

Ultra-Fast True Random Number Generator Based on Ill-Posedness Nucleation of Skyrmion Bags in Ferrimagnets

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Abstract—An ultra-fast true random number generator (TRNG) based on ill-posedness nucleation of skyrmion bags (sk-bags) with high topological charge (Q) is proposed and studied via numerical method. Under a local spin-polarized current, the ill-posed dynamics of ferrimagnetic spin texture is induced, resulting in the nucleation of sk-bag with high Q (even $|Q| > 10$). Furthermore, in combination with thermal activation, this initially complex phenomenon can be tuned to possess a defined probability distribution of Q . Based on these, we reveal for the first time that the nucleation process of Q , in contrast to the dynamic motion of topological solitons, has the potential to provide an ultra-high true random number generation with a generation rate of 20 Gbit/s, which is the fastest record compared with previous spintronic TRNGs. Finally, we verified the functionality of the device through simulation and demonstrated its advantages in terms of power consumption. Our research provides an additional possibility for developing high-performance TRNG.

Index Terms—Ferrimagnets, true random number generator, high-topological charge, skyrmion-bags.

I. INTRODUCTION

Randomness is an especially valuable concept in modern information technology such as communication and optimization algorithm implementation [1-6]. However, employing electronic equipment to produce true random number signals is quite challenging [7-8]. To accomplish this, various technical approaches have been devised, including optoelectronic devices, spintronic devices, and quantum devices, which all have the potential to generate true random number signals [9-11]. Due to the low power consumption and good compatibility with integrated circuits, spintronic random

number generator (RNG) stands out among the others as a viable technical path [12-19].

The randomness of spintronics RNG is usually based on stochastic magnetic tunnel junction (MTJ) or the dynamic motion of magnetic skyrmions. However, these spintronic devices are complicated to build and need precise handling of magnetization switching and skyrmion motion, which limits their practical application [20-23]. Additionally, because these devices are built on ferromagnetic (FM) material with a lower intrinsic frequency, they are less effective at producing random number signals with high rates [24-27]. Ferrimagnetic (FIM) materials, which consist of two antiferromagnetically coupled sublattices, have attracted extensive interest due to their distinctive dynamics near the compensation point, and have the potential to achieve ultra-fast RNGs.

In this work, we propose a method to generate ultra-fast true random number signals based on nucleation process of topological charge (Q) in FIM materials. Firstly, we demonstrate that a spin-polarized current can induce an ill-posed nucleation process of skyrmion bags (sk-bags) in ferrimagnets, meaning that small changes in the initial conditions and parameters could result in different Q values. In addition, in combination with the effects of thermal activation, the topology-related ill-posed process is converted to a stochastic process. Finally, we design an ultra-fast true random number generator (TRNG) and verify its performance. Our results show that the generation rate of the TRNG based on sk-bags nucleation process is at least three orders of magnitude faster than the fastest previous spintronic TRNGs.

II. ILL-POSEDNESS NUCLEATION OF SK-BAG

Fig. 1a shows the schematic of the nucleation of sk-bags with high Q by using a spin-polarized current, in which a FM/oxide junction is grown on the FIM film. The process is investigated numerically, and the results are shown in Fig. 1b. Here, we use the CoTb alloys as FIM film to perform the spin dynamics. The default values for the numerical experiments are given in Table I. In order to understand the mechanism for this change of Q , we propose a 1D nucleation analytical model for the FIM system, as illustrated in Fig. 1c. This model is a simplified one where we assume the distribution of magnetic moments within a one-dimensional space, and all magnetic moments are confined within the xOz plane. Additionally, we consider the presence of exchange interactions within the system, allowing

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this 1D model to reflect the laws of the actual physical scenario and generate one-dimensional topological charges. Based on these assumptions of the one-dimensional model, the FIM system can be described by two vector fields: $\mathbf{u}(x)$, the direction of the Co magnetic moments, and $\mathbf{v}(x)$, the direction of the Tb magnetic moments. Here, we assume that all magnetic moments are in the x-z plane. $\mathbf{u}(x)$ and $\mathbf{v}(x)$ can thus be denoted as $(\sin\theta, 0, \cos\theta)$ and $(\sin\varphi, 0, \cos\varphi)$, respectively, where θ and φ are the azimuthal angles of the vectors of Co and Tb elements. Based on continuous approximation method, the Landau–Lifshitz–Gilbert (LLG) equations of 1D model are thus re-written as:

$$\begin{cases} \frac{d\mathbf{u}}{dt} = -\beta_1 \frac{d^2\mathbf{v}}{dx^2} \times \mathbf{u} + \alpha_1 \mathbf{u} \times \frac{d\mathbf{u}}{dt}, \beta_1 = \frac{2\gamma_{Co}A_{ex}}{\mu_0 M_{Co}} \\ \frac{d\mathbf{v}}{dt} = -\beta_2 \frac{d^2\mathbf{u}}{dx^2} \times \mathbf{v} + \alpha_2 \mathbf{v} \times \frac{d\mathbf{v}}{dt}, \beta_2 = \frac{2\gamma_{Tb}A_{ex}}{\mu_0 M_{Tb}} \end{cases} \quad (1)$$

