	-2	0	5	-34	-11	-88	-36	-100	-12	90	-208	86	438	362	-516	-102	420	-118	416	408	-808	-160	18	930	1406
990/18	110/8	-2	0	-5	-12	11	-34	86	-4	-148	-134	-280	430	6	-136	28	658	-4	-90	96	-816	-430	1296	608	-810	706
990/19	110/7	-2	0	-5	23	11	50	-75	17	174	153	35	-277	258	-220	-210	273	-438	-475	992	927	-934	974	90	-1377	-64
990/20	330/4	-2	0	-5	-24	-11	-30	110	56	144	182	24	-234	26	-68	-224	146	116	-818	-4	-176	-826	532	-1008	-1098	42
1007/1		-3	10	11	10	33	-8	75	-19	99	-6	157	-322	-110	-157	124	-53	463	-922	76	60	-784	-429	798	-411	-1713
1008/1	168/5	0	0	2	7	52	86	30	4	120	-246	-80	-290	374	-164	464	162	180	-666	628	296	-518	1184	220	774	-1086
1008/2	168/1	0	0	2	-7	12	-66	70	92	16	122	-64	-306	-50	-20	-176	-526	540	-818	228	864	106	-736	-588	-146	-1214
1008/3	42/2	0	0	-2	7	-8	-42	2	124	76	-254	72	398	-462	-212	-264	162	-772	30	764	-236	418	-552	1036	-30	-1190
1008/4	21/1	0	0	4	7	62	-62	-84	-100	-42	10	48	-246	248	-68	324	-258	120	622	-904	-678	-642	-740	468	-200	-1266
1008/5	168/3	0	0	-4	7	-26	2	36	76	-114	-6	256	-86	-160	220	308	-258	264	606	520	-286	-530	44	1012	-768	222
1008/6	126/2	0	0	6	-7	-30	2	66	52	-114	72	196	-286	-378	-164	228	-348	348	-106	-596	-630	-1042	88	1440	1374	-34
1008/7	28/2	0	0	-6	-7	-12	-82	30	-68	216	-246	112	110	246	172	192	-558	540	110	-140	-840	-550	208	516	1398	1586
1008/8	126/2	0	0	-6	-7	30	2	-66	52	114	-72	196	-286	378	-164	-228	348	-348	-106	-596	630	-1042	88	-1440	-1374	-34
1008/9	84/1	0	0	-6	-7	36	62	-114	76	-24	-54	112	-178	-378	172	-192	402	396	254	1012	840	890	-80	-108	1638	1010
1008/10	28/1	0	0	8	7	-40	-12	58	-26	-64	62	-252	26	-6	-416	-396	450	274	-576	476	-448	-158	936	530	390	214
1008/11	56/1	0	0	-8	7	56	-28	90	-74	-96	222	100	58	-422	-512	148	642	-318	720	412	448	994	296	386	6	-138
1008/12	168/2	0	0	10	7	-52	-10	54	52	48	186	-224	94	478	316	256	66	420	342	-668	-272	-86	-1360	188	366	1554
1008/13	168/4	0	0	10	-7	-12	30	-34	-148	152	106	-304	-114	-202	-116	224	274	-660	382	-12	-552	-614	-880	-108	86	1426
1008/14	14/2	0	0	12	-7	48	56	114	-2	-120	54	-236	146	-126	376	-12	-174	138	380	484	576	-1150	-776	378	390	-1330
1008/15	14/1	0	0	14	7	-28	18	-74	-80	-112	-190	-72	-346	-162	412	24	-318	-200	-198	716	392	538	-240	-1072	-810	1354
1008/16	84/2	0	0	-14	7	4	54	14	-92	-152	106	144	158	390	508	-528	-606	-364	678	-844	-8	-422	-384	-548	-1194	-1502

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1008/17	56/2	0	0	16	7	24	-68	-54	46	176	174	116	74	10	480	-572	162	-86	-904	-660	1024	770	904	682	102	-218
1008/18	168/6	0	0	16	-7	-18	-54	128	-52	-202	-302	200	-150	-172	-164	-460	190	96	622	-744	-54	742	92	-228	116	-554
1008/19	7/1	0	0	-16	7	-8	28	-54	110	48	110	-12	-246	-182	-128	324	162	810	-488	-244	-768	-702	-440	-1302	-730	294
1008/20	21/2	0	0	18	-7	-36	-34	-42	124	0	-102	160	398	318	268	240	498	-132	398	-92	-720	-502	1024	-204	-354	-286
1008/21	42/1	0	0	-18	-7	-72	-34	-6	-92	-180	114	-56	-34	-6	-164	168	-654	-492	-250	124	36	1010	-56	228	-390	-70
1008/22	126/1	0	0	22	7	-26	-54	74	-116	58	208	252	50	126	-164	444	12	-124	-162	860	238	-146	984	-656	-954	526
1008/23	126/1	0	0	-22	7	26	-54	-74	-116	-58	-208	252	50	-126	-164	-444	-12	124	-162	860	-238	-146	984	656	954	526
1014/1	78/3	2	3	-10	8	-40	0	130	20	0	-18	184	74	362	76	452	382	-464	358	700	748	-1058	-976	1008	386	614
1014/2	78/2	2	3	16	8	38	0	-78	72	-52	242	-76	-342	336	76	-94	-450	-854	-110	908	-838	970	-352	-474	1452	562
1014/3	6/1	2	-3	-6	16	-12	0	-126	-20	168	30	88	-254	-42	-52	96	198	660	-538	-884	-792	-218	-520	492	-810	-1154
1014/4		2	-3	7	16	-64	0	-9	-72	-92	-113	-224	279	387	-260	-112	471	-380	-317	-260	-64	-1141	884	1428	282	-478
1014/5		2	-3	-8	14	30	0	46	-66	112	-170	-110	4	380	92	114	558	-74	902	646	-930	832	360	178	-204	1416
1014/6	78/1	2	-3	16	-28	-34	0	138	-108	-52	-190	176	-342	-240	-140	-454	198	154	34	656	-550	-614	8	-762	444	-1022
1014/7	78/6	-2	3	-4	-4	-2	0	-6	36	-20	-14	152	258	-84	-188	-254	366	-550	-14	-448	-926	-254	1328	-186	336	-614
1014/8	78/5	-2	-3	-6	-20	-24	0	-30	16	-72	-282	-164	-110	126	164	204	-738	-120	614	-848	-132	-218	-1096	-552	-210	1726
1014/9	1014/4	-2	-3	-7	-16	64	0	-9	72	-92	-113	224	-279	-387	-260	112	471	380	-317	260	64	1141	884	-1428	-282	478
1014/10	1014/5	-2	-3	8	-14	-30	0	46	66	112	-170	110	-4	-380	92	-114	558	74	902	-646	930	-832	360	-178	204	-1416
1014/11	78/4	-2	-3	20	32	-50	0	-30	120	-20	82	44	306	-108	-356	178	198	-94	-62	140	778	-62	-1096	462	-1224	-614
1015/1		0	-4	-5	-7	-18	-10	52	70	48	29	2	-192	-18	312	-584	642	-196	-386	356	964	1176	-230	-1268	446	172
1017/1	339/1	4	0	-17	-15	24	-7	123	-108	69	30	233	2	374	90	-288	446	585	-39	308	-315	-280	730	1378	458	-818
1020/1		0	3	5	-8	20	-42	17	124	-40	6	-176	366	250	-76	512	766	-204	-490	-292	472	634	736	-116	-54	-926
1020/2		0	3	-5	20	-20	-90	17	84	36	-102	-228	-206	-270	-132	232	-378	564	-854	844	468	-126	-420	-396	378	-398
1023/1		1	-3	-14	34	-11	16	-110	6	-112	-174	-31	-210	70	432	-298	-288	50	240	32	198	688	128	1092	-384	-898
1025/1	205/1	1	-2	0	-8	-54	-68	10	-150	64	-56	-336	-66	-41	-188	536	-172	-24	-262	-442	652	54	-104	-1236	370	-1294
1025/2	205/2	-1	-2	0	-26	-18	-2	134	-30	188	-190	192	174	41	-332	-566	718	180	-418	-286	62	378	1150	-432	-1030	254
1027/1		3	-8	5	-3	-7	13	114	32	47	153	-68	-240	-247	173	-88	219	264	-382	-123	963	-1108	-79	1083	-1380	-1670
1032/1		0	3	11	-5	-61	35	-62	27	-168	11	-284	368	20	43	-537	-452	84	-240	-194	142	854	16	1251	-1158	-423
1032/2		0	-3	-13	29	21	-87	-12	-13	92	-41	310	-334	264	-43	447	-284	220	16	278	-156	-940	-1046	-63	-826	-1087
1035/1	345/3	0	0	5	-16	48	-46	30	-46	23	-30	116	68	-54	380	-420	642	-186	-34	-124	-1026	-646	-610	612	-642	476
1035/2	345/4	-1	0	5	16	-52	-38	54	40	-23	-170	232	386	-482	132	144	-82	-100	-398	-124	428	-78	-960	1488	-470	1126
1035/3	115/1	-1	0	5	-32	-40	-66	-130	-88	-23	130	40	-334	22	-272	-24	-258	-612	-366	-496	-248	826	-296	1296	646	-1438
1035/4	115/2	-2	0	-5	-2	16	-47	24	-56	23	-85	67	104	53	-234	-285	-2	-80	-764	236	289	-225	24	-684	1370	-110
1035/5	345/1	3	0	-5	26	-54	2	72	68	23	102	-16	344	-162	-280	360	-114	768	704	560	-408	998	-550	-966	804	-310
1035/6	345/2	5	0	-5	-6	-46	-50	116	-152	-23	-206	-120	-28	118	292	344	326	-748	-584	-684	-152	118	870	1278	-228	-790
1040/1	260/1	0	2	5	4	-18	13	-54	70	66	-78	46	-358	-438	-98	300	78	114	-166	-788	198	-58	340	-1080	-6	-142
1040/2	130/1	0	2	5	-8	-6	13	114	-38	-150	114	34	146	-30	-122	-336	-570	-66	-502	-728	-582	-994	988	84	906	290
1040/3	520/1	0	2	-5	-30	12	-13	-46	-72	98	126	-56	-158	-162	98	362	-166	576	-870	738	928	234	-128	1038	730	1250
1040/4	65/1	0	-2	-5	12	-14	-13	98	26	114	58	-306	86	-374	314	-620	362	-266	634	-612	686	202	516	-48	-1230	350
1040/5	130/2	0	4	-5	8	32	-13	-86	56	-68	-202	56	66	490	-460	24	-294	480	-338	-676	-120	-210	-184	660	-286	-1202
1040/6	520/2	0	10	5	12	62	13	58	-122	26	114	-338	-342	-230	-282	-140	-418	306	-38	372	742	-554	-812	864	1146	-1390
1044/1	348/1	0	0	9	21	66	-72	25	137	112	-29	88	-375	397	87	327	-182	-449	-510	-864	-144	-370	1174	528	-986	360
1045/1		-5	-1	-5	-2	-11	-7	14	19	55	-26	261	-126	-381	387	189	-404	746	79	537	-824	169	-338	601	-762	866
1050/1		2	3	0	7	-35	-54	-32	-126	-135	21	-94	-341	56	419	-194	38	382	-128	801	-415	-608	511	-374	1234	-182
1050/2	210/2	2	3	0	7	-44	-54	-98	-60	144	-210	-208	226	-502	-484	232	530	-764	814	-60	848	958	-152	-308	-1094	-554
1050/3	210/1	2	3	0	-7	12	-2	18	56	156	-186	-52	178	-138	412	456	198	348	110	196	-936	-542	992	276	630	-110
1050/4	210/3	2	3	0	-7	12	58	-42	-4	-24	294	128	58	282	-428	-384	138	468	-250	556	624	958	632	-84	810	790
1050/5		2	3	0	-7	42	-47	3	56	-9	189	263	58	-273	307	156	-207	-507	635	556	684	-482	182	291	-810	910
1050/6		2	3	0	-7	-43	18	-92	6	-129	-111	-142	173	-128	217	-194	-622	-22	160	-189	769	-652	-773	-314	470	470
1050/7		2	3	0	-7	-69	64	114	56	-9	-33	-70	-53	504	-137	600	570	48	524	1	-93	850	-1261	-486	300	-866
1050/8		2	-3	0	7	3	-4	-54	-148	-15	-69	146	-19	-24	29	228	174	-732	-220	11	-429	-910	-889	78	-960	-550
1050/9	210/4	2	-3	0	7	28	-54	46	12	0	6	296	-134	146	-556	448	-46	748	-50	156	-1024	310	856	628	-590	1390
1050/10	210/6	2	-3	0	7	28	86	66	-48	-140	-34	-284	346	-274	4	448	94	308	510	156	336	1170	16	-772	1630	-110
1050/11	210/5	2	-3	0	-7	0	-26	-18	92	0	-6	-4	-410	174	-248	-420	-102	-588	650	-152	-168	610	-1048	684	-834	-110
1050/12		2	-3	0	-7	-9	-32	114	-16	-21	-213	50	115	-336	103	-240	-342	336	-844	-167	-1017	130	155	858	-84	-938
1050/13	210/10	-2	3	0	7	-4	42	86	-96	96	-78	80	-50	-26	32	20	382	356	-134	-888	868	70	400	1052	-634	-1202
1050/14	1050/12	-2	3	0	7	-9	32	-114	-16	21	-213	50	-115	-336	-103	240	342	336	-844	167	-1017	-130	155	-858	-84	938
1050/15	210/8	-2	3	0	7	16	-58	-34	64	16	62	60	-150	474	292	-240	662	-324	-514	372	-412	770	-560	852	1466	178
1050/16	1050/8	-2	3	0	-7	3	4	54	-148	15	-69	146	19	-24	-29	-228	-174	-732	-220	-11	-429	910	-889	-78	-960	550
1050/17	210/9	-2	3	0	-7	24	-14	-54	44	-156	174	-88	34	-138	-164	216	-318	-204	-442	316	-252	-98	-1000	-516	-522	310
1050/18	42/1	-2	3	0	-7	-72	34	-6	92	180	-114	56	34	6	-164	-168	-654	-492	-250	124	36	-1010	56	-228	390	70
1050/19	42/2	-2	-3	0	7	-8	42	2	-124	-76	254	-72	-398	462	-212	264	162	-772	30	764	-236	-418	552	-1036	30	1190
1050/20	1050/5	-2	-3	0	7	42	47	-3	56	9	189	263	-58	-273	-307	-156	207	-507	635	-556	684	482	182	-291	-810	-910

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1050/21	1050/6	-2	-3	0	7	-43	-18	92	6	129	-111	-142	-173	-128	-217	194	622	-22	160	189	769	652	-773	314	470	-470
1050/22	210/7	-2	-3	0	7	56	-54	-94	36	84	-258	-40	178	-146	-148	200	130	188	94	444	532	-770	-536	1076	-1090	-1274
1050/23	1050/7	-2	-3	0	7	-69	-64	-114	56	9	-33	-70	53	504	137	-600	-570	48	524	-1	-93	-850	-1261	486	300	866
1050/24	1050/1	-2	-3	0	-7	-35	54	32	-126	135	21	-94	341	56	-419	194	-38	382	-128	-801	-415	608	511	374	1234	182
1056/1		0	3	14	34	11	-2	48	106	24	-160	-220	354	-68	-210	16	-246	-292	-438	-124	-320	-922	-958	-824	178	14
1056/2	1056/1	0	-3	14	-34	-11	-2	48	-106	-24	-160	220	354	-68	210	-16	-246	292	-438	124	320	-922	958	824	178	14
1058/1		2	7	18	-30	-6	79	102	-36	0	33	43	-354	375	96	-129	-300	-324	120	582	147	637	-468	978	-252	-1170
1058/2	1058/1	2	7	-18	30	6	79	-102	36	0	33	43	354	375	-96	-129	300	-324	-120	-582	147	637	468	-978	252	1170
1058/3	46/2	2	-9	20	-2	52	43	50	74	0	-7	-273	4	123	152	75	-86	-444	-262	-764	-21	681	-426	-902	1272	342
1058/4	46/1	-2	-1	10	12	42	7	-20	-106	0	-227	67	-74	-497	88	215	-314	176	298	-266	-981	-411	-806	952	1332	1328
1062/1	118/1	2	0	5	-33	4	-30	14	97	134	-1	-28	290	5	192	326	537	-59	472	856	-168	-686	-919	-362	312	-514
1062/2		2	0	14	-24	-68	-12	104	52	-64	-190	8	-124	-436	-114	-124	-606	-59	292	-530	966	214	-1144	-452	438	-838
1062/3	118/3	-2	0	-5	-15	50	-66	-14	-11	172	-287	26	128	-167	-78	-38	147	59	76	514	1140	-848	359	362	-1212	836
1062/4	118/2	-2	0	13	-27	8	42	-2	-77	-98	295	-40	278	-179	-132	202	345	59	184	-356	144	814	-181	1250	600	-790
1062/5	1062/2	-2	0	-14	-24	68	-12	-104	52	64	190	8	-124	436	-114	124	606	59	292	-530	-966	214	-1144	452	-438	-838
1064/1		0	-4	14	-7	36	-50	34	19	40	-122	-240	158	-398	-428	240	-258	724	854	164	-368	-470	1240	172	-1278	-966
1065/1		-3	-3	5	-14	-58	-82	-88	44	-114	74	-56	2	-342	180	-530	106	-54	-948	-680	71	-894	-40	124	-810	1032
1071/1	357/1	1	0	6	7	-18	-8	17	86	-80	44	112	-256	270	-380	-56	-58	-194	-530	-296	-100	286	-380	1086	-1298	-166
1071/2	119/1	1	0	20	-7	-60	-68	17	-70	176	90	196	22	138	328	12	234	54	44	-596	-200	1122	480	838	-778	1142
1071/3	357/2	5	0	1	-7	-45	-83	-17	-22	-134	-210	112	331	228	307	504	555	-540	-118	-719	-40	-855	-429	1433	-35	701
1072/1	134/1	0	-10	6	34	-24	-46	-69	79	-99	183	46	-277	420	202	-189	-522	-639	-250	-67	552	-439	-140	-12	255	428
1074/1		2	-3	-22	18	-9	-12	-3	110	8	-162	153	-185	168	-322	-169	313	-490	-686	-6	-618	368	592	-908	-411	524
1074/2		-2	3	-2	1	-66	13	60	0	204	176	-120	-4	-147	-328	-57	359	-7	25	-680	-370	-1068	443	-1077	1022	-88
1078/1	22/3	2	-1	3	0	11	16	-42	-116	189	-120	163	-409	-468	110	-144	90	453	-20	-97	-465	-848	-742	-438	273	-761
1078/2	154/3	2	2	-18	0	-11	-56	-36	28	180	-54	334	386	444	-316	402	-486	282	-380	176	-324	-800	-1144	-468	870	1330
1078/3	154/4	2	-7	-3	0	-11	16	-6	-14	-51	54	-95	-193	-102	284	72	-102	63	790	-433	135	238	770	1008	639	-11
1078/4	154/5	2	10	14	0	-11	16	-108	-116	68	122	262	130	-204	-396	-166	442	-702	-196	-416	492	-408	600	1212	-1146	482
1078/5	154/2	-2	0	-2	0	11	-26	46	48	-128	-146	128	-26	-10	52	544	318	48	-466	516	-392	-754	0	-624	1590	-1018
1078/6	22/1	-2	-4	-14	0	-11	50	-130	108	-96	142	-40	382	118	220	-520	238	852	-190	-12	-112	6	304	-820	-202	1406
1078/7	154/1	-2	5	1	0	-11	8	-22	-54	213	190	-163	31	-110	4	80	-566	-645	-634	-729	431	918	-254	-904	-901	89
1078/8	22/2	-2	7	19	0	11	72	46	20	-107	120	-117	-201	228	-242	96	458	-435	668	439	-1113	72	-70	-358	-895	-409
1081/1		-1	4	2	12	-32	64	14	72	23	-108	32	32	310	-52	47	544	20	476	604	136	-766	-160	782	-78	262
1083/1	57/1	1	-3	-12	-20	-4	76	22	0	82	-242	126	180	390	308	-522	70	-188	-706	-104	432	718	-94	-1296	-846	-830
1085/1		1	10	5	-7	-2	8	42	-40	132	-120	31	100	230	302	-48	-116	428	-128	-884	-312	1158	324	718	474	-442
1086/1		2	-3	3	1	-6	2	-89	54	-190	261	-224	12	415	35	54	258	-882	-485	765	-834	-825	114	-699	-157	-1498
1088/1	68/1	0	2	8	-12	10	38	-17	-4	120	-56	164	236	70	144	48	366	504	460	768	72	-734	736	-856	906	46
1088/2	34/1	0	2	-16	24	-62	62	-17	20	-12	-80	-208	356	22	312	24	462	-240	-812	216	732	178	700	992	-390	-146
1088/3	34/2	0	2	18	-10	6	-74	17	88	-114	90	-310	-86	90	-368	-384	258	-240	-302	964	-390	722	-898	-912	1446	-1438
1088/4	68/1	0	-2	8	12	-10	38	-17	4	-120	-56	-164	236	70	-144	-48	366	-504	460	-768	-72	-734	-736	856	906	46
1088/5	34/1	0	-2	-16	-24	62	62	-17	-20	12	-80	208	356	22	-312	-24	462	240	-812	-216	-732	178	-700	-992	-390	-146
1088/6	34/2	0	-2	18	10	-6	-74	17	-88	114	90	310	-86	90	368	384	258	240	-302	-964	390	722	898	912	1446	-1438
1088/7	544/1	0	4	-8	14	8	46	-17	-116	-94	112	50	20	62	-68	-60	-162	724	388	-172	-1090	-1062	114	68	-666	-1322
1088/8	544/1	0	-4	-8	-14	-8	46	-17	116	94	112	-50	20	62	68	60	-162	-724	388	172	1090	-1062	-114	-68	-666	-1322
1088/9	544/3	0	6	-18	2	26	22	17	44	-78	-50	-170	-58	130	-68	-192	690	-388	-226	344	-90	-966	-1078	36	-298	-1006
1088/10	544/3	0	-6	-18	-2	-26	22	17	-44	78	-50	170	-58	130	68	192	690	388	-226	-344	90	-966	1078	-36	-298	-1006
1088/11	17/1	0	8	-6	-28	24	58	17	-116	-60	-30	-172	58	-342	148	288	-318	-252	-110	484	-708	362	-484	-756	-774	-382
1088/12	17/1	0	-8	-6	28	-24	58	17	116	60	-30	172	58	-342	-148	-288	-318	252	-110	-484	708	362	484	756	-774	-382
1089/1	9/1	0	0	0	-20	0	70	0	-56	0	0	308	110	0	520	0	0	0	-182	-880	0	-1190	-884	0	0	-1330
1089/2	121/1	0	0	-18	0	0	0	0	0	108	0	340	-434	0	0	36	738	720	0	-416	-612	0	0	0	-1674	-34
1089/3	363/4	1	0	-7	-4	0	-43	-41	72	-104	-273	-272	-165	403	-120	220	741	112	858	284	624	-586	-308	0	321	179
1089/4	33/1	-1	0	4	26	0	32	74	60	182	-90	-8	-66	422	-408	506	-348	200	-132	-1036	-762	542	550	-132	-570	14
1089/5	363/4	-1	0	-7	4	0	43	41	-72	-104	273	-272	-165	-403	120	220	741	112	-858	284	624	586	308	0	321	179
1089/6	363/1	3	0	12	-12	0	66	-114	-42	-18	186	-308	-146	42	366	-618	408	132	-630	-452	282	-684	-1272	-432	-954	326
1089/7	363/1	-3	0	12	12	0	-66	114	42	-18	-186	-308	-146	-42	-366	-618	408	132	630	-452	282	684	1272	432	-954	326
1089/8	363/2	4	0	13	-26	0	73	31	108	86	-207	208	45	247	450	500	441	-598	-378	494	594	-1034	-352	360	351	1079
1089/9	363/2	-4	0	13	26	0	-73	-31	-108	86	207	208	45	-247	-450	500	441	-598	378	494	594	1034	352	-360	351	1079
1089/10	33/2	-5	0	14	32	0	38	-2	-72	-68	-54	-152	174	94	528	340	438	-20	-570	-460	1092	-562	16	372	966	-526
1092/1		0	-3	11	-7	-35	-13	33	147	-61	-223	220	-291	24	-509	352	-378	404	-203	502	1108	-529	-166	162	-882	-94
1092/2		0	-3	-19	-7	-30	-13	88	67	189	7	-65	-56	-286	101	-203	267	-576	262	62	-332	231	909	617	-1417	1171
1098/1	366/1	-2	0	3	-33	19	-29	62	-4	9	-160	-276	-32	93	284	334	-428	-123	61	385	120	-535	-923	1074	476	1446
1098/2	366/2	-2	0	18	12	44	-74	2	-44	-16	270	44	-2	78	184	344	702	-628	61	80	-520	570	812	44	1106	-1054
1100/1	220/3	0	5	0	-11	-11	22	-9	89	-138	201	77	-119	-102	-260	-294	-51	270	-733	-728	-849	-830	-214	-138	633	892

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1100/2	44/1	0	5	0	26	-11	-52	-46	-96	-27	16	-293	29	-472	110	224	-754	825	-548	123	1001	1020	526	158	-1217	263
1100/3	220/1	0	-5	0	19	-11	62	-19	-131	-138	-79	217	91	158	-120	546	439	290	-373	-728	-709	-850	-1194	-58	753	-1228
1100/4	220/2	0	-8	0	-24	-11	22	-22	-28	44	110	-40	362	210	-260	460	-662	-68	606	312	360	1042	-552	-268	-966	1334
1102/1		-2	-2	11	-11	7	-42	99	19	128	-29	-38	-96	182	-357	-331	688	570	-603	-6	-248	263	-200	-372	-180	-576
1104/1	138/3	0	3	-2	34	-2	-74	-68	-88	23	-178	-240	-76	186	-28	-264	-598	-492	352	244	984	1014	438	682	-1524	-198
1104/2	138/1	0	3	-10	-32	20	-26	-46	92	-23	-194	120	-322	42	-220	192	-170	-396	934	988	552	282	888	908	1242	-30
1104/3	276/2	0	-3	2	22	14	-50	-52	20	-23	-74	-24	104	-30	-112	288	-386	204	-308	-152	720	486	-462	-742	180	786
1104/4	138/2	0	-3	2	32	48	22	42	144	23	174	304	-318	74	-192	-392	-734	-156	706	-192	-624	-406	-696	800	-102	-918
1104/5	552/1	0	-3	8	22	4	-14	-116	-30	23	-38	-60	-310	-366	-326	464	348	-44	434	406	-472	-222	642	-756	-728	-1370
1104/6	276/1	0	-3	8	-34	-36	-62	-60	-30	23	234	-140	-174	194	42	400	76	-252	-566	6	-264	-286	-486	980	1632	-1626
1104/7	552/2	0	-3	-14	-2	-58	-50	-76	-60	-23	-106	24	-256	-126	304	-32	-642	-436	-460	232	-224	-282	426	-702	764	-686
1105/1		-3	6	-5	24	-6	-13	17	6	-102	-294	118	14	-446	-10	372	-158	586	442	-900	-90	-790	56	-784	1338	-826
1105/2		-4	4	-5	-14	-50	-13	-17	-54	-68	-102	-150	226	42	-232	-54	-138	294	-170	798	730	482	-852	-58	-310	-854
1110/1		2	-3	-5	10	44	59	-46	-34	6	7	-182	37	360	101	-35	-507	821	70	612	88	622	8	1223	345	870
1110/2		-2	3	-5	1	-47	35	55	-125	213	6	-98	-37	-40	-188	304	341	-518	-382	-578	882	-713	288	-849	-1263	-1756
1113/1		4	3	12	7	60	-18	42	28	-58	154	40	-34	84	304	-424	-53	-148	-20	-498	-526	200	734	928	2	-1322
1120/1		0	4	5	-7	36	-82	6	56	-16	118	-104	-230	-30	-228	-72	-606	-40	102	-836	1020	-578	-1324	-244	722	166
1120/2		0	4	-5	-7	-24	-62	66	-84	64	118	296	390	-30	192	48	254	220	-118	264	40	802	56	-804	-198	-414
1120/3	1120/1	0	-4	5	7	-36	-82	6	-56	16	118	104	-230	-30	228	72	-606	40	102	836	-1020	-578	1324	244	722	166
1120/4	1120/2	0	-4	-5	7	24	-62	66	84	-64	118	-296	390	-30	-192	-48	254	-220	-118	-264	-40	802	-56	804	-198	-414
1126/1		2	1	8	5	12	-62	-47	-161	179	-256	198	-372	-448	-58	167	492	387	123	371	535	-256	170	-876	860	616
1126/2		2	-3	-8	-15	60	42	81	-149	-33	132	142	-328	124	234	-77	-48	-177	-261	-817	-213	-596	-1250	-364	-320	-940
1127/1	23/1	-2	5	6	0	34	57	80	70	23	245	-103	-298	-95	88	357	-414	408	-822	926	335	899	-1322	36	460	964
1136/1	71/1	0	-1	-16	1	-24	7	72	153	213	232	-149	-204	-432	-71	-273	-274	-126	-134	760	-71	-457	-112	124	837	-1424
1136/2	568/1	0	-7	6	-31	-30	-35	-6	-85	-103	-122	-65	-186	-222	-69	59	-142	-186	-110	-922	-71	-1177	582	-268	145	-638
1140/1		0	-3	5	-8	20	-58	90	19	-128	-186	136	334	-318	252	312	-274	-612	-234	660	-24	362	-1112	188	1250	-1550
1140/2		0	-3	-5	22	6	-40	2	-19	-132	176	24	-364	340	278	-372	606	196	-678	408	-960	454	-648	588	-460	-796
1143/1	127/1	1	0	15	-25	51	2	-31	-123	149	-6	10	-348	387	-80	-266	-347	656	-158	-314	-312	-646	-846	-1352	-1242	632
1144/1		0	-2	9	-21	-11	-13	86	132	133	159	98	-446	-315	-407	220	-286	279	-201	507	-556	-333	-1396	828	-1196	-1108
1146/1		2	-3	8	-4	7	-77	70	-42	-96	105	-70	-202	-229	-137	327	361	284	-668	-555	39	220	477	-324	-506	-1037
1150/1	46/1	2	1	0	12	-42	-7	-20	106	-23	-227	67	-74	-497	88	-215	-314	176	-298	-266	-981	411	806	952	-1332	1328
1150/2		2	2	0	21	47	57	-84	-5	23	285	82	-54	-53	197	-124	-148	30	-578	296	422	487	-405	397	730	-64
1150/3	230/4	2	-4	0	-3	-2	38	45	-74	-23	283	-303	-79	-407	328	-360	561	101	-268	69	-641	-994	-884	-503	1608	-1082
1150/4	230/5	2	5	0	-12	22	-19	-96	-98	-23	-227	-285	398	271	100	285	-18	-352	-478	-330	835	1127	322	-572	-504	-1712
1150/5	230/3	2	-7	0	-20	6	-47	132	146	-23	-99	-253	118	495	-272	-639	342	240	-370	-698	-357	259	542	1248	-828	-992
1150/6	230/1	-2	1	0	32	-30	-19	60	-58	-23	85	-65	34	143	332	561	422	392	-246	-894	-737	-1041	1114	936	824	868
1150/7	230/2	-2	-1	0	18	-32	47	-20	36	23	-27	-33	-56	-157	-18	-65	14	-744	552	156	699	609	-644	-512	-102	-578
1150/8	1150/2	-2	-2	0	-21	47	-57	84	-5	-23	285	82	54	-53	-197	124	148	30	-578	-296	422	-487	-405	-397	730	64
1150/9	46/2	-2	9	0	-2	-52	-43	50	-74	23	-7	-273	4	123	152	-75	-86	-444	262	-764	-21	-681	426	-902	-1272	342
1152/1	384/1	0	0	4	10	4	-26	-14	8	-148	72	18	-262	378	-432	-148	360	428	442	-692	-540	-1018	386	-108	382	298
1152/2	384/1	0	0	4	-10	-4	-26	-14	-8	148	72	-18	-262	378	432	148	360	-428	442	692	540	-1018	-386	108	382	298
1152/3	384/1	0	0	-4	10	-4	26	-14	-8	-148	-72	18	262	378	432	-148	-360	-428	-442	692	-540	-1018	386	108	382	298
1152/4	384/1	0	0	-4	-10	4	26	-14	8	148	-72	-18	262	378	-432	148	-360	428	-442	-692	540	-1018	-386	-108	382	298
1152/5	128/1	0	0	6	20	14	-54	66	-162	-172	-2	-128	-158	-202	298	408	-690	-322	298	-202	700	-418	744	-678	82	-1122
1152/6	128/1	0	0	6	-20	-14	-54	66	162	172	-2	128	-158	-202	-298	-408	-690	322	298	202	-700	-418	-744	678	82	-1122
1152/7	128/1	0	0	-6	20	-14	54	66	162	-172	2	-128	158	-202	-298	408	690	322	-298	202	700	-418	744	678	82	-1122
1152/8	128/1	0	0	-6	-20	14	54	66	-162	172	2	128	158	-202	298	-408	690	-322	-298	-202	-700	-418	-744	-678	82	-1122
1152/9	384/3	0	0	8	10	68	46	74	-16	-20	228	162	-262	-30	-264	124	-204	340	-950	436	-780	518	1010	852	686	-806
1152/10	384/3	0	0	8	-10	-68	46	74	16	20	228	-162	-262	-30	264	-124	-204	-340	-950	-436	780	518	-1010	-852	686	-806
1152/11	384/3	0	0	-8	10	-68	-46	74	16	-20	-228	162	262	-30	264	124	204	-340	950	-436	-780	518	1010	-852	686	-806
1152/12	384/3	0	0	-8	-10	68	-46	74	-16	20	-228	-162	262	-30	-264	-124	204	340	950	436	780	518	-1010	852	686	-806
1155/1		0	-3	-5	7	-11	-22	18	-112	54	6	230	122	336	104	180	-258	354	200	-610	-336	326	320	-84	-1482	-1582
1155/2		1	3	5	-7	11	54	-78	-148	200	-218	-304	126	58	532	-368	222	-204	-666	-356	312	-726	-512	-1332	-742	514
1155/3		3	3	-5	7	11	30	-102	0	-32	10	-96	-158	-226	-512	204	66	20	-118	-48	-92	-438	-1312	492	-1338	-1198
1155/4		3	-3	-5	7	-11	38	54	-4	180	162	-172	-298	-390	-412	-336	-462	156	230	-4	120	-574	524	-408	-618	614
1155/5		-3	-3	-5	7	-11	44	-72	-4	-162	102	56	-124	18	224	120	414	660	-334	314	-36	110	-316	-732	-924	1154
1155/6		-3	-3	-5	7	-11	-82	54	80	48	-234	-28	-82	270	-28	-300	582	-684	758	524	552	110	944	1368	42	1658
1156/1	68/1	0	2	8	12	10	-38	0	4	-120	-56	-164	236	-70	-144	48	-366	-504	460	-768	-72	734	-736	856	906	-46
1159/1		3	7	10	-11	-44	-73	25	-19	-55	85	-44	254	-12	-510	472	-235	345	61	381	-108	-787	-246	454	1316	596
1160/1		0	-8	-5	-6	-58	10	-18	-110	-170	-29	134	-14	-382	-448	-284	-718	-424	-382	-466	-1108	858	962	1342	1378	-1198
1168/1	146/1	0	-4	-6	8	8	-62	10	-28	-48	-126	280	-138	74	-304	-192	82	-480	310	204	-480	73	120	168	954	818
1168/2	73/1	0	8	6	34	-6	-34	90	16	-60	102	214	-286	150	322	534	-474	-786	-574	16	-192	73	988	-1242	-6	614

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1170/1	390/9	2	0	5	2	0	13	60	50	-210	228	116	386	-378	-4	312	198	-624	638	200	408	1148	824	-1332	-54	-244
1170/2		2	0	5	-3	-45	13	13	-116	73	-154	-310	255	-391	258	154	-579	412	-695	-92	75	262	367	-24	-901	-1679
1170/3	390/6	2	0	5	-14	36	-13	-68	-158	-46	8	-176	62	-30	252	120	-758	-252	398	884	80	-660	568	-1084	-1250	84
1170/4	390/10	2	0	5	-15	-39	-13	15	54	143	122	-246	-225	-469	-484	-234	-33	0	-831	772	793	-998	-681	772	465	-79
1170/5	390/11	2	0	5	24	0	13	-50	28	208	-190	248	-186	194	348	-260	-462	520	-506	772	-780	-62	736	-1464	-406	922
1170/6		2	0	5	-24	32	13	-78	-32	-158	-126	250	38	-90	-428	84	-236	188	-170	-78	-16	-1012	-1264	60	1010	372
1170/7	390/7	2	0	-5	5	35	-13	-23	-30	-63	190	330	43	473	-232	-270	193	200	-679	-12	899	154	215	1308	1019	-427
1170/8	130/1	2	0	-5	8	-6	13	-114	38	-150	-114	-34	146	30	122	-336	570	-66	-502	728	-582	-994	-988	84	-906	290
1170/9	390/8	2	0	-5	-13	15	13	75	-130	-45	138	-34	-379	-243	416	-378	3	816	-607	-700	-57	-1162	-1	-672	-969	-949
1170/10	390/5	-2	0	5	8	-12	13	42	-52	-132	-282	116	398	-174	-76	-456	-150	156	230	-592	-408	-730	728	-36	1482	1742
1170/11	390/4	-2	0	5	8	40	-13	-10	0	180	-22	-144	34	502	-76	168	422	-104	-82	-540	-512	622	104	-348	286	494
1170/12	130/2	-2	0	5	-8	32	-13	86	-56	-68	202	-56	66	-490	460	24	294	480	-338	676	-120	-210	184	660	286	-1202
1170/13	390/3	-2	0	5	-25	21	13	-123	146	-99	246	182	-295	-9	452	-390	-315	24	-727	596	-771	326	-889	96	-795	983
1170/14	390/1	-2	0	5	-28	36	13	-42	-112	168	210	-76	278	-150	-460	264	-582	204	614	-304	-1080	-934	128	-348	834	-1582
1170/15	1170/2	-2	0	-5	-3	45	13	-13	-116	-73	154	-310	255	391	258	-154	579	-412	-695	-92	-75	262	367	24	901	-1679
1170/16	390/2	-2	0	-5	-12	48	13	62	-32	8	58	-124	-162	-74	-396	164	-270	416	70	448	1092	10	328	144	502	1042
1170/17	1170/6	-2	0	-5	-24	-32	13	78	-32	158	126	250	38	90	-428	-84	236	-188	-170	-78	16	-1012	-1264	-60	-1010	372
1176/1	168/1	0	3	2	0	12	66	70	92	16	-122	-64	-306	-50	20	176	526	-540	818	-228	864	-106	736	588	-146	1214
1176/2		0	3	-2	0	-18	-33	68	-25	92	92	-25	-213	-94	-67	-278	-400	-744	734	555	-642	-973	-785	822	-424	734
1176/3	168/3	0	3	-4	0	-26	-2	36	76	-114	6	256	-86	-160	-220	-308	258	-264	-606	-520	-286	530	-44	-1012	-768	-222
1176/4		0	3	-7	0	7	52	-72	-20	-48	-243	-95	352	296	158	142	-375	-279	-246	-730	338	542	-305	-1123	426	369
1176/5	168/4	0	3	10	0	-12	-30	-34	-148	152	-106	-304	-114	-202	116	-224	-274	660	-382	12	-552	614	880	108	86	-1426
1176/6		0	3	11	0	39	32	-12	88	-92	255	35	-4	-16	-330	298	-717	217	-386	906	-34	838	1325	-1163	54	-7
1176/7		0	3	12	0	-60	44	-128	52	-160	-230	136	-318	-192	220	184	-498	-492	20	380	-264	-560	104	1508	1144	-904
1176/8	1176/2	0	-3	2	0	-18	33	-68	25	92	92	25	-213	94	-67	278	-400	744	-734	555	-642	973	-785	-822	424	-734
1176/9	168/5	0	-3	2	0	52	-86	30	4	120	246	-80	-290	374	164	-464	-162	-180	666	-628	296	518	-1184	-220	774	1086
1176/10	1176/4	0	-3	7	0	7	-52	72	20	-48	-243	95	352	-296	158	-142	-375	279	246	-730	338	-542	-305	1123	-426	-369
1176/11	168/2	0	-3	10	0	-52	10	54	52	48	-186	-224	94	478	-316	-256	-66	-420	-342	668	-272	86	1360	-188	366	-1554
1176/12	1176/6	0	-3	-11	0	39	-32	12	-88	-92	255	-35	-4	16	-330	-298	-717	-217	386	906	-34	-838	1325	1163	-54	7
1176/13	1176/7	0	-3	-12	0	-60	-44	128	-52	-160	-230	-136	-318	192	220	-184	-498	492	-20	380	-264	560	104	-1508	-1144	904
1176/14	24/1	0	-3	-14	0	-28	74	-82	-92	8	-138	-80	30	-282	4	-240	-130	-596	218	-436	856	998	-32	1508	246	-866
1176/15	168/6	0	-3	16	0	-18	54	128	-52	-202	302	200	-150	-172	164	460	-190	-96	-622	744	-54	-742	-92	228	116	554
1183/1	7/1	1	-2	-16	7	8	0	54	110	48	-110	-12	246	-182	128	-324	-162	-810	-488	-244	768	702	440	1302	-730	-294
1183/2		5	2	19	-7	50	0	77	-12	-138	251	-250	-79	219	258	-72	111	-126	-359	-286	120	-89	-1030	-826	1526	-562
1183/3	1183/2	-5	2	-19	7	-50	0	77	12	-138	251	250	79	-219	258	72	111	126	-359	286	-120	89	-1030	826	-1526	562
1190/1		2	-4	5	-7	32	22	17	-156	-128	214	-140	-34	10	176	-212	-434	76	-42	-456	-228	-54	-236	-1192	-454	98
1190/2		2	-4	-5	-7	28	-6	-17	100	72	46	-20	-334	-202	-80	-120	-474	-684	-302	624	1192	278	-712	-548	586	-1778
1196/1		0	7	21	32	9	13	-126	65	-23	-150	-64	182	66	26	-336	-264	-528	860	-175	-180	-256	218	-1071	-1302	-205
1197/1	399/2	1	0	12	7	-34	0	16	19	-6	138	-240	-218	238	-376	-352	-6	268	814	-694	-100	-422	-666	-756	446	-728
1197/2		3	0	-4	-7	60	-58	-34	-19	110	158	166	56	-54	324	184	562	-832	-190	-494	1076	826	-1072	42	1098	1368
1197/3	1197/2	-3	0	4	-7	-60	-58	34	-19	-110	-158	166	56	54	324	-184	-562	832	-190	-494	-1076	826	-1072	-42	-1098	1368
1197/4	399/1	-3	0	8	-7	-18	68	-4	-19	-118	-166	304	350	-378	-456	304	394	-844	-418	130	404	58	-178	-828	870	948
1197/5	133/1	-4	0	-6	-7	68	8	-14	-19	-188	-70	252	-186	-192	488	216	-178	500	-298	494	618	-842	10	-228	-600	-976
1200/1	150/2	0	3	0	1	-42	-67	54	115	162	-210	193	-286	12	-263	-414	-192	-690	-733	-299	228	938	160	462	-240	-511
1200/2	150/4	0	3	0	2	-70	54	-22	-24	100	216	-208	-254	-206	-292	320	-402	370	-550	-728	540	604	-792	-404	-938	56
1200/3	600/2	0	3	0	-4	28	-16	108	-32	28	-238	180	-40	422	-276	-60	220	804	-358	884	64	-152	932	1292	-1146	824
1200/4	30/2	0	3	0	-4	48	-2	114	-140	72	210	-272	334	-198	-268	216	78	-240	302	596	768	478	640	-348	210	1534
1200/5	600/4	0	3	0	5	-14	-1	-46	-19	-46	14	-133	-258	84	-167	410	-456	194	-17	653	-828	-570	552	142	-1104	-841
1200/6	300/4	0	3	0	-7	54	55	-18	25	-18	-54	271	-314	-360	-163	522	36	-126	47	-343	1080	1054	568	1422	1440	439
1200/7	120/3	0	3	0	8	-20	-22	14	-76	56	-154	-160	162	-390	388	-544	210	380	-794	-148	840	-858	-144	316	1098	-994
1200/8	12/1	0	3	0	8	-36	10	-18	100	72	-234	16	226	90	452	432	-414	684	422	332	360	-26	-512	-1188	-630	1054
1200/9	600/5	0	3	0	10	14	82	-18	136	-140	112	-72	-26	-446	396	-144	-158	342	314	-152	932	548	512	284	-810	-1304
1200/10	600/6	0	3	0	10	46	34	-66	-104	164	224	72	22	194	108	-480	-286	-426	698	328	-188	740	-1168	412	1206	1384
1200/11	300/1	0	3	0	13	-6	5	-78	-65	-138	66	-299	-214	360	-203	-78	636	-786	467	217	360	-286	-272	-498	0	-511
1200/12	600/8	0	3	0	-19	-22	-1	58	53	58	22	35	270	-468	-431	-230	0	-446	127	-811	-36	-522	-1368	-1138	144	1079
1200/13	120/6	0	3	0	20	-16	-58	-38	-4	-80	82	8	-426	-246	-524	-464	702	592	574	-172	-768	558	-408	164	-510	-514
1200/14	300/2	0	3	0	-22	14	-30	62	120	-188	96	-184	406	130	-148	-448	-414	-266	-838	-248	-1020	484	48	-548	-650	-1816
1200/15	150/3	0	3	0	-23	30	29	78	-149	-150	-234	217	146	-156	433	-30	-552	270	275	-803	-660	-646	-992	846	-1488	-319
1200/16	24/1	0	3	0	-24	28	74	-82	-92	8	-138	-80	-30	282	4	240	130	-596	-218	-436	-856	998	32	-1508	-246	-866
1200/17	15/2	0	3	0	-24	-52	-22	14	20	-168	230	288	34	122	-188	256	338	-100	742	-84	328	38	240	1212	330	-866
1200/18	30/1	0	3	0	32	60	34	-42	76	0	6	232	-134	234	-412	-360	-222	-660	-490	812	-120	-746	-152	-804	-678	-194
1200/19	120/5	0	-3	0	0	-4	-54	-114	-44	96	134	272	98	-6	12	-200	-654	-36	-442	-188	632	390	-688	1188	-694	1726

