Analytical description for current-induced vortex core displacement

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Abstract

In this paper an analytical model for current-induced vortex core displacement is developed. By using this simple model, one can solve the equations of motion analytically to determine the effects of the adiabatic and non-adiabatic spin torque terms. The final displacement direction of the vortex core due to the two torque terms mirrors their relative strengths. The resulting vortex core displacement direction combined with the amplitude of the displacement is thus a measure for both torque terms.

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For the development of novel applications such as magnetic solid-state storage devices [1], a controlled manipulation of the magnetization is required. To achieve this, injection of spin-polarized electrons is believed to be a viable alternative approach to conventional magnetic fields as field-induced magnetization dynamics suffers from unfavourable scaling. As the design rule of devices is decreased, the current density necessary in the striplines to generate the switching fields goes up and eventually reaches levels where structural degradation sets in. Spin-polarized currents on the other hand show a more favourable scaling as means of manipulating the magnetization because for a constant current density the total current and thus the power consumption goes down with decreasing features sizes.

Spin currents interact with the magnetization via the adiabatic and non-adiabatic spin-torque as theoretically predicted [2, 3]. Experimentally for instance switching of spin valves as well as in domain wall and vortex core (VC) motion were observed [4, 5]. To utilize this effect, an in-depth understanding of the torque terms needs to be developed which has previously been hampered by the difficulty of separating the spin-torque terms.

To determine in particular the non-adiabatic torque [6, 7] has proven to be difficult with a wide range of values having been determined from measurements of domain wall motion in wires [8–11]. The large variation and the strong assumptions and simplifications used in the analysis to extract the torque terms show that a reliable determination of an absolute value is not straight forward in the wire geometry.

A different geometry that has received much attention is the disc where the vortex state is present. The displacement of the vortex core under injected current entails a number of advantages compared to the study of domain wall displacement in wires, where edge roughness and other edge defects can play a major role. The vortex core is always far away from the disc edge (for reasonable excitations) and thus less influenced by defects located at the edge.

In this paper, we use an analytical model to determine the displacement direction of a vortex core under current injection. We study the vortex core motion in this simple model and find that the final equilibrium displacement direction depends on the adiabatic and non-adiabatic spin torque terms. In particular we show that this direction can be used to determine the non-adiabaticity parameter, which is key to understanding the underlying spin current transport [6, 7].

To analytically derive the final current-induced position of the vortex core, we start with the extended Landau-Lifshitz and Gilbert equation, where the third and fourth term are the adiabatic and non-adiabatic spin torque terms [6, 7]:

$$\dot{\vec{m}} = \gamma_0 \vec{H} \times \vec{m} + \alpha \vec{m} \times \dot{\vec{m}} - [\vec{u} \cdot \vec{\nabla}]\vec{m} + \beta \vec{m} \times \left([\vec{u} \cdot \vec{\nabla}]\vec{m} \right). \tag{1}$$

Thiele derived an equation of motion for a spin texture, which has been employed to describe domain wall motion and is commonly called a one-dimensional model [12]. It can also be used to describe the displacement of a single vortex core with a fixed magnetization profile and it has been extended by Thiaville to include the spin-torque terms from eq. 1 [6]:

$$\vec{F}_s(\vec{X}) + \vec{G} \times \left(\vec{u} + \frac{d\vec{X}}{dt} \right) + \bar{D} \cdot \left(\beta \vec{u} + \alpha \frac{d\vec{X}}{dt} \right) = 0, \tag{2}$$

with the vortex core position $\vec{X}$. Assuming an electron flow the in x-direction, $\vec{u}$ becomes: $\vec{u} = jPg\mu_B/(2eM_s)\vec{e}_x$ with the spin polarization of the current P and the saturation magnetization M_s. The gyrovector $\vec{G}$ points out-of-plane in the direction of the vortex core and equals $\vec{G} = pG\vec{e}_z = p2\pi M_s \mu_0 t/\gamma \vec{e}_z$ with p the direction of the vortex core (polarity ± 1) and the disk thickness t. The dissipation tensor $\bar{D}$ is defined as:

$$\bar{D} = -\frac{M_s \mu_0}{\gamma} \int dV \left(\vec{\nabla}\vartheta \vec{\nabla}\vartheta + \sin^2(\theta) \vec{\nabla}\phi \vec{\nabla}\phi \right), \tag{3}$$

with ϑ being the out-of-plane angle and ϕ the in-plane angle of the local magnetization. For a rigid vortex centered in a disk with radius r, it can be numerically evaluated shown in [13] and turns out to be a diagonal tensor with:

$$D_{zz} = 0, \quad D_{xx} = D_{yy} = D \approx -\frac{\pi M_s \mu_0 t}{\gamma} \ln(2.0r/\delta) = -fG. \tag{4}$$

The radius of the VC δ (about 10 nm in the permalloy structures usually used [14]), depends on the exchange length and slightly on the thickness of the disk t [15]. Due to the logarithm and since $d \gg \delta$ the factor f is not very sensitive to variations of the core profile, so that f can be reliably calculated. In disks the potential is radially symmetric, resulting in a force $\vec{F}_s = -\kappa \vec{X}$ that tries to push the vortex core back to the disk center. The stiffness κ is given by the disk dimension and material parameters [16].