Fig.1d illustrates the final stable state of the system after applying a spin-polarized current. This model reveals the unstable property of the sk-bag nucleation process. By analyzing equation (1), we find that this property is rooted in the equations of magnetic texture dynamics.

$$\dot{\Delta} = -\frac{\beta_1}{\alpha_1} \cos\Delta \Delta_{xx} + \frac{\beta_1\alpha_2 - \beta_2\alpha_1}{\alpha_1\alpha_2} \left[\frac{1}{4} \sin\Delta \Delta_x^2 - \frac{1}{2} \cos\Delta \Delta_{xx} \right] \quad (2)$$

Mathematically, equation (2) is a typical form of an ill-posed solution [28]. $\Delta = \theta - \varphi$ is effective magnetic moment in this FIM system, and can represent the spin texture from one aspect. As a result, the dynamics of spin texture can thus change dramatically for small perturbations, causing the final Q in the nucleated sk-bags is unstable with respect to small changes of the initial conditions.

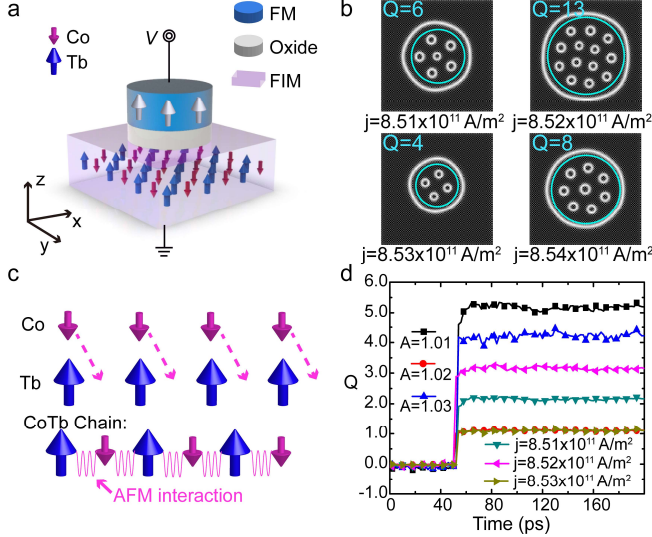


Fig. 1. (a) Schematic of the device structure for nucleation. (b) Sk-bags with different Q values nucleated by slightly varying current density. The field of view is 100 nm*100 nm. (c) Structure of the 1D model of the FIM system. (d) Change of Q induced by slight variations of the dynamic equation parameter (exchange constant) and slight variations of the initial state (initial current density).

III. GAUSSIAN RANDOM DISTRIBUTION OF SKYRMION CORES BASED ON THE ILL-POSEDNESS NUCLEATION

We next introduce thermal effects into the nucleation process in FIM system. As shown in Fig. 2a, we divide the FIM film into three regions. The local current is injected to Region I

TABLE I SIMULATION PARAMETERS

| Parameters | Value |
|---------------------------------------------------|----------------------------------------|
| Magnetization of Tb, M_{Tb} | $970 \text{ kA} \cdot \text{m}^{-1}$ |
| Magnetization of Co, M_{Co} | $650 \text{ kA} \cdot \text{m}^{-1}$ |
| Exchange constant, A_{ex} | $15 \text{ pJ} \cdot \text{m}^{-1}$ |
| Gyromagnetic ratio of Co, γ_{Co} | $1.17 \text{ MmA} \cdot \text{s}^{-1}$ |
| Gyromagnetic ratio of Tb, γ_{Tb} | $1.01 \text{ MmA} \cdot \text{s}^{-1}$ |
| Alpha damping constant, α_1 and α_2 | 0.02 |