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1200/20	150/2	0	-3	0	-1	-42	67	-54	115	-162	-210	193	286	12	263	414	192	-690	-733	299	228	-938	160	-462	-240	511
1200/21	150/4	0	-3	0	-2	-70	-54	22	-24	-100	216	-208	254	-206	292	-320	402	370	-550	728	540	-604	-792	404	-938	-56
1200/22	600/2	0	-3	0	4	28	16	-108	-32	-28	-238	180	40	422	276	60	-220	804	-358	-884	64	152	932	-1292	-1146	-824
1200/23	120/1	0	-3	0	4	-72	6	-38	-52	152	-78	-120	150	362	-484	280	670	-696	222	-4	-96	-178	632	-612	994	-1634
1200/24	600/4	0	-3	0	-5	-14	1	46	-19	46	14	-133	258	84	167	-410	456	194	-17	-653	-828	570	552	-142	-1104	841
1200/25	300/4	0	-3	0	7	54	-55	18	25	18	-54	271	314	-360	163	-522	-36	-126	47	343	1080	-1054	568	-1422	1440	-439
1200/26	600/5	0	-3	0	-10	14	-82	18	136	140	112	-72	26	-446	-396	144	158	342	314	152	932	-548	512	-284	-810	1304
1200/27	600/6	0	-3	0	-10	46	-34	66	-104	-164	224	72	-22	194	-108	480	286	-426	698	-328	-188	-740	-1168	-412	1206	-1384
1200/28	300/1	0	-3	0	-13	-6	-5	78	-65	138	66	-299	214	360	203	78	-636	-786	467	-217	360	286	-272	498	0	511
1200/29	6/1	0	-3	0	-16	-12	-38	126	-20	168	30	88	-254	42	-52	-96	-198	660	-538	884	-792	-218	520	-492	810	-1154
1200/30	120/2	0	-3	0	-16	28	26	62	68	-208	-58	-160	-270	282	76	-280	210	-196	742	836	504	1062	-768	-1052	-726	1406
1200/31	600/8	0	-3	0	19	-22	1	-58	53	-58	22	35	-270	-468	431	230	0	-446	127	811	-36	522	-1368	1138	144	-1079
1200/32	15/1	0	-3	0	20	24	-74	-54	124	-120	-78	-200	70	330	92	-24	-450	-24	-322	-196	288	430	520	156	1026	286
1200/33	120/4	0	-3	0	20	56	86	106	-4	136	-206	152	-282	-246	412	40	126	-56	-2	-388	672	-1170	-408	668	66	926
1200/34	300/2	0	-3	0	22	14	30	-62	120	188	96	-184	-406	130	148	448	414	-266	-838	248	-1020	-484	48	548	-650	1816
1200/35	150/3	0	-3	0	23	30	-29	-78	-149	150	-234	217	-146	-156	-433	30	552	270	275	803	-660	646	-992	-846	-1488	319
1200/36	60/1	0	-3	0	-28	24	70	-102	-20	-72	306	136	214	-150	-292	-72	414	744	-418	188	-480	-434	-1352	-612	-30	286
1200/37	60/2	0	-3	0	32	-36	10	78	-140	-192	6	16	34	-390	-52	408	114	-516	-58	-892	120	646	1168	-732	-1590	-194
1206/1	402/1	2	0	-14	20	-68	18	-42	76	132	22	-244	142	406	316	204	-558	380	578	-67	260	282	916	-1140	1350	-1286
1206/2	134/1	-2	0	-6	-34	-24	-46	69	-79	-99	-183	-46	-277	-420	-202	-189	522	-639	-250	67	552	-439	140	-12	-255	428
1207/1		1	1	12	13	10	-35	-17	29	109	106	-285	-64	-348	-503	385	482	-882	174	766	71	-695	-1190	492	165	620
1208/1		0	-7	12	-22	-2	2	115	-136	102	48	-189	238	-172	462	-321	-335	306	-205	375	908	644	1046	-1237	1400	-1479
1210/1		2	2	-5	24	0	-30	-126	48	-150	24	-20	-362	324	36	-378	-594	528	-360	898	552	-222	-468	-876	-714	1190
1210/2	110/2	2	4	5	-20	0	-26	42	-116	96	-270	32	-106	462	40	-504	-570	12	-590	-388	-240	-302	-8	48	282	-646
1210/3	110/1	2	4	-5	30	0	-16	112	64	36	-10	-48	-146	-278	330	476	150	732	30	-848	240	1128	-788	698	-458	134
1210/4		2	-5	-5	-11	0	12	91	55	60	-165	-195	135	30	232	-70	-265	-704	165	-740	363	-838	680	-512	105	-1190
1210/5	110/3	2	-7	5	35	0	-26	-101	-127	-58	27	-177	191	-66	-444	2	-669	386	521	96	-427	-1006	-910	818	601	-228
1210/6	110/7	-2	1	5	-23	0	-50	-75	-17	-174	153	35	-277	258	220	210	-273	438	475	992	-927	934	-974	90	1377	-64
1210/7	1210/1	-2	2	-5	-24	0	30	126	-48	-150	-24	-20	-362	-324	-36	-378	-594	528	360	898	552	222	468	876	-714	1190
1210/8	110/4	-2	-4	-5	22	0	20	20	8	-204	-122	40	278	-302	330	60	-418	188	670	-568	128	-676	876	1130	822	-434
1210/9	1210/4	-2	-5	-5	11	0	-12	-91	-55	60	165	-195	135	-30	-232	-70	-265	-704	-165	-740	363	838	-680	512	105	-1190
1210/10	110/5	-2	7	-5	-11	0	-2	9	85	-138	-45	227	-19	138	88	-534	297	-450	-287	-304	777	-962	-290	-1422	-1455	116
1210/11	110/8	-2	8	5	12	0	34	86	4	148	-134	-280	430	6	136	-28	-658	4	90	96	816	430	-1296	608	810	706
1210/12	10/1	-2	-8	5	4	0	58	-66	100	132	90	152	-34	438	-32	-204	222	420	-902	-1024	432	-362	160	-72	810	1106
1210/13	110/6	-2	-8	-5	-26	0	-92	84	-80	72	30	-208	86	378	-542	216	-18	420	718	-124	912	268	940	498	150	446
1212/1		0	3	-11	-20	50	-87	109	145	9	-10	201	-254	-218	-332	411	-468	534	-754	-970	-563	404	169	-838	-1260	-914
1215/1		1	0	5	-15	-17	28	61	28	102	104	-222	-256	398	37	-332	-521	100	-485	255	-265	-902	780	-408	-1608	-64
1215/2		1	0	5	24	61	-89	-134	-50	63	26	-105	251	-382	-392	175	454	217	-134	-369	-304	-122	-312	-96	-516	1340
1215/3		1	0	5	30	-53	46	-47	-26	-105	-283	-24	257	272	-107	136	82	-683	-710	-393	986	-650	609	330	-1518	-1180
1215/4	1215/1	-1	0	-5	-15	17	28	-61	28	-102	-104	-222	-256	-398	37	332	521	-100	-485	255	265	-902	780	408	1608	-64
1215/5	1215/2	-1	0	-5	24	-61	-89	134	-50	-63	-26	-105	251	382	-392	-175	-454	-217	-134	-369	304	-122	-312	96	516	1340
1215/6	1215/3	-1	0	-5	30	53	46	47	-26	105	283	-24	257	-272	-107	-136	-82	683	-710	-393	-986	-650	609	-330	1518	-1180
1215/7		3	0	5	-31	69	-28	-39	146	-42	-222	104	-214	228	443	-18	201	264	-115	281	687	1028	662	510	750	-304
1215/8		3	0	-5	-19	-15	-28	-45	8	42	156	134	332	150	-19	-24	117	-516	-25	-601	-27	974	-640	-636	468	-424
1215/9	1215/8	-3	0	5	-19	15	-28	45	8	-42	-156	134	332	-150	-19	24	-117	516	-25	-601	27	974	-640	636	-468	-424
1215/10	1215/7	-3	0	-5	-31	-69	-28	39	146	42	222	104	-214	-228	443	18	-201	-264	-115	281	-687	1028	662	-510	-750	-304
1215/11		4	0	5	9	-41	-14	-11	-20	-168	-226	120	-214	32	373	10	331	-212	451	-69	-1	-692	-822	-834	456	1130
1215/12	1215/11	-4	0	-5	9	41	-14	11	-20	168	226	120	-214	-32	373	-10	-331	212	451	-69	1	-692	-822	834	-456	1130
1215/13		5	0	-5	27	5	4	-25	106	150	10	-204	-250	400	-275	170	515	680	-791	975	235	424	-642	1050	30	-400
1215/14	1215/13	-5	0	5	27	-5	4	25	106	-150	-10	-204	-250	-400	-275	-170	-515	-680	-791	975	-235	424	-642	-1050	-30	-400
1216/1	608/1	0	1	8	-17	70	61	83	-19	115	-279	-72	34	108	192	-392	-131	609	-338	461	750	1177	-22	810	-476	1426
1216/2	608/1	0	-1	8	17	-70	61	83	19	-115	-279	72	34	108	-192	392	-131	-609	-338	-461	-750	1177	22	-810	-476	1426
1216/3	38/1	0	2	9	-31	-57	52	69	-19	-72	150	32	226	-258	67	579	432	330	13	856	642	-487	-700	12	-600	1424
1216/4	38/1	0	-2	9	31	57	52	69	19	72	150	-32	226	-258	-67	-579	432	-330	13	-856	-642	-487	700	-12	-600	1424
1216/5	19/1	0	5	12	11	54	-11	-93	-19	183	249	56	250	240	196	-168	-435	-195	358	961	-246	353	-34	-234	-168	758
1216/6	19/1	0	-5	12	-11	-54	-11	-93	19	-183	249	-56	250	240	-196	168	-435	195	358	-961	246	353	34	234	-168	758
1218/1		2	-3	-4	-7	-12	-28	100	24	152	-29	-42	-322	304	-92	-274	-106	-666	-110	24	-608	-896	-180	-334	1120	532
1218/2		2	-3	18	7	63	-7	114	-43	-45	-29	164	-115	-156	218	-291	-399	-519	128	-421	-288	-43	-808	417	-420	335
1224/1	408/1	0	0	-6	-24	-44	6	-17	-20	152	-270	-272	-250	-186	260	320	770	348	-210	-148	360	-646	-1168	788	1238	882
1224/2	408/2	0	0	7	4	21	-25	-17	-69	-15	-58	-298	72	369	-59	138	-262	-50	-568	124	-100	-158	710	-214	1016	-1780
1225/1	7/1	1	-2	0	0	-8	28	54	110	-48	-110	-12	246	-182	-128	324	162	-810	488	-244	-768	-702	440	-1302	-730	294
1225/2	25/1	1	-7	0	0	-43	28	-91	35	162	160	-42	-314	203	92	-196	82	280	518	141	412	763	510	-777	945	-1246

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1225/3	245/4	-1	6	0	0	-44	6	-24	114	52	146	276	210	-444	-492	-612	-50	-294	-450	668	-308	12	596	-966	408	-1200
1225/4	245/4	-1	-6	0	0	-44	-6	24	-114	52	146	-276	210	444	-492	612	-50	294	450	668	-308	-12	596	966	-408	1200
1225/5	25/1	-1	7	0	0	-43	-28	91	35	-162	160	-42	314	203	-92	196	-82	280	518	-141	412	-763	510	777	945	1246
1225/6	35/1	-1	-8	0	0	12	-78	-94	-40	-32	-50	248	434	-402	68	536	-22	560	278	164	672	82	-1000	-448	870	1026
1225/7	49/1	-2	7	0	0	-5	-14	-21	-49	159	58	-147	-219	-350	124	525	-303	105	413	-415	-432	-1113	-103	1092	329	-882
1225/8	49/1	-2	-7	0	0	-5	14	21	49	159	58	147	-219	350	124	-525	-303	-105	-413	-415	-432	1113	-103	-1092	-329	882
1225/9	245/2	-3	2	0	0	-45	-59	54	-121	-69	-162	-88	259	195	286	-45	-597	-360	392	280	48	-668	782	-768	-1194	-902
1225/10	245/2	-3	-2	0	0	-45	59	-54	121	-69	-162	88	259	-195	286	45	-597	360	-392	280	48	668	782	768	1194	902
1225/11	5/1	4	2	0	0	32	-38	26	-100	78	-50	108	-266	-22	-442	-514	-2	-500	518	-126	412	-878	600	282	150	386
1225/12	49/4	5	0	0	0	-68	0	0	0	40	-166	0	-450	0	180	0	-590	0	0	740	688	0	-1384	0	0	0
1230/1		2	3	5	16	12	82	-74	60	12	110	-288	6	41	-328	456	-278	700	-698	-44	-128	282	-320	112	-310	726
1230/2		-2	3	-5	13	-2	28	-8	-139	141	154	105	-69	41	-313	360	617	-141	-416	92	228	754	-288	-472	483	1226
1230/3		-2	-3	5	-16	51	8	-69	-22	-108	33	29	209	-41	497	-399	-318	228	-31	-898	1071	-511	-106	702	666	86
1230/4		-2	-3	-5	6	22	-22	0	-54	48	-148	-32	318	41	-332	266	318	372	250	596	-808	-22	-372	196	630	-960
1232/1	154/2	0	0	2	7	-11	26	-46	48	128	-146	128	-26	10	-52	544	318	48	466	-516	392	754	0	-624	-1590	1018
1232/2	154/3	0	2	18	-7	11	56	36	28	-180	-54	334	386	-444	316	402	-486	282	380	-176	324	800	1144	-468	-870	-1330
1232/3	77/1	0	-4	12	-7	-11	38	-48	70	-12	126	70	-358	-216	-344	-390	438	552	830	196	-648	-16	-1352	-90	1146	-70
1232/4	308/1	0	-4	-12	-7	-11	-10	24	94	180	30	94	-214	-48	-8	-30	54	-360	-178	292	-312	728	1288	-66	-390	-1510
1232/5	154/1	0	5	-1	7	11	-8	22	-54	-213	190	-163	31	110	-4	80	-566	-645	634	729	-431	-918	254	-904	901	-89
1232/6	308/2	0	7	-1	-7	-11	12	2	-82	-7	-102	171	-357	-114	344	-96	-430	201	-2	-313	579	-438	-494	-748	457	-1037
1232/7	154/4	0	-7	3	-7	11	-16	6	-14	51	54	-95	-193	102	-284	72	-102	63	-790	433	-135	-238	-770	1008	-639	11
1232/8	616/1	0	-9	15	7	-11	0	6	14	-79	-146	-221	-37	70	204	-496	266	553	-734	287	1083	802	-854	992	-351	-301
1232/9	154/5	0	10	-14	-7	11	-16	108	-116	-68	122	262	130	204	396	-166	442	-702	196	416	-492	408	-600	1212	1146	-482
1235/1		-3	4	5	2	12	13	-18	19	-72	0	-70	218	-156	-358	546	-534	-180	578	-556	342	-664	-160	0	1044	-34
1235/2		4	6	-5	-10	-16	13	18	-19	-150	-234	-212	310	134	234	174	342	-300	154	-438	-636	-454	88	1346	-30	390
1239/1		1	-3	2	-7	68	-50	-96	-20	48	-88	126	-432	520	-166	72	-576	-59	-590	82	828	-912	152	1404	1178	1332
1245/1		3	3	-5	-16	28	82	-30	48	-52	-118	-160	-362	18	-344	-120	178	-580	-210	-464	-48	-190	256	-83	-490	-686
1246/1		-2	-5	-9	7	42	80	69	-49	183	-90	65	272	-342	-265	96	243	180	-322	-430	-78	545	398	144	89	-241
1253/1		-3	-2	-2	-7	17	-78	-75	-92	48	156	-29	-153	-250	-440	227	59	-654	-562	-488	-978	-1102	-1352	628	1329	1370
1254/1		2	3	7	14	-11	36	97	-19	-40	-75	282	159	30	-437	204	106	-564	-41	439	624	684	560	314	-1615	-380
1260/1	140/2	0	0	5	7	-15	17	-123	86	-54	177	212	74	444	-46	-471	180	-144	-376	356	48	818	89	780	-1140	-169
1260/2	140/3	0	0	5	7	-28	82	46	8	128	-174	-152	-290	-50	396	296	570	272	-662	876	880	-638	-600	-624	-698	754
1260/3	420/3	0	0	5	7	36	-34	30	-16	48	126	8	74	138	-352	396	78	60	-70	-664	-156	410	344	-444	1002	290
1260/4	140/1	0	0	5	-7	7	-23	25	-62	86	29	-12	-150	-204	-178	-33	-452	-120	920	-300	-520	370	-1013	636	-292	-1381
1260/5	420/1	0	0	5	-7	-32	42	38	-36	-96	198	-220	-46	290	-152	-124	-62	-68	-614	-456	416	-826	-272	-508	-110	-874
1260/6	140/6	0	0	-5	7	15	-13	27	-154	186	-3	-328	254	-96	134	-51	-240	396	-616	296	48	-322	659	-300	-1020	-199
1260/7	420/5	0	0	-5	7	16	-14	-130	104	88	-54	28	-266	-202	348	-104	-402	100	310	-324	644	-290	744	-1044	-298	-290
1260/8	420/2	0	0	-5	7	36	-34	6	-28	-192	186	176	-418	30	-412	432	306	564	-322	716	48	-1078	-496	-468	-1314	-1438
1260/9	420/6	0	0	-5	7	-36	38	78	-52	-120	-54	80	254	6	-172	0	66	-420	-106	92	-1176	698	-1024	516	-714	-862
1260/10	420/4	0	0	-5	-7	44	-42	94	-36	-24	-54	-112	-322	22	292	-272	578	44	-26	12	280	410	-320	1252	38	1250
1260/11	140/5	0	0	-5	-7	-55	-69	-113	-126	102	81	176	254	184	-230	187	488	-388	-728	-96	-8	-994	337	-188	884	-451
1260/12	140/4	0	0	-5	-7	-68	22	30	108	-184	-166	-32	-370	-154	212	512	98	860	390	60	-840	-630	1312	436	598	914
1264/1	158/1	0	-3	-9	-9	8	-47	84	150	2	180	328	-346	-268	-328	351	498	-365	492	-784	-897	378	-79	1102	-1255	1129
1264/2	316/1	0	-6	6	0	-4	34	54	-144	8	180	-128	-232	446	86	192	-348	838	780	-904	-312	-234	-79	316	566	-806
1264/3	632/1	0	7	7	-19	44	-31	-36	-102	-154	-204	-104	-46	208	304	-187	418	113	-488	516	821	-566	79	478	-543	-1319
1265/1		1	4	-5	-9	11	72	84	27	-23	-291	-6	218	208	-134	-91	304	-745	-688	-197	-810	-1037	-388	268	228	1667
1265/2		1	4	-5	11	-11	20	44	41	23	171	126	-162	-280	-358	-449	0	-549	236	-367	-698	267	-640	-264	128	-1051
1265/3		-2	-7	-5	-24	11	-9	0	8	23	-125	-341	-46	-267	-204	209	168	-348	-686	-666	-145	-803	-442	-138	-312	-1344
1274/1	26/2	2	1	-17	0	2	-13	19	-94	-72	246	100	-11	280	241	-137	-232	386	-64	-670	55	838	1016	-420	934	1154
1274/2	182/2	2	2	5	0	-36	13	-26	47	-99	-61	23	-50	-70	-19	-191	195	-264	-310	-190	-166	-873	-1191	-259	635	-133
1274/3		2	3	-3	0	1	-13	-73	107	77	-218	-83	205	138	-348	-1	-289	-471	-497	-979	-840	457	-407	868	425	650
1274/4	1274/3	2	-3	3	0	1	13	73	-107	77	-218	83	205	-138	-348	1	-289	471	497	-979	-840	-457	-407	-868	-425	-650
1274/5	26/3	2	-4	18	0	-48	-13	-66	16	168	6	-20	254	390	-124	468	558	96	826	-160	-420	-362	776	0	-1626	1294
1274/6	182/3	2	-5	-16	0	-15	13	44	138	111	-12	-215	55	133	-180	-471	-260	-110	271	-799	912	-747	-883	924	-142	1407
1274/7	182/4	2	8	-3	0	-54	-13	96	151	33	183	331	-88	42	353	465	195	-552	-470	254	132	943	-727	1197	-753	-1037
1274/8	26/1	-2	-3	-11	0	-38	13	51	-90	-52	-190	-292	-441	-312	373	41	468	-530	-592	-206	-863	322	-460	-528	-870	346
1274/9	182/1	-2	-7	0	0	39	-13	-24	-38	39	-96	-227	425	105	344	-99	-540	-114	565	-385	-156	673	749	1044	690	-317
1275/1	51/1	1	3	0	-34	-48	-58	17	20	-58	0	-218	-184	-138	-148	516	162	-180	152	956	-538	462	390	-1268	-770	-494
1275/2	51/3	1	-3	0	2	-48	14	17	92	122	-36	-182	-76	294	428	12	234	-540	-820	-700	794	1038	858	-1052	1102	-710
1275/3	51/2	-1	3	0	8	12	26	-17	-148	-152	-66	-32	266	-6	92	288	546	420	350	-940	424	-378	288	-748	-1558	-530
1275/4	255/1	2	3	0	17	41	-8	17	-9	0	-75	68	217	-287	32	423	343	2	-20	46	112	647	646	296	486	418
1275/5		3	-3	0	13	-69	-2	-17	101	-90	30	293	103	-42	373	-183	-423	-168	230	439	222	568	701	834	-504	22

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1275/6		3	-3	0	-20	-14	-46	-17	-64	185	52	-290	15	-185	-78	356	655	459	593	-826	-713	920	-300	-79	926	-1056
1275/7	1275/5	-3	3	0	-13	-69	2	17	101	90	30	293	-103	-42	-373	183	423	-168	230	-439	222	-568	701	-834	-504	-22
1275/8	1275/6	-3	3	0	20	-14	46	17	-64	-185	52	-290	-15	-185	78	-356	-655	459	593	826	-713	-920	-300	79	926	1056
1275/9	255/2	4	3	0	8	-38	-74	-17	72	-132	-246	158	-14	-286	62	318	446	-200	-350	-770	-946	962	838	-338	942	1630
1275/10		5	-3	0	13	67	8	17	-69	48	-216	287	53	-58	-359	627	253	-56	538	563	-298	-206	355	1268	90	-1738
1275/11	1275/10	-5	3	0	-13	67	-8	-17	-69	-48	-216	287	-53	-58	359	-627	-253	-56	538	-563	-298	206	355	-1268	90	1738
1281/1		1	3	3	-7	52	-34	60	-67	-112	-173	-48	260	-110	531	174	-265	-285	-61	-99	710	-940	-545	-1342	-1524	-266
1281/2		-3	-3	-5	7	40	-46	-4	7	-12	-61	28	-20	-222	281	310	15	553	-61	-825	6	872	1285	-2	-1304	634
1281/3		-3	-3	-7	7	-58	32	-68	85	66	-41	82	-152	-186	293	152	-201	-55	-61	939	864	-502	1231	1226	926	-1496
1287/1	429/1	1	0	19	19	-11	-13	-44	-90	-163	-255	-68	-426	13	63	-124	642	605	-363	-91	-582	283	830	-1278	-480	-1306
1290/1		2	3	5	-10	-50	22	-88	-86	-2	78	-88	-214	-334	-43	-250	386	354	-240	-604	840	-68	704	-12	-986	-1570
1290/2		2	3	-5	-24	36	-54	26	104	-84	-30	-104	-206	-214	43	-276	-486	-484	322	-772	-760	110	48	-676	-446	58
1290/3		-2	3	-5	29	24	41	-60	-37	108	159	173	164	-171	43	42	-558	324	-127	-133	-330	173	467	-1080	-1404	-370
1290/4		-2	-3	-5	-20	-29	-19	-75	-68	73	-232	-267	-236	-317	-43	384	479	-672	284	-963	-948	-560	-240	561	-936	-179
1290/5		-2	-3	-5	-20	36	76	100	52	48	-242	-132	34	-102	-43	-316	-266	828	-616	392	652	1090	-560	1346	-756	-814
1295/1		1	2	5	7	-18	17	-44	150	117	-55	37	37	-348	-498	-39	-483	90	132	-609	-693	-393	-610	-168	-135	1106
1295/2		-2	1	-5	7	-12	42	38	-65	-145	214	-17	-37	330	352	215	-488	-161	73	-530	447	-134	1160	1349	-582	-324
1295/3		3	-4	-5	7	68	2	58	-20	-200	114	168	-37	-350	172	160	282	-196	-902	-720	552	26	-1060	-956	-482	426
1296/1	324/1	0	0	3	4	24	-25	-21	52	-168	-177	124	-265	426	160	540	-258	-528	-505	244	-204	-397	-200	540	-453	290
1296/2	324/1	0	0	-3	4	-24	-25	21	52	168	177	124	-265	-426	160	-540	258	528	-505	244	204	-397	-200	-540	453	290
1296/3	648/1	0	0	5	-36	64	-65	-59	28	160	57	-164	-321	246	8	84	-478	-32	415	220	884	-77	80	1268	-123	1346
1296/4	648/1	0	0	-5	-36	-64	-65	59	28	-160	-57	-164	-321	-246	8	-84	478	32	415	220	-884	-77	80	-1268	123	1346
1296/5	162/2	0	0	9	31	-15	-37	42	28	195	-111	205	-166	261	43	177	-114	159	191	421	156	182	-1133	-1083	1050	-901
1296/6	162/2	0	0	-9	31	15	-37	-42	28	-195	111	205	-166	-261	43	-177	114	-159	191	421	-156	182	-1133	1083	-1050	-901
1296/7	162/1	0	0	21	-8	-36	-49	21	112	-180	-135	-308	-1	-42	-20	-84	-174	-504	-385	-272	888	371	652	-84	21	-1246
1296/8	162/1	0	0	-21	-8	36	-49	-21	112	180	135	-308	-1	42	-20	84	174	504	-385	-272	-888	371	652	84	-21	-1246
1300/1	260/1	0	2	0	4	18	-13	54	-70	66	-78	-46	358	-438	-98	300	-78	-114	-166	-788	-198	58	-340	-1080	-6	142
1300/2	52/1	0	3	0	11	-2	13	51	150	4	-118	-116	-63	-288	293	335	708	566	904	-382	7	-518	-100	1440	1254	-1262
1300/3		0	4	0	-4	-6	13	116	-70	-102	-250	340	-30	58	384	-208	-366	-342	-558	612	-1008	-790	-824	-1108	1550	-1594
1300/4	1300/3	0	-4	0	4	-6	-13	-116	-70	102	-250	340	30	58	-384	208	366	-342	-558	-612	-1008	790	-824	1108	1550	1594
1300/5		0	5	0	16	-27	-13	-33	41	-156	-12	110	-92	-381	412	-444	-222	-540	308	-875	480	571	404	-63	-459	514
1300/6	1300/5	0	-5	0	-16	-27	13	33	41	156	-12	110	92	-381	-412	444	222	-540	308	875	480	-571	404	63	-459	-514
1302/1		2	3	12	-7	-38	58	-22	66	96	104	-31	-32	62	102	356	508	318	-390	476	184	-202	632	-964	-1350	-370
1305/1	435/2	1	0	-5	4	36	-22	2	-56	40	-29	152	34	250	-412	120	762	188	-54	-244	-600	6	-640	-664	-150	-1690
1305/2	145/1	-1	0	5	-14	-62	42	114	-70	-62	29	142	146	-162	352	444	238	-840	2	-154	-892	-38	1050	778	-1410	466
1305/3	435/1	2	0	-5	29	15	3	-121	-40	116	-29	-116	36	170	230	-231	-456	-576	342	-269	-302	-372	-348	512	-1525	-560
1305/4	435/3	-5	0	-5	16	44	78	-18	-28	-184	-29	-224	254	78	-260	-312	-574	-180	-610	-340	-296	394	-960	908	990	1234
1314/1	146/1	2	0	6	-8	8	-62	-10	28	-48	126	-280	-138	-74	304	-192	-82	-480	310	-204	-480	73	-120	168	-954	818
1314/2	438/1	-2	0	-12	16	10	40	94	160	24	-108	-268	90	-154	430	36	-56	-618	454	444	144	73	-480	-906	714	-1186
1320/1		0	3	-5	-26	11	50	136	-138	-96	96	-120	-134	432	62	-432	-194	588	-54	84	-880	-518	-310	-1052	-1350	-1874
1320/2		0	3	-5	28	-11	-12	-78	-122	80	-64	-220	-178	80	-84	-460	-482	-44	718	436	-528	496	-166	-346	1146	854
1320/3		0	-3	-5	8	11	36	86	86	-40	-228	-60	-74	-236	160	36	494	-4	830	-732	880	72	794	-1070	50	478
1323/1	900/9	0	0	0	0	0	19	0	-56	0	0	19	323	0	449	0	0	0	901	-127	0	-1190	-1387	0	0	-1853
1323/2	900/9	0	0	0	0	0	-19	0	56	0	0	-19	323	0	449	0	0	0	-901	-127	0	1190	-1387	0	0	1853
1323/3	900/17	0	0	0	0	0	89	0	56	0	0	-289	-433	0	71	0	0	0	719	1007	0	1190	503	0	0	-523
1323/4	900/17	0	0	0	0	0	-89	0	-56	0	0	289	-433	0	71	0	0	0	-719	1007	0	-1190	503	0	0	523
1323/5	189/2	0	0	21	0	21	-2	-42	-119	-147	-210	-65	-97	-399	92	252	672	-504	-632	650	-567	448	-484	462	-1407	-488
1323/6	189/2	0	0	-21	0	-21	-2	42	-119	147	210	-65	-97	399	92	-252	-672	504	-632	650	567	448	-484	-462	1407	-488
1323/7		3	0	6	0	-57	-62	12	124	156	261	109	368	-54	152	78	222	-285	712	170	396	475	-163	-27	-642	-1835
1323/8	1323/7	3	0	-6	0	-57	62	-12	-124	156	261	-109	368	54	152	-78	222	285	-712	170	396	-475	-163	27	642	1835
1323/9	189/1	3	0	12	0	12	61	117	-2	-75	3	-263	218	246	515	-318	-459	255	862	479	-117	430	-646	348	585	376
1323/10	27/1	3	0	-15	0	-15	-20	-72	-2	114	30	-101	-430	30	110	330	621	660	376	-250	-360	-785	488	-489	450	1105
1323/11	1323/7	-3	0	6	0	57	62	12	-124	-156	-261	-109	368	-54	152	78	-222	-285	-712	170	-396	-475	-163	-27	-642	1835
1323/12	1323/7	-3	0	-6	0	57	-62	-12	124	-156	-261	109	368	54	152	-78	-222	285	712	170	-396	475	-163	27	642	-1835
1323/13	189/1	-3	0	-12	0	-12	61	-117	-2	75	-3	-263	218	-246	515	318	459	-255	862	479	117	430	-646	-348	-585	376
1323/14	27/1	-3	0	15	0	15	-20	72	-2	-114	-30	-101	-430	-30	110	-330	-621	-660	376	-250	360	-785	488	489	-450	1105
1325/1	53/1	0	-1	0	-2	54	43	99	-61	-207	-99	-160	7	-414	268	-270	-53	450	182	556	693	862	119	333	1350	187
1326/1		2	3	16	-26	-70	-13	17	-132	52	22	-194	108	-240	100	-364	-462	604	-350	-596	-910	392	512	1008	-690	-916
1326/2		-2	3	16	8	58	-13	17	56	140	-110	-260	-58	496	300	14	158	-570	114	788	-474	-154	-832	1290	-764	-546
1326/3		-2	-3	8	-2	70	-13	17	-120	-76	-106	70	144	48	-500	52	18	164	-470	448	-266	308	-1192	384	-1398	656
1328/1	166/1	0	5	8	31	-3	-24	5	144	40	-71	215	-111	438	180	-534	738	79	-163	578	138	586	412	83	-1036	-1016
1330/1		2	0	5	7	-20	-58	130	-19	-24	-266	-152	258	-458	-4	-120	226	-140	-634	276	-1100	-1046	108	-636	246	58

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1330/2		2	-4	5	-7	-35	15	103	19	97	-171	-78	-314	-83	-54	364	-268	-761	-499	-525	257	430	7	-391	-194	-1264
1330/3		-2	-7	5	7	43	35	-117	19	-60	-39	-12	-236	-116	-308	107	188	-538	274	-652	800	1154	1347	468	-710	401
1332/1	444/1	0	0	4	-25	-67	57	-27	-17	107	4	-274	-37	342	52	-82	-17	420	610	110	960	205	-1330	-51	533	178
1340/1		0	-4	-5	-16	-64	42	106	124	-36	-34	-8	210	378	-20	-396	402	428	-830	-67	-888	-990	-456	-964	1290	-26
1342/1		2	7	18	20	-11	47	-27	-76	-108	72	65	89	-216	-1	-30	-357	-30	61	-376	399	-904	-58	1032	486	-205
1344/1	168/1	0	3	2	7	-12	66	-70	92	16	122	64	306	50	-20	-176	-526	-540	818	228	864	106	736	588	146	-1214
1344/2	168/5	0	3	2	7	52	-86	-30	-4	-120	-246	-80	290	-374	164	-464	162	180	666	-628	-296	-518	1184	220	-774	-1086
1344/3	42/2	0	3	-2	7	-8	42	-2	-124	-76	-254	72	-398	462	212	264	162	-772	-30	-764	236	418	-552	1036	30	-1190
1344/4	21/1	0	3	4	-7	-62	62	84	-100	-42	10	-48	246	-248	-68	324	-258	-120	-622	-904	-678	-642	740	-468	200	-1266
1344/5	168/3	0	3	-4	-7	26	-2	-36	76	-114	-6	-256	86	160	220	308	-258	-264	-606	520	-286	-530	-44	-1012	768	222
1344/6	672/1	0	3	-6	7	4	46	-82	-84	44	-70	-152	146	94	-488	-32	562	476	-34	520	-36	-654	-608	-284	-954	-1694
1344/7	84/1	0	3	-6	7	-36	-62	114	76	-24	-54	-112	178	378	172	-192	402	-396	-254	1012	840	890	80	108	-1638	1010
1344/8	168/4	0	3	10	7	12	-30	34	-148	152	106	304	114	202	-116	224	274	660	-382	-12	-552	-614	880	108	-86	1426
1344/9	168/2	0	3	10	7	-52	10	-54	-52	-48	186	-224	-94	-478	-316	-256	66	420	-342	668	272	-86	-1360	188	-366	1554
1344/10	84/2	0	3	-14	7	4	-54	-14	92	152	106	144	-158	-390	-508	528	-606	-364	-678	844	8	-422	-384	-548	1194	-1502
1344/11	168/6	0	3	16	-7	-18	54	-128	52	202	-302	200	150	172	164	460	190	96	-622	744	54	742	92	-228	-116	-554
1344/12	21/2	0	3	18	7	36	34	42	124	0	-102	-160	-398	-318	268	240	498	132	-398	-92	-720	-502	-1024	204	354	-286
1344/13	672/2	0	3	18	7	44	-58	-130	92	-84	250	72	354	334	-416	464	450	-516	-58	-656	940	178	-1072	660	1254	210
1344/14	42/1	0	3	-18	7	72	34	6	-92	-180	114	56	34	6	-164	168	-654	492	250	124	36	1010	56	-228	390	-70
1344/15	168/1	0	-3	2	-7	12	66	-70	-92	-16	122	-64	306	50	20	176	-526	540	818	-228	-864	106	-736	-588	146	-1214
1344/16	168/5	0	-3	2	-7	-52	-86	-30	4	120	-246	80	290	-374	-164	464	162	-180	666	628	296	-518	-1184	-220	-774	-1086
1344/17	42/2	0	-3	-2	-7	8	42	-2	124	76	-254	-72	-398	462	-212	-264	162	772	-30	764	-236	418	552	-1036	30	-1190
1344/18	21/1	0	-3	4	7	62	62	84	100	42	10	48	246	-248	68	-324	-258	120	-622	904	678	-642	-740	468	200	-1266
1344/19	168/3	0	-3	-4	7	-26	-2	-36	-76	114	-6	256	86	160	-220	-308	-258	264	-606	-520	286	-530	44	1012	768	222
1344/20	672/1	0	-3	-6	-7	-4	46	-82	84	-44	-70	152	146	94	488	32	562	-476	-34	-520	36	-654	608	284	-954	-1694
1344/21	84/1	0	-3	-6	-7	36	-62	114	-76	24	-54	112	178	378	-172	192	402	396	-254	-1012	-840	890	-80	-108	-1638	1010
1344/22	168/4	0	-3	10	-7	-12	-30	34	148	-152	106	-304	114	202	116	-224	274	-660	-382	12	552	-614	-880	-108	-86	1426
1344/23	168/2	0	-3	10	-7	52	10	-54	52	48	186	224	-94	-478	316	256	66	-420	-342	-668	-272	-86	1360	-188	-366	1554
1344/24	84/2	0	-3	-14	-7	-4	-54	-14	-92	-152	106	-144	-158	-390	508	-528	-606	364	-678	-844	-8	-422	384	548	1194	-1502
1344/25	168/6	0	-3	16	7	18	54	-128	-52	-202	-302	-200	150	172	-164	-460	190	-96	-622	-744	-54	742	-92	228	-116	-554
1344/26	21/2	0	-3	18	-7	-36	34	42	-124	0	-102	160	-398	-318	-268	-240	498	-132	-398	92	720	-502	1024	-204	354	-286
1344/27	672/2	0	-3	18	-7	-44	-58	-130	-92	84	250	-72	354	334	416	-464	450	516	-58	656	-940	178	1072	-660	1254	210
1344/28	42/1	0	-3	-18	-7	-72	34	6	92	180	114	-56	34	6	164	-168	-654	-492	250	-124	-36	1010	-56	228	390	-70
1350/1	270/6	2	0	0	4	-42	-20	93	59	9	-120	47	262	-126	178	144	741	444	221	538	-690	1126	665	75	1086	-1544
1350/2	54/2	2	0	0	7	-60	79	-108	11	-132	-96	20	169	-192	-488	204	360	-156	83	-47	-216	511	-529	-1128	-36	-605
1350/3	270/2	2	0	0	-8	-18	-8	15	23	63	-156	-85	-74	-246	190	288	-177	-792	-907	322	270	-254	-1123	-771	198	1192
1350/4	270/3	2	0	0	13	30	61	12	-49	18	186	-160	91	-378	268	144	570	-204	-877	187	606	-431	1151	102	-984	265
1350/5		2	0	0	14	-22	-30	-7	-81	-151	270	-113	-88	-406	-442	-56	141	274	41	328	390	626	-1215	-505	-514	1816
1350/6	270/4	2	0	0	-14	3	-47	39	32	99	51	83	-314	-108	-299	-531	-564	12	230	268	120	-1106	-739	-1086	-120	1642
1350/7	1350/5	2	0	0	-14	22	30	-7	-81	-151	-270	-113	88	406	442	-56	141	-274	41	-328	-390	-626	-1215	-505	514	-1816
1350/8		2	0	0	19	-12	-50	-126	29	18	-102	-265	-65	-240	367	-72	-636	-102	-103	52	582	-65	173	498	822	-821
1350/9	1350/8	2	0	0	-19	12	50	-126	29	18	102	-265	65	240	-367	-72	-636	102	-103	-52	-582	65	173	498	-822	821
1350/10	270/5	2	0	0	22	12	-38	-105	-157	-117	-66	-25	-314	504	-380	-252	3	318	293	322	120	-44	917	309	-1272	-1328
1350/11		2	0	0	23	-30	-34	42	-139	-192	-234	-55	191	-138	53	-366	330	396	23	452	-204	-691	-709	-1098	816	905
1350/12	1350/11	2	0	0	-23	30	34	42	-139	-192	234	-55	-191	138	-53	-366	330	-396	23	-452	204	691	-709	-1098	-816	-905
1350/13	54/1	2	0	0	-29	57	-20	-72	-106	174	210	47	-2	6	-218	474	81	-84	56	142	-360	1159	-160	735	954	-191
1350/14	270/1	2	0	0	34	48	70	-27	119	-51	30	-133	-218	-156	88	-516	-639	654	461	-182	-900	-704	-1375	915	1116	16
1350/15	270/6	-2	0	0	4	42	-20	-93	59	-9	120	47	262	126	178	-144	-741	-444	221	538	690	1126	665	-75	-1086	-1544
1350/16	54/2	-2	0	0	7	60	79	108	11	132	96	20	169	192	-488	-204	-360	156	83	-47	216	511	-529	1128	36	-605
1350/17	270/2	-2	0	0	-8	18	-8	-15	23	-63	156	-85	-74	246	190	-288	177	792	-907	322	-270	-254	-1123	771	-198	1192
1350/18	270/3	-2	0	0	13	-30	61	-12	-49	-18	-186	-160	91	378	268	-144	-570	204	-877	187	-606	-431	1151	-102	984	265
1350/19	1350/5	-2	0	0	14	22	-30	7	-81	151	-270	-113	-88	406	-442	56	-141	-274	41	328	-390	626	-1215	505	514	1816
1350/20	270/4	-2	0	0	-14	-3	-47	-39	32	-99	-51	83	-314	108	-299	531	564	-12	230	268	-120	-1106	-739	1086	120	1642
1350/21	1350/5	-2	0	0	-14	-22	30	7	-81	151	270	-113	88	-406	442	56	-141	274	41	-328	390	-626	-1215	505	-514	-1816
1350/22	1350/8	-2	0	0	19	12	-50	126	29	-18	102	-265	-65	240	367	72	636	102	-103	52	-582	-65	173	-498	-822	-821
1350/23	1350/8	-2	0	0	-19	-12	50	126	29	-18	-102	-265	65	-240	-367	72	636	-102	-103	-52	582	65	173	-498	822	821
1350/24	270/5	-2	0	0	22	-12	-38	105	-157	117	66	-25	-314	-504	-380	252	-3	-318	293	322	-120	-44	917	-309	1272	-1328
1350/25	1350/11	-2	0	0	23	30	-34	-42	-139	192	234	-55	191	138	53	366	-330	-396	23	452	204	-691	-709	1098	-816	905
1350/26	1350/11	-2	0	0	-23	-30	34	-42	-139	192	-234	-55	-191	-138	-53	366	-330	396	23	-452	-204	691	-709	1098	816	-905
1350/27	54/1	-2	0	0	-29	-57	-20	72	-106	-174	-210	47	-2	-6	-218	-474	-81	84	56	142	360	1159	-160	-735	-954	-191
1350/28	270/1	-2	0	0	34	-48	70	27	119	51	-30	-133	-218	156	88	516	639	-654	461	-182	900	-704	-1375	-915	-1116	16
1352/1	104/2	0	1	7	21	-6	0	-115	46	144	-162	-180	-13	-192	-33	-383	288	-442	-680	722	207	-274	-936	1204	966	138