When current is injected, the vortex core will be displaced according to equation (2) until the restoring force equals the spin torque. To determine the final displacement $\vec{X}$

under current injection, we look for solutions where $\frac{d\vec{X}}{dt} = 0$ and thus a steady state is obtained. Equation (2) then simplifies to:

$$x_{VC}\vec{e}_x + y_{VC}\vec{e}_y = Gu/\kappa\,(f\beta\vec{e}_x + p\vec{e}_y)\,. \tag{5}$$

A similar calculation was done by Shibata *et al.* but without including non-adiabatic contributions [17]. It becomes now obvious that the adiabatic spin-torque term is responsible for a displacement perpendicular to the electron direction (y_{VC}) while the non-adiabatic term leads to a displacement in the direction of the electron flow (x_{VC}). By measuring the angle of displacement θ with respect to the electron direction, the non-adiabaticity parameter β can therefore be directly evaluated:

$$\tan(\theta) = x_{VC}/y_{VC} = pf\beta. \tag{6}$$

For the sake of simplicity the calculation was done for harmonic potentials, but holds for any potential, as long as it is radially symmetric. It is also important to note that uncertainties in the current density, sample thickness, material parameters, etc. do not affect the displacement direction, making this relationship very robust.

To directly evaluate this expression, one needs to know p and f. To set p, one can initialize the vortex spin structure by applying a strong out-of-plane field to set the polarity of the vortex core. What remains is to calculate f. To check if the analytical calculation described above holds, we have carried out corresponding full micromagnetic simulations to determine f numerically. We use the usual parameters for permalloy [11] to determine the factor f in 30 nm thick disks with varying diameter. The result is shown in the semi logarithmic plot in Fig. 1. The predicted logarithmic dependence of the factor f on the disk diameter is very well confirmed (Fig. 1) so that the relation above can be used to determine β if the two spin torques are the governing torques and other torques, such as the Oersted field [13, 18] play a minor role. This can be achieved for instance by using contacts with lower or similar conductivity as the magnetic material.

In conclusion, we have used an analytical model for the vortex core dynamics to determine the current-induced vortex core displacement direction. The direction is found to be a directly dependent on the ratio of adiabatic and non-adiabatic torques. We have determined the proportionality factor so that potentially measuring the displacement direction allows for the determination of the non-adiabaticity parameter.

Acknowledgments

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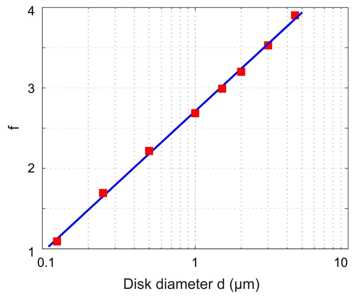
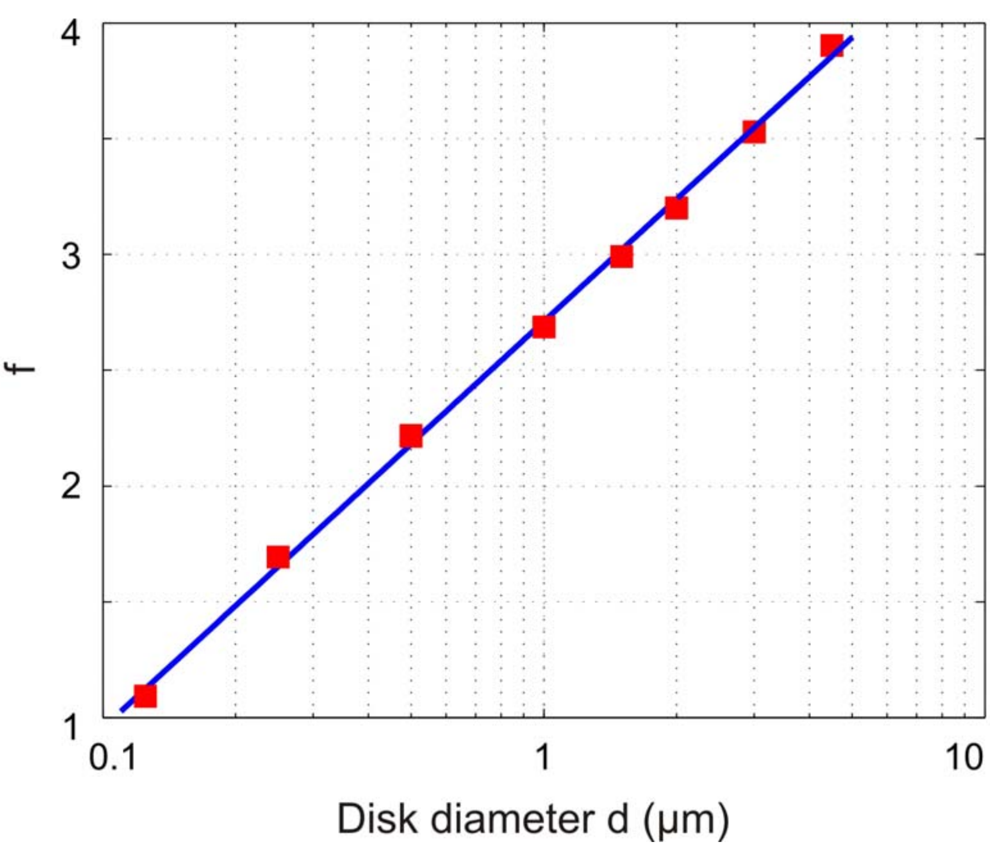


FIG. 1: (Color online) The factor f is plotted as a function of the disk diameter for 30 nm thick disks. Red squares correspond to the numerical results and the blue line is a logarithmic fit.



4
3
2
1
f
0.1
1
10
Disk diameter d (μm)