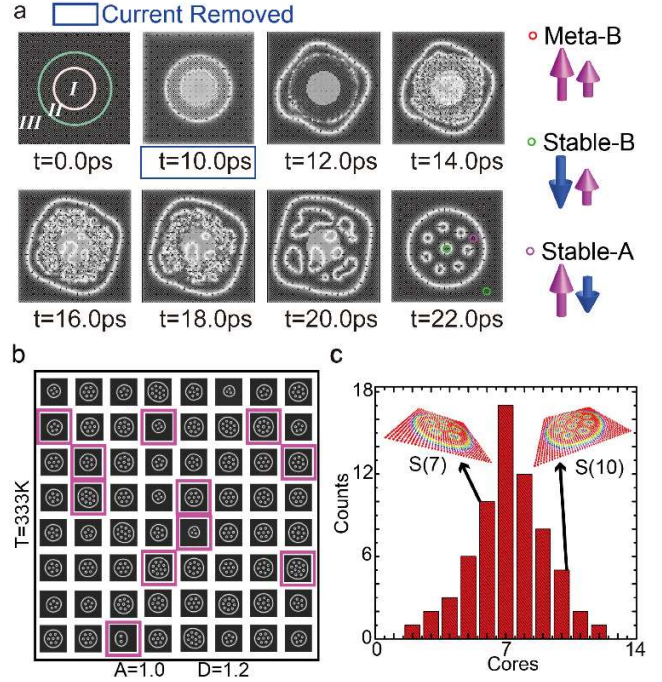


Fig. 2. (a) Whole process of the sk-bag nucleation. Different colors of circles indicate different states of magnetic configurations. (b) Stochastic behavior of Q (i.e. core number). (c) Frequency distribution of sk-bags with different Q values in the 64 simulations.

directly. Region 2 can be considered as having no local current injection, but it is directly affected by Region I. Region 3 is the rest of the film. In our simulation, we apply the current at $t=0.0$ ps and withdraw it at $t=10.0$ ps. In this process, the FIM sublattice exhibits three states, which we note as stable-A, stable-B, and meta-B. Among them, meta-B is a metastable state, i.e., after the withdrawal of the current, it starts to disintegrate due to exchange interaction and Dzyaloshinskii–Moriya interaction (DMI) interaction, resulting in nucleation of sk-bags [29-32]. To verify that the nucleation results exhibit a certain probability distribution, we perform 64 simulations with the same parameters as shown in Fig. 2b. In Fig. 2c, we count the nucleation results, and the value of skyrmions cores number is consistent with the Gaussian distribution. From the above studies, the disintegration of meta-B state determines the nucleation results. Therefore, we can choose the ratio of DMI constant D to exchange constant A as the characteristic parameter to adjust the nucleation result under the same current. As shown in Fig. 3a and Fig. 3b, we conduct large numbers of experiments at different D/A ratios, and the nucleation results follow a Gaussian distribution, and D/A can adjust the expected value and maximum value of the skyrmion cores number. At

the same time, we also study the effect of temperature. As shown in Fig. 3c and Fig. 3d, we find that as a source of randomness, under the same conditions, temperature will enhance the nucleation of sk-bag with high Q .

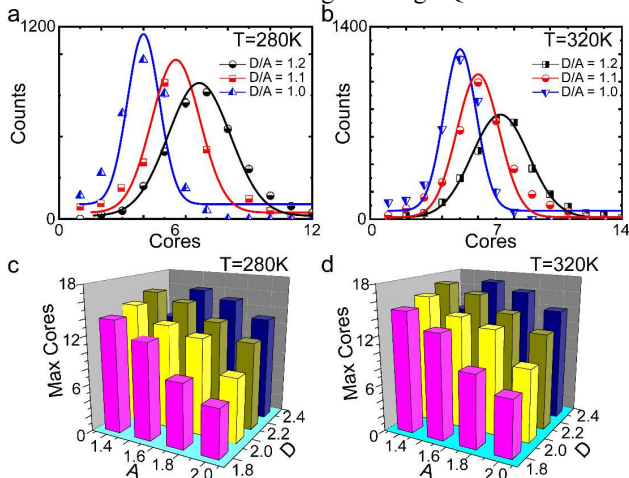


Fig. 3. (a)-(b) Distributions of Q in sk-bag at different temperatures (i.e. 280K and 320K). The results indicate that an increasing D/A ratio leads to large mathematical expectation of Q . (c)-(d) Maximum value of Q varying with the D/A ratio at different temperatures (i.e. 280K and 320K).