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1352/2	8/1	0	-4	2	-24	44	0	50	-44	-56	198	160	162	198	52	-528	-242	668	550	-188	-728	-154	-656	-236	-714	478
1352/3	104/1	0	5	-19	3	2	0	77	58	76	-6	292	-207	-240	-317	375	-692	-214	-488	-782	1057	-1174	892	-704	-6	-830
1352/4		0	8	9	4	20	0	75	40	-68	199	236	-355	393	-424	208	-161	44	799	-740	304	301	-412	1488	54	638
1352/5	1352/4	0	8	-9	-4	-20	0	75	-40	-68	199	-236	355	-393	-424	-208	-161	-44	799	740	-304	-301	-412	-1488	-54	-638
1358/1		-2	-8	20	-7	36	44	22	-62	128	152	-80	96	462	104	480	278	-338	-686	-1042	4	22	336	-642	206	-97
1359/1	453/1	3	0	19	-32	7	-57	66	-90	195	267	21	190	-9	462	220	360	676	-166	101	-789	-188	-770	-952	1335	1058
1360/1	680/2	0	2	-5	22	18	-70	17	128	-194	210	222	-438	26	-220	-164	-434	-728	426	680	-394	-638	42	324	-266	898
1360/2	680/1	0	-2	5	-2	14	-30	17	-72	-178	238	-34	286	242	-52	52	-306	-240	478	-56	-722	218	930	332	982	1234
1360/3	340/1	0	-2	5	-2	30	-62	17	56	110	206	-114	-194	-430	-4	68	206	-496	-290	-8	798	314	-366	1276	86	-1006
1360/4	170/1	0	-4	-5	4	12	-58	17	52	-84	-246	-68	-358	-78	412	-408	750	420	-190	-596	-324	1010	-164	-588	-486	-718
1360/5	340/2	0	5	5	-2	-12	-13	17	-35	-30	-249	229	-124	-66	262	75	-543	225	-535	-386	-231	-547	376	-768	-537	1367
1360/6	85/1	0	5	-5	22	-60	-31	17	61	78	69	31	56	-6	538	465	723	753	35	322	99	-1123	-488	852	1215	-601
1360/7	85/2	0	7	5	22	64	73	-17	49	-110	155	197	-372	-262	-258	13	-653	333	-355	-814	-47	-437	384	736	511	537
1360/8	170/2	0	-7	5	10	-24	41	-17	103	6	-45	-5	-196	210	58	171	3	-645	197	46	975	-637	-272	72	-609	-847
1360/9	680/3	0	8	-5	-12	34	-30	17	-32	-168	-62	-274	-314	282	-162	22	354	480	98	374	958	-762	830	-978	-578	-1006
1360/10	85/3	0	-10	5	22	30	-46	17	-104	-42	-66	-194	206	-126	388	540	78	-432	-610	-848	174	362	-398	-828	630	-1486
1365/1		1	-3	5	7	-52	13	82	-28	48	-154	200	-386	186	188	144	-466	780	230	76	192	-230	-1152	-1428	234	-158
1365/2		1	-3	-5	-7	48	-13	-86	-4	-192	-134	-204	146	-306	-280	76	366	-364	-694	744	-80	-58	-384	972	1278	1078
1365/3		-1	3	5	-7	48	-13	110	44	-36	-250	264	-190	42	-208	-124	-162	460	338	300	-712	530	384	-756	1122	1666
1365/4		-1	3	-5	7	4	13	-106	68	-156	-86	212	-50	270	-76	384	-650	564	350	160	504	-698	-96	-84	-570	622
1365/5		-1	3	-5	-7	60	13	-50	68	12	-30	-292	-106	46	428	-120	-258	-220	-546	552	0	1038	16	-476	886	-218
1365/6		-3	3	5	7	-20	13	-6	-4	-16	94	120	-146	-54	276	568	302	-196	-650	404	-944	802	-512	-1100	-6	1290
1370/1		2	4	5	-20	-28	22	2	-92	160	38	-68	110	-390	-452	376	-498	-620	-698	484	-876	-214	-300	-252	-1126	770
1380/1		0	-3	-5	-16	30	-28	78	-52	-23	-252	200	146	438	-46	588	438	-168	-586	-94	-30	-466	-520	708	-600	-340
1385/1		-3	1	5	17	-4	-48	87	20	5	-37	264	-366	-207	24	-381	-570	102	194	-9	450	-95	1264	612	-1357	1423
1386/1	154/1	2	0	1	-7	11	-8	-22	54	-213	-190	163	31	-110	4	80	566	-645	634	-729	-431	-918	-254	-904	-901	-89
1386/2	154/2	2	0	-2	-7	-11	26	46	-48	128	146	-128	-26	-10	52	544	-318	48	466	516	392	754	0	-624	1590	1018
1386/3	462/4	2	0	7	7	-11	-67	-30	-7	-28	-121	-310	-71	180	-108	-71	-128	429	22	-803	-468	-117	-96	1122	1146	-92
1386/4	462/3	2	0	-11	-7	-11	-37	46	15	92	-205	142	-431	8	448	-149	672	615	322	-411	968	-227	0	1302	870	-1736
1386/5	462/2	2	0	14	-7	-11	38	-54	40	-8	170	92	294	258	-52	76	322	-260	22	-436	368	-2	-200	952	70	-1086
1386/6	462/1	2	0	21	7	11	65	54	65	-132	-39	-178	-439	-96	272	375	-612	507	758	-1087	0	-673	-700	-1218	1350	-808
1386/7	462/6	-2	0	-1	-7	11	-43	-100	-87	58	223	88	37	-128	-458	341	342	105	190	-579	-128	-161	-396	420	798	1414
1386/8	154/4	-2	0	-3	7	11	-16	-6	14	51	-54	95	-193	-102	284	72	102	63	-790	-433	-135	-238	770	1008	639	11
1386/9	462/7	-2	0	-3	7	11	41	-6	-43	-120	-111	266	-79	-216	284	-213	216	-393	350	821	264	-865	-484	-1158	-330	980
1386/10	462/5	-2	0	4	-7	11	62	120	118	188	-62	-322	-198	-48	32	326	482	-400	70	-124	712	304	-1016	-430	-442	-966
1386/11	462/9	-2	0	13	-7	-11	-67	-8	21	194	221	88	-347	-292	-458	-221	642	-273	-530	561	-604	703	552	144	-750	-1370
1386/12	154/5	-2	0	14	7	11	-16	-108	116	-68	-122	-262	130	-204	-396	-166	-442	-702	196	-416	-492	408	600	1212	-1146	-482
1386/13	462/8	-2	0	17	7	11	-21	104	-161	-194	-9	-180	-363	108	-386	-333	122	-537	-950	-83	-180	177	-220	-1112	394	826
1386/14	154/3	-2	0	-18	7	11	56	-36	-28	-180	54	-334	386	444	-316	402	486	282	380	176	324	800	-1144	-468	870	-1330
1392/1	174/3	0	3	-8	-19	9	17	-7	36	182	29	102	-166	-406	80	173	-68	-222	106	-681	286	358	-516	-922	-1129	-1016
1392/2	174/5	0	3	-10	-7	63	-7	-89	78	52	-29	-192	200	166	356	-353	-154	-258	520	15	764	244	-186	1018	553	1294
1392/3	174/1	0	3	-14	21	-37	-87	119	50	-48	-29	-332	324	462	132	331	-222	-250	-308	-29	928	488	-190	522	745	-1566
1392/4	174/2	0	3	21	-19	38	-12	109	65	-108	-29	-72	-311	377	167	-349	338	155	802	856	-932	-222	-110	-168	-810	144
1392/5	174/4	0	-3	0	17	23	-63	19	8	-42	-29	198	-110	-514	404	-517	584	182	430	-365	34	-54	-236	-258	213	156
1392/6	348/1	0	-3	-9	-21	66	-72	-25	-137	112	29	-88	-375	-397	-87	327	182	-449	-510	864	-144	-370	-1174	528	986	360
1392/7	174/6	0	-3	-18	29	49	-15	101	110	-84	29	-132	-404	10	224	313	-374	394	-56	475	-1120	420	1018	-1230	-45	270
1394/1		2	-1	-6	1	12	28	17	46	-106	217	40	-394	41	277	-294	-6	-191	170	-236	-1065	594	-821	-629	-918	-1634
1395/1		1	0	5	5	-59	8	42	107	141	-252	-31	76	-60	-27	0	437	-688	-138	-1030	-279	-537	1057	726	-339	1392
1395/2	465/3	1	0	-5	7	41	-18	8	-95	25	-264	31	-220	96	-147	490	-117	586	-230	-334	1059	-187	1077	520	-1361	54
1395/3		1	0	-5	7	-43	60	14	67	-113	-24	31	224	-48	69	136	159	376	250	-238	-357	-457	-477	-1166	-1091	-1584
1395/4	1395/3	-1	0	5	7	43	60	-14	67	113	24	31	224	48	69	-136	-159	-376	250	-238	357	-457	-477	1166	1091	-1584
1395/5	155/1	-1	0	5	16	-2	-48	94	-140	68	-300	31	296	138	-318	224	-312	-160	-128	716	-912	182	-180	1418	1190	126
1395/6	1395/1	-1	0	-5	5	59	8	-42	107	-141	252	-31	76	60	-27	0	-437	688	-138	-1030	279	-537	1057	-726	339	1392
1395/7	465/2	3	0	5	-24	38	-18	44	-136	62	296	-31	-278	210	476	276	-10	632	-732	316	-984	-12	1196	-24	738	238
1395/8	465/1	4	0	-5	22	20	-72	80	40	112	258	31	-364	114	468	-206	384	40	586	398	804	8	-600	172	838	-1026
1400/1	280/1	0	1	0	-7	-39	17	15	74	14	-237	-180	318	-348	22	193	208	452	340	408	528	554	539	-164	-576	827
1400/2	56/2	0	2	0	7	24	68	-54	-46	-176	-174	-116	-74	-10	480	572	162	-86	-904	-660	1024	-770	-904	-682	-102	218
1400/3	280/3	0	4	0	-7	20	10	14	12	-104	-122	224	-158	378	-404	-112	-270	324	-186	-156	-360	102	-912	-1068	-1590	-866
1400/4		0	5	0	7	11	-46	127	117	80	34	-292	-376	507	32	-134	612	780	-426	-207	702	1185	54	-309	-339	182
1400/5	280/4	0	-5	0	7	-39	19	37	-18	90	99	-32	-46	-248	-178	-429	652	40	-36	348	72	1190	699	116	-704	-223
1400/6	1400/4	0	-5	0	-7	11	46	-127	117	-80	34	-292	376	507	-32	134	-612	780	-426	207	702	-1185	54	309	-339	-182
1400/7	56/1	0	-6	0	7	56	28	90	74	96	-222	-100	-58	422	-512	-148	642	-318	720	412	448	-994	-296	-386	-6	138

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1400/8	280/2	0	-7	0	-7	9	-23	-41	34	6	131	4	-26	-260	190	-167	368	324	-164	-200	784	410	1211	1132	-72	707
1404/1		0	0	6	23	-45	13	132	-160	-123	-141	-235	-169	306	197	-312	249	-735	104	32	-876	-364	200	195	1185	-1636
1404/2	1404/1	0	0	-6	23	45	13	-132	-160	123	141	-235	-169	-306	197	312	-249	735	104	32	876	-364	200	-195	-1185	-1636
1406/1		2	1	6	-13	30	-1	-87	19	33	27	38	37	-162	-394	144	-291	-633	218	-307	438	-787	170	-204	-690	176
1410/1		2	-3	5	2	-30	43	-70	-106	41	-50	106	-76	-24	-493	47	-418	383	-149	436	315	-879	1141	900	-63	-1050
1410/2		2	-3	-5	-12	34	8	-58	40	-136	182	-110	-82	228	288	-47	-338	564	218	-304	-392	-842	-28	-464	-746	-98
1414/1		2	-10	-15	-7	0	-93	17	-33	-33	-300	-237	-126	-120	-328	-397	-476	378	534	-120	-393	2	701	-1454	0	-322
1420/1		0	-1	5	-31	30	-9	78	-31	-47	-146	-149	-426	-174	165	163	-258	738	-18	-406	-71	1051	262	308	-519	1094
1421/1	203/2	1	4	-14	0	-28	-70	14	-140	72	29	-208	254	-186	-444	160	270	684	-86	-708	280	-506	480	1060	-810	-1314
1421/2	203/1	4	-8	-2	0	2	26	80	-128	0	29	-160	-274	36	246	244	114	420	-188	624	1120	352	438	676	336	216
1422/1		2	0	-4	-12	-14	86	-70	36	-78	-110	152	-194	382	-68	-132	-262	544	438	-220	-1076	-774	-79	-366	-1092	46
1422/2	158/1	2	0	9	9	8	-47	-84	-150	2	-180	-328	-346	268	328	351	-498	-365	492	784	-897	378	79	1102	1255	1129
1422/3	474/1	2	0	-18	9	35	7	-111	66	-133	279	-220	-76	-2	463	-432	42	418	-372	622	-438	-297	79	-167	-932	-1625
1422/4	1422/1	-2	0	4	-12	14	86	70	36	78	110	152	-194	-382	-68	132	262	-544	438	-220	1076	-774	-79	366	1092	46
1422/5	474/2	-2	0	6	-27	-23	-53	-45	-102	-35	177	-268	200	386	-497	-168	486	-718	444	838	-42	831	79	827	-868	-1481
1424/1	89/1	0	-2	2	4	56	-16	-30	50	92	204	-324	-20	270	-86	0	534	206	-672	576	352	-338	336	-630	89	-1506
1424/2	89/2	0	7	11	-8	32	-4	39	59	83	66	279	-350	-78	-71	258	597	572	-420	30	-230	-497	714	420	89	1833
1425/1		1	3	0	-4	-68	82	86	19	-18	30	-298	-34	52	482	-114	362	-210	-718	-904	-988	-488	-530	1032	-880	246
1425/2	57/1	1	-3	0	20	-4	76	-22	-19	-82	242	-126	180	-390	-308	522	70	188	-706	-104	-432	-718	94	1296	846	-830
1425/3	1425/1	-1	-3	0	4	-68	-82	-86	19	18	30	-298	34	52	-482	114	-362	-210	-718	904	-988	488	-530	-1032	-880	-246
1425/4		2	-3	0	-9	-62	38	-76	-19	-42	-259	-120	-230	455	-340	224	-61	-119	-113	468	995	-271	318	-336	-945	-872
1425/5	1425/4	-2	3	0	9	-62	-38	76	-19	42	-259	-120	230	455	340	-224	61	-119	-113	-468	995	271	318	336	-945	872
1425/6	285/1	3	3	0	-32	-12	10	30	19	48	150	224	-254	-54	196	504	-78	132	230	-740	-120	-122	1184	-612	1050	1006
1428/1		0	3	-2	7	20	-58	17	12	120	-138	160	446	-198	516	16	-514	228	246	76	520	-454	48	940	970	-1374
1428/2		0	3	3	7	21	-43	-17	-130	102	-66	-232	-211	324	-169	-408	-321	12	398	581	-180	-217	-487	-687	417	659
1428/3		0	-3	9	7	-51	-31	-17	-163	153	102	200	146	135	209	-108	168	24	-58	-664	-72	566	980	-930	864	980
1430/1		-2	4	5	-32	-11	13	42	-28	176	206	-328	-290	314	-188	-600	398	-612	-322	196	912	562	-96	1180	298	-1118
1431/1		3	0	-9	-4	-15	2	-24	17	48	24	8	-379	-177	464	579	53	327	-142	1019	708	452	-721	1386	-1548	-85
1431/2	1431/1	-3	0	9	-4	15	2	24	17	-48	-24	8	-379	177	464	-579	-53	-327	-142	1019	-708	452	-721	-1386	1548	-85
1435/1		4	8	-5	7	60	56	4	-36	22	-142	204	-18	41	362	8	-88	336	134	-44	848	-750	-824	-126	2	-1352
1440/1	160/1	0	0	5	6	-60	50	30	40	-178	-166	20	10	250	142	-214	-490	800	250	-774	-100	-230	-1320	-982	-874	-310
1440/2	160/1	0	0	5	-6	60	50	30	-40	178	-166	-20	10	250	-142	214	-490	-800	250	774	100	-230	1320	982	-874	-310
1440/3	480/2	0	0	5	8	4	-6	2	16	-60	142	176	-214	278	68	116	350	684	-394	-108	-96	-398	-136	436	750	82
1440/4	480/2	0	0	5	-8	-4	-6	2	-16	60	142	-176	-214	278	-68	-116	350	-684	-394	108	96	-398	136	-436	750	82
1440/5	480/1	0	0	5	12	20	-58	70	-92	-112	-66	-108	-58	-66	-388	408	-474	540	14	-276	96	-790	308	1036	-1210	1426
1440/6	480/1	0	0	5	-12	-20	-58	70	92	112	-66	108	-58	-66	388	-408	-474	-540	14	276	-96	-790	-308	-1036	-1210	1426
1440/7		0	0	5	30	-50	-88	-74	140	-80	234	0	116	72	280	-120	498	870	650	420	-1020	-322	160	980	1124	1114
1440/8	1440/7	0	0	5	-30	50	-88	-74	-140	80	234	0	116	72	-280	120	498	-870	650	-420	1020	-322	-160	-980	1124	1114
1440/9	480/4	0	0	-5	4	40	-90	70	-40	108	-166	40	-130	310	268	-556	370	240	-130	-876	-840	250	880	-188	726	-1550
1440/10	480/4	0	0	-5	-4	-40	-90	70	40	-108	-166	-40	-130	310	-268	556	370	-240	-130	876	840	250	-880	188	726	-1550
1440/11	480/5	0	0	-5	12	-24	38	6	-104	100	-230	56	190	-202	148	124	-206	-128	190	204	-440	1210	-816	-1412	214	1202
1440/12	480/5	0	0	-5	-12	24	38	6	104	-100	-230	-56	190	-202	-148	-124	-206	128	190	-204	440	1210	816	1412	214	1202
1440/13	480/6	0	0	-5	16	-24	-14	18	36	-104	250	-28	-54	-354	228	-408	-262	64	374	300	-1016	274	788	396	-786	-1086
1440/14	480/6	0	0	-5	-16	24	-14	18	-36	104	250	28	-54	-354	-228	408	-262	-64	374	-300	1016	274	-788	-396	-786	-1086
1440/15	1440/7	0	0	-5	30	50	-88	74	140	80	-234	0	116	-72	280	120	-498	-870	650	420	1020	-322	160	-980	-1124	1114
1440/16	1440/7	0	0	-5	-30	-50	-88	74	-140	-80	-234	0	116	-72	-280	-120	-498	870	650	-420	-1020	-322	-160	980	-1124	1114
1440/17	480/3	0	0	-5	32	-64	-6	-38	-116	120	122	164	146	238	-148	184	-470	216	806	-732	-264	-638	596	884	-930	322
1440/18	480/3	0	0	-5	-32	64	-6	-38	116	-120	122	-164	146	238	148	-184	-470	-216	806	732	264	-638	-596	-884	-930	322
1444/1		0	7	2	-11	22	-7	-33	0	-131	-119	-182	-322	308	118	264	385	-833	688	-749	504	-275	924	770	-210	-588
1444/2	1444/1	0	-7	2	-11	22	7	-33	0	-131	119	182	322	-308	118	264	-385	833	688	749	-504	-275	-924	770	210	588
1445/1		-1	8	5	14	-20	-58	0	-80	-118	126	70	-134	100	-272	-464	-642	180	-110	-924	90	828	1334	-552	1490	1376
1445/2	1445/1	-1	-8	-5	-14	20	-58	0	-80	118	-126	-70	134	-100	-272	-464	-642	180	110	-924	-90	-828	-1334	-552	1490	-1376
1445/3	85/1	3	5	5	22	-60	-31	0	-61	78	-69	31	-56	6	-538	-465	723	-753	-35	-322	99	1123	-488	-852	1215	601
1445/4	85/2	3	7	-5	22	64	73	0	-49	-110	-155	197	372	262	258	-13	-653	-333	355	814	-47	437	384	-736	511	-537
1445/5	85/3	3	-10	-5	22	30	-46	0	104	-42	66	-194	-206	126	-388	-540	78	432	610	848	174	-362	-398	828	630	1486
1445/6		-3	2	5	5	17	1	0	-76	-7	86	178	-6	266	-138	305	-197	213	-112	-794	1190	982	-162	-718	-107	714
1445/7	1445/6	-3	-2	-5	-5	-17	1	0	-76	7	-86	-178	6	-266	-138	305	-197	213	112	-794	-1190	-982	162	-718	-107	-714
1445/8	5/1	-4	-2	5	-6	-32	-38	0	100	78	50	108	-266	-22	442	-514	2	500	518	126	-412	878	-600	282	-150	-386
1449/1		3	0	3	7	-30	11	78	74	23	9	56	-169	225	47	255	-102	24	722	884	90	578	-484	756	1416	1019
1449/2	1449/1	-3	0	-3	7	30	11	-78	74	-23	-9	56	-169	-225	47	-255	102	-24	722	884	-90	578	-484	-756	-1416	1019
1450/1	290/4	2	-2	0	32	12	-22	-104	12	-212	-29	52	0	-354	-526	58	-314	-692	730	140	696	404	116	172	-42	-176
1450/2	58/1	2	-7	0	2	37	-27	-24	-88	28	-29	-143	360	386	-381	103	431	288	-840	180	706	-716	931	-1188	-642	-486

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1450/3	290/3	-2	0	0	20	-52	42	22	28	-36	29	24	266	-38	-88	-188	194	-460	-314	-896	-416	606	992	24	-774	-1626
1450/4	290/2	-2	2	0	-12	-48	2	24	-48	-8	-29	-328	280	-154	206	102	-546	548	50	480	816	-124	296	408	1558	-304
1450/5	290/1	-2	-2	0	24	12	18	44	-100	68	-29	172	-156	22	18	114	518	100	282	204	-768	-32	-20	-1332	-530	804
1450/6	58/2	-2	7	0	18	27	57	44	152	152	-29	-173	120	-314	-339	357	59	-572	-420	-660	726	-1004	361	168	58	1206
1452/1		0	3	4	19	0	-54	-4	-1	-214	-102	-225	27	-174	-76	234	-26	-354	-403	-317	-96	-117	1249	946	-916	541
1452/2	1452/1	0	3	4	-19	0	54	4	1	-214	102	-225	27	174	76	234	-26	-354	403	-317	-96	117	-1249	-946	-916	541
1452/3		0	3	-9	20	0	-37	-9	-64	72	63	152	-181	-333	128	-468	-117	504	-442	-772	216	650	-196	1368	-513	-1357
1452/4	1452/3	0	3	-9	-20	0	37	9	64	72	-63	152	-181	333	-128	-468	-117	504	442	-772	216	-650	196	-1368	-513	-1357
1452/5	132/4	0	3	10	-8	0	-18	-46	-40	44	-186	-72	-114	-174	416	-156	-62	-348	446	-956	-444	-306	664	124	602	1522
1452/6	12/1	0	3	-18	-8	0	10	-18	100	72	234	-16	-226	-90	-452	432	414	-684	-422	332	-360	-26	-512	1188	-630	-1054
1452/7	132/1	0	-3	0	-2	0	88	66	40	6	54	8	-106	-354	124	546	-408	552	-404	-4	126	166	874	-444	1002	-802
1452/8	132/2	0	-3	-12	-14	0	-56	-42	-116	-30	-198	-88	350	-198	-56	-594	-204	-312	-620	356	-462	-482	238	-492	954	-1426
1452/9	132/3	0	-3	22	20	0	-22	-110	-48	72	142	184	-194	482	80	392	-34	-108	-382	84	-1040	606	1292	-356	-406	1090
1455/1		-1	3	5	-2	48	-46	-114	-66	174	-34	100	-446	382	-440	-212	194	542	602	-254	798	-290	-476	-978	1478	97
1456/1	364/3	0	0	17	7	50	-13	-108	9	19	23	173	156	426	59	311	-165	856	-626	-334	-472	-253	1051	-405	915	719
1456/2	182/2	0	2	-5	7	36	-13	26	47	99	-61	23	-50	70	19	-191	195	-264	310	190	166	873	1191	-259	-635	133
1456/3	364/2	0	-4	-19	-7	-38	-13	120	7	9	171	-313	160	354	197	-67	-617	-40	-90	-490	540	275	233	-1291	627	-89
1456/4	182/3	0	-5	16	-7	15	-13	-44	138	-111	-12	-215	55	-133	180	-471	-260	-110	-271	799	-912	747	883	924	142	-1407
1456/5	182/1	0	-7	0	-7	-39	13	24	-38	-39	-96	-227	425	-105	-344	-99	-540	-114	-565	385	156	-673	-749	1044	-690	317
1456/6	364/1	0	-7	-8	7	57	-13	44	110	-21	-28	71	43	-113	-212	175	-348	-546	529	-527	448	63	-135	1340	-866	-1163
1456/7	182/4	0	8	3	-7	54	13	-96	151	-33	183	331	-88	-42	-353	465	195	-552	470	-254	-132	-943	727	1197	753	1037
1462/1		2	5	-15	4	8	-23	17	84	-44	-30	126	-281	-4	43	-553	233	-621	-529	-679	-941	-645	1250	581	-136	-248
1467/1	489/1	3	0	17	-1	-18	-47	-54	144	-27	-67	-243	-98	81	-149	-584	198	-302	-270	-746	922	-218	820	-355	1600	-1193
1470/1	210/10	2	3	5	0	-4	42	86	96	-96	-78	-80	50	26	-32	20	-382	-356	134	888	868	70	400	1052	634	-1202
1470/2		2	3	5	0	-4	-77	-26	-121	-166	6	235	-419	-128	-291	-442	276	-706	442	531	1036	427	1317	-1188	-38	-866
1470/3		2	3	5	0	-15	-77	96	37	-99	240	166	335	-21	-40	639	153	684	-488	608	198	-338	-736	0	1290	1456
1470/4	210/9	2	3	5	0	24	-14	-54	-44	156	174	88	-34	138	164	216	318	204	442	-316	-252	-98	-1000	-516	522	310
1470/5		2	3	-5	0	-6	-19	12	119	-12	-252	251	359	54	-37	-246	552	408	386	-811	-54	173	1061	1206	672	818
1470/6	210/8	2	3	-5	0	16	-58	-34	-64	-16	62	-60	150	-474	-292	-240	-662	324	514	-372	-412	770	-560	852	-1466	178
1470/7		2	3	-5	0	-19	33	64	-141	-51	216	290	-109	457	184	313	-319	44	-368	216	-314	602	112	712	-1018	584
1470/8		2	3	-5	0	-20	-26	26	42	-194	-42	-274	2	250	-296	328	-148	-488	-272	8	-684	-310	-584	-404	266	678
1470/9	1470/5	2	-3	5	0	-6	19	-12	-119	-12	-252	-251	359	-54	-37	246	552	-408	-386	-811	-54	-173	1061	-1206	-672	-818
1470/10	1470/7	2	-3	5	0	-19	-33	-64	141	-51	216	-290	-109	-457	184	-313	-319	-44	368	216	-314	-602	112	-712	1018	-584
1470/11	1470/8	2	-3	5	0	-20	26	-26	-42	-194	-42	274	2	-250	-296	-328	-148	488	272	8	-684	310	-584	404	-266	-678
1470/12	30/2	2	-3	5	0	-48	-2	114	-140	72	210	-272	-334	198	-268	-216	-78	-240	-302	596	-768	478	-640	348	-210	1534
1470/13	210/7	2	-3	5	0	56	-54	-94	-36	-84	-258	40	-178	146	148	200	-130	-188	-94	-444	532	-770	-536	1076	1090	-1274
1470/14	1470/2	2	-3	-5	0	-4	77	26	121	-166	6	-235	-419	128	-291	442	276	706	-442	531	1036	-427	1317	1188	38	866
1470/15	1470/3	2	-3	-5	0	-15	77	-96	-37	-99	240	-166	335	21	-40	-639	153	-684	488	608	198	338	-736	0	-1290	-1456
1470/16		-2	3	5	0	-4	-62	70	-6	-70	-162	62	218	-130	232	-304	-380	-376	-56	-952	708	-682	-632	-244	1198	-1206
1470/17	210/1	-2	3	5	0	12	-2	18	-56	-156	-186	52	-178	138	-412	456	-198	-348	-110	-196	-936	-542	992	276	-630	-110
1470/18		-2	3	5	0	-32	15	-70	15	-42	90	-85	113	164	169	326	-44	-782	658	1071	344	431	397	680	1534	-1234
1470/19		-2	3	5	0	33	-37	60	119	75	-144	-46	-199	-135	260	183	411	492	-460	980	-306	-934	-604	108	-546	1808
1470/20		-2	3	-5	0	-2	-47	0	39	80	56	19	131	-310	-265	218	296	92	-870	-255	-426	1161	-299	-1022	-236	-862
1470/21	210/3	-2	3	-5	0	12	58	-42	4	24	294	-128	-58	-282	428	-384	-138	-468	250	-556	624	958	632	-84	-810	790
1470/22	210/2	-2	3	-5	0	-44	-54	-98	60	-144	-210	208	-226	502	484	232	-530	764	-814	60	848	958	-152	-308	1094	-554
1470/23	210/5	-2	-3	5	0	0	-26	-18	-92	0	-6	4	410	-174	248	-420	102	588	-650	152	-168	610	-1048	684	834	-110
1470/24	1470/20	-2	-3	5	0	-2	47	0	-39	80	56	-19	131	310	-265	-218	296	-92	870	-255	-426	-1161	-299	1022	236	862
1470/25	210/6	-2	-3	5	0	28	86	66	48	140	-34	284	-346	274	-4	448	-94	-308	-510	-156	336	1170	16	-772	-1630	-110
1470/26	1470/16	-2	-3	-5	0	-4	62	-70	6	-70	-162	-62	218	130	232	304	-380	376	56	-952	708	682	-632	244	-1198	1206
1470/27	210/4	-2	-3	-5	0	28	-54	46	-12	0	6	-296	134	-146	556	448	46	-748	50	-156	-1024	310	856	628	590	1390
1470/28	1470/18	-2	-3	-5	0	-32	-15	70	-15	-42	90	85	113	-164	169	-326	-44	782	-658	1071	344	-431	397	-680	-1534	1234
1470/29	1470/19	-2	-3	-5	0	33	37	-60	-119	75	-144	46	-199	135	260	-183	411	-492	460	980	-306	934	-604	-108	546	-1808
1470/30	30/1	-2	-3	-5	0	-60	34	-42	76	0	6	232	134	-234	-412	360	222	-660	490	812	120	-746	152	804	678	-194
1472/1	46/1	0	1	10	-12	42	-7	20	-106	23	227	67	-74	-497	88	215	-314	-176	298	-266	-981	-411	806	952	-1332	-1328
1472/2	46/1	0	-1	10	12	-42	-7	20	106	-23	227	-67	-74	-497	-88	-215	-314	176	298	266	981	-411	-806	-952	-1332	-1328
1472/3	184/2	0	4	-22	8	20	-22	98	12	23	10	192	106	186	-332	-544	-390	716	-110	-836	-280	-486	288	-180	650	-1262
1472/4	184/2	0	-4	-22	-8	-20	-22	98	-12	-23	10	-192	106	186	332	544	-390	-716	-110	836	280	-486	-288	180	650	-1262
1472/5	23/1	0	5	6	-8	-34	57	-80	70	23	-245	103	298	95	-88	-357	414	408	-822	-926	335	-899	-1322	36	-460	-964
1472/6	23/1	0	-5	6	8	34	57	-80	-70	-23	-245	-103	298	95	88	357	414	-408	-822	926	-335	-899	1322	-36	-460	-964
1472/7	184/1	0	8	4	4	26	-70	94	54	23	86	144	172	-42	386	80	108	164	400	398	320	-810	204	102	1018	-1370
1472/8	184/1	0	-8	4	-4	-26	-70	94	-54	-23	86	-144	172	-42	-386	-80	108	-164	400	-398	-320	-810	-204	-102	1018	-1370
1472/9	46/2	0	9	20	2	52	-43	-50	74	-23	7	-273	4	123	152	75	-86	444	-262	-764	-21	681	426	-902	-1272	-342

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1472/10	46/2	0	-9	20	-2	-52	-43	-50	-74	23	7	273	4	123	-152	-75	-86	-444	-262	764	21	681	-426	902	-1272	-342
1476/1	492/1	0	0	-5	-26	-34	-85	-97	-79	-186	168	271	-2	-41	268	-84	-378	-337	-358	279	-837	705	-384	-1293	347	694
1476/2	492/2	0	0	12	10	41	58	53	56	162	-1	-15	-363	-41	-91	195	670	192	193	-646	-891	-35	-426	-728	-294	188
1479/1		0	3	12	-1	-36	-53	17	39	160	29	-94	219	-373	-239	409	-733	-432	381	-598	-921	-251	508	726	884	1654
1479/2		-1	-3	-14	4	12	-62	-17	110	-62	-29	-278	-266	102	178	-336	-372	30	422	-776	-838	108	210	-562	250	-1036
1482/1		-2	-3	-14	-26	2	-13	-11	19	-127	130	87	79	407	-507	314	378	105	647	699	232	-322	140	-942	210	-1031
1482/2		-2	-3	-19	-1	-9	-13	-63	-19	-120	-100	-2	-188	-322	149	-339	368	-572	-641	-754	-204	-341	-398	-964	-914	1460
1484/1		0	4	-3	7	15	-28	33	32	-27	-96	221	-160	-30	305	-384	53	-153	-862	-292	852	-802	-487	480	-1569	-403
1484/2		0	-6	4	-7	-40	-92	-66	-98	40	-30	-72	-302	402	-432	-252	53	266	-12	-460	200	-786	-344	-770	290	-98
1485/1		3	0	5	-1	-11	74	30	-133	-69	120	-193	-115	3	362	-96	-678	-117	605	-241	-834	-427	68	-132	252	-1522
1485/2		3	0	5	-16	11	26	54	62	42	-162	-226	107	315	-307	-438	-96	-18	-385	-1024	-633	-943	-559	324	588	326
1485/3		3	0	-5	-25	-11	-16	-9	20	33	-177	-178	209	231	-7	-303	-414	735	326	182	540	542	155	768	-756	1217
1485/4	1485/3	-3	0	5	-25	11	-16	9	20	-33	177	-178	209	-231	-7	303	414	-735	326	182	-540	542	155	-768	756	1217
1485/5	1485/1	-3	0	-5	-1	11	74	-30	-133	69	-120	-193	-115	-3	362	96	678	117	605	-241	834	-427	68	132	-252	-1522
1485/6	1485/2	-3	0	-5	-16	-11	26	-54	62	-42	162	-226	107	-315	-307	438	96	18	-385	-1024	633	-943	-559	-324	-588	326
1488/1	186/1	0	3	3	7	0	2	120	115	138	-168	-31	-376	-159	448	-264	564	135	416	268	579	92	430	-342	522	1001
1488/2	186/5	0	3	-7	3	18	-52	-60	119	20	178	31	58	-285	-230	-164	180	-351	288	-324	65	-386	758	-814	594	-1615
1488/3	93/1	0	3	-9	34	-33	65	-21	97	84	48	-31	146	-378	-182	501	-402	-102	209	835	105	542	-1109	597	-1638	-1483
1488/4	744/1	0	3	14	24	4	46	-30	116	-168	254	31	22	-70	212	-256	-122	-660	750	804	-792	-934	-576	44	970	1346
1488/5	186/6	0	-3	-1	6	-39	89	27	23	68	64	31	-206	-138	-218	379	630	366	-279	-123	-121	-674	965	-493	570	-1003
1488/6	186/2	0	-3	-11	-9	30	-16	-60	11	16	-130	31	-266	-273	22	188	-156	9	312	-324	-647	730	-538	-518	714	113
1488/7	186/3	0	-3	-11	22	-63	15	95	11	-108	56	-31	230	378	-102	157	-466	-270	591	513	-647	-262	175	443	714	485
1488/8	186/4	0	-3	15	-17	-24	2	-48	115	-30	264	-31	-160	-51	-128	-480	132	-309	-280	604	159	-652	838	690	-534	329
1488/9	186/7	0	-3	-21	19	12	-34	-72	7	30	-84	-31	380	9	268	480	276	-309	-712	-116	783	1040	-386	-54	-1446	1625
1494/1	166/1	-2	0	-8	-31	-3	-24	-5	-144	40	71	-215	-111	-438	-180	-534	-738	79	-163	-578	138	586	-412	83	1036	-1016
1498/1		2	5	2	-7	-33	31	12	-103	-83	-266	130	-259	453	-504	504	381	-264	-87	-178	280	152	-379	-396	-1021	-1404
1498/2		2	-7	-12	7	-47	21	16	37	15	106	306	281	-349	-386	-352	-271	218	455	796	-1044	350	951	968	345	-890
1504/1		0	4	8	16	-42	20	-66	-54	92	-52	-212	194	-170	-278	47	-30	56	-526	-226	264	74	-480	-620	-522	1346
1504/2	1504/1	0	-4	8	-16	42	20	-66	54	-92	-52	212	194	-170	278	-47	-30	-56	-526	226	-264	74	480	620	-522	1346
1510/1		2	1	5	-4	9	-31	30	-97	63	177	-31	-226	-300	62	-186	-522	-297	-70	-439	-6	101	-220	237	312	-1300
1512/1		0	0	4	7	12	27	-61	-62	13	19	-329	42	-310	491	-290	373	-111	-254	-297	-93	-254	-614	1380	-1	-488
1512/2	1512/1	0	0	-4	7	-12	27	61	-62	-13	-19	-329	42	310	491	290	-373	111	-254	-297	93	-254	-614	-1380	1	-488
1512/3		0	0	7	7	57	42	-70	79	169	-50	145	279	-13	-268	-68	112	-288	-8	-894	189	88	-236	-318	803	-8
1512/4	1512/3	0	0	-7	7	-57	42	70	79	-169	50	145	279	13	-268	68	-112	288	-8	-894	-189	88	-236	318	-803	-8
1512/5		0	0	16	7	48	-39	11	-74	25	265	91	-54	-22	173	310	-113	639	478	-831	-729	178	-1046	60	191	1576
1512/6	1512/5	0	0	-16	7	-48	-39	-11	-74	-25	-265	91	-54	22	173	-310	113	-639	478	-831	729	178	-1046	-60	-191	1576
1518/1		2	3	3	5	11	-49	-123	-136	-23	-159	155	-241	-354	-178	555	-66	-504	848	-31	1155	-478	1025	642	1584	-1738
1518/2		2	3	-3	1	-11	17	-79	-126	23	45	-119	75	-148	-198	-329	442	-288	394	211	415	232	457	-530	-1428	-20
1518/3		2	-3	-2	-8	11	-74	62	36	-23	134	-176	262	82	28	56	182	180	-38	-348	1096	-78	472	1268	-406	1258
1518/4		2	-3	-6	-19	-11	23	-30	-16	23	-132	-79	65	435	-34	-186	-570	39	326	755	252	2	740	1209	258	-1438
1518/5		-2	3	6	-21	-11	35	-26	16	23	84	241	-45	173	-362	-50	-298	-291	-270	417	856	826	940	733	250	1810
1519/1	217/1	-1	8	-4	0	66	78	-78	106	-28	88	31	152	18	-506	484	364	-770	222	-220	-512	646	-380	832	1402	-414
1520/1	380/1	0	-1	-5	-19	-20	-77	-11	19	-79	-303	-214	-250	-230	402	-48	-417	-99	332	319	1088	-373	-102	-934	498	-1386
1520/2	190/2	0	2	5	-8	-44	0	-74	-19	-84	266	-136	424	470	236	240	36	-736	650	830	216	254	1220	688	102	-1280
1520/3	190/1	0	-2	5	12	20	-4	-34	19	-40	-150	200	-156	-218	-248	180	72	48	-134	-334	520	438	-980	156	670	1124
1520/4	190/3	0	4	5	20	44	42	-86	-19	164	-162	312	226	34	432	-580	506	-364	518	-924	-320	-542	1208	1120	-1022	1166
1520/5	95/4	0	-4	5	32	12	-42	114	-19	-160	214	144	94	-6	308	-184	-274	-276	-826	-52	344	-166	688	-996	1578	786
1520/6	95/1	0	-4	-5	22	12	8	-66	-19	30	-6	64	-16	54	-182	-594	396	564	-706	628	984	14	328	294	918	-1564
1520/7	95/2	0	5	-5	1	24	-31	33	-19	-27	111	94	-70	-510	34	192	-75	-45	-28	-371	-384	-73	1234	-366	-1578	-538
1520/8	95/3	0	-7	5	-11	36	65	-87	-19	129	231	-110	-142	-330	-74	336	501	-633	-88	-119	204	407	-1262	-270	-30	1406
1520/9	760/1	0	8	-5	30	-20	-12	54	-19	-114	178	296	-164	438	162	-74	288	324	590	728	-464	-906	-712	102	-1202	-616
1521/1	9/1	0	0	0	-20	0	0	0	-56	0	0	-308	-110	0	-520	0	0	0	182	880	0	-1190	884	0	0	1330
1521/2	39/1	0	0	-12	-2	-36	0	78	-74	96	-18	214	286	-384	524	300	-558	576	74	-38	-456	682	704	-888	-1020	-110
1521/3	507/2	1	0	-7	-10	22	0	-37	30	162	113	196	13	-285	-246	462	537	-576	-635	202	1086	-805	884	-518	-194	-1202
1521/4	507/2	-1	0	7	10	-22	0	-37	-30	162	113	-196	-13	285	-246	-462	537	576	-635	-202	-1086	805	884	518	194	1202
1521/5	507/4	3	0	-9	-2	30	0	111	46	6	105	100	-17	-231	-514	-162	-639	600	233	-926	-930	253	-1324	810	498	-1358
1521/6	169/4	3	0	-9	-15	-48	0	-45	-6	162	144	-264	-303	-192	97	111	414	522	376	36	357	1098	-830	-438	-438	852
1521/7	507/4	-3	0	9	2	-30	0	111	-46	6	105	-100	17	231	-514	162	-639	-600	233	926	930	-253	-1324	-810	-498	1358
1521/8	169/4	-3	0	9	15	48	0	-45	6	162	144	264	303	192	97	-111	414	-522	376	-36	-357	-1098	-830	438	438	-852
1521/9	169/1	4	0	17	-20	-32	0	13	-30	-78	-197	74	227	-165	-156	-162	-93	-864	145	-862	654	-215	-76	628	-266	-238
1521/10	169/1	-4	0	-17	20	32	0	13	30	-78	-197	-74	-227	165	-156	162	-93	864	145	862	-654	215	-76	-628	266	238
1521/11	13/1	-5	0	-7	13	-26	0	-77	126	96	82	-196	131	336	-201	-105	432	-294	-56	-478	9	-98	1304	-308	-1190	-70

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1522/1		2	-2	3	-8	38	-30	99	-64	-56	-105	310	37	-5	-326	-441	608	-61	-12	-467	-506	-388	-339	-544	-147	78
1526/1		-2	-4	-10	7	-36	-44	52	-6	108	138	140	-14	-40	-396	-54	278	-830	422	956	832	-134	388	-348	1358	870
1530/1	170/1	2	0	5	-4	12	-58	-17	-52	-84	246	68	-358	78	-412	-408	-750	420	-190	596	-324	1010	164	-588	486	-718
1530/2	510/6	2	0	5	11	-49	10	-17	-109	-22	-169	-98	-57	-253	216	-409	283	-324	-178	904	-100	-503	666	-172	-404	1314
1530/3	510/7	2	0	5	-28	68	-42	-17	60	-204	-234	-228	346	-162	372	-136	114	612	-542	-812	-204	-126	-660	-588	-378	-350
1530/4	510/3	2	0	-5	3	-27	-34	17	127	-58	81	58	-69	-379	52	-423	329	-660	-490	-580	-4	717	-602	-1052	-772	890
1530/5	510/4	2	0	-5	-6	0	56	17	-116	-112	-270	220	66	44	-182	216	-418	618	878	-598	-1066	276	1000	-656	-1438	-1432
1530/6	510/8	2	0	-5	11	15	-28	17	-109	-108	-129	-16	-193	195	248	-153	51	42	-64	-34	1056	-439	-1330	-504	198	-1138
1530/7		2	0	-5	17	57	68	17	-73	-114	273	-100	185	225	-94	-405	255	684	-394	-1078	-816	335	-268	684	-828	806
1530/8	510/5	2	0	-5	-23	-17	-12	-17	-65	-44	291	-188	321	503	-148	-311	449	-198	368	218	56	-761	-530	1112	942	370
1530/9	1530/7	-2	0	5	17	-57	68	-17	-73	114	-273	-100	185	-225	-94	405	-255	-684	-394	-1078	816	335	-268	-684	828	806
1530/10	510/2	-2	0	5	-25	-1	-26	-17	-49	110	35	-302	-369	-145	96	575	127	-408	50	316	-100	985	18	68	-896	-78
1530/11	170/2	-2	0	-5	-10	-24	41	17	-103	6	45	5	-196	-210	-58	171	-3	-645	197	-46	975	-637	272	72	609	-847
1530/12	510/1	-2	0	-5	-25	61	-54	17	-93	66	-75	30	19	-135	-228	-599	-363	480	-278	-416	-420	813	-18	852	504	1018
1531/1		-3	-6	-12	8	67	-76	33	-132	1	70	-194	-126	126	-287	591	-372	231	901	181	863	698	-86	-296	-959	1766
1533/1		0	3	-8	-7	42	25	-124	-47	159	-153	-282	49	-133	233	384	-405	113	324	-234	-625	73	-20	615	1205	-586
1534/1		-2	-9	-7	-17	-30	13	90	121	-144	157	-314	254	173	52	-154	-329	59	-690	-524	284	772	651	-992	-822	-1182
1540/1		0	2	5	-7	11	8	8	112	-172	-102	-6	2	96	204	254	450	-886	-812	72	-1084	-812	64	-848	-758	-738
1540/2		0	-8	5	-7	11	-42	58	-48	68	-222	-56	-278	206	284	624	50	324	-602	92	256	-222	404	772	-558	582
1540/3		0	-8	5	-7	-11	-86	-74	-92	68	174	-232	118	-14	-332	-476	-522	-292	190	312	-536	-178	184	-900	-646	-1046
1542/1		2	-3	-16	-22	-35	-59	-31	-56	-43	-249	-257	-32	142	6	-70	-722	333	166	-780	480	621	-59	-272	-339	454
1542/2		-2	-3	9	14	36	53	96	74	150	-84	305	248	315	20	-39	-309	414	317	-169	-216	-313	-541	204	-270	-958
1545/1		-3	3	5	-30	-69	-63	-58	-26	-124	-276	-84	-256	33	34	531	-260	-444	398	238	-386	-551	311	681	-1076	296
1547/1		1	-8	-6	-7	16	13	17	-116	-72	-22	-320	-366	-318	-532	376	38	76	-222	-700	-464	506	-560	-684	890	-1350
1548/1	516/1	0	0	-5	-1	39	-67	0	-11	180	55	190	-86	-176	43	-207	-172	844	-324	-146	-72	16	758	-701	238	-527
1550/1	62/1	2	2	0	11	-18	82	6	25	-58	180	31	146	47	12	136	232	715	-518	436	387	-678	660	382	-800	1631
1550/2	310/5	2	2	0	-20	44	20	68	-68	-182	-6	-31	208	202	-422	-608	-264	684	474	-680	-16	376	-1200	-1230	6	-1190
1550/3	310/4	2	2	0	24	-22	-24	-86	-68	104	-72	-31	-232	70	-158	-432	-440	24	-428	-20	864	-218	252	398	-302	-222
1550/4	310/3	2	-7	0	4	-4	2	-79	133	-176	186	-31	-191	-35	391	-242	237	33	360	-398	1031	-173	-522	-537	-240	826
1550/5		2	8	0	9	21	67	26	-142	-86	166	-31	359	345	511	-87	-613	-822	-355	472	681	322	-572	983	-1230	-354
1550/6	310/2	-2	1	0	18	-30	2	25	-51	148	64	31	69	-11	-25	50	-75	-483	-596	604	-65	-593	-328	-933	838	-350
1550/7	62/2	-2	8	0	35	-46	-20	-8	97	-28	-206	-31	282	367	562	148	84	-301	-236	-60	699	814	670	650	1566	615
1550/8	1550/5	-2	-8	0	-9	21	-67	-26	-142	86	166	-31	-359	345	-511	87	613	-822	-355	-472	681	-322	-572	-983	-1230	354
1550/9	310/1	-2	10	0	0	60	-16	112	-124	146	-62	-31	-140	474	-94	156	28	-348	-646	404	-912	380	-992	1066	-354	-414
1557/1	519/1	1	0	-6	-12	-30	-60	-5	-59	-92	-161	-39	-324	-42	-334	-44	-72	440	71	-562	525	495	-1304	443	702	938
1557/2	519/2	-2	0	-9	-24	-9	18	-14	-116	-32	208	-261	-363	30	-211	-104	-387	-436	-820	407	-765	-693	58	-514	162	-1384
1560/1		0	3	-5	2	-52	-13	96	50	-106	164	208	-170	462	-484	-248	62	-676	-674	44	-104	-112	-712	684	-1278	-248
1560/2		0	3	-5	24	-8	-13	-102	-104	180	-34	32	-126	-462	-484	456	-158	-280	-850	-396	160	-178	608	1476	394	-1106
1560/3		0	-3	5	10	24	13	16	114	-78	176	56	-198	438	76	136	-158	-792	118	-100	848	-516	352	196	-726	700
1560/4		0	-3	5	-32	60	13	-86	12	24	62	104	-210	-438	-212	16	-242	540	-602	-4	-112	930	1168	-260	-6	-566
1560/5		0	-3	-5	-18	28	-13	-72	-146	42	-100	32	246	-210	308	-336	-626	-580	-130	-444	712	200	-808	708	-1118	1216
1561/1		-1	4	18	-7	2	-70	94	54	-160	-2	-232	294	270	-56	136	-298	-684	254	-478	-672	-1022	-856	-442	1314	-166
1562/1		-2	4	19	-33	-11	15	55	32	174	-118	130	-343	217	-110	210	-282	628	-655	578	-71	54	-626	720	1157	474
1565/1		1	-8	5	22	20	58	114	-4	150	258	-86	-226	-294	-74	378	606	854	-482	-630	324	-958	-824	384	-318	1774
1568/1		0	0	4	0	0	92	104	0	0	130	0	-214	-472	0	0	518	0	468	0	0	-592	0	0	176	1816
1568/2	1568/1	0	0	-4	0	0	-92	-104	0	0	130	0	-214	472	0	0	518	0	-468	0	0	592	0	0	-176	-1816
1568/3	32/1	0	0	-22	0	0	18	94	0	0	-130	0	214	230	0	0	518	0	-830	0	0	-1098	0	0	1670	-594
1568/4	224/1	0	2	0	0	20	20	50	-10	-72	-134	180	-270	250	92	236	150	-570	200	176	-640	-250	-640	-882	-1074	-270
1568/5		0	2	14	0	-20	-6	20	102	-124	-78	236	66	268	132	516	-354	438	486	804	248	768	192	294	80	1404
1568/6	1568/5	0	2	-14	0	20	6	-20	102	124	-78	236	66	-268	-132	516	-354	438	-486	-804	-248	-768	-192	294	-80	-1404
1568/7	224/1	0	-2	0	0	-20	20	50	10	72	-134	-180	-270	250	-92	-236	150	570	200	-176	640	-250	640	882	-1074	-270
1568/8	1568/5	0	-2	14	0	20	-6	20	-102	124	-78	-236	66	268	-132	-516	-354	-438	486	-804	-248	768	-192	-294	80	1404
1568/9	1568/5	0	-2	-14	0	-20	6	-20	-102	-124	-78	-236	66	-268	132	-516	-354	-438	-486	804	248	-768	192	-294	-80	-1404
1568/10		0	8	4	0	-40	-36	40	-72	-176	162	-16	-54	-472	-72	144	486	-648	-684	216	-608	1008	1008	216	-1040	-936
1568/11	1568/10	0	8	-4	0	40	36	-40	-72	176	162	-16	-54	472	72	144	486	-648	684	-216	608	-1008	-1008	216	1040	936
1568/12	32/2	0	8	10	0	40	50	30	40	-48	-34	320	310	-410	-152	-416	-410	-200	-30	-776	-400	630	1120	552	326	110
1568/13	1568/10	0	-8	4	0	40	-36	40	72	176	162	16	-54	-472	72	-144	486	648	-684	-216	608	1008	-1008	-216	-1040	-936
1568/14	1568/10	0	-8	-4	0	-40	36	-40	72	-176	162	16	-54	472	-72	-144	486	648	684	216	-608	-1008	1008	-216	1040	936
1568/15	32/2	0	-8	10	0	-40	50	30	-40	48	-34	-320	310	-410	152	416	-410	200	-30	776	400	630	-1120	-552	326	110
1570/1		-2	-2	-5	-25	35	-42	78	-105	60	142	19	36	-70	355	420	197	384	-720	-744	-797	491	-616	1329	663	555
1573/1	13/1	5	-7	-7	13	0	-13	-77	126	-96	82	196	-131	-336	201	-105	-432	-294	56	478	9	-98	-1304	308	-1190	70
1575/1	105/1	0	0	0	-7	-42	-20	66	38	12	258	146	-434	282	-20	-72	336	360	-682	-812	-810	124	1136	156	1038	-1208

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1575/2	35/1	1	0	0	-7	-12	78	-94	40	32	50	-248	434	-402	68	536	22	560	-278	164	-672	-82	-1000	-448	870	-1026
1575/3	7/1	-1	0	0	7	8	-28	54	-110	48	110	12	246	-182	-128	324	-162	-810	-488	-244	768	702	440	-1302	-730	-294
1575/4	525/2	2	0	0	-7	21	24	22	16	25	-167	10	-133	168	-97	400	182	-488	28	-967	285	-838	-469	406	-324	-114
1575/5	525/2	-2	0	0	7	21	-24	-22	16	-25	-167	10	133	168	97	-400	-182	-488	28	967	285	838	-469	-406	-324	114
1575/6	525/4	3	0	0	7	6	41	-27	-4	-75	123	-205	-262	-57	407	60	-327	-33	-427	-628	-300	98	686	-1401	-714	494
1575/7	315/3	3	0	0	-7	-60	-38	-84	110	120	-162	236	376	126	34	-6	582	-492	-880	826	666	826	-592	792	-1002	-1442
1575/8	525/4	-3	0	0	-7	6	-41	27	-4	75	123	-205	262	-57	-407	-60	327	-33	-427	628	-300	-98	686	1401	-714	-494
1575/9	21/2	-3	0	0	-7	36	34	42	-124	0	-102	-160	-398	318	268	240	-498	132	398	-92	720	502	-1024	-204	-354	286
1575/10	315/3	-3	0	0	-7	60	-38	84	110	-120	162	236	376	-126	34	6	-582	492	-880	826	-666	826	-592	-792	1002	-1442
1575/11	21/1	4	0	0	7	-62	62	84	100	-42	10	-48	246	248	-68	324	258	-120	622	-904	678	642	740	468	-200	1266
1575/12	105/2	5	0	0	-7	-12	-30	-134	-92	112	58	-224	146	-18	-340	208	-754	-380	718	-412	960	-1066	896	436	1038	702
1584/1	132/1	0	0	0	-2	-11	-88	66	40	6	54	-8	-106	-354	124	546	408	552	404	4	126	-166	874	444	-1002	-802
1584/2	66/1	0	0	0	-14	11	80	-30	-56	-126	222	16	-106	-114	52	246	264	264	92	796	426	-1174	-842	852	1062	-1282
1584/3	22/3	0	0	3	10	11	-16	-42	-116	189	120	163	-409	-468	-110	144	-90	-453	20	97	-465	848	742	438	273	761
1584/4	33/1	0	0	4	26	11	-32	-74	60	-182	90	8	-66	-422	-408	-506	-348	-200	132	1036	762	-542	550	-132	-570	14
1584/5	264/2	0	0	6	8	-11	-30	18	56	-100	-26	136	-178	-110	-288	116	398	196	-782	-292	180	-398	-56	548	-282	-142
1584/6	264/1	0	0	6	14	11	6	108	98	-32	8	40	50	8	486	40	-710	-604	322	476	216	502	862	592	-354	446
1584/7	88/2	0	0	7	6	-11	-40	78	-36	7	-8	-183	227	36	-322	-184	6	-99	164	695	-987	-248	242	-1494	905	-1031
1584/8	44/1	0	0	7	26	-11	52	-46	96	27	-16	293	-29	472	110	-224	-754	825	-548	123	1001	-1020	-526	-158	1217	-263
1584/9	198/1	0	0	8	22	-11	-54	26	38	-64	-294	-36	-390	-138	242	132	-388	-732	430	-520	420	-594	-506	380	256	418
1584/10	198/1	0	0	-8	22	11	-54	-26	38	64	294	-36	-390	138	242	-132	388	732	430	-520	-420	-594	-506	-380	-256	418
1584/11	88/1	0	0	-9	-2	-11	0	38	-44	175	264	-159	-173	220	542	-264	-682	421	308	-177	365	-528	-686	698	-967	-1127
1584/12	132/4	0	0	-10	-8	-11	18	-46	-40	44	-186	72	-114	-174	416	-156	62	-348	-446	956	-444	306	664	-124	-602	1522
1584/13	66/2	0	0	-10	-16	11	10	10	144	-84	-218	176	46	26	488	404	-194	444	202	84	-764	354	-1312	-1252	1222	-1358
1584/14	132/2	0	0	12	-14	11	56	-42	-116	-30	-198	88	350	-198	-56	-594	204	-312	620	-356	-462	482	238	492	-954	-1426
1584/15	396/1	0	0	12	-26	-11	-34	126	-110	180	-18	292	-238	426	-146	-528	408	-324	-550	-824	-552	-850	-866	660	768	-286
1584/16	264/3	0	0	-12	-22	11	-48	54	-100	58	-262	-248	-130	26	-216	22	-620	-424	340	620	810	-1118	214	988	6	590
1584/17	396/1	0	0	-12	-26	11	-34	-126	-110	-180	18	292	-238	-426	-146	528	-408	324	-550	-824	552	-850	-866	-660	-768	-286
1584/18	33/2	0	0	14	32	-11	-38	2	-72	68	54	152	174	-94	528	-340	438	20	570	460	-1092	562	16	372	966	-526
1584/19	22/1	0	0	-14	8	-11	-50	-130	108	-96	-142	-40	382	118	-220	520	-238	-852	190	12	-112	-6	-304	820	-202	-1406
1584/20	264/4	0	0	18	28	11	-18	34	-80	128	-162	312	-290	146	-256	432	490	836	230	-900	520	-798	484	-812	-74	-1790
1584/21	22/2	0	0	19	-14	11	-72	46	20	-107	-120	-117	-201	228	242	-96	-458	435	-668	-439	-1113	-72	70	358	-895	409
1584/22	132/3	0	0	-22	20	11	22	-110	-48	72	142	-184	-194	482	80	392	34	-108	382	-84	-1040	-606	1292	356	406	1090
1590/1		2	3	-5	28	-24	10	126	-116	72	26	160	58	-254	-148	360	-53	-264	622	212	600	-710	784	212	-382	-414
1590/2		2	-3	-5	16	46	42	28	-94	20	216	156	-226	-354	60	390	53	286	376	-900	-552	100	-48	-1188	510	570
1590/3		2	-3	-5	32	-66	74	-36	50	84	72	44	-130	30	44	-426	53	558	-232	92	-312	212	1088	348	-1410	602
1590/4		-2	-3	5	-6	-44	-16	-12	28	184	58	-22	-16	168	256	-456	-53	-172	124	556	604	-550	-686	1296	-950	-646
1596/1		0	-3	-8	7	22	-8	-120	19	190	-246	340	194	-314	-320	600	-230	284	398	-546	276	226	-58	704	-426	948
1600/1	32/1	0	0	0	0	0	-18	94	0	0	130	0	214	-230	0	0	518	0	-830	0	0	-1098	0	0	-1670	-594
1600/2	800/2	0	0	0	0	0	92	104	0	0	-130	0	-396	230	0	0	-572	0	830	0	0	592	0	0	1670	1816
1600/3	800/2	0	0	0	0	0	-92	-104	0	0	-130	0	396	230	0	0	572	0	830	0	0	-592	0	0	1670	-1816
1600/4	200/1	0	1	0	6	19	12	75	91	-174	272	-230	-182	117	372	52	-402	-312	-170	763	-52	981	1054	351	799	-962
1600/5	200/1	0	1	0	6	-19	-12	-75	-91	-174	272	230	182	117	372	52	402	312	-170	763	52	-981	-1054	351	799	962
1600/6	100/1	0	1	0	-26	45	-44	117	-91	18	-144	-26	214	-459	-460	468	-558	-72	118	251	-108	299	898	927	351	386
1600/7	100/1	0	1	0	-26	-45	44	-117	91	18	-144	26	-214	-459	-460	468	558	72	118	251	108	-299	-898	927	351	-386
1600/8	200/1	0	-1	0	-6	19	-12	-75	91	174	272	-230	182	117	-372	-52	402	-312	-170	-763	-52	-981	1054	-351	799	962
1600/9	200/1	0	-1	0	-6	-19	12	75	-91	174	272	230	-182	117	-372	-52	-402	312	-170	-763	52	981	-1054	-351	799	-962
1600/10	100/1	0	-1	0	26	45	44	-117	-91	-18	-144	-26	-214	-459	460	-468	558	-72	118	-251	-108	-299	898	-927	351	-386
1600/11	100/1	0	-1	0	26	-45	-44	117	91	-18	-144	26	214	-459	460	-468	-558	72	118	-251	108	299	-898	-927	351	386
1600/12	160/1	0	2	0	6	60	50	30	40	178	-166	-20	10	-250	-142	214	490	-800	-250	774	-100	230	1320	-982	874	310
1600/13	5/1	0	2	0	-6	-32	-38	-26	-100	78	50	-108	266	22	442	514	2	-500	518	126	412	878	600	282	-150	-386
1600/14	50/3	0	2	0	-26	28	12	64	60	58	-90	-128	236	242	362	-226	-108	20	-542	-434	-1128	-632	-720	-478	-490	-1456
1600/15	50/3	0	2	0	-26	-28	-12	-64	-60	58	-90	128	-236	242	362	-226	108	-20	-542	-434	1128	632	720	-478	-490	1456
1600/16	5/1	0	-2	0	6	32	-38	-26	100	-78	50	108	266	22	-442	-514	2	500	518	-126	-412	878	-600	-282	-150	-386
1600/17	160/1	0	-2	0	-6	-60	50	30	-40	-178	-166	20	10	-250	142	-214	490	800	-250	-774	100	230	-1320	982	874	310
1600/18	50/3	0	-2	0	26	28	-12	-64	60	-58	-90	-128	-236	242	-362	226	108	20	-542	434	-1128	632	-720	478	-490	1456
1600/19	50/3	0	-2	0	26	-28	12	64	-60	-58	-90	128	236	242	-362	226	-108	-20	-542	434	1128	-632	720	478	-490	-1456
1600/20	20/1	0	4	0	16	60	86	-18	-44	-48	186	176	254	186	-100	-168	-498	252	58	-1036	168	-506	272	948	-1014	766
1600/21	40/3	0	4	0	-16	-36	-42	110	116	-16	-198	240	-258	442	-292	-392	142	348	570	692	168	134	784	564	1034	382
1600/22	8/1	0	4	0	24	-44	22	-50	44	-56	-198	160	-162	-198	-52	528	-242	-668	-550	-188	-728	-154	656	-236	714	478
1600/23	40/3	0	-4	0	16	36	-42	110	-116	16	-198	-240	-258	442	292	392	142	-348	570	-692	-168	134	-784	-564	1034	382
1600/24	20/1	0	-4	0	-16	-60	86	-18	44	48	186	-176	254	186	100	168	-498	-252	58	1036	-168	-506	-272	-948	-1014	766

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1600/25	8/1	0	-4	0	-24	44	22	-50	-44	56	-198	-160	-162	-198	52	-528	-242	668	-550	188	728	-154	-656	236	714	478
1600/26	200/2	0	5	0	-2	39	-84	-61	151	58	-192	18	138	229	-164	212	-578	-336	-858	-209	780	-403	230	-1293	-1369	382
1600/27	200/2	0	5	0	-2	-39	84	61	-151	58	-192	-18	-138	229	-164	212	578	336	-858	-209	-780	403	-230	-1293	-1369	-382
1600/28	800/6	0	5	0	10	15	-8	-21	-105	10	20	-230	54	-195	-300	480	-322	-560	730	255	-40	317	-830	75	-705	-1434
1600/29	800/6	0	5	0	10	-15	8	21	105	10	20	230	-54	-195	-300	480	322	560	730	255	40	-317	830	75	-705	1434
1600/30	200/2	0	-5	0	2	39	84	61	151	-58	-192	18	-138	229	164	-212	578	-336	-858	209	780	403	230	1293	-1369	-382
1600/31	200/2	0	-5	0	2	-39	-84	-61	-151	-58	-192	-18	138	229	164	-212	-578	336	-858	209	-780	-403	-230	1293	-1369	382
1600/32	800/6	0	-5	0	-10	15	8	21	-105	-10	20	-230	-54	-195	300	-480	322	-560	730	-255	-40	-317	-830	-75	-705	1434
1600/33	800/6	0	-5	0	-10	-15	-8	-21	105	-10	20	230	54	-195	300	-480	-322	560	730	-255	40	317	830	-75	-705	-1434
1600/34	40/2	0	6	0	-34	16	58	70	4	-134	242	-100	-438	-138	-178	22	162	-268	-250	-422	852	-306	456	-434	-726	-1378
1600/35	40/2	0	-6	0	34	-16	58	70	-4	134	242	100	-438	-138	178	-22	162	268	-250	422	-852	-306	-456	434	-726	-1378
1600/36	25/1	0	7	0	-6	43	-28	-91	35	-162	-160	42	-314	-203	92	-196	82	280	518	141	412	763	510	777	-945	-1246
1600/37	25/1	0	7	0	-6	-43	28	91	-35	-162	-160	-42	314	-203	92	-196	-82	-280	518	141	-412	-763	-510	777	-945	1246
1600/38	50/1	0	7	0	34	27	28	21	35	78	120	-182	-146	357	-148	84	-702	-840	238	461	708	-133	-650	-903	735	1106
1600/39	50/1	0	7	0	34	-27	-28	-21	-35	78	120	182	146	357	-148	84	702	840	238	461	-708	133	650	-903	735	-1106
1600/40	25/1	0	-7	0	6	43	28	91	35	162	-160	42	314	-203	-92	196	-82	280	518	-141	412	-763	510	-777	-945	1246
1600/41	25/1	0	-7	0	6	-43	-28	-91	-35	162	-160	-42	-314	-203	-92	196	82	-280	518	-141	-412	763	-510	-777	-945	-1246
1600/42	50/1	0	-7	0	-34	27	-28	-21	35	-78	120	-182	146	357	148	-84	702	-840	238	-461	708	133	-650	903	735	-1106
1600/43	50/1	0	-7	0	-34	-27	28	21	-35	-78	120	182	-146	357	148	-84	-702	840	238	-461	-708	-133	650	903	735	1106
1600/44	10/1	0	8	0	-4	12	-58	-66	-100	132	90	-152	-34	-438	-32	-204	222	420	-902	1024	-432	-362	160	-72	810	-1106
1600/45	32/2	0	8	0	-16	40	-50	30	-40	-48	34	320	310	410	152	416	-410	200	-30	776	400	630	-1120	552	-326	110
1600/46	10/1	0	-8	0	4	-12	-58	-66	100	-132	90	152	-34	-438	32	204	222	-420	-902	-1024	432	-362	-160	72	810	-1106
1600/47	32/2	0	-8	0	16	-40	-50	30	40	48	34	-320	310	410	-152	-416	-410	-200	-30	-776	-400	630	1120	-552	-326	110
1600/48	200/6	0	9	0	-26	59	28	-5	-109	194	32	10	-198	117	388	68	-18	-392	710	-253	-612	549	414	-121	-81	1502
1600/49	200/6	0	9	0	-26	-59	-28	5	109	194	32	-10	198	117	388	68	18	392	710	-253	612	-549	-414	-121	-81	-1502
1600/50	200/6	0	-9	0	26	59	-28	5	-109	-194	32	10	198	117	-388	-68	18	-392	710	253	-612	-549	414	121	-81	-1502
1600/51	200/6	0	-9	0	26	-59	28	-5	109	-194	32	-10	-198	117	-388	-68	-18	392	710	253	612	549	-414	121	-81	1502
1600/52	40/1	0	10	0	18	16	-6	6	124	-42	-142	-188	202	54	66	-38	738	-564	262	-554	140	-882	-1160	642	-854	478
1600/53	40/1	0	-10	0	-18	-16	-6	6	-124	42	-142	188	202	54	-66	38	738	564	262	554	-140	-882	1160	-642	-854	478
1608/1		0	3	-17	-19	-60	-28	-90	-70	-209	188	-161	-167	273	-435	496	419	29	-210	67	-108	-1077	-488	-841	-1404	-1004
1610/1		2	-1	5	7	45	-39	-107	4	-23	79	-298	140	-216	-50	-565	78	-338	-390	-296	792	182	-1245	724	126	1785
1610/2		-2	-5	5	7	-6	-21	-8	70	23	165	-329	-142	-125	296	-77	-338	-392	-298	890	291	577	-454	-36	524	-56
1610/3		-2	-5	5	7	-39	-43	69	92	23	-297	254	-340	216	-34	-33	366	510	-430	-1024	-72	38	-25	756	678	1649
1610/4		-2	-5	5	-7	-50	-33	98	-18	-23	285	69	-132	-87	138	-565	380	-180	700	76	-861	505	962	406	-530	-742
1610/5		-2	8	5	-7	-50	84	-58	-96	-23	-118	-126	-236	30	-148	-240	120	834	154	-496	-224	-1068	-442	380	1030	-846
1611/1	537/1	1	0	12	11	26	83	110	30	6	-186	130	-124	-63	-398	437	141	677	155	-780	-720	22	-407	367	1108	-1118
1616/1	202/1	0	2	3	30	-22	-51	-13	71	41	204	-97	-434	-240	440	-497	122	-590	-728	-862	-627	280	335	328	994	674
1616/2	202/2	0	8	18	13	12	-16	117	-143	-42	-9	16	440	-84	283	-354	-273	612	83	-974	1062	272	-50	1056	720	-1531
1617/1	33/1	-1	3	4	0	11	32	-74	60	-182	-90	8	-66	-422	408	506	348	200	-132	-1036	762	542	-550	132	-570	-14
1617/2	231/4	2	3	-11	0	11	5	118	105	-68	-195	-214	33	376	-168	-61	24	-625	558	173	168	-973	-1072	-1458	198	352
1617/3	231/3	-2	-3	-1	0	-11	-7	14	45	-88	-69	-22	57	380	48	385	-672	469	342	-139	132	-145	1244	-522	-822	-272
1617/4	231/1	3	-3	14	0	-11	-2	74	0	-148	26	-112	-98	10	208	-460	258	204	-178	-924	-748	230	-456	228	198	-562
1617/5	231/5	-3	3	4	0	11	-50	28	-30	112	130	146	-302	-4	-548	-86	-246	-120	638	-132	-692	152	768	-1098	1158	-1618
1617/6	231/2	5	-3	6	0	-11	-70	-126	80	-200	134	244	-314	-278	-372	84	182	756	-694	820	160	2	40	-760	102	862
1617/7	33/2	-5	-3	14	0	-11	38	2	-72	68	-54	152	174	-94	-528	340	-438	-20	-570	-460	-1092	-562	-16	-372	966	526
1620/1		0	0	5	-7	30	-22	-48	68	-111	-87	20	200	-69	-232	-243	498	-66	359	-1063	618	-532	410	-693	1599	50
1620/2	1620/1	0	0	-5	-7	-30	-22	48	68	111	87	20	200	69	-232	243	-498	66	359	-1063	-618	-532	410	693	-1599	50
1624/1		0	5	-15	-7	61	3	108	-60	-174	29	233	-226	-368	137	369	-193	-432	-290	838	344	48	1285	592	898	976
1629/1	181/1	3	0	18	20	33	20	102	-34	-3	216	-133	-376	-264	263	93	462	279	155	-52	-477	-943	992	-1122	-744	-1222
1630/1		-2	4	-5	-23	20	-27	123	-36	87	234	-88	-119	333	178	514	-244	30	-662	-463	23	-574	-952	-270	-348	-1142
1638/1	182/1	2	0	0	7	-39	13	-24	38	-39	96	227	425	105	344	-99	540	-114	-565	-385	156	-673	749	1044	690	317
1638/2		2	0	3	-7	23	-13	-133	103	-113	141	-90	-233	230	-337	-96	134	-838	295	468	-54	-553	-694	1010	-390	-102
1638/3	546/5	2	0	-12	7	22	-13	2	-88	80	22	-92	118	-324	84	134	194	-210	-470	-292	66	-506	-776	778	920	-490
1638/4	182/4	-2	0	-3	7	54	13	96	-151	-33	-183	-331	-88	42	353	465	-195	-552	470	254	-132	-943	-727	1197	-753	1037
1638/5	1638/2	-2	0	-3	-7	-23	-13	133	103	113	-141	-90	-233	-230	-337	96	-134	838	295	468	54	-553	-694	-1010	390	-102
1638/6	182/2	-2	0	5	-7	36	-13	-26	-47	99	61	-23	-50	-70	-19	-191	-195	-264	310	-190	166	873	-1191	-259	635	133
1638/7	546/4	-2	0	9	7	18	13	-60	-43	-9	249	-79	-412	-222	-295	-411	237	384	-466	-1042	288	-691	1001	-39	339	713
1638/8	546/1	-2	0	-9	-7	-62	-13	16	79	155	-51	243	412	406	-103	-429	169	-320	-614	258	264	-121	-967	679	-1059	-21
1638/9	546/2	-2	0	12	7	50	-13	58	-40	64	110	124	-50	-84	-12	82	442	618	-278	20	390	-2	-680	-322	-968	1022
1638/10	546/3	-2	0	14	-7	-8	13	98	-28	52	2	-168	-146	514	-236	216	66	84	446	292	-100	450	392	292	402	314
1638/11	182/3	-2	0	-16	7	15	-13	44	-138	-111	12	215	55	133	-180	-471	260	-110	-271	-799	-912	747	-883	924	-142	-1407
1640/1		0	-2	5	14	0	-44	116	-16	-96	-62	128	130	-41	-448	-306	-48	60	282	438	960	-806	-416	-1332	158	1032