IV. IMPLEMENTATION OF ULTRA-FAST TRNG AND ITS PERFORMANCE

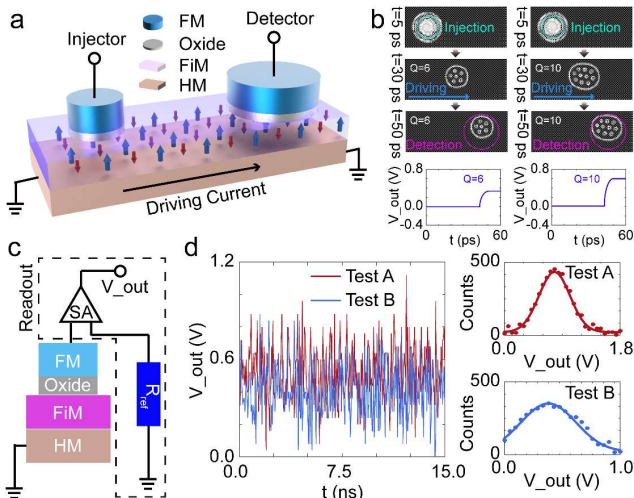


Fig. 4. (a) Schematic of TRNG based on sk-bag nucleation. (b) Working principle of TRNG. Detector on the right side can read out the resistance of sk-bag, and sk-bags with various Q correspond to different outputs. (c) Readout circuit of the device. (d) Random signal generation function is demonstrated.

Finally, a TRNG is realized using the intrinsic unpredictability resulting from ill-posed sk-bag nucleation. Its schematic is illustrated in Fig. 4a, where the injector uses current to produce sk-bags at the device's left end. An MTJ is introduced to read out the resistance signal of the sk-bag at the right end [18]. In addition, we also exhibit the device's working process in Fig. 4b for sk-bag with different Q . Firstly, by utilizing an injector on the left side of the device to inject localized spin currents, a skyrmion bag with a Q is nucleated at the device's left end. The randomness of the topological charge carried by the skyrmion bag effectively encodes true random

information. Then, by passing a driving current through the heavy metal layer, the skyrmion bag can be driven from the left end of the device to the right end. At the right end of the device, there is a detection area where the skyrmion bag can be detected via the magnetoresistance effect once it arrives. As shown in Fig. 4c, depending on the different configurations of the magnetic tunnel junction (MTJ), the random signal represented by the quantity Q of the skyrmion bag can be detected with a sense amplifier (SA). Based on the operational principles of the device described above, the speed at which the device generates

TABLE II PERFORMANCE COMPARISON

| Technology | Static power | Dynamic power |
|--------------------------|--------------|---------------|
| MTJ-based TRNG [36] | no | 196.6 pJ/bit |
| Skyrmion-based TRNG [37] | no | 9.5 pJ/bit |
| CMOS-based TRNG [35] | 0.9 nW | 82.4 pJ/bit |
| This work | no | 0.009 pJ/bit |

signals depends on both the nucleation speed of the skyrmion bag and its motion velocity. Therefore, the FIM material's ultrafast magnetic dynamics properties allow for the generation of signals in less than 50 ps which implies a bitstream density of 20 Gbit/s. This result is not only an improvement of at least three orders of magnitude over the previous fastest spintronics-based TRNGs, but even comparable to the generation rate of quantum TRNGs [33]. In addition, we have also compared the power consumption performance of our device with other RNGs in table II. The comparative data, as presented in tables within our documentation, illustrate that our device exhibits a significant advantage in terms of power consumption relative to other random number generators. In Fig. 4d, we obtain different random signal sequences by adjusting the magnitude of the test current and then verify their Gaussian distribution using numerical experiments. The generated random number sequence is also fed into several NIST tests, and all the tests are passed as shown in Table III [34]. Therefore, our work proposes a novel paradigm of TRNG with simple structure and low power, which paves the way towards the application of spintronics devices in emerging computing.

TABLE III RESULTS OF RANDOMNESS TEST

| TEST | P-value | Pass/Fail |
|---------------------|----------|-----------|
| Approximate Entropy | 0.867912 | Pass |
| Random Excursions | 0.831170 | Pass |
| Frequency | 0.731178 | Pass |
| Runs | 0.582143 | Pass |
| FFT | 0.064321 | Pass |

V. CONCLUSION

In summary, we investigate in depth an electrically stimulated nucleation process of sk-bags with high Q . The ill-posedness in the evolution of spin texture in a FIM system is discovered and a method to implement ultra-fast TRNG through sk-bags is proposed. NIST tests further validate the performance of this TRNG device. Our research brings new insights into the mechanism of spin dynamics in FIM system and lays the foundation for using the sk-bag to fabricate ultra-fast spintronic computing devices.

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