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1645/1		5	4	-5	-7	-16	-38	-94	-124	204	30	-184	-210	150	112	47	494	388	-626	-544	-672	-186	848	364	1106	114
1650/1		2	3	0	1	11	-68	-99	-25	-63	150	122	-119	-303	-308	-189	-168	-435	-328	-344	267	952	515	-618	1380	1681
1650/2	330/10	2	3	0	-2	11	16	-96	-112	-180	-102	-208	-110	-90	10	180	618	-36	-286	928	48	520	-412	618	-234	-422
1650/3	330/9	2	3	0	-20	11	-26	-6	-28	48	-162	128	-86	66	-344	-312	-486	-84	494	-716	-432	-206	440	-192	-294	-1082
1650/4		2	3	0	30	-11	9	4	59	75	167	-135	-294	302	-31	-124	-272	-238	166	658	265	802	-44	27	-675	-591
1650/5	330/6	2	-3	0	6	11	40	-80	56	-44	178	-16	146	414	-158	44	-166	44	402	-744	1056	-1136	-468	-182	678	1082
1650/6	330/5	2	-3	0	6	-11	-48	52	-76	132	134	-192	-30	-334	-334	572	10	-220	-302	-392	-704	184	16	1446	-114	642
1650/7	66/1	2	-3	0	-14	11	-80	-30	56	126	-222	-16	106	114	52	-246	264	264	92	796	426	1174	842	-852	-1062	1282
1650/8	330/8	2	-3	0	16	11	50	70	-44	96	-122	184	-134	-86	12	264	194	-716	182	436	-104	134	-648	628	-102	-418
1650/9	330/7	2	-3	0	16	-11	-38	-18	44	-168	54	8	130	-174	-164	-528	-510	780	-82	-92	336	574	56	-1044	426	-1298
1650/10	330/3	-2	3	0	-2	-11	28	36	-64	-12	-126	-280	298	54	-62	444	-366	108	146	-848	48	628	-676	-342	-570	178
1650/11	66/2	-2	3	0	-16	11	-10	10	-144	84	218	-176	-46	-26	488	-404	-194	444	202	84	-764	-354	1312	1252	-1222	1358
1650/12	330/4	-2	3	0	24	11	30	110	56	144	-182	24	234	-26	68	-224	146	-116	-818	4	176	826	532	-1008	1098	-42
1650/13	1650/1	-2	-3	0	-1	11	68	99	-25	63	150	122	119	-303	308	189	168	-435	-328	344	267	-952	515	618	1380	-1681
1650/14	330/1	-2	-3	0	-10	-11	-44	-124	-56	-100	42	-120	-86	222	-54	-76	162	-68	-734	552	-320	-292	676	-422	-490	-174
1650/15	1650/4	-2	-3	0	-30	-11	-9	-4	59	-75	167	-135	294	302	31	124	272	-238	166	-658	265	-802	-44	-27	-675	591
1650/16	330/2	-2	-3	0	34	11	88	-36	-100	-12	-90	-208	-86	-438	-362	-516	-102	-420	-118	-416	-408	808	-160	18	-930	-1406
1656/1	184/1	0	0	4	-4	-26	70	-94	54	23	86	-144	-172	42	386	80	108	-164	-400	398	320	-810	-204	-102	-1018	-1370
1656/2	552/1	0	0	-8	-22	4	-14	116	30	23	38	60	-310	366	326	464	-348	-44	434	-406	-472	-222	-642	-756	728	-1370
1656/3	552/2	0	0	14	2	-58	-50	76	60	-23	106	-24	-256	126	-304	-32	642	-436	-460	-232	-224	-282	-426	-702	-764	-686
1656/4	184/2	0	0	-22	8	20	22	-98	-12	-23	10	192	-106	-186	332	544	-390	716	110	836	280	-486	288	-180	-650	-1262
1664/1		0	5	19	-7	-72	13	-119	66	-118	236	-40	135	30	107	-579	42	644	-278	-118	-553	-836	-944	-140	-1186	-892
1664/2	1664/1	0	5	-19	7	-72	-13	-119	66	118	-236	40	-135	30	107	579	-42	644	278	-118	553	-836	944	-140	-1186	-892
1664/3	1664/1	0	-5	19	7	72	13	-119	-66	118	236	40	135	30	-107	579	42	-644	-278	118	553	-836	944	140	-1186	-892
1664/4	1664/1	0	-5	-19	-7	72	-13	-119	-66	-118	-236	-40	-135	30	-107	-579	-42	-644	278	118	-553	-836	-944	140	-1186	-892
1665/1		1	0	5	-27	-41	-71	-83	-95	69	-232	-132	37	-358	52	52	-305	-254	730	84	-76	469	-210	-213	-951	-1012
1665/2	555/1	-1	0	-5	25	-3	22	-35	-118	-28	-83	153	-37	-73	155	136	-201	-198	497	700	-1044	1040	-828	1434	-1310	131
1665/3	555/2	-1	0	-5	25	57	-26	-107	-46	-124	157	-75	-37	443	-265	-512	459	570	-259	-428	1020	-232	-468	114	-134	-889
1665/4	1665/1	-1	0	-5	-27	41	-71	83	-95	-69	232	-132	37	358	52	-52	305	254	730	84	76	469	-210	213	951	-1012
1666/1	34/1	-2	2	-16	0	62	62	17	20	-12	80	208	-356	-22	-312	-24	-462	-240	-812	-216	732	-178	700	992	390	146
1666/2	34/2	-2	2	18	0	-6	-74	-17	88	-114	-90	310	86	-90	368	384	-258	-240	-302	-964	-390	-722	-898	-912	-1446	1438
1668/1		0	-3	-12	22	41	31	73	151	180	72	-110	-293	-416	187	277	-105	488	340	-674	644	278	686	384	-264	-1548
1671/1		-1	3	18	5	-30	-46	-46	106	-45	-247	-84	-77	331	-252	-24	-676	439	-39	-652	-962	-33	-11	-327	750	290
1672/1		0	-8	6	-4	11	-86	34	-19	56	106	48	-62	-458	76	240	-38	-368	634	400	-624	-286	-24	92	-502	-110
1674/1		2	0	-4	-13	-15	48	19	-29	27	91	31	296	-384	-426	460	-542	-210	84	-219	52	80	-286	-763	789	-1594
1674/2		2	0	18	-1	-27	-70	-57	-61	-21	-165	31	-178	-186	-232	-426	-414	144	386	-127	-420	1076	710	-195	1281	446
1674/3		2	0	-18	-29	-55	-58	53	-119	-95	-77	-31	-324	-338	26	242	78	-778	628	447	-738	-418	436	-467	-705	1122
1674/4	1674/1	-2	0	4	-13	15	48	-19	-29	-27	-91	31	296	384	-426	-460	542	210	84	-219	-52	80	-286	763	-789	-1594
1674/5	1674/3	-2	0	18	-29	55	-58	-53	-119	95	77	-31	-324	338	26	-242	-78	778	628	447	738	-418	436	467	705	1122
1674/6	1674/2	-2	0	-18	-1	27	-70	57	-61	21	165	31	-178	186	-232	426	414	-144	386	-127	420	1076	710	195	-1281	446
1680/1	210/8	0	3	5	7	-16	58	34	-64	16	62	-60	150	474	292	-240	-662	324	-514	372	412	-770	560	852	1466	-178
1680/2	210/2	0	3	5	7	44	54	98	60	144	-210	208	-226	-502	-484	232	-530	764	814	-60	-848	-958	152	-308	-1094	554
1680/3	105/2	0	3	5	-7	-12	30	-134	92	-112	-58	224	-146	18	-340	-208	-754	-380	718	-412	960	1066	-896	-436	-1038	-702
1680/4	210/3	0	3	5	-7	-12	-58	42	4	-24	294	-128	-58	282	-428	-384	-138	-468	-250	556	-624	-958	-632	-84	810	-790
1680/5	420/2	0	3	5	-7	36	-34	-6	28	-192	-186	-176	-418	-30	412	432	-306	564	-322	-716	48	-1078	496	-468	1314	-1438
1680/6	105/1	0	3	5	-7	-42	20	66	-38	-12	-258	-146	434	-282	-20	72	336	360	-682	-812	-810	-124	-1136	-156	-1038	1208
1680/7	840/5	0	3	-5	7	0	-54	74	20	160	-246	-84	306	-370	88	-460	686	684	186	904	-912	-26	320	-732	1150	1526
1680/8	210/10	0	3	-5	7	4	-42	-86	96	96	-78	-80	50	-26	32	20	-382	-356	-134	-888	-868	-70	-400	1052	-634	1202
1680/9	420/1	0	3	-5	7	-32	42	-38	36	-96	-198	220	-46	-290	152	-124	62	-68	-614	456	416	-826	272	-508	110	-874
1680/10	840/6	0	3	-5	7	58	4	-42	78	-72	102	90	-390	442	204	120	-300	104	302	-836	74	148	552	-1428	454	424
1680/11	210/1	0	3	-5	-7	-12	2	-18	-56	156	-186	52	-178	-138	412	456	-198	-348	110	196	936	542	-992	276	630	110
1680/12	210/9	0	3	-5	-7	-24	14	54	-44	-156	174	88	-34	-138	-164	216	318	204	-442	316	252	98	1000	-516	-522	-310
1680/13	840/2	0	-3	5	7	-20	54	82	116	-88	-186	128	190	250	-36	-384	-82	124	390	-524	-344	186	-272	388	714	-510
1680/14	840/3	0	-3	5	7	-22	-44	-110	22	36	-122	186	306	-330	-20	64	504	560	-418	452	146	-236	-536	92	-574	184
1680/15	210/4	0	-3	5	7	-28	54	-46	-12	0	6	-296	134	146	-556	448	46	-748	-50	156	1024	-310	-856	628	-590	-1390
1680/16	840/4	0	-3	5	7	44	22	66	132	168	54	-144	-354	-22	156	240	-354	76	-154	628	-8	1018	-96	-348	218	-1598
1680/17	420/4	0	-3	5	7	44	-42	-94	36	-24	54	112	-322	-22	-292	-272	-578	44	-26	-12	280	410	320	1252	-38	1250
1680/18	420/5	0	-3	5	-7	16	-14	130	-104	88	54	-28	-266	202	-348	-104	402	100	310	324	644	-290	-744	-1044	298	-290
1680/19	840/1	0	-3	5	-7	16	-62	-14	56	136	-154	116	6	-150	20	-152	-78	-124	166	-140	-204	-210	984	-628	138	-1202
1680/20	420/6	0	-3	5	-7	-36	38	-78	52	-120	54	-80	254	-6	172	0	-66	-420	-106	-92	-1176	698	1024	516	714	-862
1680/21	210/6	0	-3	-5	7	-28	-86	-66	48	-140	-34	284	-346	-274	4	448	-94	-308	510	156	-336	-1170	-16	-772	1630	110
1680/22	210/7	0	-3	-5	7	-56	54	94	-36	84	-258	40	-178	-146	-148	200	-130	-188	94	444	-532	770	536	1076	-1090	1274

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1680/23	210/5	0	-3	-5	-7	0	26	18	-92	0	-6	4	410	174	-248	-420	102	588	650	-152	168	-610	1048	684	-834	110
1680/24	420/3	0	-3	-5	-7	36	-34	-30	16	48	-126	-8	74	-138	352	396	-78	60	-70	664	-156	410	-344	-444	-1002	290
1682/1	58/1	2	-7	5	-2	-37	27	-24	88	-28	0	143	360	-386	-381	103	-431	288	840	-180	706	-716	-931	1188	642	-486
1682/2	58/2	-2	7	-15	-18	-27	-57	44	-152	-152	0	173	120	314	-339	357	-59	-572	420	660	726	-1004	-361	-168	-58	1206
1683/1	187/1	-1	0	-6	-24	11	-58	-17	28	24	-222	-112	-394	-410	-204	-240	386	-564	-530	108	392	-406	1008	-236	-1354	-1774
1683/2	561/1	-1	0	-10	12	-11	-26	-17	-28	28	6	68	354	86	308	128	-414	-860	850	340	996	218	828	-468	-1322	-230
1683/3	561/2	3	0	-18	8	11	-58	-17	-88	48	-30	-112	-34	-450	-88	156	546	264	362	572	648	254	-952	828	-498	-46
1690/1		2	2	-5	24	60	0	54	-84	-132	150	150	354	240	178	324	-48	-696	-758	156	210	-258	880	-1332	684	66
1690/2		2	-2	-5	5	33	0	-42	-25	72	192	-304	-133	30	-34	327	327	-144	-580	-832	354	-670	-676	162	849	1322
1690/3	130/1	2	-2	-5	-8	-6	0	114	-38	150	114	34	-146	30	122	-336	-570	-66	-502	-728	-582	994	-988	84	-906	-290
1690/4		2	4	-5	-23	57	0	48	-5	-180	120	34	-227	-6	-214	-549	-537	300	-322	772	894	-272	-454	1068	-681	-1100
1690/5		2	5	-5	19	-23	0	7	-137	-187	-277	-80	-189	163	463	-492	138	759	-209	673	291	702	-620	-300	-481	307
1690/6		2	-5	-5	-7	-51	0	81	155	-57	171	224	281	-309	-499	-336	-162	-321	-613	383	747	-502	1184	-480	-789	29
1690/7	1690/1	-2	2	5	-24	-60	0	54	84	-132	150	-150	-354	-240	178	-324	-48	696	-758	-156	-210	258	880	1332	-684	-66
1690/8	1690/2	-2	-2	5	-5	-33	0	-42	25	72	192	304	133	-30	-34	-327	327	144	-580	832	-354	670	-676	-162	-849	-1322
1690/9	1690/4	-2	4	5	23	-57	0	48	5	-180	120	-34	227	6	-214	549	-537	-300	-322	-772	-894	272	-454	-1068	681	1100
1690/10	130/2	-2	-4	5	8	32	0	-86	56	68	-202	56	-66	-490	460	24	-294	480	-338	-676	-120	210	184	660	286	1202
1690/11	1690/5	-2	5	5	-19	23	0	7	137	-187	-277	80	189	-163	463	492	138	-759	-209	-673	-291	-702	-620	300	481	-307
1690/12	1690/6	-2	-5	5	7	51	0	81	-155	-57	171	-224	-281	309	-499	336	-162	321	-613	-383	-747	502	1184	480	789	-29
1690/13	10/1	-2	-8	-5	4	-12	0	66	100	132	-90	-152	34	438	32	204	222	-420	902	1024	-432	-362	-160	-72	-810	-1106
1692/1	564/1	0	0	12	36	16	-58	38	106	-148	260	70	2	132	-30	-47	-194	-276	794	614	-872	382	524	-596	-494	46
1692/2	564/2	0	0	21	3	-55	-4	-56	-2	43	-131	-308	125	-6	-552	47	62	-594	-550	908	362	-968	-229	-1066	1346	805
1694/1	154/2	2	0	2	7	0	-26	46	48	-128	146	-128	-26	-10	-52	-544	318	-48	-466	516	-392	-754	0	-624	-1590	1018
1694/2	154/1	2	-5	-1	7	0	8	-22	-54	213	-190	163	31	-110	-4	-80	-566	645	-634	-729	431	918	254	-904	901	-89
1694/3	14/1	2	8	-14	7	0	-18	-74	-80	-112	-190	72	-346	-162	412	24	318	-200	198	-716	392	-538	-240	1072	810	1354
1694/4	14/2	-2	-2	-12	-7	0	-56	114	-2	-120	54	236	146	-126	376	-12	174	138	-380	-484	576	1150	-776	-378	-390	-1330
1694/5	154/3	-2	-2	18	-7	0	-56	-36	28	180	54	-334	386	444	316	-402	-486	-282	-380	176	-324	-800	1144	-468	-870	-1330
1694/6	154/4	-2	7	3	-7	0	16	-6	-14	-51	-54	95	-193	-102	-284	-72	-102	-63	790	-433	135	238	-770	1008	-639	11
1694/7	154/5	-2	-10	-14	-7	0	16	-108	-116	68	-122	-262	130	-204	396	166	442	702	-196	-416	492	-408	-600	1212	1146	-482
1700/1	68/1	0	2	0	12	-10	38	17	4	-120	56	164	236	70	144	-48	366	-504	-460	768	72	734	736	-856	906	-46
1700/2	340/1	0	-2	0	-2	-30	62	-17	-56	110	206	114	194	-430	-4	68	-206	496	-290	-8	-798	-314	366	1276	86	1006
1700/3	340/2	0	5	0	-2	12	13	-17	35	-30	-249	-229	124	-66	262	75	543	-225	-535	-386	231	547	-376	-768	-537	-1367
1702/1		2	3	7	-22	20	10	49	79	-23	290	30	37	186	193	189	-424	60	-137	-292	903	342	504	-448	561	1626
1705/1		-3	4	5	8	-11	38	-102	104	144	-102	31	-238	42	236	468	402	492	-70	-136	168	-46	-376	-1140	1314	1082
1705/2		5	4	-5	-11	-11	-52	-25	-10	-19	-8	31	-337	82	-121	464	50	741	-866	87	490	-922	-1142	1211	282	129
1710/1	570/6	2	0	-5	-2	16	-10	-36	19	-124	174	-74	94	240	-276	-540	-146	-606	450	180	456	14	550	-1442	-212	-830
1710/2	190/2	2	0	-5	8	-44	0	74	19	-84	-266	136	424	-470	-236	240	-36	-736	650	-830	216	254	-1220	688	-102	-1280
1710/3	190/1	2	0	-5	-12	20	-4	34	-19	-40	150	-200	-156	218	248	180	-72	48	-134	334	520	438	980	156	-670	1124
1710/4	570/7	2	0	-5	26	-54	32	-78	19	-12	-204	-256	-340	156	326	132	-90	360	-838	-16	-888	854	-640	84	-828	1424
1710/5	570/2	-2	0	5	-8	20	-82	18	19	88	186	-248	262	-246	288	168	302	-72	-546	-804	-240	602	-800	116	-766	790
1710/6	570/3	-2	0	5	-24	-32	2	-106	-19	-152	-90	52	306	-62	-268	-456	318	-300	502	-644	608	-198	260	1248	-110	-574
1710/7	570/5	-2	0	5	-34	-28	-6	-8	19	204	-262	298	346	296	340	204	-462	-194	-46	-20	-1080	-922	-1382	-10	-180	514
1710/8	570/4	-2	0	-5	-4	12	-46	102	19	84	-222	8	-214	126	-160	-36	318	516	-346	-700	480	338	248	-720	30	614
1710/9	190/3	-2	0	-5	-20	44	42	86	19	164	162	-312	226	-34	-432	-580	-506	-364	518	924	-320	-542	-1208	1120	1022	1166
1710/10	570/1	-2	0	-5	-34	18	-48	34	-19	128	80	112	-124	208	42	144	378	-440	-118	496	-72	-738	920	-832	-440	-864
1716/1		0	3	18	28	11	13	46	-36	-28	22	-96	354	-170	-260	-96	26	772	710	-624	-680	-794	240	740	-1170	-246
1716/2		0	-3	2	0	11	13	0	18	-58	-222	94	56	338	-280	232	636	-508	802	104	140	-736	448	-964	-1146	170
1716/3		0	-3	2	-36	11	13	-54	72	176	-42	-68	326	86	260	-56	-354	716	-458	-328	-184	722	-560	-748	-1434	26
1716/4		0	-3	-9	-25	-11	13	-120	50	-165	-207	152	-70	-321	-367	-408	42	-87	-469	167	-558	-697	-406	-726	1272	-766
1716/5		0	-3	10	-12	-11	13	114	144	-80	78	-260	38	-26	92	-208	222	436	550	8	208	386	-440	28	-258	170
1722/1		2	3	7	-7	-48	23	-107	-19	66	-216	-147	-152	41	-38	56	68	-17	400	-1019	459	-897	192	-1289	1345	-1240
1722/2		2	-3	-9	7	36	-7	-21	119	-18	48	-295	344	-41	-106	384	-372	-141	-628	767	225	-529	488	795	687	200
1725/1	345/3	0	3	0	16	-48	46	30	-46	23	30	116	-68	54	-380	-420	642	186	-34	124	1026	646	-610	612	642	-476
1725/2	345/4	-1	-3	0	-16	52	38	54	40	-23	170	232	-386	482	-132	144	-82	100	-398	124	-428	78	-960	1488	470	-1126
1725/3		3	3	0	3	16	-6	-93	22	-23	249	118	-333	104	-34	-611	-404	-846	-590	-574	-305	277	864	-421	-91	782
1725/4	345/1	3	-3	0	-26	54	-2	72	68	23	-102	-16	-344	162	280	360	-114	-768	704	-560	408	-998	-550	-966	-804	310
1725/5	1725/3	-3	-3	0	-3	16	6	93	22	23	249	118	333	104	34	611	404	-846	-590	574	-305	-277	864	421	-91	-782
1725/6		4	-3	0	25	-47	13	-12	43	23	221	160	74	293	-141	324	-212	652	82	-868	-200	381	-207	981	-1288	982
1725/7	1725/6	-4	3	0	-25	-47	-13	12	43	-23	221	160	-74	293	141	-324	212	652	82	868	-200	-381	-207	-981	-1288	-982
1725/8		5	3	0	11	67	-19	30	89	23	-285	-184	-228	489	95	-50	-88	-734	-484	964	366	1071	1085	-183	-618	824
1725/9	345/2	5	-3	0	6	46	50	116	-152	-23	206	-120	28	-118	-292	344	326	748	-584	684	152	-118	870	1278	228	790
1725/10		5	-3	0	29	37	89	-30	19	-23	-205	316	-92	-331	-195	-510	-172	106	916	-284	586	879	-1005	-57	-1498	1076

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1725/11	1725/10	-5	3	0	-29	37	-89	30	19	23	-205	316	92	-331	195	510	172	106	916	284	586	-879	-1005	57	-1498	-1076
1725/12	1725/8	-5	-3	0	-11	67	19	-30	89	-23	-285	-184	228	489	-95	50	88	-734	-484	-964	366	-1071	1085	183	-618	-824
1727/1		0	-2	8	-1	-11	-55	-101	-80	161	-104	-98	-335	-162	201	-22	238	741	-146	-86	260	-1082	-172	672	-1639	812
1728/1	108/3	0	0	0	17	0	-89	0	-107	0	0	308	433	0	520	0	0	0	901	-1007	0	-271	503	0	0	1853
1728/2	108/3	0	0	0	-17	0	-89	0	107	0	0	-308	433	0	-520	0	0	0	901	1007	0	-271	-503	0	0	1853
1728/3	108/1	0	0	0	37	0	19	0	-163	0	0	-308	-323	0	-520	0	0	0	-719	-127	0	-919	1387	0	0	-523
1728/4	108/1	0	0	0	-37	0	19	0	163	0	0	308	-323	0	520	0	0	0	-719	127	0	-919	-1387	0	0	-523
1728/5	216/2	0	0	1	9	17	44	-56	-94	-50	-30	139	174	-318	-242	-630	547	236	-328	614	296	433	56	1225	-1506	1391
1728/6	216/2	0	0	1	-9	-17	44	-56	94	50	-30	-139	174	-318	242	630	547	-236	-328	-614	-296	433	-56	-1225	-1506	1391
1728/7	216/2	0	0	-1	9	-17	44	56	-94	50	30	139	174	318	-242	630	-547	-236	-328	614	-296	433	56	-1225	1506	1391
1728/8	216/2	0	0	-1	-9	17	44	56	94	-50	30	-139	174	318	242	-630	-547	236	-328	-614	296	433	-56	1225	1506	1391
1728/9	54/1	0	0	3	29	-57	-20	72	106	-174	-210	47	-2	6	-218	-474	81	84	-56	142	-360	-1159	-160	735	954	191
1728/10	54/1	0	0	3	-29	57	-20	72	-106	174	-210	-47	-2	6	218	474	81	-84	-56	-142	360	-1159	160	-735	954	191
1728/11	54/1	0	0	-3	29	57	-20	-72	106	174	210	47	-2	-6	-218	474	-81	-84	-56	142	360	-1159	-160	-735	-954	191
1728/12	54/1	0	0	-3	-29	-57	-20	-72	-106	-174	210	-47	-2	-6	218	-474	-81	84	-56	-142	-360	-1159	160	735	-954	191
1728/13	216/1	0	0	4	3	28	11	-44	-29	-172	192	116	69	-384	-328	-156	-392	412	425	-257	1000	-359	877	-328	1572	-1483
1728/14	216/1	0	0	4	-3	-28	11	-44	29	172	192	-116	69	-384	328	156	-392	-412	425	257	-1000	-359	-877	328	1572	-1483
1728/15	216/1	0	0	-4	3	-28	11	44	-29	172	-192	116	69	384	-328	156	392	-412	425	-257	-1000	-359	877	328	-1572	-1483
1728/16	216/1	0	0	-4	-3	28	11	44	29	-172	-192	-116	69	384	328	-156	392	412	425	257	1000	-359	-877	-328	-1572	-1483
1728/17	108/2	0	0	9	1	-63	28	-72	98	126	-126	259	-386	450	-34	-54	-693	-180	280	-586	504	161	-440	-999	-882	-721
1728/18	108/2	0	0	9	-1	63	28	-72	-98	-126	-126	-259	-386	450	34	54	-693	180	280	586	-504	161	440	999	-882	-721
1728/19	108/2	0	0	-9	1	63	28	72	98	-126	126	259	-386	-450	-34	54	693	180	280	-586	-504	161	-440	999	882	-721
1728/20	108/2	0	0	-9	-1	-63	28	72	-98	126	126	-259	-386	-450	34	-54	693	-180	280	586	504	161	440	-999	882	-721
1728/21	54/2	0	0	12	7	-60	79	108	11	-132	96	-20	169	-192	488	204	360	-156	-83	47	216	-511	529	1128	-36	605
1728/22	54/2	0	0	12	-7	60	79	108	-11	132	96	20	169	-192	-488	-204	360	156	-83	-47	-216	-511	-529	-1128	-36	605
1728/23	54/2	0	0	-12	7	60	79	-108	11	132	-96	-20	169	192	488	-204	-360	156	-83	47	-216	-511	529	-1128	36	605
1728/24	54/2	0	0	-12	-7	-60	79	-108	-11	-132	-96	20	169	192	-488	204	-360	-156	-83	-47	216	-511	-529	1128	36	605
1728/25	27/1	0	0	15	25	15	-20	-72	2	114	30	-101	430	30	110	-330	621	660	376	-250	-360	785	-488	-489	450	-1105
1728/26	27/1	0	0	15	-25	-15	-20	-72	-2	-114	30	101	430	30	-110	330	621	-660	376	250	360	785	488	489	450	-1105
1728/27	27/1	0	0	-15	25	-15	-20	72	2	-114	-30	-101	430	-30	110	330	-621	-660	376	-250	360	785	-488	489	-450	-1105
1728/28	27/1	0	0	-15	-25	15	-20	72	-2	114	-30	101	430	-30	-110	-330	-621	660	376	250	-360	785	488	-489	-450	-1105
1728/29	864/1	0	0	19	13	-65	56	-108	-58	-66	118	-145	-190	-430	-530	-74	-295	-628	-360	-146	388	753	1136	-153	850	391
1728/30	864/1	0	0	19	-13	65	56	-108	58	66	118	145	-190	-430	530	74	-295	628	-360	146	-388	753	-1136	153	850	391
1728/31	864/1	0	0	-19	13	65	56	108	-58	66	-118	-145	-190	430	-530	74	295	628	-360	-146	-388	753	1136	153	-850	391
1728/32	864/1	0	0	-19	-13	-65	56	108	58	-66	-118	145	-190	430	530	-74	295	-628	-360	146	388	753	-1136	-153	-850	391
1729/1		-1	-2	-19	-7	-43	-13	-86	19	-92	100	47	-141	-28	478	254	-512	-135	2	664	-138	-142	-155	378	-110	854
1729/2		-2	4	16	-7	46	-13	-49	19	-115	-88	233	-261	-479	-107	376	-460	-9	101	7	48	-1138	124	588	-970	-1705
1734/1	102/4	2	3	-5	-12	-37	19	0	37	3	86	142	296	121	3	402	174	270	520	-780	-84	302	-178	698	1512	500
1734/2	102/3	2	3	12	22	48	2	0	20	54	-84	-62	-44	138	428	-516	174	-852	-908	-508	426	574	-110	-1308	798	1690
1734/3	102/1	-2	3	3	-20	51	-61	0	-43	219	150	-290	-56	-15	83	426	-378	-210	448	-124	-900	1078	-722	-78	-144	268
1734/4	6/1	-2	3	-6	16	-12	38	0	20	-168	-30	88	-254	-42	-52	-96	198	-660	538	884	-792	-218	520	-492	810	-1154
1734/5		-2	3	-12	-11	-18	-7	0	59	168	-222	127	265	462	-19	-264	60	-438	457	23	36	358	412	-228	-426	-1685
1734/6	102/2	-2	-3	5	32	-27	-69	0	-83	117	-94	-198	244	-169	227	-382	686	450	700	540	276	298	182	282	-1468	1140
1734/7	1734/5	-2	-3	12	11	18	-7	0	59	-168	222	-127	-265	-462	-19	-264	60	-438	-457	23	-36	-358	-412	-228	-426	1685
1740/1		0	3	5	16	40	-60	-122	-50	-42	-29	-156	-146	-472	-56	-248	-238	-356	-320	458	272	-906	1196	720	-120	1458
1740/2		0	-3	5	-8	28	60	-62	-26	102	-29	240	-14	260	-368	184	-46	4	52	146	-304	-318	584	-672	-1308	966
1743/1		-3	3	9	7	9	2	36	-43	33	-120	-130	-337	30	-76	-252	9	-549	-685	-481	-228	-1132	116	83	855	1454
1755/1		1	0	-5	20	-50	-13	112	-39	108	-197	146	-407	327	-38	341	574	-576	326	-796	363	696	57	-860	-1085	-1304
1755/2		1	0	-5	-31	25	-13	124	-60	69	67	-97	-11	462	-227	-256	253	123	-664	812	672	-1176	-336	493	-281	796
1755/3	1755/1	-1	0	5	20	50	-13	-112	-39	-108	197	146	-407	-327	-38	-341	-574	576	326	-796	-363	696	57	860	1085	-1304
1755/4	1755/2	-1	0	5	-31	-25	-13	-124	-60	-69	-67	-97	-11	-462	-227	256	-253	-123	-664	812	-672	-1176	-336	-493	281	796
1755/5		3	0	-5	11	48	13	-63	20	33	-78	-214	344	321	47	-114	576	276	-700	-34	225	839	227	-789	1305	1262
1755/6		3	0	-5	-25	-72	13	-123	-148	93	258	-46	8	297	167	-186	216	-708	476	-322	-399	-85	-373	-333	-1431	-1234
1755/7	1755/5	-3	0	5	11	-48	13	63	20	-33	78	-214	344	-321	47	114	-576	-276	-700	-34	-225	839	227	789	-1305	1262
1755/8	1755/6	-3	0	5	-25	72	13	123	-148	-93	-258	-46	8	-297	167	186	-216	708	476	-322	399	-85	-373	333	1431	-1234
1760/1		0	4	5	0	-11	-38	82	44	44	234	-4	-346	98	-292	364	406	716	418	140	1172	1042	-1264	92	250	114
1760/2	1760/1	0	-4	5	0	11	-38	82	-44	-44	234	4	-346	98	292	-364	406	-716	418	-140	-1172	1042	1264	-92	250	114
1760/3		0	5	-5	-25	11	54	1	119	-50	-69	289	-355	294	-328	-446	73	198	-263	-424	523	774	-962	534	-479	-592
1760/4	1760/3	0	-5	-5	25	-11	54	1	-119	50	-69	-289	-355	294	328	446	73	-198	-263	424	-523	774	962	-534	-479	-592
1764/1		0	0	0	0	0	19	0	-107	0	0	289	323	0	71	0	0	0	-182	-127	0	271	-1387	0	0	1330
1764/2	1764/1	0	0	0	0	0	-19	0	107	0	0	-289	323	0	71	0	0	0	182	-127	0	-271	-1387	0	0	-1330
1764/3		0	0	0	0	0	89	0	-163	0	0	-19	-433	0	449	0	0	0	182	1007	0	-919	503	0	0	-1330

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1764/4	1764/3	0	0	0	0	0	-89	0	163	0	0	19	-433	0	449	0	0	0	-182	1007	0	919	503	0	0	1330
1764/5	588/1	0	0	4	0	20	4	24	-44	-72	38	-184	-30	-216	-164	520	146	460	-628	556	-592	-1024	-104	-324	896	920
1764/6	588/1	0	0	-4	0	20	-4	-24	44	-72	38	184	-30	216	-164	-520	146	-460	628	556	-592	1024	-104	324	-896	-920
1764/7	28/2	0	0	6	0	12	82	-30	-68	-216	-246	112	110	-246	-172	192	-558	540	-110	140	840	550	-208	516	-1398	-1586
1764/8	84/1	0	0	6	0	-36	-62	114	76	24	-54	112	-178	378	-172	-192	402	396	-254	-1012	-840	-890	80	-108	-1638	-1010
1764/9	28/1	0	0	-8	0	40	12	-58	-26	64	62	-252	26	6	416	-396	450	274	576	-476	448	158	-936	530	-390	-214
1764/10	84/2	0	0	14	0	-4	-54	-14	-92	152	106	144	158	-390	-508	-528	-606	-364	-678	844	8	422	384	-548	1194	1502
1764/11	12/1	0	0	-18	0	-36	10	18	100	-72	234	16	-226	90	452	432	-414	-684	-422	332	360	-26	512	-1188	-630	1054
1764/12	196/1	0	0	20	0	-44	-44	-72	100	120	-218	-280	-30	-120	220	-88	-110	-580	380	-980	112	-640	-488	-660	-320	248
1764/13	196/1	0	0	-20	0	-44	44	72	-100	120	-218	280	-30	120	220	88	-110	580	-380	-980	112	640	-488	660	320	-248
1770/1		2	3	5	0	-28	-62	-30	20	-44	-210	-196	106	-342	292	-188	398	-59	-18	-484	56	38	-920	-1304	1294	-682
1770/2		2	3	5	-24	17	-53	-39	25	-38	230	-108	-429	42	7	-204	-273	-59	-538	-279	112	-933	-735	2	415	946
1770/3		2	3	-5	11	-23	37	-119	-150	72	-70	-158	-219	37	237	-384	52	59	402	-1004	-23	-78	795	-233	-1220	1296
1770/4		2	-3	-5	10	-41	17	37	-11	-62	0	32	-91	326	-143	-28	-93	-59	-660	303	656	-739	593	878	-1119	-750
1770/5		2	-3	-5	-16	21	5	-21	-7	18	66	152	-115	-198	233	-168	621	-59	-574	-313	-840	1061	-391	-162	27	1478
1770/6		-2	3	5	-8	-52	74	50	-164	68	-90	-40	122	-230	400	172	-162	59	222	-96	-944	182	-616	-464	-150	-1050
1770/7		-2	3	5	12	45	11	-83	-111	130	-174	268	-61	474	383	-420	707	-59	410	457	160	-429	-863	-1494	1467	1554
1770/8		-2	3	5	-17	38	29	21	22	105	-64	195	386	-298	-328	-87	-513	-59	308	-350	-111	641	-940	1443	-559	110
1770/9		-2	-3	5	-15	54	-13	129	-114	209	48	23	-146	202	160	-119	127	59	-740	94	-277	963	436	1167	-481	-1006
1771/1		-1	-4	6	-7	-11	20	48	2	-23	170	-38	-418	-470	368	118	218	-420	342	-568	-12	258	-736	286	-1520	376
1775/1	71/1	-1	-1	0	1	24	-7	-72	-153	213	232	149	204	-432	-71	-273	274	126	-134	760	71	457	112	124	837	1424
1776/1	222/2	0	3	0	16	-48	50	60	-20	-162	-264	-332	37	330	-368	504	354	-222	-322	532	888	-922	-1328	696	-1488	806
1776/2	222/3	0	3	-2	0	-28	-42	90	28	48	-42	152	37	-342	500	224	-426	-628	262	60	-504	-1190	-552	-4	-110	-846
1776/3	111/1	0	3	-4	1	13	73	99	105	-133	300	-62	-37	-198	-68	-354	-7	-220	322	706	-672	893	-910	-243	995	1234
1776/4	444/1	0	3	-4	25	-67	57	27	17	107	-4	274	-37	-342	-52	-82	17	420	610	-110	960	205	1330	-51	-533	178
1776/5	111/3	0	-3	2	28	-20	10	-78	150	-82	-222	154	-37	-306	-386	-12	-46	-658	250	748	-324	-130	230	-216	-118	898
1776/6	111/2	0	-3	-8	13	35	-35	-3	-15	-47	-12	94	-37	54	244	-282	619	-8	250	478	96	-955	410	579	37	-2
1776/7	222/1	0	-3	-16	24	-8	-78	12	16	198	-72	-280	37	-30	-244	-56	-654	-38	526	516	552	-842	-588	-368	1136	726
1782/1		2	0	7	-8	11	58	-124	-6	-132	-286	-317	-149	-74	322	611	425	531	268	-513	-629	-48	112	-88	-1534	1501
1782/2	1782/1	-2	0	-7	-8	-11	58	124	-6	132	286	-317	-149	74	322	-611	-425	-531	268	-513	629	-48	112	88	1534	1501
1779/1		-1	-3	-10	-19	-20	9	83	-65	84	-88	-282	-306	-470	-332	169	-102	-564	-725	874	-470	259	92	849	-924	479
1780/1		0	2	5	30	-60	-42	38	150	-98	-154	-62	358	-398	-22	-100	-198	-234	-354	316	-708	-54	-1172	-462	89	1342
1785/1		1	3	5	7	-40	-70	17	-64	-104	34	88	338	426	100	128	-546	312	-74	36	704	586	568	-432	562	-622
1785/2		1	3	5	-7	-12	-42	17	76	120	6	32	-194	90	100	-96	-658	4	-410	-244	-360	-646	-720	156	-1622	386
1785/3		5	3	5	7	44	22	17	-132	-56	230	200	-130	314	-276	-168	-530	820	550	-204	16	-838	-472	-396	714	-1246
1793/1		-3	2	-5	13	-11	-49	20	-22	-165	179	131	-58	-483	-109	396	576	182	-420	652	-604	346	264	-935	-1104	-1273
1794/1		2	3	-2	14	56	-13	46	-74	-23	146	216	44	192	196	78	242	666	-386	14	-918	-342	-504	-484	354	-1032
1794/2		2	3	-6	24	-20	13	-86	-132	-23	-282	-172	-166	-62	-132	364	702	200	-210	-92	-636	2	-320	388	734	-146
1794/3		2	-3	6	30	34	13	100	144	-23	78	-16	20	-86	204	136	-198	-340	-696	-1052	-1128	-682	1234	502	-100	-686
1794/4		-2	3	-8	-14	22	13	58	-26	23	-294	248	-416	-86	520	244	-578	500	598	746	800	-154	-712	978	-44	256
1800/1	120/5	0	0	0	0	-4	-54	114	44	96	-134	-272	98	6	-12	-200	654	-36	-442	188	632	390	688	1188	694	1726
1800/2	200/2	0	0	0	2	-39	84	61	151	58	-192	-18	-138	-229	-164	212	-578	336	858	-209	780	-403	-230	1293	1369	382
1800/3	360/4	0	0	0	-2	34	68	-38	4	152	46	-260	312	-48	200	104	-414	2	-38	244	-708	378	-852	844	1380	-514
1800/4	360/4	0	0	0	-2	-34	68	38	4	-152	-46	-260	312	48	200	-104	414	-2	-38	244	708	378	-852	-844	-1380	-514
1800/5	200/2	0	0	0	-2	-39	-84	-61	151	-58	-192	-18	138	-229	164	-212	578	336	858	209	780	403	-230	-1293	1369	-382
1800/6	600/2	0	0	0	4	28	-16	-108	32	28	238	-180	-40	-422	276	-60	-220	804	-358	-884	64	-152	-932	1292	1146	824
1800/7	600/2	0	0	0	-4	28	16	108	32	-28	238	-180	40	-422	-276	60	220	804	-358	884	64	152	-932	-1292	1146	-824
1800/8	120/1	0	0	0	-4	-72	6	38	52	152	78	120	150	-362	484	280	-670	-696	222	4	-96	-178	-632	-612	-994	-1634
1800/9	600/4	0	0	0	5	-14	1	-46	19	46	-14	133	258	-84	-167	-410	-456	194	-17	653	-828	570	-552	-142	1104	841
1800/10	600/4	0	0	0	-5	-14	-1	46	19	-46	-14	133	-258	-84	167	410	456	194	-17	-653	-828	-570	-552	142	1104	-841
1800/11	200/1	0	0	0	6	19	-12	-75	-91	174	272	-230	182	-117	-372	-52	-402	-312	170	-763	52	981	1054	351	-799	-962
1800/12	200/1	0	0	0	-6	19	12	75	-91	-174	272	-230	-182	-117	372	52	402	-312	170	763	52	-981	1054	-351	-799	962
1800/13	120/3	0	0	0	-8	-20	-22	-14	76	56	154	160	162	390	-388	-544	-210	380	-794	148	840	-858	144	316	-1098	-994
1800/14	600/5	0	0	0	10	14	-82	-18	-136	140	-112	72	26	446	396	144	-158	342	314	-152	932	-548	-512	-284	810	1304
1800/15	600/6	0	0	0	10	46	-34	-66	104	-164	-224	-72	-22	-194	108	480	-286	-426	698	328	-188	-740	1168	-412	-1206	-1384
1800/16	600/5	0	0	0	-10	14	82	18	-136	-140	-112	72	-26	446	-396	-144	158	342	314	152	932	548	-512	284	810	-1304
1800/17	600/6	0	0	0	-10	46	34	66	104	164	-224	-72	22	-194	-108	-480	286	-426	698	-328	-188	740	1168	412	-1206	1384
1800/18	72/1	0	0	0	12	64	-58	-32	-136	128	-144	20	18	-288	200	-384	-496	-128	-458	496	512	602	1108	-704	-960	-206
1800/19	72/1	0	0	0	12	-64	-58	32	-136	-128	144	20	18	288	200	384	496	128	-458	496	-512	602	1108	704	960	-206
1800/20	120/2	0	0	0	16	28	26	-62	-68	-208	58	160	-270	-282	-76	-280	-210	-196	742	-836	504	1062	768	-1052	726	1406
1800/21	40/3	0	0	0	-16	-36	42	-110	-116	16	-198	240	258	-442	292	392	142	348	-570	-692	-168	134	784	564	-1034	382
1800/22	40/1	0	0	0	18	16	6	-6	-124	42	-142	-188	-202	-54	-66	38	738	-564	-262	554	-140	-882	-1160	642	854	478

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1800/23	360/2	0	0	0	18	34	-12	102	164	-48	146	100	-328	-288	-120	-16	126	642	602	-436	652	-1062	388	444	-820	766
1800/24	360/2	0	0	0	18	-34	-12	-102	164	48	-146	100	-328	288	-120	16	-126	-642	602	-436	-652	-1062	388	-444	820	766
1800/25	600/8	0	0	0	19	-22	-1	-58	-53	58	-22	-35	270	468	431	-230	0	-446	127	811	-36	-522	1368	-1138	-144	1079
1800/26	600/8	0	0	0	-19	-22	1	58	-53	-58	-22	-35	-270	468	-431	230	0	-446	127	-811	-36	522	1368	1138	-144	-1079
1800/27	120/6	0	0	0	-20	-16	-58	38	4	-80	-82	-8	-426	246	524	-464	-702	592	574	172	-768	558	408	164	510	-514
1800/28	120/4	0	0	0	-20	56	86	-106	4	136	206	-152	-282	246	-412	40	-126	-56	-2	388	672	-1170	408	668	-66	926
1800/29	24/1	0	0	0	24	28	74	82	92	8	138	80	-30	-282	-4	240	-130	-596	-218	436	-856	998	-32	-1508	246	-866
1800/30	8/1	0	0	0	-24	44	-22	50	44	-56	-198	-160	162	198	-52	528	-242	668	550	-188	-728	-154	-656	236	-714	478
1800/31	200/6	0	0	0	26	59	28	-5	109	194	32	10	-198	-117	388	68	18	-392	-710	-253	612	-549	414	121	81	-1502
1800/32	200/6	0	0	0	-26	59	-28	5	109	-194	32	10	198	-117	-388	-68	-18	-392	-710	253	612	549	414	-121	81	1502
1800/33	40/2	0	0	0	34	-16	-58	-70	4	-134	242	100	438	138	-178	22	162	268	250	-422	852	-306	-456	434	726	-1378
1800/34	360/3	0	0	0	-34	18	-12	106	-44	-56	270	204	-120	80	-536	536	-542	-174	186	-332	-132	602	-548	492	-1052	-482
1800/35	360/3	0	0	0	-34	-18	-12	-106	-44	56	-270	204	-120	-80	-536	-536	542	174	186	-332	132	602	-548	-492	1052	-482
1805/1	95/1	0	-4	-5	-22	-12	-8	-66	0	-30	6	64	16	-54	182	594	-396	564	-706	628	984	14	328	-294	-918	1564
1805/2		1	5	5	22	9	-54	-54	0	-92	134	252	236	243	496	502	-62	-681	-142	-55	974	695	736	-63	-726	1167
1805/3	1805/2	-1	-5	5	22	9	54	-54	0	-92	-134	-252	-236	-243	496	502	62	681	-142	55	-974	695	-736	-63	726	-1167
1805/4	95/2	-3	5	-5	-1	-24	31	33	0	27	-111	94	70	510	-34	-192	75	-45	-28	-371	-384	-73	1234	366	1578	538
1805/5	95/3	-3	-7	5	11	-36	-65	-87	0	-129	-231	-110	142	330	74	-336	-501	-633	-88	-119	204	407	-1262	270	30	-1406
1805/6	5/1	4	-2	-5	6	32	38	26	0	-78	50	108	-266	-22	442	-514	-2	-500	-518	-126	-412	-878	-600	282	150	-386
1805/7		5	-5	5	-19	50	-55	51	0	-147	165	70	-210	80	-558	-464	-455	-225	500	-105	1140	-703	-700	-918	-870	-1380
1805/8	95/4	-5	-4	5	-32	-12	42	114	0	160	-214	144	-94	6	-308	184	274	-276	-826	-52	344	-166	688	996	-1578	-786
1805/9	1805/7	-5	5	5	-19	50	55	51	0	-147	-165	-70	210	-80	-558	-464	455	225	500	105	-1140	-703	700	-918	870	1380
1806/1		2	3	-4	-7	19	-51	85	-118	67	-130	-33	-394	-27	43	192	225	320	-684	-89	70	-866	-1080	181	-54	1813
1806/2		2	-3	14	-7	0	86	66	124	-12	-310	284	-106	-278	43	-168	-354	684	618	612	-728	-602	-248	-252	674	258
1813/1	259/1	3	2	-2	0	20	74	-76	136	-16	86	238	-37	-86	-468	-264	-298	-420	-838	-244	-152	-514	204	-314	-680	712
1815/1	165/1	0	-3	-5	-2	0	22	-72	-122	72	-96	-112	266	96	382	360	318	660	430	380	168	-218	706	-1068	-6	686
1815/2		1	3	5	9	0	-77	129	-53	-124	153	-2	-155	122	-430	-30	476	-472	-512	686	211	-434	-218	-295	-616	-1334
1815/3		1	-3	-5	-33	0	-31	33	113	-44	-51	-50	-239	218	46	594	628	-260	548	-382	5	-598	974	-1147	-1172	-1190
1815/4	1815/2	-1	3	5	-9	0	77	-129	53	-124	-153	-2	-155	-122	430	-30	476	-472	512	686	211	434	218	295	-616	-1334
1815/5	15/2	-1	3	5	24	0	-22	14	20	-168	-230	-288	-34	-122	188	256	-338	100	-742	-84	-328	38	240	-1212	330	866
1815/6	165/2	-1	3	-5	-36	0	-2	-66	-140	-68	-150	-128	-314	118	-172	-324	82	-740	-122	-124	-988	-2	-1100	868	-470	1186
1815/7	1815/3	-1	-3	-5	33	0	31	-33	-113	-44	51	-50	-239	-218	-46	594	628	-260	-548	-382	5	598	-974	1147	-1172	-1190
1815/8	15/1	-3	-3	-5	-20	0	-74	-54	124	-120	78	200	-70	-330	-92	-24	450	24	322	-196	-288	430	520	-156	1026	-286
1816/1		0	-4	-12	-11	-39	-88	40	159	139	133	170	-2	182	-133	-12	273	-823	-390	-142	-37	659	24	-894	229	-618
1818/1	202/1	-2	0	-3	-30	-22	-51	13	-71	41	-204	97	-434	240	-440	-497	-122	-590	-728	862	-627	280	-335	328	-994	674
1818/2	606/1	-2	0	4	-17	50	70	93	-143	-88	-263	-282	178	412	-473	572	-391	-190	493	44	278	626	6	52	-152	-559
1818/3	202/2	-2	0	-18	-13	12	-16	-117	143	-42	9	-16	440	84	-283	-354	273	612	83	974	1062	272	50	1056	-720	-1531
1820/1		0	-4	5	-7	34	-13	4	40	-88	30	-270	310	-152	-94	168	-404	568	-698	-552	432	1070	-184	-572	-360	234
1820/2		0	-4	-5	-7	16	-13	106	-80	-172	150	96	-62	130	428	-600	418	712	-938	636	-720	878	800	-548	1146	894
1820/3		0	-4	-5	-7	-68	-13	22	88	164	38	292	-342	382	-272	408	-282	-576	-98	76	-888	822	520	-1164	278	-338
1820/4		0	-8	-5	-7	-40	-13	62	28	-152	-122	-168	426	198	-292	264	302	396	878	-108	916	-698	640	-300	-1490	446
1820/5		0	-10	5	7	-32	-13	92	-14	-86	38	40	-162	-416	-148	-24	468	110	358	-140	650	86	824	-788	-24	674
1825/1	365/2	3	7	0	1	-39	16	130	-136	50	-123	-254	-41	-215	-162	-99	676	691	-304	-601	-803	73	-593	312	1169	-1586
1825/2	365/1	3	-10	0	16	48	88	-9	137	-57	213	-97	97	201	421	-192	-690	186	-304	-146	420	-73	-196	612	-399	-1280
1825/3	73/1	-3	8	0	34	6	34	-90	-16	-60	102	-214	286	150	322	534	474	786	-574	16	192	-73	-988	-1242	-6	-614
1827/1		1	0	12	7	21	-51	88	-117	-15	-29	20	-243	-100	-208	215	217	111	-218	-195	656	-803	-884	43	1236	-917
1827/2	1827/1	-1	0	-12	7	-21	-51	-88	-117	15	29	20	-243	100	-208	-215	-217	-111	-218	-195	-656	-803	-884	-43	-1236	-917
1827/3	203/2	-1	0	-14	7	28	70	14	140	-72	-29	208	254	-186	-444	160	-270	684	86	-708	-280	506	480	1060	-810	1314
1827/4	609/1	3	0	-12	7	60	32	-24	-160	144	-29	-106	-286	-180	260	126	246	174	302	-136	1008	992	512	1218	-540	-100
1827/5	203/1	-4	0	-2	7	-2	-26	80	128	0	-29	160	-274	36	246	244	-114	420	188	624	-1120	-352	438	676	336	-216
1827/6	609/2	-5	0	-4	7	-16	58	46	-64	42	29	-56	26	-306	318	-112	186	-120	368	-348	-266	-844	-1026	296	-294	-60
1830/1		2	-3	5	12	-30	18	-36	-100	170	-244	-304	-262	-178	276	296	180	-230	61	380	-266	154	-1212	-396	-108	-74
1830/2		-2	3	5	-36	40	-26	26	92	-120	-142	48	130	-222	32	136	-50	168	61	160	-12	-814	88	-900	326	-702
1830/3		-2	3	-5	8	6	69	-59	-119	-14	271	-179	-332	-279	-461	-159	614	624	-61	593	608	-668	-25	834	-1246	-1316
1830/4		-2	-3	5	-26	64	-50	-24	-4	62	30	-236	-56	470	-162	436	-384	312	-61	-830	108	886	-452	844	450	-1134
1830/5		-2	-3	-5	14	-6	-30	-14	76	-68	220	-236	-116	390	-362	576	-34	162	-61	170	-22	-314	-352	324	-40	466
1836/1		0	0	6	20	15	47	17	-148	-147	-177	-250	-184	-153	-133	-96	150	-336	290	-301	765	134	-460	-618	-264	614
1836/2	1836/1	0	0	-6	20	-15	47	-17	-148	147	177	-250	-184	153	-133	96	-150	336	290	-301	-765	134	-460	618	264	614
1840/1	230/1	0	1	5	32	30	19	-60	58	-23	85	65	-34	143	332	561	-422	-392	-246	-894	737	1041	-1114	936	824	-868
1840/2	230/2	0	-1	-5	18	32	-47	20	-36	23	-27	33	56	-157	-18	-65	-14	744	552	156	-699	-609	644	-512	-102	578
1840/3	115/2	0	3	5	2	16	-47	-24	56	23	85	-67	104	-53	234	-285	2	-80	-764	-236	289	-225	-24	-684	-1370	-110
1840/4	230/4	0	-4	-5	-3	2	-38	-45	74	-23	283	303	79	-407	328	-360	-561	-101	-268	69	641	994	884	-503	1608	1082

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1840/5	115/1	0	-4	-5	32	-40	-66	130	88	-23	-130	-40	-334	-22	272	-24	258	-612	-366	496	-248	826	296	1296	-646	-1438
1840/6	230/5	0	5	-5	-12	-22	19	96	98	-23	-227	285	-398	271	100	285	18	352	-478	-330	-835	-1127	-322	-572	-504	1712
1840/7	230/3	0	-7	5	-20	-6	47	-132	-146	-23	-99	253	-118	495	-272	-639	-342	-240	-370	-698	357	-259	-542	1248	-828	992
1842/1		2	3	-14	9	-36	12	11	119	-146	-30	-248	306	-334	500	-462	305	-38	-428	-404	-507	-568	521	47	-411	-1325
1842/2		2	-3	-12	-25	-45	-34	-39	-109	-30	-132	50	-97	-453	-436	270	243	-132	176	-256	-549	152	428	-300	-882	-1054
1845/1	205/1	1	0	-5	8	54	68	10	-150	64	56	-336	66	41	188	536	-172	24	-262	442	-652	-54	-104	-1236	-370	1294
1845/2		1	0	-5	17	36	-40	-62	3	-71	200	-3	-123	41	377	-184	323	393	-838	910	-1048	108	-266	240	863	-1406
1845/3	1845/2	-1	0	5	17	-36	-40	62	3	71	-200	-3	-123	-41	377	184	-323	-393	-838	910	1048	108	-266	-240	-863	-1406
1845/4	205/2	-1	0	5	26	18	2	134	-30	188	190	192	-174	-41	332	-566	718	-180	-418	286	-62	-378	1150	-432	1030	-254
1845/5	615/1	-1	0	-5	11	18	-68	-36	-15	73	210	-213	81	-41	247	284	273	-235	-28	-944	228	-978	620	988	225	666
1845/6	615/2	3	0	-5	16	12	-26	86	-84	96	-62	96	-162	-41	-100	72	-142	-4	-362	-596	-256	-102	-392	-1012	838	-150
1845/7	615/3	5	0	-5	-10	6	-2	-84	-54	40	180	-240	-126	-41	-20	-382	102	-40	-262	-356	-792	594	-4	-692	906	-1272
1848/1		0	3	6	-7	-11	-42	-102	76	128	230	224	-226	-46	132	-544	-482	4	-554	-356	-480	202	-976	-964	1314	434
1850/1	74/1	2	5	0	7	-63	28	-6	-70	6	-42	-292	-37	351	-32	-357	-57	432	-340	1012	-609	-539	818	-1299	-390	-1772
1850/2		2	-10	0	32	52	-62	-16	-85	-189	98	-92	-37	-249	433	-422	63	37	-590	222	-154	-259	-1207	-64	630	-672
1850/3	370/3	-2	-1	0	25	9	76	24	-40	72	60	26	-37	267	382	-267	-171	396	-898	676	-21	691	-394	-309	-918	766
1850/4	370/2	-2	2	0	-2	-72	-2	66	38	36	-90	-70	-37	-438	-272	198	354	-498	542	-2	408	358	722	174	-102	574
1850/5	74/2	-2	5	0	19	5	-6	72	-44	-182	10	-244	37	-225	2	-221	659	156	-620	-416	-1125	641	-484	-1239	1304	560
1850/6	370/1	-2	-6	0	-3	5	16	-115	110	-6	-111	-79	37	171	-361	428	527	112	-323	464	-366	-712	176	180	446	1407
1850/7	1850/2	-2	10	0	-32	52	62	16	-85	189	98	-92	37	-249	-433	422	-63	37	-590	-222	-154	259	-1207	64	630	672
1854/1	618/1	-2	0	8	12	57	-38	94	-127	27	-117	233	341	424	182	50	-396	420	-464	-542	276	-176	462	1108	1297	1055
1855/1		-3	-8	-5	-7	-20	-72	-14	52	-132	106	-46	134	280	48	-228	-53	-506	-626	-332	-288	-584	-196	-728	-34	698
1856/1	58/1	0	7	-5	2	37	-27	24	-88	28	29	143	360	386	381	103	431	288	840	-180	-706	716	-931	1188	-642	486
1856/2	928/1	0	7	13	16	45	-61	-102	68	194	29	149	-400	280	-263	509	605	578	718	260	738	652	-917	-678	-1008	-1764
1856/3	58/2	0	7	15	-18	-27	57	-44	-152	-152	29	-173	120	-314	-339	-357	59	572	420	-660	726	1004	361	168	58	-1206
1856/4	58/1	0	-7	-5	-2	-37	-27	24	88	-28	29	-143	360	386	-381	-103	431	-288	840	180	706	716	931	-1188	-642	486
1856/5	928/1	0	-7	13	-16	-45	-61	-102	-68	-194	29	-149	-400	280	263	-509	605	-578	718	-260	-738	652	917	678	-1008	-1764
1856/6	58/2	0	-7	15	18	27	57	-44	152	152	29	173	120	-314	339	357	59	-572	420	660	-726	1004	-361	-168	58	-1206
1860/1		0	-3	5	-8	-20	-34	-6	12	120	246	31	310	-518	92	-88	-738	268	366	220	-512	-758	160	1348	18	1634
1862/1	38/1	-2	2	9	0	57	52	-69	-19	-72	-150	-32	-226	258	-67	-579	-432	330	13	-856	642	487	-700	12	600	-1424
1863/1		1	0	3	-26	-32	57	49	56	23	89	100	-121	-202	106	390	594	258	-609	686	-550	-1037	-902	48	-691	1406
1863/2	1863/1	-1	0	-3	-26	32	57	-49	56	-23	-89	100	-121	202	106	-390	-594	-258	-609	686	550	-1037	-902	-48	691	1406
1866/1		2	3	6	-3	-28	21	-41	-66	-157	-237	-242	8	261	-50	-176	-212	485	150	644	1103	83	-832	-210	-504	504
1870/1		2	-8	5	-16	11	26	-17	-76	-36	150	-256	134	522	-172	-108	-114	708	134	80	48	182	-880	156	858	-1294
1870/2		-2	1	5	26	-11	43	17	37	6	-113	205	-204	250	-274	115	405	621	-351	-654	881	433	392	1124	695	-81
1872/1	234/3	0	0	2	26	-52	-13	48	-18	52	224	-310	-18	330	-328	-616	-324	-188	-110	-118	656	-178	-836	-60	870	1238
1872/2	156/2	0	0	2	32	-68	13	14	-4	72	-102	136	-386	-250	140	-296	-526	332	-410	-596	-880	506	640	1380	-1450	-446
1872/3	234/3	0	0	-2	26	52	-13	-48	-18	-52	-224	-310	-18	-330	-328	616	324	188	-110	-118	-656	-178	-836	60	-870	1238
1872/4	78/6	0	0	-4	-4	2	-13	6	36	-20	14	152	-258	-84	188	254	-366	550	-14	-448	926	254	-1328	186	336	614
1872/5	156/1	0	0	6	4	36	13	-66	-56	96	-222	-260	-106	90	-44	168	-30	348	-346	256	-168	-814	-200	1236	-318	-502
1872/6	78/5	0	0	-6	-20	24	13	30	16	-72	282	-164	110	126	-164	-204	738	120	614	-848	132	218	1096	552	-210	-1726
1872/7	13/1	0	0	7	13	-26	13	-77	126	-96	82	-196	-131	-336	201	-105	432	-294	-56	-478	9	98	-1304	-308	1190	70
1872/8	104/2	0	0	7	21	6	13	115	46	144	162	-180	13	-192	33	383	-288	442	-680	722	-207	274	936	-1204	966	-138
1872/9	78/3	0	0	-10	8	40	13	-130	20	0	18	184	-74	362	-76	-452	-382	464	358	700	-748	1058	976	-1008	386	-614
1872/10	26/1	0	0	-11	-19	-38	-13	51	-90	-52	190	-292	-441	-312	-373	-41	-468	530	592	206	-863	-322	460	528	-870	-346
1872/11	39/1	0	0	12	-2	-36	13	78	-74	-96	-18	214	-286	384	-524	300	-558	576	74	-38	-456	-682	-704	-888	1020	110
1872/12	52/1	0	0	13	11	-2	-13	51	-150	-4	118	116	63	288	293	-335	708	566	904	-382	7	518	100	-1440	-1254	1262
1872/13	78/2	0	0	16	8	-38	-13	78	72	-52	-242	-76	342	336	-76	94	450	854	-110	908	838	-970	352	474	1452	-562
1872/14	78/1	0	0	16	-28	34	-13	-138	-108	-52	190	176	342	-240	140	454	-198	-154	34	656	550	614	-8	762	444	1022
1872/15	26/2	0	0	-17	35	2	13	19	-94	-72	-246	100	-11	280	-241	137	232	-386	64	670	55	-838	-1016	420	934	-1154
1872/16	26/3	0	0	18	-20	-48	13	-66	16	168	-6	-20	254	390	124	-468	-558	-96	-826	160	-420	362	-776	0	-1626	-1294
1872/17	104/1	0	0	-19	3	-2	-13	-77	58	76	6	292	207	-240	317	-375	692	214	-488	-782	-1057	1174	-892	704	-6	830
1872/18	78/4	0	0	20	32	50	-13	30	120	-20	-82	44	-306	-108	356	-178	-198	94	-62	140	-778	62	1096	-462	-1224	614
1876/1		0	-1	-11	-7	-16	-57	128	68	125	97	69	-31	363	80	6	186	-770	426	-67	-631	-962	-990	2	-492	-62
1878/1		2	-3	21	0	-54	-35	61	-76	-132	-30	-112	-366	-74	-148	-96	-751	239	376	-274	290	364	-663	-168	-341	1169
1880/1		0	7	5	-27	-21	-37	62	-89	96	48	273	134	178	358	47	96	432	-519	524	820	-673	1040	207	366	-1122
1884/1		0	-3	-6	-3	52	-7	-135	86	77	-240	18	281	-462	463	168	510	533	816	-218	568	-978	-704	492	519	-908
1887/1		0	3	15	9	41	24	-17	55	22	137	-86	37	432	-153	384	-59	-571	451	-194	-813	-298	838	-1016	666	-849
1887/2		0	3	15	31	-69	-20	-17	-99	-154	225	266	37	124	-43	296	425	683	-209	26	969	758	90	1008	622	691
1890/1		2	0	5	7	9	-19	-108	11	-126	-66	-148	-346	-147	-139	201	249	-582	344	305	-912	-151	-832	873	609	686
1890/2		2	0	5	7	-46	-8	79	-121	-49	-132	-5	-38	-26	114	-448	-609	56	-635	30	1090	-338	477	-579	-326	-920
1890/3		2	0	5	7	68	52	-95	-19	23	234	-239	424	-62	522	248	-549	-706	-329	-1038	1030	-38	-471	291	-836	754

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1890/4		2	0	5	-7	25	13	-29	-98	-133	-217	-237	186	24	-31	-201	-518	684	224	204	-160	-752	691	1128	-1180	-1456
1890/5		2	0	5	-7	28	-44	-47	-95	131	-22	-171	-48	-30	-262	-360	343	150	-685	114	-370	-1070	841	-1077	404	914
1890/6		2	0	5	-7	-37	73	-60	9	-194	-74	76	-178	-251	11	199	109	358	-672	-1017	-552	165	880	431	-1039	446
1890/7		2	0	-5	7	-18	35	-57	104	-171	-165	-313	128	-114	47	174	291	21	86	1067	-135	-1066	-220	-1020	-51	-1846
1890/8	1890/7	-2	0	5	7	18	35	57	104	171	165	-313	128	114	47	-174	-291	-21	86	1067	135	-1066	-220	1020	51	-1846
1890/9	1890/1	-2	0	-5	7	-9	-19	108	11	126	66	-148	-346	147	-139	-201	-249	582	344	305	912	-151	-832	-873	-609	686
1890/10	1890/2	-2	0	-5	7	46	-8	-79	-121	49	132	-5	-38	26	114	448	609	-56	-635	30	-1090	-338	477	579	326	-920
1890/11	1890/3	-2	0	-5	7	-68	52	95	-19	-23	-234	-239	424	62	522	-248	549	706	-329	-1038	-1030	-38	-471	-291	836	754
1890/12	1890/4	-2	0	-5	-7	-25	13	29	-98	133	217	-237	186	-24	-31	201	518	-684	224	204	160	-752	691	-1128	1180	-1456
1890/13	1890/5	-2	0	-5	-7	-28	-44	47	-95	-131	22	-171	-48	30	-262	360	-343	-150	-685	114	370	-1070	841	1077	-404	914
1890/14	1890/6	-2	0	-5	-7	37	73	60	9	194	74	76	-178	251	11	-199	-109	-358	-672	-1017	552	165	880	-431	1039	446
1899/1	633/1	4	0	-3	30	-3	-23	114	19	-104	246	88	321	266	93	147	642	-20	-330	-526	-585	-330	137	-1068	756	950
1900/1	380/1	0	-1	0	-19	20	77	11	-19	-79	-303	214	250	-230	402	-48	417	99	332	319	-1088	373	102	-934	498	1386
1904/1	119/1	0	6	-20	7	-60	-68	-17	70	176	-90	-196	22	-138	-328	12	-234	54	44	596	-200	1122	-480	838	778	1142
1905/1		-5	3	-5	-29	6	-44	-140	9	-134	160	-70	-240	-336	-482	89	-78	-105	-830	-956	-841	-223	782	1291	1293	338
1911/1	39/1	0	3	12	0	-36	-13	78	-74	-96	18	214	-286	384	524	-300	558	-576	-74	38	-456	682	704	888	1020	-110
1911/2	273/2	-1	-3	5	0	-1	-13	-19	117	-141	-131	128	55	0	-201	96	510	156	845	-470	324	373	-526	-266	250	-322
1911/3	273/3	-1	-3	-9	0	-57	13	37	-107	-183	191	240	-379	84	-313	-296	-414	-40	-65	-1086	-208	-635	-582	-798	726	-1498
1911/4	273/1	-4	3	0	0	-6	13	4	52	6	14	48	-190	-180	356	-536	210	-244	-470	240	854	82	-876	-504	660	-1318
1911/5		-5	3	-3	0	9	-13	22	56	-42	109	-75	-256	132	-208	280	381	-43	-508	612	238	134	-957	-927	-312	577
1911/6	1911/5	-5	-3	3	0	9	13	-22	-56	-42	109	75	-256	-132	-208	-280	381	43	508	612	238	-134	-957	927	312	-577
1918/1		2	4	-2	-7	20	18	-10	-22	-80	-26	-170	346	-48	-188	-606	-486	-310	916	-140	704	258	496	-592	944	-372
1920/1		0	3	5	-2	30	2	-54	-106	18	-138	-292	270	-466	32	74	-302	-518	86	448	328	258	288	-236	1254	-790
1920/2		0	3	5	-4	6	-44	-84	8	48	90	166	156	166	-460	-448	-470	26	-206	-548	-392	30	750	-4	-186	530
1920/3		0	3	5	8	-50	-48	36	24	8	-118	178	-160	254	-148	-96	258	-278	-154	-212	-1112	-1162	138	84	-746	-1590
1920/4		0	3	5	10	-22	26	14	-34	-190	-162	-268	-362	-170	16	434	594	-170	130	-1024	280	282	-160	-732	-746	-534
1920/5	1920/1	0	3	-5	2	30	-2	-54	-106	-18	138	292	-270	-466	32	-74	302	-518	-86	448	-328	258	-288	-236	1254	-790
1920/6	1920/2	0	3	-5	4	6	44	-84	8	-48	-90	-166	-156	166	-460	448	470	26	206	-548	392	30	-750	-4	-186	530
1920/7	1920/3	0	3	-5	-8	-50	48	36	24	-8	118	-178	160	254	-148	96	-258	-278	154	-212	1112	-1162	-138	84	-746	-1590
1920/8	1920/4	0	3	-5	-10	-22	-26	14	-34	190	162	268	362	-170	16	-434	-594	-170	-130	-1024	-280	282	160	-732	-746	-534
1920/9	1920/1	0	-3	5	2	-30	2	-54	106	-18	-138	292	270	-466	-32	-74	-302	518	86	-448	-328	258	-288	236	1254	-790
1920/10	1920/2	0	-3	5	4	-6	-44	-84	-8	-48	90	-166	156	166	460	448	-470	-26	-206	548	392	30	-750	4	-186	530
1920/11	1920/3	0	-3	5	-8	50	-48	36	-24	-8	-118	-178	-160	254	148	96	258	278	-154	212	1112	-1162	-138	-84	-746	-1590
1920/12	1920/4	0	-3	5	-10	22	26	14	34	190	-162	268	-362	-170	-16	-434	594	170	130	1024	-280	282	160	732	-746	-534
1920/13	1920/1	0	-3	-5	-2	-30	-2	-54	106	18	138	-292	-270	-466	-32	74	302	518	-86	-448	328	258	288	236	1254	-790
1920/14	1920/2	0	-3	-5	-4	-6	44	-84	-8	48	-90	166	-156	166	460	-448	470	-26	206	548	-392	30	750	4	-186	530
1920/15	1920/3	0	-3	-5	8	50	48	36	-24	8	118	178	160	254	148	-96	-258	278	154	212	-1112	-1162	138	-84	-746	-1590
1920/16	1920/4	0	-3	-5	10	22	-26	14	34	-190	162	-268	362	-170	-16	434	-594	170	-130	1024	280	282	-160	732	-746	-534
1922/1	62/2	2	8	-3	-35	46	-20	-8	97	-28	206	0	282	367	562	-148	84	-301	236	60	699	814	-670	650	-1566	-615
1922/2	62/1	-2	2	1	-11	18	82	6	25	-58	-180	0	146	47	12	-136	232	715	518	-436	387	-678	-660	382	800	-1631
1925/1	385/3	0	2	0	-7	11	-80	84	68	198	-198	56	286	78	-260	402	534	-180	-622	1018	-900	-956	-424	-636	-378	-758
1925/2	385/2	0	2	0	-7	-11	52	-48	68	66	66	-340	-242	-54	-524	-390	-522	744	830	-170	-636	-296	1160	684	-642	562
1925/3	385/5	-1	-2	0	-7	11	-22	-6	70	-182	-20	32	-76	352	-132	624	-592	720	442	164	452	698	-950	628	30	-656
1925/4	385/4	-1	-10	0	-7	11	-54	-86	-98	82	4	112	-196	120	-148	-464	488	-368	-614	836	948	554	50	484	-690	1368
1925/5		3	4	0	7	11	23	42	5	-123	-249	185	152	-336	-61	-360	-72	-72	-790	644	813	-976	-298	465	621	-1735
1925/6		3	4	0	-7	11	-61	-14	145	157	255	213	-408	112	-341	-192	376	376	218	644	477	368	598	1277	481	-811
1925/7	385/1	-3	-4	0	7	11	46	-106	-140	128	210	-252	78	442	356	72	-466	316	-682	-224	-528	142	148	-1112	-254	-1694
1925/8	1925/6	-3	-4	0	7	11	61	14	145	-157	255	213	408	112	341	192	-376	376	218	-644	477	-368	598	-1277	481	811
1925/9	1925/5	-3	-4	0	-7	11	-23	-42	5	123	-249	185	-152	-336	61	360	72	-72	-790	-644	813	976	-298	-465	621	1735
1925/10	77/1	-3	-4	0	-7	11	-38	48	-70	-12	126	-70	358	-216	-344	-390	-438	-552	830	196	648	16	1352	-90	1146	70
1926/1	642/3	2	0	-7	26	52	10	32	14	123	134	-122	20	-105	363	-75	-338	215	-326	-439	150	-50	-869	1038	783	1710
1926/2		2	0	-17	-26	-50	-21	-123	-67	-74	-120	148	-43	-68	-542	-46	-96	848	583	-314	-405	-10	384	-1102	-1312	-904
1926/3	642/1	-2	0	7	-8	30	-13	9	-11	100	-132	-230	-231	362	414	4	158	84	283	-868	-597	-124	-1036	-1126	1536	1074
1926/4	642/2	-2	0	-9	-34	4	-77	121	-11	64	-4	170	249	-20	-236	380	108	340	151	670	-945	-208	-416	-364	-534	312
1926/5	1926/2	-2	0	17	-26	50	-21	123	-67	74	120	148	-43	68	-542	46	96	-848	583	-314	405	-10	384	1102	1312	-904
1932/1		0	3	8	7	25	-62	72	-113	-23	-250	-278	-174	51	-460	293	87	131	83	-776	-220	-528	354	186	-62	-1274
1932/2		0	-3	6	7	72	-28	-120	86	23	-270	-130	2	270	-460	-174	-402	108	-70	860	84	-802	512	-846	-324	-124
1932/3		0	-3	-19	7	-8	37	-40	-164	23	265	-90	-13	435	85	-609	568	558	-230	260	654	-492	-48	724	1026	-699
1935/1		3	0	5	4	8	-70	138	-86	86	286	-84	28	-208	-43	-522	-206	-60	-850	-756	-672	-1004	300	1170	1002	-1322
1935/2	1935/1	-3	0	-5	4	-8	-70	-138	-86	-86	-286	-84	28	208	-43	522	206	60	-850	-756	672	-1004	300	-1170	-1002	-1322
1936/1	88/2	0	1	-7	-6	0	40	78	36	-7	-8	-183	227	36	322	184	-6	99	-164	695	987	248	-242	-1494	-905	-1031
1936/2	22/3	0	-1	-3	-10	0	16	-42	116	-189	120	163	-409	-468	110	-144	90	453	-20	97	465	-848	-742	438	-273	761

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1936/3	968/2	0	2	13	10	0	27	-27	-38	-150	-285	198	57	-227	64	-390	-267	-280	50	546	-772	178	1058	378	-1185	-733
1936/4	968/2	0	2	13	-10	0	-27	27	38	-150	285	198	57	227	-64	-390	-267	-280	-50	546	-772	-178	-1058	-378	-1185	-733
1936/5	8/1	0	4	-2	24	0	-22	-50	44	56	-198	160	-162	198	52	-528	-242	668	-550	-188	-728	-154	-656	236	714	-478
1936/6	242/2	0	-4	3	8	0	-83	-123	-112	-36	21	-128	107	201	308	492	-345	-204	-470	760	-900	742	92	-864	-645	299
1936/7	242/2	0	-4	3	-8	0	83	123	112	-36	-21	-128	107	-201	-308	492	-345	-204	470	760	-900	-742	-92	864	-645	299
1936/8	22/1	0	-4	14	-8	0	50	-130	-108	96	-142	-40	382	118	220	-520	238	852	-190	12	112	6	304	820	202	-1406
1936/9	44/1	0	5	-7	-26	0	-52	-46	-96	-27	-16	293	-29	472	-110	224	754	-825	548	123	-1001	1020	526	-158	-1217	-263
1936/10	242/1	0	-5	-15	36	0	12	-84	60	-105	120	-205	115	-420	168	180	270	429	-600	65	237	12	840	-288	255	-1375
1936/11	242/1	0	-5	-15	-36	0	-12	84	-60	-105	-120	-205	115	420	-168	180	270	429	600	65	237	-12	-840	288	255	-1375
1936/12	22/2	0	7	-19	14	0	72	46	-20	107	-120	-117	-201	228	-242	96	458	-435	668	-439	1113	72	-70	358	895	409
1936/13	88/1	0	-7	9	2	0	0	38	44	-175	264	-159	-173	220	-542	264	682	-421	-308	-177	-365	528	686	698	967	-1127
1936/14	121/1	0	-8	18	0	0	0	0	0	108	0	-340	-434	0	0	36	-738	720	0	416	-612	0	0	0	1674	-34
1938/1		2	-3	7	-33	34	44	-17	-19	-62	-154	106	-182	-172	10	493	277	456	206	-327	49	-1068	-106	15	-825	-545
1944/1		0	0	8	9	-58	67	-38	-49	26	-54	263	-51	-456	199	-78	-442	308	-914	-76	430	322	-875	994	84	809
1944/2		0	0	8	-21	32	-23	-8	11	56	96	-67	-141	264	-281	192	128	248	-74	464	-1040	-1178	-185	424	-216	29
1944/3	1944/1	0	0	-8	9	58	67	38	-49	-26	54	263	-51	456	199	78	442	-308	-914	-76	-430	322	-875	-994	-84	809
1944/4	1944/2	0	0	-8	-21	-32	-23	8	11	-56	-96	-67	-141	-264	-281	-192	-128	-248	-74	464	1040	-1178	-185	-424	216	29
1944/5		0	0	11	30	-4	16	70	5	53	-15	74	-156	366	292	-201	557	-364	-518	-7	-473	-359	1024	1078	1044	-193
1944/6	1944/5	0	0	-11	30	4	16	-70	5	-53	15	74	-156	-366	292	201	-557	364	-518	-7	473	-359	1024	-1078	-1044	-193
1947/1		-3	3	-13	-30	11	-16	19	-101	-80	-42	-128	-197	-234	-96	-135	635	-59	-310	-932	-864	-833	-172	-1141	-1335	-1630
1950/1	390/7	2	3	0	-5	-35	13	-23	-30	-63	-190	330	-43	-473	232	-270	193	-200	-679	12	-899	-154	215	1308	-1019	427
1950/2	390/6	2	3	0	14	-36	13	-68	-158	-46	-8	-176	-62	30	-252	120	-758	252	398	-884	-80	660	568	-1084	1250	-84
1950/3	78/1	2	3	0	-28	34	13	-138	108	52	-190	-176	-342	240	140	-454	-198	-154	34	656	550	-614	8	-762	-444	-1022
1950/4	390/9	2	-3	0	-2	0	-13	60	50	-210	-228	116	-386	378	4	312	198	624	638	-200	-408	-1148	824	-1332	54	244
1950/5	78/2	2	-3	0	8	-38	13	78	-72	52	242	76	-342	-336	-76	-94	450	854	-110	908	838	970	-352	-474	-1452	562
1950/6	78/3	2	-3	0	8	40	-13	-130	-20	0	-18	-184	74	-362	-76	452	-382	464	358	700	-748	-1058	-976	1008	-386	614
1950/7	390/8	2	-3	0	13	-15	-13	75	-130	-45	-138	-34	379	243	-416	-378	3	-816	-607	700	57	1162	-1	-672	969	949
1950/8	390/10	2	-3	0	15	39	13	15	54	143	-122	-246	225	469	484	-234	-33	0	-831	-772	-793	998	-681	772	-465	79
1950/9	390/11	2	-3	0	-24	0	-13	-50	28	208	190	248	186	-194	-348	-260	-462	-520	-506	-772	780	62	736	-1464	406	-922
1950/10	390/5	-2	3	0	-8	12	-13	42	-52	-132	282	116	-398	174	76	-456	-150	-156	230	592	408	730	728	-36	-1482	-1742
1950/11	390/4	-2	3	0	-8	-40	13	-10	0	180	22	-144	-34	-502	76	168	422	104	-82	540	512	-622	104	-348	-286	-494
1950/12	390/2	-2	3	0	12	-48	-13	62	-32	8	-58	-124	162	74	396	164	-270	-416	70	-448	-1092	-10	328	144	-502	-1042
1950/13	78/5	-2	3	0	-20	24	-13	30	-16	72	-282	164	-110	-126	-164	204	738	120	614	-848	132	-218	-1096	-552	210	1726
1950/14	390/3	-2	3	0	25	-21	-13	-123	146	-99	-246	182	295	9	-452	-390	-315	-24	-727	-596	771	-326	-889	96	795	-983
1950/15	78/4	-2	3	0	32	50	13	30	-120	20	82	-44	306	108	356	178	-198	94	-62	140	-778	-62	-1096	462	1224	-614
1950/16	78/6	-2	-3	0	-4	2	13	6	-36	20	-14	-152	258	84	188	-254	-366	550	-14	-448	926	-254	1328	-186	-336	-614
1950/17	390/1	-2	-3	0	28	-36	-13	-42	-112	168	-210	-76	-278	150	460	264	-582	-204	614	304	1080	934	128	-348	-834	1582
1953/1	217/1	1	0	-4	7	-66	-78	-78	-106	28	-88	-31	152	18	-506	484	-364	-770	-222	-220	512	-646	-380	832	1402	414
1960/1	280/1	0	1	-5	0	-39	17	15	-74	-14	-237	180	-318	348	-22	193	-208	-452	-340	-408	528	554	539	-164	576	827
1960/2	280/3	0	4	-5	0	20	10	14	-12	104	-122	-224	158	-378	404	-112	270	-324	186	156	-360	102	-912	-1068	1590	-866
1960/3	40/3	0	-4	-5	0	36	42	110	116	16	198	-240	-258	-442	-292	-392	142	348	570	692	168	134	784	-564	-1034	382
1960/4	280/4	0	-5	5	0	-39	19	37	18	-90	99	32	46	248	178	-429	-652	-40	36	-348	72	1190	699	116	704	-223
1960/5	40/2	0	6	5	0	16	-58	70	-4	-134	-242	-100	-438	138	178	-22	162	268	-250	422	-852	-306	-456	-434	726	-1378
1960/6		0	7	-5	0	58	82	50	64	-111	103	-130	376	-307	-197	120	-508	600	-165	-633	840	606	-1316	61	-187	406
1960/7	1960/6	0	-7	5	0	58	-82	-50	-64	-111	103	130	376	307	-197	-120	-508	-600	165	-633	840	-606	-1316	-61	187	-406
1960/8	280/2	0	-7	-5	0	9	-23	-41	-34	-6	131	-4	26	260	-190	-167	-368	-324	164	200	784	410	1211	1132	72	707
1960/9	40/1	0	-10	5	0	-16	6	6	124	42	142	188	202	-54	66	-38	738	-564	262	-554	140	-882	-1160	-642	854	478
1962/1	654/1	2	0	6	-19	12	20	-75	86	-111	78	-169	-58	-315	389	-333	342	327	293	-862	-6	-1063	932	-822	636	-1393
1962/2	654/2	2	0	14	-11	-12	-12	-79	-10	-43	-210	167	-66	313	-267	-329	-538	-205	-523	634	378	273	-1340	-478	780	1159
1962/3	654/3	2	0	-20	-19	-60	-50	-31	8	-47	-220	-37	-278	-327	-483	-441	54	403	497	-572	1058	313	994	-1036	-378	1171
1968/1	492/1	0	3	5	26	-34	-85	97	79	-186	-168	-271	-2	41	-268	-84	378	-337	-358	-279	-837	705	384	-1293	-347	694
1968/2	492/2	0	-3	-12	-10	41	58	-53	-56	162	1	15	-363	41	91	195	-670	192	193	646	-891	-35	426	-728	294	188
1968/3	246/1	0	-3	-14	28	-1	16	-107	138	32	99	35	149	41	339	-511	-58	136	-335	-682	-389	-323	-10	834	526	-330
1968/4	984/1	0	-3	-16	-18	1	82	-119	16	-110	-225	-167	81	-41	65	-225	-322	-764	61	-830	-535	349	-538	-436	810	260
1970/1		2	-1	-5	-8	44	-33	-70	-121	106	-54	-58	86	215	298	494	78	-427	-469	551	876	-639	316	500	-96	214
1976/1		0	-1	21	-5	64	-13	-3	19	-178	200	0	-41	158	-9	-49	198	758	778	-564	-249	192	1316	864	-520	-1130
1978/1		-2	-7	-12	-24	-16	-1	66	96	23	-97	-181	-278	-277	43	-581	156	-108	-568	-510	377	599	-912	-292	-734	594
1980/1	220/1	0	0	5	-19	11	-62	-19	-131	-138	79	217	-91	-158	120	546	439	-290	-373	728	709	850	-1194	-58	-753	1228
1980/2	660/1	0	0	-5	0	-11	-42	14	-52	-96	26	-144	126	-58	364	328	50	284	-794	-316	280	-358	784	-324	1398	-894
1980/3	220/3	0	0	-5	11	11	-22	-9	89	-138	-201	77	119	102	260	-294	-51	-270	-733	728	849	830	-214	-138	-633	-892
1980/4	220/2	0	0	-5	24	11	-22	-22	-28	44	-110	-40	-362	-210	260	460	-662	68	606	-312	-360	-1042	-552	-268	966	-1334
1984/1	62/1	0	2	-1	-11	18	82	-6	-25	58	-180	31	146	47	12	-136	232	-715	518	436	387	678	660	382	-800	-1631

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
1984/2	62/1	0	-2	-1	11	-18	82	-6	25	-58	-180	-31	146	47	-12	136	232	715	518	-436	-387	678	-660	-382	-800	-1631
1984/3	62/2	0	8	3	-35	46	-20	8	-97	28	206	-31	282	367	562	-148	84	301	236	-60	699	-814	670	650	1566	-615
1984/4	62/2	0	-8	3	35	-46	-20	8	97	-28	206	31	282	367	-562	148	84	-301	236	60	-699	-814	-670	-650	1566	-615
1989/1	663/1	-4	0	10	-10	-18	-13	17	-74	132	-210	-230	-46	114	36	-446	754	50	-226	582	370	826	272	-162	186	-790
1989/2	221/1	-5	0	-6	-20	-70	-13	17	96	98	48	-312	262	-360	-460	168	-666	-780	-392	-24	320	-748	90	-1280	-782	-524
1992/1		0	3	-17	-14	-57	-64	-24	5	-117	-194	-92	-9	-400	-28	-88	111	-115	-93	-9	-438	506	-80	-83	717	-800
1995/1		0	-3	5	7	-2	-46	-110	-19	78	-104	256	-138	-392	416	258	-368	256	-814	-684	-488	646	-624	750	480	-530
1995/2		1	3	-5	7	42	-58	36	-19	112	-130	-98	-24	-338	52	-154	-458	-340	842	426	112	882	-420	712	1030	-184
1995/3		1	-3	5	7	-25	21	-77	19	-33	171	102	-2	487	-262	-364	-576	-29	-719	59	1145	454	-69	651	1322	-524
1995/4		1	-3	-5	7	30	6	-72	19	92	-134	122	-232	-158	-352	-134	-306	156	486	454	-720	-986	1336	376	-558	-184
1995/5		1	-3	-5	-7	-16	10	26	19	-176	42	-140	258	222	-104	188	654	612	154	664	0	-618	1104	764	-1426	-1642
1995/6		1	-3	-5	-7	-33	27	-93	19	113	-43	234	-218	171	-2	188	688	51	-781	307	323	470	-715	-681	954	432
1995/7		3	3	-5	7	-19	-1	44	-19	15	-136	13	72	470	503	204	473	-511	-637	312	-247	-299	1008	591	-804	394
1995/8		3	-3	-5	7	-51	-7	-36	19	105	-276	-295	416	-138	-547	264	513	-471	-367	272	-693	785	-160	1371	1224	-250
1995/9		3	-3	-5	7	-72	-70	-78	19	-168	186	20	-214	30	524	-72	-558	684	-682	20	-336	-790	596	-960	-498	-1006
1995/10		-4	-3	-5	7	40	-84	-22	19	-168	-234	132	178	212	-232	376	-306	236	46	-106	70	-986	246	-204	-1268	-404
2025/1		1	0	0	15	40	28	-77	-140	-48	50	-84	20	287	226	-539	526	-224	238	198	-604	679	-537	-378	1029	1337
2025/2	2025/1	1	0	0	-15	-40	-28	-77	-140	-48	-50	-84	-20	-287	-226	-539	526	224	238	-198	604	-679	-537	-378	-1029	-1337
2025/3	2025/1	-1	0	0	15	-40	28	77	-140	48	-50	-84	20	-287	226	539	-526	224	238	198	604	679	-537	378	-1029	1337
2025/4	2025/1	-1	0	0	-15	40	-28	77	-140	48	50	-84	-20	287	-226	539	-526	-224	238	-198	-604	-679	-537	378	1029	-1337
2025/5	405/1	5	0	0	-9	-8	-43	122	-59	213	224	-36	-206	413	392	311	377	337	40	-348	62	1214	-294	-534	-810	928
2025/6	405/1	-5	0	0	-9	8	-43	-122	-59	-213	-224	-36	-206	-413	392	-311	-377	-337	40	-348	-62	1214	-294	534	810	928
2160/1	135/3	0	0	5	0	-10	-80	7	113	81	-220	189	170	-130	-10	-160	631	560	229	-750	-890	-890	27	-429	-750	-1480
2160/2	270/6	0	0	5	4	42	20	-93	-59	9	-120	-47	-262	-126	178	144	-741	-444	221	538	690	-1126	-665	75	1086	1544
2160/3	135/1	0	0	5	6	47	-5	-131	56	-3	-157	-225	-70	140	-397	347	4	-748	-338	-492	-32	970	1257	102	-1488	974
2160/4	270/2	0	0	5	-8	-18	8	15	-23	-63	156	85	74	246	190	-288	-177	-792	-907	322	270	254	1123	771	-198	-1192
2160/5	270/3	0	0	5	13	-30	-61	-12	49	18	186	160	-91	-378	268	144	-570	204	-877	187	-606	431	-1151	102	-984	-265
2160/6	270/4	0	0	5	-14	-3	47	-39	-32	99	51	-83	314	-108	-299	-531	564	-12	230	268	-120	1106	739	-1086	-120	-1642
2160/7	540/1	0	0	5	-17	30	-61	-120	43	-90	-90	-8	317	-30	220	180	-630	840	599	-107	210	-421	-353	1350	1020	-997
2160/8	540/2	0	0	5	22	-9	17	75	4	183	-129	187	-34	-264	-443	609	228	60	-454	244	444	398	349	1038	-852	914
2160/9	270/5	0	0	5	22	-12	38	105	157	-117	-66	25	314	504	-380	-252	-3	-318	293	322	-120	44	-917	309	-1272	1328
2160/10	270/1	0	0	5	34	48	-70	-27	-119	51	-30	133	218	156	88	516	-639	654	461	-182	-900	704	1375	-915	-1116	-16
2160/11	135/3	0	0	-5	0	10	-80	-7	113	-81	220	189	170	130	-10	160	-631	-560	229	-750	890	-890	27	429	750	-1480
2160/12	270/6	0	0	-5	4	-42	20	93	-59	-9	120	-47	-262	126	178	-144	741	444	221	538	-690	-1126	-665	-75	-1086	1544
2160/13	135/1	0	0	-5	6	-47	-5	131	56	3	157	-225	-70	-140	-397	-347	-4	748	-338	-492	32	970	1257	-102	1488	974
2160/14	270/2	0	0	-5	-8	18	8	-15	-23	63	-156	85	74	-246	190	288	177	792	-907	322	-270	254	1123	-771	198	-1192
2160/15	270/3	0	0	-5	13	30	-61	12	49	-18	-186	160	-91	378	268	-144	570	-204	-877	187	606	431	-1151	-102	984	-265
2160/16	270/4	0	0	-5	-14	3	47	39	-32	-99	-51	-83	314	108	-299	531	-564	12	230	268	120	1106	739	1086	120	-1642
2160/17	540/1	0	0	-5	-17	-30	-61	120	43	90	90	-8	317	30	220	-180	630	-840	599	-107	-210	-421	-353	-1350	-1020	-997
2160/18	540/2	0	0	-5	22	9	17	-75	4	-183	129	187	-34	264	-443	-609	-228	-60	-454	244	-444	398	349	-1038	852	914
2160/19	270/5	0	0	-5	22	12	38	-105	157	117	66	25	314	-504	-380	252	3	318	293	322	120	44	-917	-309	1272	1328
2160/20	270/1	0	0	-5	34	-48	-70	27	-119	-51	30	133	218	-156	88	-516	639	-654	461	-182	900	704	1375	915	1116	-16
2304/1	256/1	0	0	0	0	18	0	-90	106	0	0	0	0	522	-290	0	0	-846	0	-70	0	430	0	1350	1026	-1910
2304/2	256/1	0	0	0	0	-18	0	-90	-106	0	0	0	0	522	290	0	0	846	0	70	0	430	0	-1350	1026	-1910
2304/3	256/7	0	0	4	0	0	92	-94	0	0	-284	0	396	-230	0	0	-572	0	-468	0	0	1098	0	0	1670	-594
2304/4	256/7	0	0	-4	0	0	-92	-94	0	0	284	0	-396	-230	0	0	572	0	468	0	0	1098	0	0	1670	-594
2304/5	768/1	0	0	8	12	-12	20	-62	-108	72	128	204	-228	-22	204	-600	-256	-828	-84	-348	-456	-822	1356	108	-938	1278
2304/6	768/1	0	0	8	-12	12	20	-62	108	-72	128	-204	-228	-22	-204	600	-256	828	-84	348	456	-822	-1356	-108	-938	1278
2304/7	768/1	0	0	-8	12	12	-20	-62	108	72	-128	204	228	-22	-204	-600	256	828	84	348	-456	-822	1356	-108	-938	1278
2304/8	768/1	0	0	-8	-12	-12	-20	-62	-108	-72	-128	-204	228	-22	204	600	256	-828	84	-348	456	-822	-1356	108	-938	1278
2304/9	256/3	0	0	12	32	-8	20	98	88	32	172	-256	-92	-102	296	320	76	408	-636	-552	-416	138	-64	392	582	238
2304/10	256/3	0	0	12	-32	8	20	98	-88	-32	172	256	-92	-102	-296	-320	76	-408	-636	552	416	138	64	-392	582	238
2304/11	256/3	0	0	-12	32	8	-20	98	-88	32	-172	-256	92	-102	-296	320	-76	-408	636	552	-416	138	-64	-392	582	238
2304/12	256/3	0	0	-12	-32	-8	-20	98	88	-32	-172	256	92	-102	296	-320	-76	408	636	-552	416	138	64	392	582	238
2304/13	1568/1	0	0	22	0	0	92	104	0	0	130	0	-396	472	0	0	-518	0	468	0	0	-1098	0	0	176	594
2304/14	1568/1	0	0	22	0	0	-92	-104	0	0	130	0	396	-472	0	0	-518	0	-468	0	0	-1098	0	0	-176	594
2304/15	1568/1	0	0	-22	0	0	92	-104	0	0	-130	0	-396	-472	0	0	518	0	468	0	0	-1098	0	0	-176	594
2304/16	1568/1	0	0	-22	0	0	-92	104	0	0	-130	0	396	472	0	0	518	0	-468	0	0	-1098	0	0	176	594
2400/1	96/1	0	3	0	4	20	-70	-90	140	192	-134	100	170	-110	-532	56	430	-20	270	524	-80	-330	1060	1188	1274	590
2400/2	480/4	0	3	0	4	40	90	70	40	-108	166	-40	130	-310	268	556	370	240	-130	-876	-840	-250	-880	188	-726	1550
2400/3	480/2	0	3	0	-8	-4	6	2	16	-60	-142	176	214	-278	-68	116	350	-684	-394	108	96	398	-136	436	-750	-82
2400/4	480/1	0	3	0	12	20	58	70	92	112	66	108	58	66	-388	-408	-474	540	14	-276	96	790	-308	-1036	1210	-1426

level/no.	twist of	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
2400/5	96/4	0	3	0	12	-60	42	-10	-132	-48	226	252	362	-94	-228	-408	-346	300	-466	204	-1056	-330	-612	564	-1510	-594
2400/6	480/5	0	3	0	-12	24	-38	6	-104	100	230	56	-190	202	-148	124	-206	128	190	-204	440	-1210	-816	-1412	-214	-1202
2400/7	480/6	0	3	0	-16	24	14	18	36	-104	-250	-28	54	354	-228	-408	-262	-64	374	-300	1016	-274	788	396	786	1086
2400/8		0	3	0	-18	30	38	-70	12	72	-64	-312	138	-374	-468	132	446	510	754	384	-924	-340	72	-156	-290	-376
2400/9	2400/8	0	3	0	-18	-30	-38	70	-12	72	-64	312	-138	-374	-468	132	-446	-510	754	384	924	340	-72	-156	-290	376
2400/10	480/3	0	3	0	32	-64	6	-38	116	-120	-122	-164	-146	-238	-148	-184	-470	216	806	-732	-264	638	-596	-884	930	-322
2400/11	96/2	0	3	0	-36	36	-54	22	-36	-144	50	108	-214	-446	252	72	22	684	-466	-180	-576	54	972	-684	346	1134
2400/12	96/1	0	-3	0	-4	-20	-70	-90	-140	-192	-134	-100	170	-110	532	-56	430	20	270	-524	80	-330	-1060	-1188	1274	590
2400/13	480/4	0	-3	0	-4	-40	90	70	-40	108	166	40	130	-310	-268	-556	370	-240	-130	876	840	-250	880	-188	-726	1550
2400/14	480/2	0	-3	0	8	4	6	2	-16	60	-142	-176	214	-278	68	-116	350	684	-394	-108	-96	398	136	-436	-750	-82
2400/15	480/5	0	-3	0	12	-24	-38	6	104	-100	230	-56	-190	202	148	-124	-206	-128	190	204	-440	-1210	816	1412	-214	-1202
2400/16	480/1	0	-3	0	-12	-20	58	70	-92	-112	66	-108	58	66	388	408	-474	-540	14	276	-96	790	308	1036	1210	-1426
2400/17	96/4	0	-3	0	-12	60	42	-10	132	48	226	-252	362	-94	228	408	-346	-300	-466	-204	1056	-330	612	-564	-1510	-594
2400/18	480/6	0	-3	0	16	-24	14	18	-36	104	-250	28	54	354	228	408	-262	64	374	300	-1016	-274	-788	-396	786	1086
2400/19	2400/8	0	-3	0	18	30	-38	70	12	-72	-64	-312	-138	-374	468	-132	-446	510	754	-384	-924	340	72	156	-290	376
2400/20	2400/8	0	-3	0	18	-30	38	-70	-12	-72	-64	312	138	-374	468	-132	446	-510	754	-384	924	-340	-72	156	-290	-376
2400/21	480/3	0	-3	0	-32	64	6	-38	-116	120	-122	164	-146	-238	148	184	-470	-216	806	732	264	638	596	884	930	-322
2400/22	96/2	0	-3	0	36	-36	-54	22	36	144	50	-108	-214	-446	-252	-72	22	-684	-466	180	576	54	-972	684	346	1134
2430/1		2	0	-5	-1	-39	56	51	-70	-102	246	-40	-214	-132	-187	150	339	432	341	-529	-309	428	-1222	-1146	54	20
2430/2		2	0	-5	8	42	-7	-66	-151	42	-240	185	146	156	47	-84	-426	-522	-721	-205	1014	-859	1109	-1290	-180	-547
2430/3		2	0	-5	23	-24	-73	12	65	42	24	-157	-79	-300	-205	18	492	-192	-430	-988	1086	-34	329	-708	-498	-1789
2430/4		2	0	-5	-25	9	-40	99	146	42	-306	-112	146	-108	245	-546	267	336	797	191	-669	-892	-46	558	-114	-844
2430/5		2	0	-5	-25	12	11	-12	29	30	132	191	-163	-288	203	186	-288	-468	-310	452	-354	-370	-331	60	-258	335
2430/6	2430/1	-2	0	5	-1	39	56	-51	-70	102	-246	-40	-214	132	-187	-150	-339	-432	341	-529	309	428	-1222	1146	-54	20
2430/7	2430/2	-2	0	5	8	-42	-7	66	-151	-42	240	185	146	-156	47	84	426	522	-721	-205	-1014	-859	1109	1290	180	-547
2430/8	2430/3	-2	0	5	23	24	-73	-12	65	-42	-24	-157	-79	300	-205	-18	-492	192	-430	-988	-1086	-34	329	708	498	-1789
2430/9	2430/4	-2	0	5	-25	-9	-40	-99	146	-42	306	-112	146	108	245	546	-267	-336	797	191	669	-892	-46	-558	114	-844
2430/10	2430/5	-2	0	5	-25	-12	11	12	29	-30	-132	191	-163	288	203	-186	288	468	-310	452	354	-370	-331	-60	258	335

Appendix D

Weight two newforms

The following table contains coefficients a_p for all primes $p \leq 97$ of all weight two newforms with rational coefficients for $\Gamma_0(N)$ with $N \leq 228$. They have been computed with the help of W. Stein's package HECKE which is included in the MAGMA computer algebra system ([112]). W. Stein has set up a web page containing larger tables ([97]). For all newforms occurring in this thesis I add Stein's notation.

level	in [97]	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
11	$11A1$	-2	-1	1	-2	1	4	-2	0	-1	0	7	3	-8	-6	8	-6	5	12	-7	-3	4	-10	-6	15	-7
14	$14A1$	-1	-2	0	1	0	-4	6	2	0	-6	-4	2	6	8	-12	6	-6	8	-4	0	2	8	-6	-6	-10
15		-1	-1	1	0	-4	-2	2	4	0	-2	0	-10	10	4	8	-10	-4	-2	12	-8	10	0	12	-6	2
17		-1	0	-2	4	0	-2	1	-4	4	6	4	-2	-6	4	0	6	-12	-10	4	-4	-6	12	-4	10	2
19		0	-2	3	-1	3	-4	-3	1	0	6	-4	2	-6	-1	-3	12	-6	-1	-4	6	-7	8	12	12	8
20	$20A1$	0	-2	-1	2	0	2	-6	-4	6	6	-4	2	6	-10	-6	-6	12	2	2	-12	2	8	6	-6	2
21	$21A1$	-1	1	-2	-1	4	-2	-6	4	0	-2	0	6	2	-4	0	6	12	-2	4	0	-6	-16	-12	-14	18
24	$24A1$	0	-1	-2	0	4	-2	2	-4	-8	6	8	6	-6	4	0	-2	4	-2	-4	8	10	-8	-4	-6	2
26		-1	1	-3	-1	6	1	-3	2	0	6	-4	-7	0	-1	3	0	-6	8	14	-3	2	8	12	-6	-10
26	$26B1$	1	-3	-1	1	-2	-1	-3	6	-4	2	4	3	0	-5	13	12	-10	-8	-2	-5	-10	-4	0	6	14
27	$27A1$	0	0	0	-1	0	5	0	-7	0	0	-4	11	0	8	0	0	0	-1	5	0	-7	17	0	0	-19
30	$30A1$	-1	1	-1	-4	0	2	6	-4	0	-6	8	2	-6	-4	0	-6	0	-10	-4	0	2	8	12	18	2
32	$32A1$	0	0	-2	0	0	6	2	0	0	-10	0	-2	10	0	0	14	0	-10	0	0	-6	0	0	10	18
33		1	-1	-2	4	1	-2	-2	0	8	-6	-8	6	-2	0	8	6	-4	6	-4	0	-14	-4	12	-6	2
34		1	-2	0	-4	6	2	-1	-4	0	0	-4	-4	6	8	0	-6	0	-4	8	0	2	8	0	-6	14
35		0	1	-1	1	-3	5	3	2	-6	3	-4	2	-12	-10	9	12	0	8	-4	0	2	-1	12	-12	-1
36		0	0	0	-4	0	2	0	8	0	0	-4	-10	0	8	0	0	0	14	-16	0	-10	-4	0	0	14
37		-2	-3	-2	-1	-5	-2	0	0	2	6	-4	-1	-9	2	-9	1	8	-8	8	9	-1	4	-15	4	4
37		0	1	0	-1	3	-4	6	2	6	-6	-4	1	-9	8	3	-3	12	8	-4	-15	11	-10	9	6	8
38		-1	1	0	-1	-6	5	3	1	3	9	-4	2	0	8	0	-3	9	-10	5	-6	-7	-10	-6	-12	-10
38		1	-1	-4	3	2	-1	3	-1	-1	-5	-8	-2	-8	4	8	-1	15	2	3	2	9	-10	-6	0	-2
39		1	-1	2	-4	4	1	2	0	0	-10	4	-2	6	-12	0	6	12	-2	-8	0	2	8	4	-2	10
40		0	0	1	-4	4	-2	2	4	4	-2	-8	6	-6	-8	4	6	-4	-2	8	0	-6	0	-16	-6	-14
42		1	-1	-2	-1	-4	6	2	-4	8	-2	0	-10	-6	-4	0	6	4	6	4	8	10	0	-4	-6	-14
43		-2	-2	-4	0	3	-5	-3	-2	-1	-6	-1	0	5	-1	4	-5	-12	2	-3	2	2	-8	15	-4	7
44		0	1	-3	2	-1	-4	6	8	-3	0	5	-1	0	-10	0	-6	3	-4	-1	15	-4	2	6	-9	-7
45		1	0	-1	0	4	-2	-2	4	0	2	0	-10	-10	4	-8	10	4	-2	12	8	10	0	-12	6	2
46		-1	0	4	-4	2	-2	-2	-2	1	2	0	-4	6	10	0	-4	12	-8	-10	0	6	-12	14	-6	6

level	in [97]	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
48	48A1	0	1	-2	0	-4	-2	2	4	8	6	-8	6	-6	-4	0	-2	-4	-2	4	-8	10	8	4	-6	2
49		1	0	0	0	4	0	0	0	8	2	0	-6	0	-12	0	-10	0	0	4	16	0	8	0	0	0
50	50A1	-1	1	0	2	-3	-4	-3	5	6	0	2	2	-3	-4	12	6	0	2	-13	12	11	-10	-9	15	2
50	50B1	1	-1	0	-2	-3	4	3	5	-6	0	2	-2	-3	4	-12	-6	0	2	13	12	-11	-10	9	15	-2
51		0	1	3	-4	-3	-1	-1	-1	9	6	2	-4	-3	-7	-6	-6	6	8	-4	12	2	-10	-6	0	-16
52		0	0	2	-2	-2	-1	6	-6	8	2	10	-6	-6	4	-2	6	-10	-2	10	10	2	-4	-6	-6	2
53		-1	-3	0	-4	0	-3	-3	-5	7	-7	4	5	6	-2	-2	-1	-2	-8	-12	1	-4	-1	-1	-14	1
54		-1	0	3	-1	-3	-4	0	2	-6	6	5	2	-6	-10	6	9	12	8	14	0	-7	8	-3	-18	-1
54		1	0	-3	-1	3	-4	0	2	6	-6	5	2	6	-10	-6	-9	-12	8	14	0	-7	8	3	18	-1
55		1	0	1	0	-1	2	6	-4	4	6	-8	-2	2	4	-12	-2	4	-10	-16	8	14	8	-4	10	10
56		0	2	-4	1	0	0	-2	-2	8	2	4	-6	-2	8	-4	-10	6	4	-12	0	-14	-8	6	10	-2
56		0	0	2	-1	-4	2	-6	8	0	6	8	-2	2	-4	-8	6	0	-6	-4	-8	10	16	8	-6	-6
57		-2	-1	-3	-5	1	2	-1	-1	-4	-2	-6	0	0	-1	-9	10	-8	-1	8	-12	-11	16	12	-6	-10
57		1	1	-2	0	0	6	-6	-1	4	2	8	-10	-2	-4	12	-6	-12	-2	-4	0	10	0	16	-2	10
57		-2	1	1	3	-3	-6	3	-1	4	-10	2	8	-8	-1	3	-6	0	7	8	12	-11	0	4	10	-2
58		-1	-3	-3	-2	-1	3	-4	-8	0	-1	3	-8	-2	7	11	1	-4	4	-4	-2	-12	-7	0	-6	-6
58		1	-1	1	-2	-3	-1	8	0	4	-1	-3	8	2	-11	13	-11	0	-8	-12	2	4	15	4	-10	-2
61		-1	-2	-3	1	-5	1	4	-4	-9	-6	0	8	5	-8	4	6	9	-1	-7	-8	-11	3	4	-4	-14
62		1	0	-2	0	0	2	-6	4	8	2	-1	10	-6	8	-8	-6	-12	-6	-12	8	10	-8	8	-6	2
63		1	0	2	-1	-4	-2	6	4	0	2	0	6	-2	-4	0	-6	-12	-2	4	0	-6	-16	12	14	18
64		0	0	2	0	0	-6	2	0	0	10	0	2	10	0	0	-14	0	10	0	0	-6	0	0	10	18
65		-1	-2	-1	-4	2	-1	2	-6	-6	2	-10	-2	-6	10	4	2	6	2	-4	6	-6	-12	-16	2	-2
66		-1	1	0	2	-1	-4	-6	-4	6	6	8	-10	6	8	-6	0	0	8	-4	6	2	14	-12	-6	14
66		1	-1	2	-4	-1	-6	2	4	4	6	0	6	-6	4	-12	2	12	-14	4	-12	-6	-4	4	10	-14
66		1	1	-4	-2	1	4	-2	0	-6	10	-8	-2	2	4	-2	4	0	-8	-12	2	-6	10	4	10	-2
67		2	-2	2	-2	-4	2	3	7	9	-5	-10	-1	0	-2	-1	10	9	-2	1	0	-7	-8	4	7	0
69		1	1	0	-2	4	-6	4	2	-1	2	4	2	2	10	0	-12	-12	-6	-10	8	-14	10	12	-16	-10
70		1	0	-1	-1	4	-6	2	0	0	6	8	-10	2	4	8	-2	-8	-14	-12	-16	2	-8	8	10	2
72	72A1	0	0	2	0	-4	-2	-2	-4	8	-6	8	6	6	4	0	2	-4	-2	-4	-8	10	-8	4	6	2
73		1	0	2	2	-2	-6	2	8	4	2	-2	-6	6	-2	6	10	-6	-14	8	0	1	-4	-14	-6	-10
75		2	-1	0	-3	2	1	2	-5	6	10	-3	2	-8	1	2	-4	-10	7	-3	-8	-14	0	6	0	17
75		1	1	0	0	-4	2	-2	4	0	-2	0	10	10	-4	-8	10	-4	-2	-12	-8	-10	0	-12	-6	-2
75		-2	1	0	3	2	-1	-2	-5	-6	10	-3	-2	-8	-1	-2	4	-10	7	3	-8	14	0	-6	0	-17
76		0	2	-1	-3	5	-4	-3	-1	8	-2	4	10	10	1	-1	-4	6	-13	-12	2	9	8	-12	12	-8
77		0	-3	-1	-1	-1	-4	2	-6	-5	10	1	-5	-2	-8	8	-6	3	-2	-3	1	10	6	12	-15	-5
77		1	2	-2	-1	1	4	4	0	-4	-6	10	-6	4	12	-10	-6	2	0	8	-12	-8	8	0	-6	-10
77		0	1	3	1	-1	-4	-6	2	3	-6	5	11	6	8	0	-6	-9	-10	5	9	2	-10	12	-3	-1
78		-1	-1	2	4	-4	1	2	-8	0	6	-4	-2	-10	4	8	-10	4	-2	-16	-8	2	8	12	14	10
79		-1	-1	-3	-1	-2	3	-6	4	2	-6	-10	-2	-10	4	7	8	-3	-4	8	15	2	-1	-6	-7	-19
80		0	0	1	4	-4	-2	2	-4	-4	-2	8	6	-6	8	-4	6	4	-2	-8	0	-6	0	16	-6	-14
80		0	2	-1	-2	0	2	-6	4	-6	6	4	2	6	10	6	-6	-12	2	-2	12	2	-8	-6	-6	2
82		-1	-2	-2	-4	-2	4	-2	6	-8	0	-8	2	-1	-12	4	-4	8	-14	-2	8	10	4	12	-14	6
83		-1	-1	-2	-3	3	-6	5	2	-4	-7	5	-11	-2	-8	0	6	5	5	-2	2	0	14	-1	0	-8
84		0	-1	4	-1	2	-6	-4	-4	2	-2	0	2	0	-4	12	-6	-8	6	-8	14	-2	12	-4	0	-2
84		0	1	0	1	-6	2	0	-4	-6	6	8	2	12	-4	12	-6	0	-10	8	6	-10	-4	-12	12	-10
85		1	2	-1	-2	2	2	1	0	6	-6	-10	2	10	4	12	-10	8	-14	8	-2	-14	-14	4	6	2
88		0	-3	-3	-2	-1	0	-6	4	1	-8	-7	-1	4	6	-8	2	-1	4	-5	3	16	2	-2	15	-7
89		-1	-1	-1	-4	-2	2	3	-5	7	0	-9	-2	0	-7	-12	-3	4	6	12	-10	7	-6	12	-1	9
89		1	2	-2	2	-4	2	6	-2	2	-6	6	10	-6	2	12	-6	-10	-6	12	4	10	-12	-6	1	-18
90		-1	0	1	2	6	-4	-6	-4	0	-6	-4	8	0	8	0	-6	6	2	-4	-12	-10	-4	12	12	2
90		1	0	-1	2	-6	-4	6	-4	0	6	-4	8	0	8	0	6	-6	2	-4	12	-10	-4	-12	-12	2
90		1	0	1	-4	0	2	-6	-4	0	6	8	2	6	-4	0	6	0	-10	-4	0	2	8	-12	-18	2

level	in [97]	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
91		-2	0	-3	-1	-6	-1	4	5	3	-5	-3	-4	-6	-1	7	-9	8	-10	-6	-8	-13	3	15	3	7
91		0	-2	-3	1	0	1	-6	-7	3	-9	5	2	-6	-1	3	-9	0	-10	14	-6	11	-1	3	15	-1
92		0	1	0	2	0	-1	-6	2	-1	-3	5	8	3	8	9	6	-12	14	8	-15	-7	-10	6	0	-10
92		0	-3	-2	-4	2	-5	4	-2	1	-7	-3	2	-9	-8	9	2	0	-2	14	-3	-3	-6	8	12	0
94		1	0	0	0	2	-4	-2	-2	4	4	4	2	6	6	-1	2	12	2	2	8	-14	-16	-16	-10	-14
96	96$A1$	0	1	2	-4	4	-2	-6	-4	0	2	4	-2	2	4	8	10	-4	6	4	-16	-6	4	12	10	-14
96	96$B1$	0	-1	2	4	-4	-2	-6	4	0	2	-4	-2	2	-4	-8	10	4	6	-4	16	-6	-4	-12	10	-14
98		-1	2	0	0	0	4	-6	-2	0	-6	4	2	-6	8	12	6	6	-8	-4	0	-2	8	6	6	10
99		-1	0	-4	-2	-1	-2	2	-6	4	-6	4	-6	-10	6	-8	0	4	-6	8	0	-2	-10	12	0	2
99		1	0	4	-2	1	-2	-2	-6	-4	6	4	-6	10	6	8	0	-4	-6	8	0	-2	-10	-12	0	2
99		-1	0	2	4	-1	-2	2	0	-8	6	-8	6	2	0	-8	-6	4	6	-4	0	-14	-4	-12	6	2
99		2	0	-1	-2	-1	4	2	0	1	0	7	3	8	-6	-8	6	-5	12	-7	3	4	-10	6	-15	-7
100		0	2	0	-2	0	-2	6	-4	-6	6	-4	-2	6	10	6	6	12	2	-2	-12	-2	8	-6	-6	-2
101		0	-2	-1	-2	-2	1	3	-5	1	-4	-9	-2	8	-8	7	-2	-14	4	2	13	8	-9	-4	14	2
102		-1	-1	-4	-2	0	-6	-1	4	6	-4	-6	-4	-10	-4	4	-2	12	-4	-12	-6	2	10	-12	-2	6
102		-1	1	0	2	0	2	-1	-4	-6	0	-10	8	6	-4	12	6	-12	8	-4	6	2	-10	12	-18	14
102		1	1	-2	0	-4	-2	1	4	0	-10	8	-2	10	12	0	6	12	-10	-12	0	10	-8	4	-6	-14
104		0	1	-1	5	-2	-1	-3	-2	4	-6	-4	11	8	-1	9	-12	6	0	6	7	-2	12	-16	-10	-10
105		1	1	1	1	0	-6	2	-8	8	-2	4	-2	-6	4	8	10	4	-2	4	-12	-2	8	-4	-6	-18
106		-1	-1	-4	0	-4	1	5	-7	1	5	-4	1	-10	-10	-6	-1	-6	4	4	15	-8	1	-3	2	17
106		-1	2	1	-2	5	-4	3	-4	-3	-6	7	-6	2	7	4	1	7	2	16	12	-12	-7	-14	17	3
106		1	1	0	-4	0	5	-3	-1	3	9	-4	5	6	-10	6	-1	6	8	-4	-3	-4	-13	3	18	-7
106		1	-2	3	2	-3	-4	3	-4	-9	6	5	-10	6	-1	0	-1	15	-10	-4	12	8	11	-6	9	-13
108		0	0	0	5	0	-7	0	-1	0	0	-4	-1	0	8	0	0	0	-13	11	0	17	-13	0	0	5
109		1	0	3	2	1	0	-8	-5	7	-5	6	2	2	-4	9	12	12	-5	-12	-6	-5	8	-2	1	1
110		-1	1	-1	5	1	2	3	-7	-6	-3	-7	-7	6	8	6	-3	-6	-1	8	3	2	-10	-6	9	-4
110		1	1	-1	-1	-1	2	-3	-1	6	-9	5	5	-6	8	6	9	6	5	8	-9	-10	14	-6	-15	8
110		1	-1	1	3	1	-6	-7	5	-6	5	-3	3	2	4	-2	-1	-10	7	8	7	14	10	-6	-15	-12
112		0	-2	-4	-1	0	0	-2	2	-8	2	-4	-6	-2	-8	4	-10	-6	4	12	0	-14	8	-6	10	-2
112		0	0	2	1	4	2	-6	-8	0	6	-8	-2	2	4	8	6	0	-6	4	8	10	-16	-8	-6	-6
112		0	2	0	-1	0	-4	6	-2	0	-6	4	2	6	-8	12	6	6	8	4	0	2	-8	6	-6	-10
113		-1	2	2	0	0	2	-6	6	-6	-6	-4	2	-2	6	6	10	6	6	2	-6	2	10	-4	-14	-14
114		-1	-1	0	4	4	0	-2	1	-2	-6	6	-8	10	-12	10	2	4	-10	0	-16	-2	10	-16	-2	-10
114		1	-1	2	0	-4	2	-6	-1	-4	-2	4	10	10	4	-4	-10	12	14	-12	8	-6	-4	12	-6	10
114		1	1	0	-4	0	-4	6	1	-6	6	2	-4	6	-4	6	6	-12	14	8	0	14	-10	-12	-6	-10
115		2	0	-1	1	2	-2	3	-2	1	7	-5	11	1	0	0	11	-13	-8	5	5	6	-12	9	4	-14
116		0	1	3	-4	3	5	-6	-4	-6	-1	5	8	0	-1	-3	3	6	2	8	6	-16	11	6	-12	8
116		0	2	-2	4	-6	2	2	-6	4	-1	-6	2	2	10	-2	10	0	10	-12	8	10	-6	16	2	10
116		0	-3	3	4	-1	-3	2	4	-6	-1	9	-8	-8	-5	-7	-5	-10	10	8	-2	0	-1	6	12	0
117		-1	0	-2	-4	-4	1	-2	0	0	10	4	-2	-6	-12	0	-6	-12	-2	-8	0	2	8	-4	2	10
118		-1	-1	-3	-1	-2	-2	-2	3	0	-1	10	-12	7	-6	-6	-11	-1	-12	10	4	12	-15	-14	4	0
118		-1	2	2	-3	1	3	-1	-8	8	-4	-4	-1	5	-9	2	12	1	10	4	-15	10	11	-11	-6	14
118		1	-1	1	3	2	-6	-2	-5	4	-5	2	8	7	-6	-2	9	-1	-8	-2	12	4	5	14	0	8
118		1	2	-2	-3	-1	-3	7	4	4	4	-4	-7	-11	9	10	0	-1	-2	4	9	-14	11	-13	18	2
120		0	1	-1	4	0	-6	-2	4	-8	-6	0	-6	10	-4	8	10	0	6	-4	0	-14	16	12	2	2
120		0	1	1	0	-4	6	-6	-4	0	-2	-8	-2	-6	12	8	6	12	14	4	8	-6	-8	-12	10	2
121		0	-1	-3	0	0	0	0	0	-9	0	-5	7	0	0	-12	6	-15	0	13	-3	0	0	0	-9	17
121		1	2	1	-2	0	1	-5	6	2	9	-2	-3	-5	0	2	9	8	6	2	12	-2	-10	6	-9	-13
121		-1	2	1	2	0	-1	5	-6	2	-9	-2	-3	5	0	2	9	8	-6	2	12	2	10	-6	-9	-13
121		2	-1	1	2	0	-4	2	0	-1	0	7	3	8	6	8	-6	5	-12	-7	-3	-4	10	6	15	-7
122		-1	-2	1	-5	-3	-3	0	0	5	6	0	-12	-3	-8	12	-2	-9	-1	7	-16	-3	1	-12	12	2
123		0	-1	-2	-4	5	-4	-5	-2	4	1	-5	-7	-1	7	7	-14	-12	-3	-2	-3	13	-2	-2	18	-14

level	in [97]	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
123		-2	1	-4	-2	-3	-6	3	0	-6	5	7	-7	1	-1	3	-6	0	-3	-2	-3	-11	10	-16	-10	-12
124		0	0	1	3	6	-4	0	-5	-4	2	-1	-2	-9	2	4	12	9	12	-12	5	-14	10	2	6	-7
124		0	-2	-3	-1	-6	2	6	-1	-6	0	1	-10	-9	8	0	0	-3	-10	-4	-15	14	8	6	12	-7
126		-1	0	2	-1	4	6	-2	-4	-8	2	0	-10	6	-4	0	-6	-4	6	4	-8	10	0	4	6	-14
126		1	0	0	1	0	-4	-6	2	0	6	-4	2	-6	8	12	-6	6	8	-4	0	2	8	6	6	-10
128		0	-2	-2	-4	2	-2	-2	-2	4	6	0	-10	-6	-6	-8	6	-14	-2	-10	12	14	-8	6	-2	-2
128		0	2	2	-4	-2	2	-2	2	4	-6	0	10	-6	6	-8	-6	14	2	10	12	14	-8	-6	-2	-2
128		0	2	-2	4	-2	-2	-2	2	-4	6	0	-10	-6	6	8	6	14	-2	10	-12	14	8	-6	-2	-2
128		0	-2	2	4	2	2	-2	-2	-4	-6	0	10	-6	-6	8	-6	-14	2	-10	-12	14	8	6	-2	-2
129		0	-1	-2	-2	-5	3	-3	2	-1	0	-5	8	-7	-1	-8	3	12	-8	-15	-14	12	-16	15	10	11
129		1	1	2	0	0	-2	-6	4	-4	-6	8	6	2	-1	4	-2	0	14	12	8	2	-8	0	14	-14
130		-1	-2	1	-4	-6	1	-6	2	6	-6	2	2	-6	2	-12	6	6	2	-4	-6	-10	-4	0	-6	2
130		1	2	-1	-4	-2	-1	2	6	6	2	-6	-2	10	-10	-12	2	10	2	-12	10	10	-4	0	-14	14
130		1	0	1	0	0	1	2	-8	-4	-2	-4	6	10	0	8	6	8	-2	4	-12	10	-8	12	10	-14
131		0	-1	-2	-1	0	-3	4	-2	-2	0	-2	-8	-3	3	10	-9	1	-15	-6	10	4	-8	4	-11	12
132		0	-1	2	2	-1	6	-4	-2	-8	0	0	-6	0	10	0	14	-12	-14	4	0	6	2	16	-14	-2
132		0	1	2	-2	1	-2	4	-6	0	-8	-8	10	8	-2	-8	-2	12	10	12	8	6	-2	16	-14	-2
135		-2	0	-1	-3	-2	-5	-8	1	6	2	0	5	-10	4	4	-2	-8	7	-9	2	-5	-3	6	-12	-13
135		2	0	1	-3	2	-5	8	1	-6	-2	0	5	10	4	-4	2	8	7	-9	-2	-5	-3	-6	12	-13
136		0	2	0	0	2	-6	-1	4	4	0	-8	-4	6	8	-8	10	0	12	8	12	2	-4	16	10	-18
136		0	-2	-2	-2	-6	2	1	0	6	-10	2	6	-6	-8	0	-10	-8	14	4	2	-14	-10	8	-10	2
138		-1	-1	-2	-2	-6	-2	0	0	-1	6	8	0	10	-12	-8	2	-12	4	-12	0	-10	-6	14	0	-6
138		-1	1	0	2	0	2	0	2	-1	-6	-4	-10	-6	2	0	12	12	-10	14	0	2	-10	0	12	-10
138		1	-1	2	0	0	-2	2	-8	-1	-2	-8	2	10	8	8	2	-4	2	8	0	-6	8	-16	18	10
139		1	2	-1	3	5	-7	-6	-2	2	9	9	2	-6	-4	8	0	6	4	5	5	-6	-5	7	7	-12
140		0	3	-1	-1	-5	-3	-1	6	6	-9	-4	2	-4	10	-1	4	-8	-8	12	8	2	13	-4	4	-13
140		0	1	1	1	3	-1	-3	2	-6	-9	8	-10	0	2	-3	0	12	8	8	0	14	5	-12	12	17
141		0	-1	-1	-3	-3	-4	8	-6	3	-1	4	1	-10	-8	-1	10	-10	2	4	-6	-8	-3	-18	-2	5
141		-1	-1	0	4	0	6	-6	2	4	8	6	-6	-8	-6	1	2	12	2	-2	0	-10	-4	4	-10	-18
141		-1	1	2	0	4	-2	2	0	0	-6	-4	-10	-2	8	-1	-2	-4	14	-8	16	2	8	-4	18	-14
141		2	1	-1	-3	1	-2	2	6	3	3	2	-7	10	-10	-1	4	8	-10	10	-14	-10	17	8	6	1
141		-2	1	-3	-3	-5	2	-6	-6	9	1	-2	1	6	2	1	0	-12	-2	2	-2	-2	-15	-4	10	1
142		-1	-1	-2	-1	-2	-3	-6	5	-1	6	1	6	-6	5	-3	-6	2	-6	-14	-1	-17	10	4	9	-6
142		-1	0	2	0	6	4	6	-8	-4	-2	-8	10	-2	-8	-4	0	10	-8	2	1	-2	0	-4	6	14
142		-1	3	2	-3	-6	-5	6	1	5	-2	-5	-2	10	1	-1	6	-2	-2	2	1	7	-6	-4	9	2
142		1	1	0	-1	0	-1	0	-1	3	0	5	-4	0	-1	9	6	6	2	8	-1	-1	8	12	-3	-16
142		1	-3	-4	-3	0	1	0	-5	-7	-8	7	4	4	-5	-13	-6	10	-2	-4	1	7	0	-4	-3	-4
143		0	-1	-1	-2	-1	-1	-4	2	7	-2	-3	-11	10	-4	-4	2	-1	-2	-1	-9	-16	8	0	-7	-13
144		0	0	2	0	4	-2	-2	4	-8	-6	-8	6	6	-4	0	2	4	-2	4	8	10	8	-4	6	2
144		0	0	0	4	0	2	0	-8	0	0	4	-10	0	-8	0	0	0	14	16	0	-10	4	0	0	14
145		-1	0	-1	-2	-6	2	-2	-2	2	-1	2	10	2	8	-12	-6	-8	-6	2	-12	-6	-10	-14	18	2
147		-1	-1	2	0	4	2	6	-4	0	-2	0	6	-2	-4	0	6	-12	2	4	0	6	-16	12	14	-18
147		2	-1	2	0	-2	-1	0	-1	0	4	-9	3	10	5	6	12	12	-10	-5	-6	3	-1	-6	-16	6
147		2	1	-2	0	-2	1	0	1	0	4	9	3	-10	5	-6	12	-12	10	-5	-6	-3	-1	6	16	-6
148		0	-1	-4	-3	5	0	-6	2	-6	-6	4	1	-9	4	-7	9	-4	-8	-12	3	-5	6	-1	2	0
150		-1	-1	0	2	2	6	2	0	-4	0	-8	2	2	-4	-8	6	10	2	-8	12	-4	0	-4	-10	-8
150		1	-1	0	4	0	-2	-6	-4	0	-6	8	-2	-6	4	0	6	0	-10	4	0	-2	8	-12	18	-2
150		1	1	0	-2	2	-6	-2	0	4	0	-8	-2	2	4	8	-6	10	2	8	12	4	0	4	-10	8
152		0	-2	-1	-3	-3	-4	5	-1	0	2	8	-10	6	-7	-9	-8	14	-5	0	-6	-15	-4	4	0	16
152		0	1	0	3	2	1	-5	1	-1	-3	4	2	-8	-8	-8	9	1	14	13	10	9	-10	10	-12	14
153		-2	0	-1	-2	-3	-5	-1	-1	-7	6	4	10	9	1	-12	-12	6	2	4	-8	0	-6	4	2	8
153		2	0	1	-2	3	-5	1	-1	7	-6	4	10	-9	1	12	12	-6	2	4	8	0	-6	-4	-2	8

level	in [97]	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
153		1	0	2	4	0	-2	-1	-4	-4	-6	4	-2	6	4	0	-6	12	-10	4	4	-6	12	4	-10	2
153		0	0	-3	-4	3	-1	1	-1	-9	-6	2	-4	3	-7	6	6	-6	8	-4	-12	2	-10	6	0	-16
154		-1	0	-4	-1	-1	2	-4	-6	4	-2	-2	10	4	-8	2	6	-12	-14	-12	-8	4	0	-6	-6	-14
154		-1	2	2	-1	1	-4	0	4	4	2	-10	-6	0	-4	10	-14	10	-8	8	-4	4	16	4	10	6
154		1	0	2	-1	-1	2	2	0	-8	-2	-8	-2	10	4	8	6	0	10	-12	16	-14	0	0	-6	10
155		0	-1	-1	0	-4	-6	5	-1	8	-10	-1	1	-3	-7	-6	5	11	-12	-2	9	-9	-10	9	0	-14
155		-1	2	-1	4	4	0	-8	4	2	-6	1	-4	-6	-6	8	-12	-4	10	8	0	-4	0	2	14	-18
155		-2	-1	1	-2	2	-6	-7	-5	4	0	1	-7	-3	9	-2	9	-5	-8	8	-3	-1	0	-11	10	18
156		0	-1	-4	-2	-4	1	2	-2	0	-6	-10	10	8	4	-4	-10	-8	-14	2	16	-10	-16	0	-4	-2
156		0	1	0	2	0	1	-6	2	0	-6	2	2	-12	-4	0	6	12	2	-10	12	14	8	12	0	-10
158		-1	-1	-1	-3	4	-7	-4	-6	6	4	8	10	-8	-8	-3	2	1	0	-4	-11	-6	-1	6	-15	1
158		-1	1	3	-1	0	5	0	2	-6	0	-4	2	-12	8	-9	6	-9	8	-4	-9	2	1	18	9	17
158		1	-1	1	3	2	-1	-2	0	-6	-10	2	-2	2	4	3	4	5	12	8	-13	-6	-1	-6	-15	13
158		1	2	-2	0	-4	2	-2	0	0	8	8	4	-10	-2	0	-8	14	0	8	8	6	-1	12	6	10
158		1	-3	-3	-3	-2	-5	6	0	-2	6	-10	-10	2	4	-3	-12	-1	12	-8	-3	-6	1	14	-7	-11
160	160$A1$	0	-2	-1	-2	-4	-6	2	8	-6	-2	4	2	-10	-2	-2	2	0	2	-6	-12	10	-8	-10	-6	10
160		0	2	-1	2	4	-6	2	-8	6	-2	-4	2	-10	2	2	2	0	2	6	12	10	8	10	-6	10
161		-1	0	2	1	4	6	-2	4	-1	-2	-4	-2	-6	12	-12	-10	0	2	12	8	-14	8	-4	6	-10
162		-1	0	-3	-4	0	-1	-3	-4	0	9	-4	-1	6	8	-12	-6	0	-1	-4	-12	11	-16	-12	-3	2
162		-1	0	0	2	3	2	3	-1	6	-6	-4	-4	-9	-1	6	-12	-3	8	5	12	11	-4	-12	-6	5
162		1	0	0	2	-3	2	-3	-1	-6	6	-4	-4	9	-1	-6	12	3	8	5	-12	11	-4	12	6	5
162		1	0	3	-4	0	-1	3	-4	0	-9	-4	-1	-6	8	12	6	0	-1	-4	12	11	-16	12	3	2
163		0	0	-4	2	-6	4	0	-6	6	-4	-6	-8	3	7	1	-9	-2	3	-2	-5	-2	-8	5	-14	-11
166		-1	-1	-2	1	-5	-2	-3	-2	4	-3	1	1	6	8	12	-14	-3	-7	2	-14	-4	-6	-1	4	12
168		0	-1	2	1	0	6	-2	4	-4	-10	-8	6	-2	-4	8	-10	12	-2	12	-12	-14	-8	12	-2	10
168		0	1	2	-1	0	-2	6	-4	-4	6	-8	-10	-10	12	-8	6	4	-10	12	4	2	8	4	6	10
170		-1	-2	-1	2	6	2	1	8	-6	-6	2	2	-6	-4	12	6	0	2	8	-6	2	-10	12	6	2
170		-1	3	-1	2	-4	-3	1	3	-6	9	-3	-8	-6	6	-13	-9	15	7	-2	9	-3	0	12	-9	7
170		-1	1	1	2	0	5	-1	-1	6	-9	-1	-4	-6	2	-9	-9	3	-7	14	3	11	8	0	-9	-7
170		-1	-2	1	-2	-2	-6	1	-8	-2	6	-2	6	2	-4	4	-10	0	-10	8	14	10	-14	-4	6	-14
170		1	1	-1	2	0	-1	-1	-1	-6	-3	5	8	6	-10	-3	-3	3	11	2	9	11	8	-12	15	-7
171		-1	0	2	0	0	6	6	-1	-4	-2	8	-10	2	-4	-12	6	12	-2	-4	0	10	0	-16	2	10
171		2	0	-1	3	3	-6	-3	-1	-4	10	2	8	8	-1	-3	6	0	7	8	-12	-11	0	-4	-10	-2
171		2	0	3	-5	-1	2	1	-1	4	2	-6	0	0	-1	9	-10	8	-1	8	12	-11	16	-12	6	-10
171		0	0	-3	-1	-3	-4	3	1	0	-6	-4	2	6	-1	3	-12	6	-1	-4	-6	-7	8	-12	-12	8
172		0	-2	0	-4	-3	-1	-3	2	-3	6	5	8	-3	1	-12	-9	-12	-10	11	6	-10	8	-15	0	-1
174		-1	-1	3	-3	6	0	7	5	-8	1	-8	-3	-5	3	9	-2	-11	-6	0	0	-10	-2	0	10	0
174		-1	1	2	0	-4	6	-2	4	0	-1	-4	-6	6	-12	-8	-6	8	10	-4	-8	2	4	0	14	18
174		-1	1	-3	5	6	-4	3	-1	0	-1	-4	-1	-9	-7	-3	-6	3	-10	-4	12	2	14	0	-6	8
174		1	-1	1	1	6	-4	-7	-3	4	-1	0	-7	5	-5	-5	10	3	10	0	-4	10	-6	16	-10	-8
174		1	1	-1	1	-2	0	-3	-1	-4	1	4	3	-7	9	-1	-2	-3	6	12	16	-10	10	0	6	0
175		0	-1	0	-1	-3	-5	-3	2	6	3	-4	-2	-12	10	-9	-12	0	8	4	0	-2	-1	-12	-12	1
175		2	1	0	-1	-3	1	7	0	6	-5	2	2	2	-4	-3	6	10	-8	2	-8	6	-5	-4	0	7
175		-2	-1	0	1	-3	-1	-7	0	-6	-5	2	-2	2	4	3	-6	10	-8	-2	-8	-6	-5	4	0	-7
176		0	3	-3	2	1	0	-6	-4	-1	-8	7	-1	4	-6	8	2	1	4	5	-3	16	-2	2	15	-7
176		0	1	1	2	-1	4	-2	0	1	0	-7	3	-8	6	-8	-6	-5	12	7	3	4	10	6	15	-7
176		0	-1	-3	-2	1	-4	6	-8	3	0	-5	-1	0	10	0	-6	-3	-4	1	-15	-4	-2	-6	-9	-7
178		-1	2	2	0	0	-4	2	-2	8	0	0	0	-10	-2	-8	6	10	-4	-8	8	-2	8	14	1	-2
178		1	1	3	-4	-6	2	3	5	-3	0	5	-10	0	-1	12	9	12	-10	-4	-6	-1	-10	-12	-1	17
179		2	0	3	-4	4	-1	1	-3	6	3	-8	2	12	-11	1	0	-5	14	-9	0	10	10	17	-1	-14
180		0	0	1	2	0	2	6	-4	-6	-6	-4	2	-6	-10	6	6	-12	2	2	12	2	8	-6	6	2
182		-1	1	4	-1	-1	1	4	2	-7	-8	3	7	-7	-8	3	0	-6	-13	7	4	9	-13	-16	-6	11

level	in [97]	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
182		-1	3	0	1	-5	-1	-4	2	5	4	1	7	-9	-12	-7	-4	-6	13	11	0	7	-17	4	14	5
182		1	0	2	-1	4	-1	-6	0	8	-10	-8	6	-6	4	-8	6	8	10	4	-8	2	8	0	18	2
182		1	3	-4	-1	1	-1	0	-6	-7	-4	7	9	-3	4	7	0	-10	1	1	16	5	11	0	-6	-1
182		1	1	0	1	-3	1	0	2	-3	0	5	-7	3	8	-3	-12	6	-1	5	12	11	-1	12	-18	17
184		0	-1	-2	-4	-2	7	-4	-6	-1	5	3	2	-9	8	-1	-6	-8	-10	2	-13	-3	6	0	-4	-8
184		0	0	0	4	6	-2	6	-6	1	-6	0	-8	6	-2	-8	-8	4	-4	2	-8	6	12	10	10	-18
184		0	3	0	-2	0	-5	-6	6	1	9	3	-8	3	-8	7	-2	4	-10	8	7	9	-6	-14	16	6
184		0	-1	-4	2	-4	-5	-2	6	1	1	-9	-4	3	8	-5	6	-4	-10	-4	-5	-15	-6	6	-8	10
185		1	-2	-1	-2	0	-2	2	2	-8	2	-6	-1	10	-4	-10	-6	-6	2	-14	0	2	-6	18	2	-10
185		-2	1	-1	-5	3	-2	-4	-4	-2	2	0	-1	7	-10	11	-3	0	-4	16	-15	11	-12	-3	-4	8
185		0	-1	1	-3	-5	4	-4	-8	4	4	2	1	-5	-6	9	3	-8	-10	-4	5	-15	-14	11	-2	10
186		-1	-1	-1	2	3	3	1	7	0	4	1	-10	-6	6	-5	-2	6	3	-3	7	-10	-1	17	6	5
186		-1	1	3	-2	5	-7	-1	7	4	-8	-1	-6	-2	-10	-1	6	-10	1	-3	3	14	-11	7	-6	-3
186		1	1	1	-2	-3	-1	3	-5	4	0	1	-2	2	-6	-7	14	10	7	-7	-3	-6	15	-1	10	13
187		2	0	4	-5	-1	4	1	2	-2	-3	4	-2	-3	-2	3	9	-3	-10	7	2	-3	0	14	1	-10
187		0	1	3	2	1	2	-1	2	-3	-6	-7	-7	12	-10	0	6	-3	8	-7	-9	2	8	6	15	11
189		-2	0	-1	-1	-4	-2	3	-8	-6	-4	6	-3	1	11	9	6	-15	4	-8	-12	6	-1	-9	2	12
189		0	0	3	1	6	-4	3	2	-6	-6	-4	-7	-3	-1	9	-6	9	-10	-4	0	2	-1	3	6	-10
189		2	0	1	-1	4	-2	-3	-8	6	4	6	-3	-1	11	-9	-6	15	4	-8	12	6	-1	9	-2	12
189		0	0	-3	1	-6	-4	-3	2	6	6	-4	-7	3	-1	-9	6	-9	-10	-4	0	2	-1	-3	-6	-10
190		-1	-1	-1	-1	0	-3	-7	-1	-5	-5	10	2	2	6	0	9	-7	-4	7	0	-9	-10	-2	-10	-18
190		1	-3	-1	-5	-4	-1	-3	1	7	-3	-2	-2	-6	6	0	-13	-9	-12	-3	0	11	-2	-10	2	-2
190		1	1	1	-1	0	-1	-3	1	3	-3	2	-10	6	2	0	3	3	8	-7	12	-13	14	6	6	-10
192		0	-1	-2	-4	-4	2	-6	4	0	-2	4	2	2	-4	8	-10	4	-6	-4	-16	-6	4	-12	10	-14
192		0	1	2	0	-4	2	2	4	-8	-6	8	-6	-6	-4	0	2	-4	2	4	8	10	-8	4	-6	2
192		0	1	-2	4	4	2	-6	-4	0	-2	-4	2	2	4	-8	-10	-4	-6	4	16	-6	-4	12	10	-14
192		0	-1	2	0	4	2	2	-4	8	-6	-8	-6	-6	4	0	2	4	2	-4	-8	10	8	-4	-6	2
194		1	0	4	-4	4	-4	6	-6	-4	0	0	-8	-2	-8	0	6	6	10	6	0	-10	8	-2	14	-1
195		2	-1	1	3	-1	-1	-1	-2	-3	-2	-6	11	-5	4	-10	11	8	13	12	-5	10	-3	-12	-15	17
195		2	1	-1	-1	5	-1	-7	-6	3	2	2	7	9	-8	10	5	0	5	-4	9	-6	-3	-4	11	-11
195		-1	1	1	0	4	1	2	-4	8	-2	-8	6	-6	-4	-8	6	-12	-2	-4	0	-6	16	-4	10	18
195		2	1	1	-3	-5	1	5	2	-1	10	-2	-3	-9	-4	10	9	0	-11	-4	15	6	-11	8	-11	-9
196		0	1	3	0	-3	2	3	-1	3	-6	-7	-1	6	-4	-9	3	9	-1	-7	0	-1	-13	12	15	-10
196		0	-1	-3	0	-3	-2	-3	1	3	-6	7	-1	-6	-4	9	3	-9	1	-7	0	1	-13	-12	-15	10
197		-2	0	0	-3	4	-2	-8	-3	-3	7	-10	7	9	1	-11	10	0	5	-10	8	6	2	-7	-8	-2
198		-1	0	0	2	1	2	6	2	0	6	-4	2	-6	-10	-12	12	-12	-10	8	-12	14	2	12	0	2
198		-1	0	4	-2	-1	4	2	0	6	-10	-8	-2	-2	4	2	-4	0	-8	-12	-2	-6	10	-4	-10	-2
198		-1	0	-2	-4	1	-6	-2	4	-4	-6	0	6	6	4	12	-2	-12	-14	4	12	-6	-4	-4	-10	-14
198		1	0	0	2	-1	2	-6	2	0	-6	-4	2	6	-10	12	-12	12	-10	8	12	14	2	-12	0	2
198		1	0	0	2	1	-4	6	-4	-6	-6	8	-10	-6	8	6	0	0	8	-4	-6	2	14	12	6	14
200		0	2	0	2	-4	4	0	-4	-2	2	0	4	2	-6	-6	-4	-12	-10	14	8	8	16	2	6	16
200		0	-3	0	2	1	4	5	1	-2	-8	10	-6	-3	4	4	6	8	10	-1	-12	3	6	-13	-9	-14
200		0	0	0	4	4	2	-2	4	-4	-2	-8	-6	-6	8	-4	-6	-4	-2	-8	0	6	0	16	-6	14
200		0	3	0	-2	1	-4	-5	1	2	-8	10	6	-3	-4	-4	-6	8	10	1	-12	-3	6	13	-9	14
200		0	-2	0	-2	-4	-4	0	-4	2	2	0	-4	2	6	6	4	-12	-10	-14	8	-8	16	-2	6	-16
201		1	-1	-3	-3	0	4	2	-2	-7	-8	-1	-3	-9	9	0	1	-9	14	-1	-4	11	-16	5	0	16
201		-2	-1	0	0	-6	4	-7	-5	-1	1	-4	3	0	-6	9	10	3	2	-1	-16	-7	8	-4	-15	4
201		-1	1	-1	-5	-4	-4	6	-2	-3	4	-7	5	-3	7	8	-5	3	-2	1	-12	-13	-8	1	4	-12
202		-1	0	2	1	4	0	5	1	6	-5	0	-8	-4	-5	6	3	-12	-1	2	-10	-16	-2	16	0	13
203		1	2	2	1	-4	-2	4	2	0	-1	-2	2	0	0	-10	6	12	-4	12	-8	-4	12	-16	12	12
203		-2	-1	-4	1	2	4	-2	5	9	-1	-8	8	-3	-6	-7	9	0	2	3	7	-1	0	14	15	3
203		-1	-1	1	1	-5	-5	-4	-4	6	1	7	-10	0	-9	7	3	0	14	-6	8	-16	-9	16	-6	0

level	in [97]	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97
204		0	-1	-1	4	3	3	-1	1	3	-10	6	-4	5	-1	-2	-14	-6	8	-12	12	2	-14	6	16	0
204		0	1	1	0	5	-5	1	1	-3	2	2	-8	-5	-9	6	-6	6	-4	12	-12	-2	10	-2	12	16
205		-1	2	-1	2	6	2	2	-6	-4	10	0	-6	1	-4	-2	-14	12	-10	-2	-2	6	-2	0	10	10
205		1	2	1	2	0	-4	4	0	-8	2	0	-6	-1	8	2	8	-12	2	10	8	-6	-8	12	14	-8
205		-1	0	1	-4	0	-2	-6	0	-8	6	0	6	1	4	-4	6	-4	14	-8	-12	-6	-4	4	-6	-6
206		-1	2	4	0	-6	-2	2	-4	0	-6	8	8	2	2	-8	-12	12	10	-2	0	10	0	-4	2	14
207		-1	0	0	-2	-4	-6	-4	2	1	-2	4	2	-2	10	0	12	12	-6	-10	-8	-14	10	-12	16	-10
208		0	-1	-1	-5	2	-1	-3	2	-4	-6	4	11	8	1	-9	-12	-6	0	-6	-7	-2	-12	16	-10	-10
208		0	0	2	2	2	-1	6	6	-8	2	-10	-6	-6	-4	2	6	10	-2	-10	-10	2	4	6	-6	2
208		0	3	-1	-1	2	-1	-3	-6	4	2	-4	3	0	5	-13	12	10	-8	2	5	-10	4	0	6	14
208		0	-1	-3	1	-6	1	-3	-2	0	6	4	-7	0	1	-3	0	6	8	-14	3	2	-8	-12	-6	-10
209		0	1	-3	-4	1	2	0	1	3	-6	-7	-7	0	-10	0	6	3	-10	11	15	8	-16	0	9	-1
210	210A1	-1	-1	-1	-1	-4	-2	-6	0	-8	10	-8	2	-2	8	4	10	4	-6	0	-12	-6	-8	-4	14	2
210		-1	1	1	1	0	2	-6	8	0	6	-4	-10	-6	-4	0	-6	-12	-10	-4	12	-10	8	12	-6	-10
210	210C1	1	-1	1	1	4	-2	2	-4	-8	6	-8	-2	2	-12	-8	6	4	-2	12	8	-14	0	12	2	10
210		1	1	-1	1	0	2	-6	-4	0	-6	-4	2	6	8	-12	6	-12	2	8	0	14	-16	12	6	14
210		1	1	1	-1	-4	-2	2	4	-8	-2	0	6	-6	-4	0	-10	12	14	-12	-8	10	16	-12	10	2
212		0	2	2	0	-4	-2	2	2	-2	2	2	10	2	-4	-12	-1	-12	10	-2	6	10	10	-6	-10	14
212		0	-1	-2	-2	2	-7	-3	5	-3	9	-8	-3	2	4	10	1	-2	-10	4	-9	-6	5	-11	-10	-3
213		1	1	2	2	0	-2	0	0	0	-2	-10	-6	0	-4	12	-4	12	10	2	-1	-10	4	-4	6	-2
214		-1	1	-4	-2	-3	-1	6	1	-7	-6	4	-9	-5	12	8	7	-6	1	-10	6	-4	-7	4	-15	-6
214		-1	-2	-1	4	-6	-4	-6	-2	5	0	-2	0	-11	-9	11	10	-3	-8	5	0	8	11	4	-15	-12
214		1	1	0	2	-3	-1	6	-7	9	-6	-4	-1	3	8	0	-9	6	-7	14	6	-4	-7	12	9	14
214		1	-2	-3	-4	-2	4	-2	-2	1	-4	-10	12	-11	1	-1	6	-5	4	-5	-12	-16	7	-16	9	12
215		0	0	-1	-2	-1	-1	-3	-2	-1	4	3	-8	5	-1	0	-5	12	-4	-3	6	-8	0	-9	-6	-17
216		0	0	-4	-3	-4	1	4	-1	-4	0	-4	-9	0	-8	12	8	-4	-5	11	-8	1	-5	-8	-12	5
216		0	0	-1	3	5	4	-8	2	2	6	-7	-6	-6	-2	6	5	-4	-8	-10	-8	1	16	-11	6	-1
216		0	0	1	3	-5	4	8	2	-2	-6	-7	-6	6	-2	-6	-5	4	-8	-10	8	1	16	11	-6	-1
216		0	0	4	-3	4	1	-4	-1	4	0	-4	-9	0	-8	-12	-8	4	-5	11	8	1	-5	8	12	5
218		1	-2	-3	-4	3	-4	-6	5	3	-3	-4	-4	0	-10	-3	12	12	-7	-4	-12	-1	-16	6	-3	-19
219		1	-1	-4	2	-4	-2	0	-4	0	8	6	-2	-10	-6	-8	-12	4	-14	8	-8	-1	8	16	-14	-2
219		-2	-1	-1	2	-4	-2	-3	-1	0	-10	-6	1	2	6	7	3	1	-5	-13	10	-1	-1	-11	-2	-11
219		0	1	-3	-4	0	-4	3	-1	6	-6	-10	-7	0	2	-3	9	-9	-1	-13	12	1	11	15	-18	5
220		0	-2	1	-4	-1	-4	0	-4	-6	-6	8	2	6	8	6	-6	-12	2	-10	-12	-16	8	0	6	14
220		0	2	1	0	1	0	-4	-4	6	2	0	-6	-10	4	10	2	-4	-14	2	4	-4	-8	12	6	6
221		1	2	2	2	-6	-1	1	4	6	-6	-2	2	-6	0	-4	14	4	2	0	-10	10	14	12	-18	2
221		-1	0	4	-2	6	-1	1	8	4	-6	-2	-8	0	4	0	-6	0	-10	-8	2	0	0	-4	-2	-4
222		-1	-1	2	0	-4	6	6	8	0	-6	4	1	-6	-8	8	6	-4	-2	-12	0	10	-12	-4	-10	-6
222		-1	-1	-4	3	5	3	3	-7	9	0	-2	1	6	4	-10	3	-4	-2	6	-12	13	-6	5	11	6
222		-1	1	4	-1	-1	-3	3	-5	5	4	-10	-1	-6	4	2	-11	-12	10	14	0	-11	-10	-9	11	10
222		1	-1	0	3	1	1	-3	3	-1	-4	-6	-1	-10	12	-6	-1	0	2	2	0	-3	14	9	-3	-10
222		1	1	0	-1	3	-1	-3	-7	3	0	2	1	-6	-4	6	9	0	-10	2	12	5	2	3	-3	2
224		0	-2	0	-1	-4	-4	-2	-6	8	2	-4	10	-10	4	4	-2	10	-8	-8	0	-6	-16	2	18	-2
224		0	2	0	1	4	-4	-2	6	-8	2	4	10	-10	-4	-4	-2	-10	-8	8	0	-6	16	-2	18	-2
225		0	0	0	-5	0	-5	0	-1	0	0	-7	10	0	-5	0	0	0	-13	-5	0	10	-4	0	0	-5
225		0	0	0	5	0	5	0	-1	0	0	-7	-10	0	5	0	0	0	-13	5	0	-10	-4	0	0	5
225		-1	0	0	0	4	2	2	4	0	2	0	10	-10	-4	8	-10	4	-2	-12	8	-10	0	12	6	-2
225		2	0	0	3	-2	-1	2	-5	6	-10	-3	-2	8	-1	2	-4	10	7	3	8	14	0	6	0	-17
225		-2	0	0	-3	-2	1	-2	-5	-6	-10	-3	2	8	1	-2	4	10	7	-3	8	-14	0	-6	0	17
226		1	-2	-4	0	-4	-2	-2	-2	4	-4	8	-8	-6	6	-12	10	-6	-6	2	-8	-14	8	16	-14	-2
228		0	-1	2	0	2	2	6	-1	2	4	-8	-2	-8	-8	2	-4	0	2	12	-4	6	-16	6	0	-2
228		0	-1	-3	1	-5	-6	-5	1	4	6	6	-8	-8	9	1	2	-8	11	0	-4	-11	-8	-4	10	-10

Bibliography

[1] Aczel, A. D., *Fermat's Last Theorem: Unlocking the Secret of an Ancient Mathematical Problem*, Four Walls Eight Windows (1996).

[2] Ahlgren, S., *The Points of a Certain Fivefold over Finite Fields and the Twelfth Power of the Eta Function*, Finite Fields and Their Applications **8**, no. 1 (2002), pp. 18–33.

[3] Ahlgren, S., Ono, K., *The modularity of a certain Calabi–Yau threefold*, Monatshefte für Mathematik **129** (2000), pp. 177–190.

[4] Ahlgren, S., Ono, K., Penniston, D., *Zeta functions of an infinite family of K3 surfaces*, American J. of Math. **124**, no. 2 (2002), pp. 353–368.

[5] Barth, W., *A Quintic Surface with 15 three-divisble Cusps*, preprint (2000).

[6] Barth, W., Nieto, I., *Abelian surfaces of type (1,3) and quartic surfaces with 16 skew lines*, J. Alg. Geometry **3**, no. 2 (1994), pp. 173–222.

[7] Barth, W., Sarti, A., *Polyhedral Groups and Pencils of K3-Surfaces with Maximal Picard Number*, Asian J. of Math. **7**, no. 4 (2003), pp. 519–538.

[8] Batyrev, V., *Birational Calabi–Yau n-folds have equal Betti numbers*, in *Proceedings Warwick Euroconference 1996*, eds. Hulek, K., Catanese, F., Peters, C., Reid, M., London Math. Soc. Lecture Note Ser. **264** (1999), Cambridge Univ. Press, pp. 1–11.

[9] Batyrev, V., Borisov, L., *Dual Cones and Mirror Symmetry for Generalized Calabi-Yau Manifolds*, Mirror Symmetry II, eds. Greene, B., Yau, S.-T., International Press, Cambridge (1997), pp. 71–86.

[10] Batyrev, V., Hosono, S., Lewis, J. D., Lian, B. H., Yau, S.-T., Yui, N., Zagier, D., *Calabi–Yau Varieties and Mirror Symmetry*, report for PIMS workshop at BIRS, Dec. 6-11, 2003, `http://www.pims.math.ca/birs/workshops/2003/03w5061/report03w5061.pdf`.

[11] Batyrev, V., van Straten, D., *Generalized hypergeometric functions and rational curves on Calabi–Yau complete intersections in toric geometry*, Comm. Math. Phys. **168** (1995), pp. 493–533.

[12] Beauville, A., *Les familles stables de courbes elliptiques sur* $\mathbb{P}^1$ *admettant quatre fibres singulières*, C. R. Acad. Sc. Paris **294** (1982), pp. 657–660.

[13] Bernardara, M., *Calabi–Yau complete intersections with infinitely many lines*, preprint (2004), math.AG/0402454.

[14] Beukers, F., Stienstra, J., *On the Picard-Fuchs equation and the formal Brauer group of certain elliptic K3-surfaces*, Math. Ann. **271** (1985), pp. 269–304.

[15] Breuil, C., Conrad, B., Diamond, F., Taylor, R., *On the modularity of elliptic curves over* $\mathbb{Q}$*: wild 3-adic exercises*, J. of the AMS **14**, no. 4 (2001), pp. 843-939.

[16] Brieskorn, E., *Über die Auflösung gewisser Singularitäten von holomorphen Abbildungen*, Math. Ann. **166** (1966), pp. 76–102.

[17] Candelas, P., Dale, A. M., Lütken, C. A., Schimmrigk, R., *Complete intersection Calabi–Yau manifolds*, Nucl. Physics **B298** (1988), pp. 493–525.

[18] Candelas, P., de la Ossa, X., Rodriguez-Villegas, F., *Calabi–Yau manifolds over finite fields I*, preprint (2000), hep-th/0012233.

[19] Candelas, P., de la Ossa, X., Rodriguez-Villegas, F., *Calabi–Yau manifolds over finite fields II*, in *Proceedings of the Workshop on "Calabi–Yau Varieties and Mirror Symmetry"*, Fields Institute, Toronto, July 23-29, 2001, eds. Yui, N., Lewis, J. D., Fields Inst. Comm. Series **38** (2003), AMS, pp. 121–157.

[20] Candelas, P., Horowitz, G. T., Strominger, A., Witten, E., *Vacuum configurations for superstrings*, Nucl. Physics **B258** (1985), pp. 46–74.

[21] Catanese, F., Ceresa, G., *Constructing Sextic Surfaces with a Given Number d of Nodes*, J. Pure Appl. Alg. **23** (1982), pp. 1–12.

[22] Clemens, C. H., *Double Solids*, Adv. in Math. **47** (1983), pp. 107–230.

[23] Consani, K., Scholten, J., *Arithmetic on a quintic threefold*, Internat. J. Math. **12**, no. 3 (2001), pp. 943–972.

[24] Coxeter, H. S. M., *Regular Polytopes*, 3rd edition, Dover Publications, New York (1973).

[25] Cynk, S., *Defect of a nodal hypersurface*, Manuscripta Math. **104** (2001), pp. 325–331.

[26] Cynk, S., *Double coverings of octic arrangements with isolated singularities*, Adv. Theor. Math. Phys. **3** (1999), pp. 217–225.

[27] Cynk, S., *Cohomologies of a double covering of a non–singular algebraic 3-fold*, Math. Z. **240**, no. 4 (2002), pp 731–743.

[28] Cynk, S., Meyer, C., *Geometry and arithmetic of certain double octic Calabi–Yau manifolds*, preprint (2003), math.AG/0304121, to appear in Canadian Math. Bull.

[29] Cynk, S., Szemberg, T., *Double covers and Calabi–Yau varieties*, Banach Center Publ. **44** (1998), pp. 93–101.

[30] Cynk, S., van Straten, D., *Infinitesimal deformations of double covers of smooth algebraic varieties*, preprint (2003), math.AG/0303329, to appear in Math. Nachr.

[31] Deligne, P., *Formes modulaires et représentations ℓ-adiques*, Sem. Bourbaki **355** (1968/69), Lect. Notes **349** (1971), Springer, pp. 139–172.

[32] Deligne, P., Serre, J. P., *Formes modulaires de poids 1*, Ann. Sci. Éc. Norm. Sup. **7** (1974), pp. 507–530.

[33] Dieulefait, L., *Computing the Level of a Modular Rigid Calabi–Yau Threefold*, Exp. Math. **13**, no. 2 (2004), pp. 165–169.

[34] Dieulefait, L., *From potential modularity to modularity for integral Galois representations and rigid Calabi–Yau threefolds*, preprint (2004), math.NT/0409102.

[35] Dieulefait, L., Manoharmayum, J., *Modularity of rigid Calabi–Yau threefolds over $\mathbb{Q}$*, in *Proceedings of the Workshop on "Calabi–Yau Varieties and Mirror Symmetry"*, Fields Institute, Toronto, July 23-29, 2001, eds. Yui, N., Lewis, J. D., Fields Inst. Comm. Series **38** (2003), AMS, pp. 159–166.

[36] Dimca, A., *On the homology and cohomology of complete intersections with isolated singularities*, Compositio Mathematica **58** (1986), pp. 321–339.

[37] Dolgachev, I. V., *Lectures on modular forms*, `http://www.math.lsa.umich.edu/~idolga/`.

[38] Elkies, N., *Complete cubic parametrization of the Fermat cubic surface* $w^3+x^3+y^3+z^3=0$, `http://www.math.harvard.edu/~elkies/4cubes.html`.

[39] Endraß, S., *A Projective Surface of Degree Eight with 168 Nodes*, J. Alg. Geometry **6** (1997), pp. 325–334.

[40] Endraß, S., *On the divisor class group of double solids*, Manuscripta Math. **99** (1999), pp. 341–358.

[41] Esnault, H., Viehweg, E., *Lectures on vanishing theorems*, DMV Seminar **20** (1992), Birkhäuser. `http://www.uni-essen.de/~mat903/books.html`.

[42] Faltings, G., *Endlichkeitssätze für abelsche Varietäten über Zahlkörpern*, Invent. math. **73** (1983), pp. 349–366.

[43] Fontaine, M., Mazur, B., *Geometric Galois Representations*, in *Elliptic Curves, Modular Forms and Fermat's Last Theorem*, Hongkong 1993, eds. Coates, J., Yau, S. T., Ser. Number Theory **1**, International Press (1995), pp. 41–78.

[44] Freitag, E., Kiehl, R., *Etale Cohomology and the Weil Conjecture*, Ergebnisse der Mathematik und ihrer Grenzgebiete **13** (1988), Springer.

[45] Greuel, G.-M., Pfister, G., Schönemann, H., SINGULAR 2.0*, A Computer Algebra System for Polynomial Computations*, Centre for Computer Algebra, University of Kaiserslautern (2001), `http://www.singular.uni-kl.de`.

[46] Haessig, C. D., *Equalities, congruences, and quotients of zeta functions in Arithmetic Mirror Symmetry*, preprint (2005), math.NT/0501115.

[47] Hartshorne, R., *Algebraic Geometry*, Graduate texts in mathematics **52** (1977), Springer.

[48] Hirzebruch, F., *Some examples of threefolds with trivial canonical bundle*, in Collected Works Vol. **II** (1995), Springer, pp. 757–770.

[49] Hulek, K., Spandaw, J., *Counting points on Calabi–Yau threefolds - Some computational aspects*, eds. Ciliberto, C. et al., Applications of Algebraic Geometry to Coding Theory, Physics and Computation, Kluwer Academic Publishers (2001), pp. 195–205.

[50] Hulek, K., Spandaw, J., van Geemen, B., van Straten, D., *The modularity of the Barth-Nieto quintic and its relatives*, Adv. Geom. **1** (2001), pp. 263–289.

[51] Hulek, K., Verrill, H. A., *On modularity of rigid and nonrigid Calabi–Yau varieties associated to the root lattice* A_4, preprint (2003), math.AG/0304169, to appear in Nagoya Math. Journal.

[52] Hulek, K., Verrill, H. A., *On the modularity of Calabi–Yau threefolds containing elliptic ruled surfaces*, preprint (2005), math.AG/0502158, to appear in *Calabi–Yau Varieties and Mirror Symmetry*, proceedings of PIMS workshop at BIRS, Dec. 6-11, 2003.

[53] Inose, K., Shioda, T., *On singular K3 surfaces*, in *Complex Analysis and Algebraic Geometry*, eds. Baily, W., Shioda, T. (1997), Kinokuniya Shoten and Cambridge University Press, pp. 119–136.

[54] Jones, J., *Tables of number fields with prescribed ramification*, `http://math.la.asu.edu/∼jj/numberfields`.

[55] Kimura, K., *A rational map between two threefolds*, preprint (2004), math.AG/0410259.

[56] Klemm, A, Theisen, S., *Considerations of one modulus Calabi-Yau compactifications: Picard-Fuchs equations, Kähler Potentials and Mirror Maps*, Nucl. Physics **B389** (1993), pp. 153–180.

[57] Klemm, A, Theisen, S., *Mirror Maps and Instanton Sums for Complete Intersections in Weighted Projective Space*, Modern Physics Letters A **9**, no. 20 (1994), pp. 1807–1817.

[58] Knapp, A. W., *Elliptic Curves*, Princeton Mathematical Notes **40** (1993), Princeton University Press.

[59] Kuwata, M., Top, J., *A singular K3 surface related to sums of consecutive cubes*, Indagationes Mathematicae **11**, no. 3 (2000), pp. 419–435.

[60] Lauder, A. G. B., *Counting solutions to equations in many variables over finite fields*, Foundations of Computational Mathematics **4**, no. 3 (2004), pp. 221–267.

[61] Libgober, A., Teitelbaum, J., *Lines on Calabi–Yau complete intersections, mirror symmetry, and Picard-Fuchs equations*, Int. Math. Research Notices **1** (1993), pp. 29–39.

[62] Livné, R., *Cubic exponential sums and Galois representations*, Current trends in arithmetical algebraic geometry (Arcata, Calif., 1985), Contemp.Math. **67**, Amer.Math.Soc., Providence, R.I. (1987), pp. 247–261.

[63] Livné, R., *Motivic orthogonal two-dimensional representations of* $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$, Israel J. of Math. **92**, no. 1-3 (1995), pp. 148–156.

[64] Livné, R., Yui, N., *The modularity of certain non-rigid Calabi–Yau threefolds*, preprint (2003), math.AG/0304497.

[65] Long, L., *L-Series of Certain Elliptic Surfaces*, Canadian Math. Bull. **46**, no. 4 (2003), pp. 546–558.

[66] Lovejoy, J., Penniston, D., *3-regular partitions and a modular K3 surface*, in *q-Series with Applications to Combinatorics, Number Theory, and Physics*, Contemporary Mathematics **291** (2001), pp. 177-182.

[67] Martin, I., *Multiplicative* η*-quotients*, Trans. Amer. Math. Soc. **348**, no. 12 (1996), pp. 4825–4856.

[68] Meyer, C., *Die L-Reihen einiger symmetrischer Quintiken*, diploma thesis, Mainz (2000).

[69] Meyer, C., *Modular quintics in* $\mathbb{P}^4$, Math. Nachr. **259** (2003), pp. 66–73.

[70] Mihăilescu, P., *Primary cyclotomic units and a proof of Catalan's conjecture*, J. Reine Angew. Math. **572** (2004), pp. 167–195.

[71] Milne, J. S., *Étale cohomology*, Princeton Mathematical Series **33** (1980), Princeton University Press.

[72] Milne, J. S., *Modular Functions and Modular Forms*, `http://www.jmilne.org/math/CourseNotes/math678.html`.

[73] Morrison, D. R., *Mirror Symmetry and Rational Curves on Quintic Threefolds: A Guide for Mathematicians*, J. of the AMS **6**, no. 1 (1993), pp. 223–247.

[74] Mortenson, E., *Modularity of a certain Calabi–Yau threefold and combinatorial congruences*, to appear in Ramanujan Journal.

[75] Nygaard, N., van Geemen, B., *On the Geometry and Arithmetic of Some Siegel Modular Threefolds*, J. Number Theory **53** (1995), pp. 45–87.

[76] Peters, C., Stienstra, J., *A pencil of K3-surfaces related to Apéry's recurrence for* $\zeta(3)$ *and Fermi surfaces for potential zero*, in *Arithmetic of complex manifolds*, Erlangen 1988, eds. Barth, W., Lange, H., Lect. Notes **1399** (1989), Springer, pp. 48–59.

[77] Peters, C., Top, J., van der Vlugt, M., *The Hasse zeta function of a K3 surface related to the number of words of weight 5 in the Melas codes*, J. Reine Angew. Math. **432** (1992), pp. 151–176.

[78] Reid, M., *Update on 3-folds*, Proc. ICM 2002, no. **3**, also available as preprint (2003), math.AG/0206157.

[79] Ribet, K. A., *Galois representations and modular forms*, Bull. of the AMS **32**, no. 4 (1995), pp. 375–402.

[80] Rodriguez-Villegas, F., *Hypergeometric Families of Calabi–Yau Manifolds.* in *Proceedings of the Workshop on "Calabi–Yau Varieties and Mirror Symmetry"*, Fields Institute, Toronto, July 23-29, 2001, eds. Yui, N., Lewis, J. D., Fields Inst. Comm. Series **38** (2003), AMS, pp. 223–232.

[81] Saito, M., Yui, N., *The modularity conjecture for rigid Calabi–Yau threefolds over* $\mathbb{Q}$, J. of Math. Kyoto Univ. **41**, no. 2 (2001), pp. 403–419.

[82] Sarti, A., *Pencils of Symmetric Surfaces in* $\mathbb{P}_3$, J. of Algebra **246** (2001), pp. 429–452.

[83] Sarti, A., *Symmetric Surfaces with Many Singularities*, Comm. in Algebra **32**, no. 10 (2004), pp. 3745–3770.

[84] Schoen, C., *On fiber products of rational elliptic surfaces with section*, Math. Z. **197**, no. 2 (1988), pp. 177–199.

[85] Schoen, C., *On the computation of the cycle class map for nullhomologous cycles over the algebraic closure of a finite field*, Ann. Sci. Éc. Norm. Sup. **28**, no. 1 (1995), pp. 1–50.

[86] Schoen, C., *On the geometry of a special determinantal hypersurface associated to the Mumford-Horrocks vector bundle*, J. Reine Angew. Math **364** (1986), pp. 85–111.

[87] Schütt, M., *Die Modularität von starren 3-dimensionalen Calabi–Yau-Varietäten*, diploma thesis, Hannover (2004).

[88] Schütt, M., *New examples of modular rigid Calabi–Yau threefolds*, Collect. Math. 55, **2** (2004), pp. 219-228.

[89] Schütt, M., *On the modularity of three Calabi–Yau threefolds with bad reduction at 11*, preprint (2004), math.AG/0405450, to appear in Canadian Math. Bull.

[90] Segre, B., *Sul massimo numero di nodi delle superficie algebraiche*, Atti Acc. Ligure **10**, no. 1 (1952), pp. 15–22.

[91] Serre, J. P., *Abelian* ℓ*-Adic Representations and Elliptic Curves*, W. A. Benjamin (1968).

[92] Serre, J. P., *A course in arithmetic*, Graduate Texts in Mathematics **7** (1973), Springer.

[93] Serre, J. P., *Résumé des cours de 1984-1985*, Annuaire du Collège de France (1985), pp. 85–90.

[94] Serre, J. P., *Sur les représentations modulaires de degré 2 de* $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$, Duke Math. J. **54**, no. 1 (1987), pp. 179–230.

[95] Singh, S., *Fermat's Last Theorem*, Fourth Estate (1997).

[96] Shioda, T., *On elliptic modular surfaces*, J. Math. Soc. Japan **24**, no. 1 (1972), pp. 20–59.

[97] Stein, W. A., *The Modular Forms Database* (2004), `http://modular.fas.harvard.edu/Tables`.

[98] Top, J., van Geemen, B., *An isogeny of K3 surfaces*, preprint (2003), math.AG/0309272.

[99] Van Geemen, B., Werner, J., *New examples of threefolds with* $c_1 = 0$, Math. Z. **203** (1990), pp. 211–225.

[100] Van Geemen, B., Werner, J., *Nodal quintics in* $\mathbb{P}^4$, in *Arithmetic of complex manifolds*, Erlangen 1988, eds. Barth, W., Lange, H., Lect. Notes **1399** (1989), Springer, pp. 48–59.

[101] Van Straten, D., *A quintic hypersurface in* $\mathbb{P}^4$ *with 130 nodes*, Topology **32**, no. 4 (1993), pp. 857–864.

[102] Varchenko, A., *On semi-continuity of the spectrum and an upperbound for the number of singular points of projective hypersurfaces*, Dokl. Akda. Nauk. USSR **270** (1983), pp. 735-739.

[103] Verrill, H. A., *Root Lattices and Pencils of Varieties*, J. of Math. Kyoto Univ. **36**, no. 2 (1996), pp. 421–446.

[104] Verrill, H. A., *The L-series of certain rigid Calabi–Yau threefolds*, J. Number Theory **81** (2000), pp. 310–334.

[105] Wan, D., *Mirror Symmetry for Zeta Functions*, preprint (2004), math.AG/0411464.

[106] Werner, J., *Kleine Auflösungen spezieller dreidimensionaler Varietäten*, Bonner mathematische Schriften **186** (1987).

[107] Yi, Y. C., *On the modularity of a rigid Calabi–Yau manifold*, J. of Math. Kyoto Univ. **44**, no. 1 (2004), pp. 119–127.

[108] Yui, N., *Arithmetic of Calabi–Yau varieties*, in *Mathematisches Institut Universität Göttingen, Seminars 2004*, ed. Tschinkel, Y. (2004), pp. 9–29, `http://www.math.princeton.edu/~ytschink/.goettingen/SS04/protokolle/dvi/book.pdf`.

[109] Yui, N., *Arithmetic of certain Calabi–Yau varieties and mirror symmetry*, in *Arithmetic algebraic geometry*, Park City, UT 1999, IAS/Park City Math. Series **9** (2001), AMS, pp. 507–569.

[110] Yui, N., *The arithmetic of certain Calabi–Yau varieties over number fields*, In: *The Arithmetic and Geometry of Algebraic Cycles*, Proceedings of the NATO Advanced Study Institute, Banff, Canada, June 7–19, 1998, NATO ASI Series, Series C, Mathematical and Physical Sciences **548**, Kluwer (2000), pp. 515–560.

[111] Yui, N., *Update on the modularity of Calabi–Yau varieties*, with appendix by Verrill, H. A., in *Proceedings of the Workshop on "Calabi–Yau Varieties and Mirror Symmetry"*, Fields Institute, Toronto, July 23-29, 2001, eds. Yui, N., Lewis, J. D., Fields Inst. Comm. Series **38** (2003), AMS, pp. 307–362.

[112] *The Magma Computational Algebra System for Algebra, Number Theory and Geometry*, `http://magma.maths.usyd.edu.au/`.